MATH191: Problem Sheet 3

Due *Monday 15th October*

1. Find the general solution of the equation

$$3\cos\theta + 4\sin\theta = 2.$$

2. In a) and b), convert from polar to Cartesian coordinates. In c) and d), convert from Cartesian to polar coordinates.

a) $(r,\theta) = (2,\pi/4)$; b) $(r,\theta) = (1,\pi)$; c) $(x,y) = (1,\sqrt{3})$; d) $(x,y) = (-1,1)$.

3. Let

$$f(x) = \frac{\cos(x) - 1}{x^2}.$$

Calculate $f(x)$ for each of the values $x = 0.1$, 0.01, -0.1, and -0.01. What does this suggest the value of $\lim_{x\to 0} f(x)$ to be? (If you don't recognize the number you're getting, try hitting the `1/x` (or x^{-1}) button on your calculator.)

Repeat the question with your calculator set on degrees.

4. For each of the following functions $f(x)$, evaluate the limit

$$\lim_{x\to a} f(x)$$

for the given value of a, or explain why the limit doesn't exist.

a) $f(x) = x^2 - 2,\ a = 1$; b) $f(x) = \dfrac{x^2-1}{x-1},\ a = 1$; c) $f(x) = \dfrac{x^2-5}{x-2},\ a = 2$;

d) $\lim\limits_{x\to\pm\infty} \dfrac{x^2-2}{(x-1)^2},$; e) $f(x) = \dfrac{\sin 5x}{x},\ a = 0$ f) $f(x) = \dfrac{|x|}{x}, a = 0.$

(In part e), you may use the fact that $\lim_{x\to 0} \frac{\sin x}{x} = 1$ In part f) if you are unsure what $f(x)$ looks like, try calculating $f(x)$ for a few positive and negative values of x.)

I will collect solutions at the lecture on Monday 15th October. Any solutions which are not handed in then, or by 5pm that day in the envelope outside Office 120 in the Theoretical Physics Wing of the Maths Building will not be marked.