MATH181 Homework Sheet 4

(Due 31 October 2011)

For help, look at Stroud chapter 6 and chapter 9.

1. A particle has the position x at time t given by

$(a)\quad x(t) = A\cos bt$.

Calculate the particle's velocity, $v(t)$, and acceleration, $a(t)$, as a function of t. (A and b are constants.)

Repeat this for particles with the following $x(t)$.

$(b)\qquad x(t) = Ae^{-t} - Be^{-3t}$

$(c)\qquad x(t) = A(1+kt)e^{-kt}$

$(d)\qquad x(t) = b\sin(\omega t)\ e^{-kt}$

(A, B, b, k and ω are all constants.)

2. Find all the stationary points of the following functions, and show whether they are maxima, minima, or neither. Find all the points of inflexion.

$(i)\qquad x^4 - 2x^2$

$(ii)\qquad \sin x$

$(iii)\qquad x^3 - 3x^2 + 3x$

$(iv)\qquad \cosh x$

(Remember the definitions $\cosh x = \frac{1}{2}\left(e^x + e^{-x}\right)$; $\quad \sinh x = \frac{1}{2}\left(e^x - e^{-x}\right)$.)

3. Sketch the function $g(x) = x^4 - 2x^2$ (the function from question 2.(i)) showing all the zeroes, turning points and points of inflexion.

4. Calculate the first and second derivatives of e^{x^3}.

5. A relativistic space-ship travels so that the astronaut experiences a constant acceleration of G. Its position at time t is

$$x = \frac{c}{G}\sqrt{(c^2 + G^2t^2)}\ -\ \frac{c^2}{G}$$

By differentiating this expression find

(i) the velocity $v = \dfrac{dx}{dt}$

(ii) the acceleration $a = \dfrac{d^2x}{dt^2}$

as seen by a stationary observer.

Don't panic! You should be able to answer these two parts without knowing about relativity.

(iii) How does v behave at if t is small, and if t is large? Why should you expect this on physical grounds?