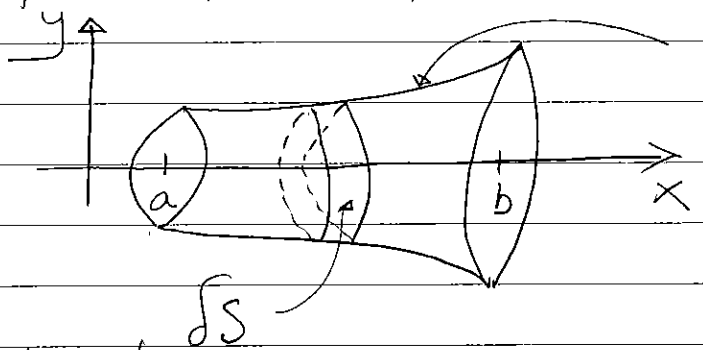
Applications of integration:

iv) Area of surface of revolution.



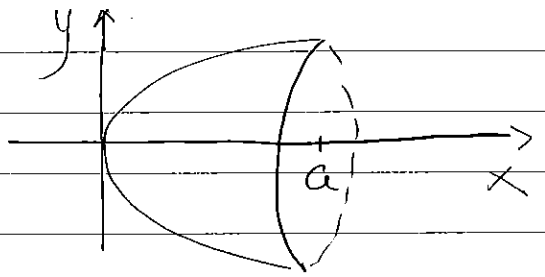
for the band

$$\delta S = 2\pi y \cdot \delta s \quad \text{(better than } \delta x\text{)}$$

$$\delta s = \sqrt{(\delta x)^2 + (\delta y)^2}$$

$$\Rightarrow S = \int_a^b \delta S = \int_a^b 2\pi y \sqrt{(dx)^2 + (dy)^2} = \int_a^b 2\pi y \sqrt{1 + y'^2}\, dx$$

Example



$$y^2 = \sqrt{4ax}$$

$$y = +2\sqrt{ax}$$

$$\frac{dy}{dx} = 2\sqrt{a}\left(\frac{1}{2}x^{-1/2}\right) = \frac{\sqrt{a}}{\sqrt{x}}$$

$$\Rightarrow S = 2\pi \int_0^a \left(2\sqrt{ax}\right)\left(1 + \frac{a}{x}\right)^{1/2} dx$$

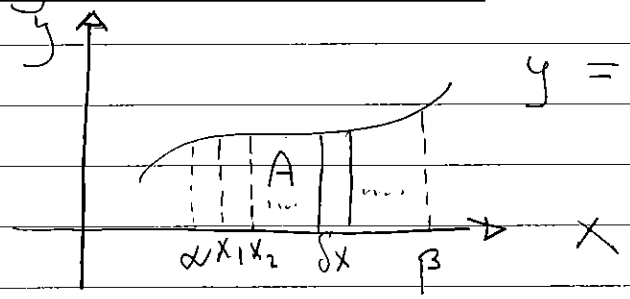
$$= 4\pi\sqrt{a} \int_0^a (x+a)^{1/2}\, dx$$

$$= 4\pi\sqrt{a}\left[\frac{2}{3}(x+a)^{3/2}\right]_0^a = \frac{8\pi\sqrt{a}}{3}\left((2a)^{3/2} - a^{3/2}\right)$$

$$= \frac{8\pi a^2}{3}\left(2^{3/2} - 1\right)$$

MATH 181.22

Integration Revision



what is the area A?

First break the area into small segments

$$\beta - \alpha = N(\delta x)$$

Then the area $\simeq f(x_1)\times\delta x + f(x_2)\times\delta x + \cdots + f(x_N)\times\delta x$.

$$\simeq \sum_{n=1}^{N} f(x_n)\cdot\delta x$$

as we let $\delta x \to 0$

$$A = \int_{\alpha}^{\beta} f(x)\,dx$$

what are the properties of $f(x)$?

Suppose that $\sum_{n=1}^{N} f(x_n)\delta x = \sum_{n=1}^{N} \delta F(x_n)$

$$= \sum_{n=1}^{N} \frac{\delta F(x_n)}{\delta x}\delta x$$

Then $A = \int_{\alpha}^{\beta} dF(x) = F(\beta) - F(\alpha)$

$\Rightarrow \frac{dF(x)}{dx} = f(x)$ → integration is ANTIDIFFENTIATION!

MATH 181. 23

Problem: what is $\int (x^3 + xe^{x^2})\,dx$?

Answer: $= \frac{1}{4}x^4 + \frac{1}{2}e^{x^2} + C$

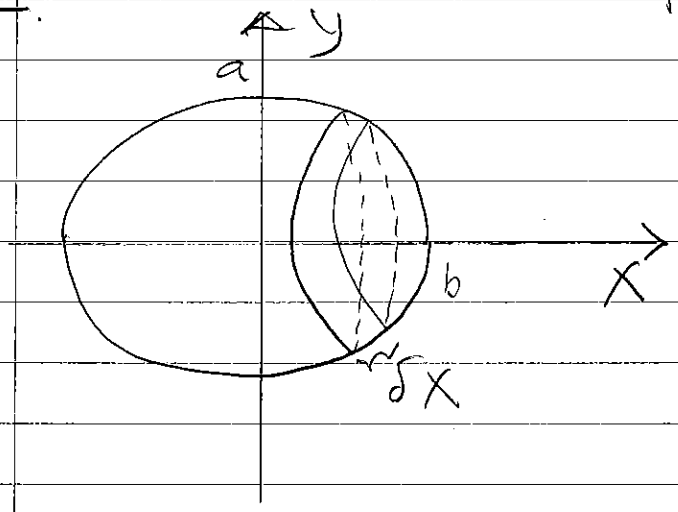
Example. Evaluate $\int_\alpha^\beta \frac{x}{x^2+a^2}\,dx$

This is of the form of the log.

$$\int_\alpha^\beta \frac{x}{x^2+a^2}\,dx \underset{\substack{y=x^2 \\ \frac{1}{2}dy=dx}}{=} \frac{1}{2}\int_\alpha^\beta \frac{dy}{y+a^2} = \frac{1}{2}\Big[\ln(x^2+a^2)\Big]_{\text{put back } y=x^2}^{\beta}{}_{\alpha} =$$

$$= \frac{1}{2}\Big[\ln(\beta^2+a^2) - \ln(\alpha^2+a^2)\Big] = \frac{1}{\alpha}\ln\left(\frac{\beta^2+a^2}{\alpha^2+a^2}\right)$$

Example: What is the volume of a ellipsoid?



$$\frac{x^2}{b^2} + \frac{y^2}{a^2} = 1$$

Volume of the disk is $\delta V = \pi y^2 \delta x$.

$$V = \int_{-b}^{b} \pi \cdot a^2\left(1 - \frac{x^2}{b^2}\right)^{1/2} \cdot dx$$

$$= \pi a^2\left[x - \frac{x^3}{3b^2}\right]_{-b}^{b} = 2\pi a^2\left[b - \frac{b}{3}\right] =$$

$$= \frac{4\pi}{3}a^2 \cdot b.$$