MATH181 Homework Sheet 7

Due 21st November 2011

See Stroud, Chapter 10, Chapter 11.

1. Given that $f(x, y) = y \sin(xy)$ find the partial derivatives

$$\frac{\partial f}{\partial x}, \quad \frac{\partial f}{\partial y}, \quad \frac{\partial^2 f}{\partial x^2}, \quad \frac{\partial^2 f}{\partial y \partial x}, \quad \frac{\partial^2 f}{\partial x \partial y}, \quad \text{and} \quad \frac{\partial^2 f}{\partial y^2}$$

2. A hill has a height given by

$$h(x, y) = Ae^{-x^2-2y^2} \ .$$

Find the gradient of h.

A child at the point $(2, 1)$ drops a ball. Which direction will the ball roll initially?

3. Show that if $V(x, y) = e^{2x} \cos 2y$, then

$$\frac{\partial^2 V}{\partial x^2} + \frac{\partial^2 V}{\partial y^2} = 0 \ .$$

4. Show that $F(x, t) = a \sin^3(x - ct)$, is a solution of the wave equation:

$$\frac{\partial^2}{\partial t^2} F = c^2 \frac{\partial^2}{\partial x^2} F \ .$$

5. A cylinder of ice is slowly melting. At a given time the cylinder has a radius of 100 cm and a height of 200 cm. What is its volume? Because of melting, its radius is decreasing by 1 cm hr^{-1} and its height is decreasing by 2 cm hr^{-1}. At what rate is its volume changing?

6. The air temperature at height z over the point (x, y) is given by

$$T(x, y, z) = (40 + x^2 - 3y + xy)e^{-z} - 10$$

A migrating bird is at the point (0,0,1), flying with velocity $(1, 3, \frac{1}{10})$. Use the partial derivative chain rule to find the rate at which the bird feels the temperature changing.