MATH181 Homework Sheet 8

Due 28th November 2011

Look at Stroud, Chapters 19, 22, 23.

1. Calculate the gradient of

$$F(x, y) = 3x^2 + 3y^2 - y^3$$

and find its stationary points. Classify these points by using the second derivatives, $F_{xx}, F_{yy}, F_{x,y}$, and F_{yx}.

2. Evaluate the repeated integral

$$\int_1^3 dx \int_0^2 dy \, (3y^2 - 2xy)$$

On a sketch of the $x - y$ plane, shade in the region this integral covers. What shape is it?

3. A rectangular swimming pool covers the area $0 < x < 10,\quad 0 < y < 20$ and has a depth given by $d(x, y) = 1 + \frac{1}{10}y$. By doing a repeated integral, find out how much water is needed to fill the pool. [All measurements are in metres.]

4. The tea room in theoretical physics has a curved ceiling with height

$$h(x, y) = 4 + \frac{xy}{4}.$$

The room is a square, with $-2 < x < 2,\quad -2 < y < 2$. What is the volume of the room? What is the average height of the ceiling? [All measurements are in metres.]

5. Integrate the function

$$g(x, y) = 2x^2 + y$$

over the region A bounded by the curves $\quad y = x \quad$ and $\quad y = x^2$.

6. The temperature in a metal cube $0 < x < L,\quad 0 < y < L,\quad 0 < z < L$, is

$$T(x, y, z) = 50 + 20 \, \sin\left(\frac{\pi x}{L}\right) \sin\left(\frac{\pi y}{L}\right) \sin\left(\frac{\pi z}{L}\right).$$

What is the average temperature in the cube?

7. Use polar coordinates to integrate

$$F(x, y) = x^2 + y^2 + \sqrt{x^2 + y^2}$$

over the area A enclosed by the curve $x^2 + y^2 = 4$