

MODULI FIXING IN HETEROtic STRING VACUA

Alon E. Faraggi



- AEF, NPB 728 (2005) 83.
- AEF, Elisa Manno and Cristina Timirgaziu, EPJC 50 (2007) 701.
- Gerald Cleaver, AEF, E. Manno and C. Timirgaziu, PRD 78 (2008) 046009

UK BSM 08, University of Sussex, 23 September 2008

Elements of string unification:

Classically $g^{\alpha\beta} \longrightarrow \eta^{\alpha\beta}$

Quantum $D = 26$ (Bosonic) $D = 10$ (Fermionic)

Heterotic-string $D_L = 10$ $D_R = 26$

REAL WORLD $D = 4$

\Rightarrow Bosonic $\rightarrow 4_{L+R} + 22_L + 22_R$

\Rightarrow Fermionic $\rightarrow 4_{L+R} + 6_L + 6_R$

\Rightarrow Heterotic-string $\rightarrow 4_{L+R} + (6_L + 6_R) + 16_R$

6D IM 16D w $R_J = \sqrt{2}$

Moduli \rightarrow size & shape of internal 6D manifold

Additionally. Twisted moduli. SUSY moduli.

How are the moduli fixed?

Realistic string vacua \longleftrightarrow phenomenological guide?

DATA → STANDARD MODEL

$$\text{SU}(3) \times \text{SU}(2) \times \text{U}(1)_Y \longrightarrow \text{SU}(5) \longrightarrow \text{SO}(10)$$

$$\left[\begin{pmatrix} v \\ e \end{pmatrix} + D_L^c \right] + \left[U_L^c + \begin{pmatrix} u \\ d \end{pmatrix} + E_L^c \right] + N_L^c$$
$$\bar{5} \quad + \quad 10 \quad + \quad 1 \quad = \quad \frac{16}{16}$$

STANDARD MODEL -> UNIFICATION

ADDITIONAL EVIDENCE:

Logarithmic running, proton longevity, neutrino masses

PRIMARY GUIDES:

3 generations

SO(10) embedding

Realistic free fermionic models

'Phenomenology of the Standard Model and string unification'

- Top quark mass $\sim 175\text{--}180\text{GeV}$ PLB 274 (1992) 47
- Generation mass hierarchy NPB 407 (1993) 57
- CKM mixing NPB 416 (1994) 63 (with Halyo)
- Stringy seesaw mechanism PLB 307 (1993) 311 (with Halyo)
- Gauge coupling unification NPB 457 (1993) 409 (with Dienes)
- Proton stability NPB 428 (1994) 111
- Squark degeneracy NPB 526 (1998) 21 (with Pati)
- Minimal Superstring Standard Model PLB 455 (1999) 135
(with Cleaver & Nanopoulos)
- Moduli fixing NPB 728 (2005) 83

Free Fermionic Construction

Left-Movers: $\psi_{1,2}^\mu, \chi_i, y_i, \omega_i$ ($i = 1, \dots, 6$)

Right-Movers

$$\bar{\phi}_{A=1,\dots,44} = \begin{cases} \bar{y}_i, \bar{\omega}_i & i = 1, \dots, 6 \\ \bar{\eta}_i & i = 1, 2, 3 \\ \bar{\psi}_{1,\dots,5} \\ \bar{\phi}_{1,\dots,8} \end{cases}$$

$$f \longrightarrow -e^{i\pi\alpha(f)} f$$

$$V \longrightarrow V$$

$$Z = \sum_{\substack{\text{all spin} \\ \text{structures}}} c(\vec{\alpha}) Z(\vec{\beta})$$

Models \longleftrightarrow Basis vectors + one-loop phases

The NAHE set:

$$1 = \{\psi^\mu, \chi^{1,\dots,6}, y^{1,\dots,6}, \omega^{1,\dots,6} \mid \bar{y}^{1,\dots,6}, \bar{\omega}^{1,\dots,6}, \bar{\eta}^{1,2,3}, \bar{\psi}^{1,\dots,5}, \bar{\phi}^{1,\dots,8}\}$$

$$S = \{\psi^\mu, \chi^{1,\dots,6}\}, \quad N = 4 \text{ Vacua}$$

$$b_1 = \{\chi^{34}, \chi^{56}, y^{34}, y^{56} \mid \bar{y}^{34}, \bar{y}^{56}, \bar{\eta}^1, \bar{\psi}^{1,\dots,5}\}, \quad N = 4 \rightarrow N = 2$$

$$b_2 = \{\chi^{12}, \chi^{56}, y^{12}, \omega^{56} \mid \bar{y}^{12}, \bar{\omega}^{56}, \bar{\eta}^2, \bar{\psi}^{1,\dots,5}\}, \quad N = 2 \rightarrow N = 1$$

$$b_3 = \{\chi^{12}, \chi^{34}, \omega^{12}, \omega^{34} \mid \bar{\omega}^{12}, \bar{\omega}^{34}, \bar{\eta}^3, \bar{\psi}^{1,\dots,5}\}, \quad N = 2 \rightarrow N = 1$$

$Z_2 \times Z_2$ orbifold compactification

\implies Gauge group $SO(10) \times SO(6)^{1,2,3} \times E_8$

$$\text{NAHE} \oplus (\xi_2 = \{\bar{\psi}_{1,\dots,5}, \bar{\eta}^1, \bar{\eta}^2, \bar{\eta}^3\} = 1) \rightarrow \{1, S, \xi_1, \xi_2, b_1, b_2\}$$

Gauge group: $SO(4)^3 \times E_6 \times U(1)^2 \times E_8$ and 24 generations.

toroidal compactification $(6_L + 6_R)$ g_{ij}, b_{ij}

$$g_{ij} = \begin{pmatrix} 2 & -1 & 0 & 0 & 0 & 0 \\ -1 & 2 & -1 & 0 & 0 & 0 \\ 0 & -1 & 2 & -1 & 0 & 0 \\ 0 & 0 & -1 & 2 & -1 & -1 \\ 0 & 0 & 0 & -1 & 2 & 0 \\ 0 & 0 & 0 & -1 & 0 & 2 \end{pmatrix} \quad b_{ij} = \begin{cases} g_{ij} & i \neq j \\ 0 & i = j \\ -g_{ij} & i \neq j \end{cases}$$

$R_i \rightarrow$ the free fermionic point \rightarrow G.G. $SO(12) \times E_8 \times E_8$

mod out by a $Z_2 \times Z_2$ with standard embedding

$\Rightarrow SO(4)^3 \times E_6 \times U(1)^2 \times E_8$ with 24 generations

Exact correspondence

In the realistic free fermionic models

replace $X = \{\bar{\psi}^{1,\dots,5}, \bar{\eta}^1, \bar{\eta}^2, \bar{\eta}^3\} = 1$

with $2\gamma = \{\bar{\psi}^{1,\dots,5}, \bar{\eta}^1, \bar{\eta}^2, \bar{\eta}^3, \bar{\phi}^{1,\dots,4}\} = 1$

Then $\{\vec{1}, \vec{S}, \vec{\xi}_1 = \vec{1} + \vec{b}_1 + \vec{b}_2 + \vec{b}_3, 2\gamma\} \rightarrow N=4 \text{ SUSY and}$

$$SO(12) \times SO(16) \times SO(16)$$

apply $b_1 \times b_2 \rightarrow Z_2 \times Z_2 \rightarrow N=1 \text{ SUSY and}$

$$SO(4)^3 \times SO(10) \times U(1)^3 \times SO(16)$$

$$b_1, \quad b_2, \quad b_3 \quad \Rightarrow \quad (3 \times 8) \cdot 16 \text{ of } SO(10)_O$$

$$b_1 + 2\gamma, b_2 + 2\gamma, b_3 + 2\gamma \quad \Rightarrow \quad (3 \times 8) \cdot 16 \text{ of } SO(16)_H$$

This will be important for the twisted moduli.

Classification of fermionic $Z_2 \times Z_2$ orbifolds (FKNR, FKR)

Basis vectors: consistent modular blocks 4,8 periodic fermions

$$1 = \{\psi^\mu, \chi^{1,\dots,6}, y^{1,\dots,6}, \omega^{1,\dots,6} \mid \bar{y}^{1,\dots,6}, \bar{\omega}^{1,\dots,6}, \bar{\eta}^{1,2,3}, \bar{\psi}^{1,\dots,5}, \bar{\phi}^{1,\dots,8}\}$$

$$S = \{\psi^\mu, \chi^{1,\dots,6}\},$$

$$z_1 = \{\bar{\phi}^{1,\dots,4}\},$$

$$z_2 = \{\bar{\phi}^{5,\dots,8}\},$$

$$e_i = \{y^i, \omega^i | \bar{y}^i, \bar{\omega}^i\}, \quad i = 1, \dots, 6, \quad N = 4 \text{ Vacua}$$

$$b_1 = \{\chi^{34}, \chi^{56}, y^{34}, y^{56} | \bar{y}^{34}, \bar{y}^{56}, \bar{\eta}^1, \bar{\psi}^{1,\dots,5}\}, \quad N = 4 \rightarrow N = 2$$

$$b_2 = \{\chi^{12}, \chi^{56}, y^{12}, y^{56} | \bar{y}^{12}, \bar{y}^{56}, \bar{\eta}^2, \bar{\psi}^{1,\dots,5}\}, \quad N = 2 \rightarrow N = 1$$

Vector bosons: NS, $z_{1,2}$, $z_1 + z_2$, $x = 1 + s + \sum e_i + z_1 + z_2$

impose: Gauge group $SO(10) \times U(1)^3 \times$ hidden

Independent phases $c_{v_j}^{[vi]} = \exp[i\pi(v_i|v_j)]$: upper block

	1	S	e_1	e_2	e_3	e_4	e_5	e_6	z_1	z_2	b_1	b_2
1	-1	-1	\pm									
S		-1	-1	-1	-1	-1	-1	-1	-1	-1	1	1
e_1			\pm									
e_2				\pm								
e_3					\pm							
e_4						\pm						
e_5							\pm	\pm	\pm	\pm	\pm	\pm
e_6								\pm	\pm	\pm	\pm	\pm
z_1										\pm	\pm	\pm
z_2											\pm	\pm
b_1												\pm
b_2												

Apriori 55 independent coefficients $\rightarrow 2^{55}$ distinct vacua

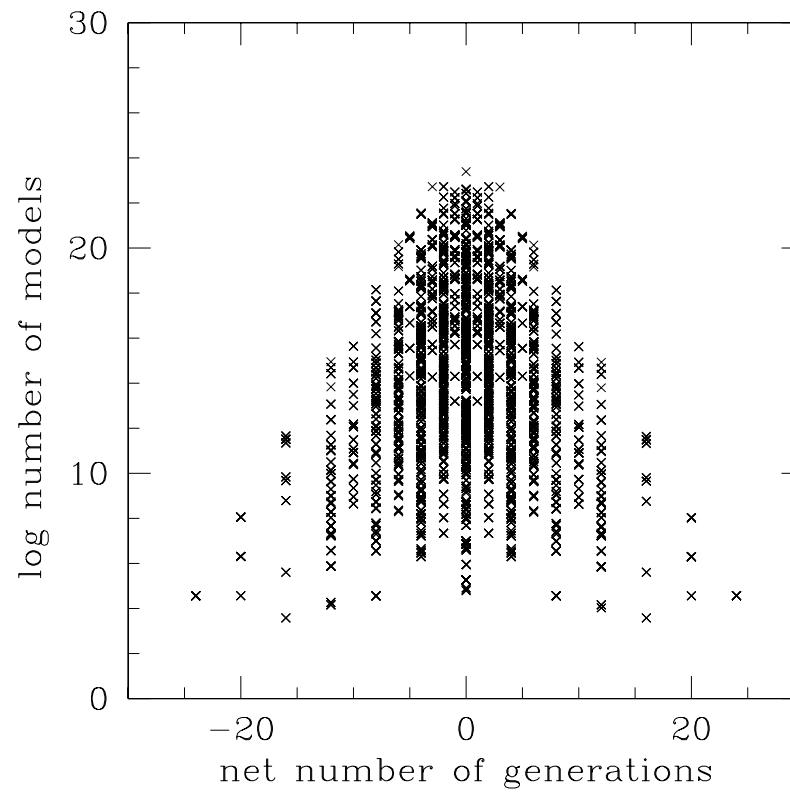
Impose: Gauge group $SO(10) \times U(1)^3 \times SO(8)^2$

\rightarrow 40 independent coefficients

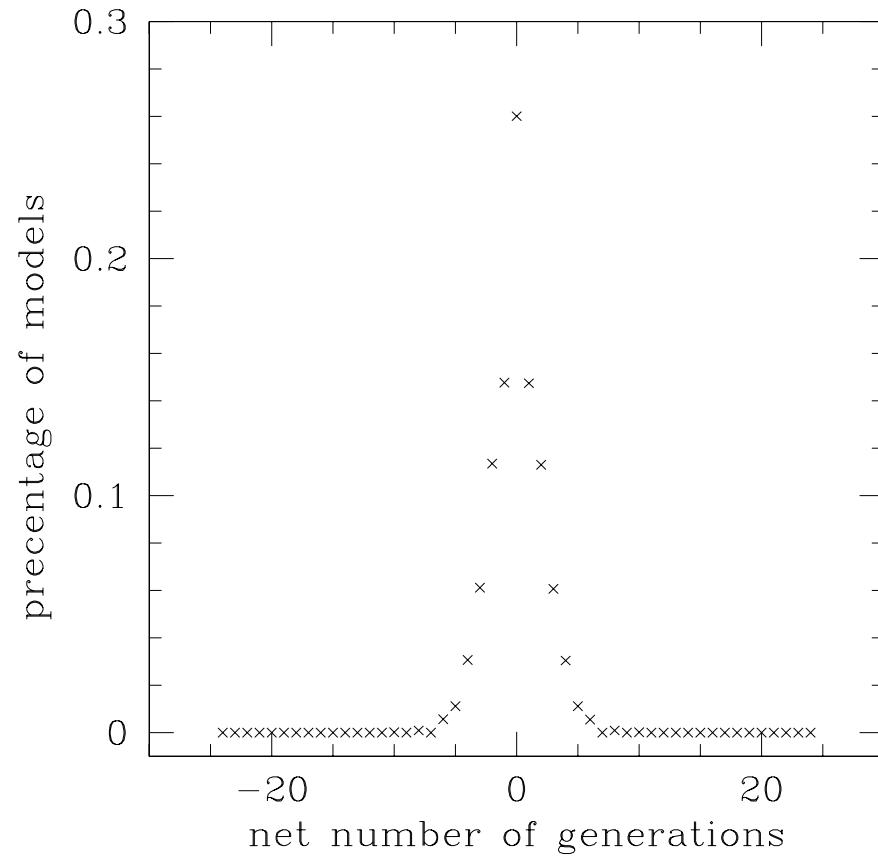
RESULTS:

FKR I: Random sampling of phases. $SO(10) \times U(1)^3 \times$ hidden

FKR II: Complete classification. $SO(10) \times U(1)^3 \times SO(8)^2$



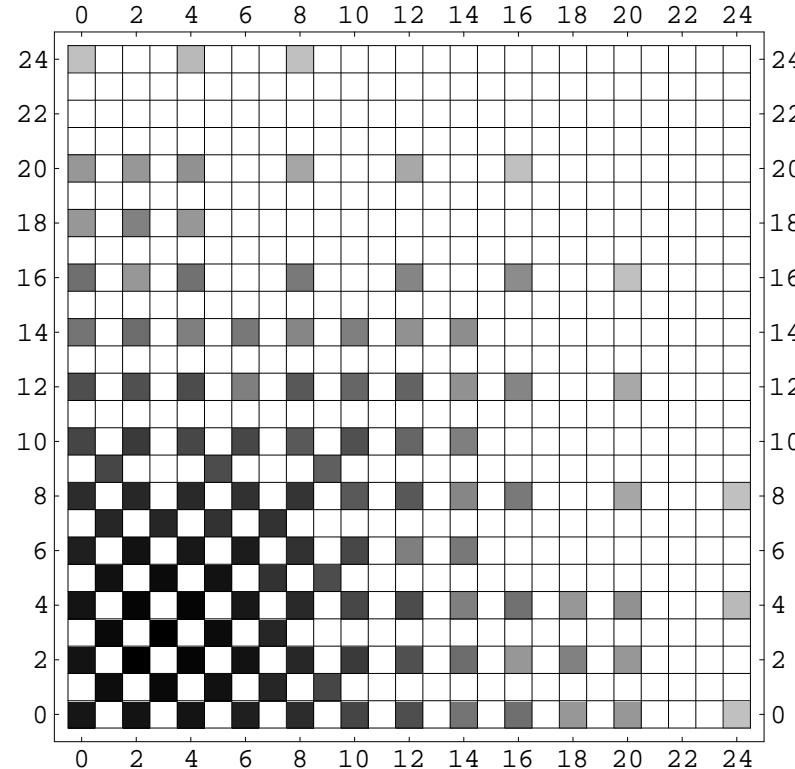
RESULTS ARE SIMILAR



7×10^9 models $\sim 15\%$ with 3 gen FKRI

Spinor–vector duality:

Invariance under exchange of $\#(16 + \overline{16}) < - > \#(10)$



Symmetric under exchange of rows and columns

Self-dual: $\#(16 + \overline{16}) = \#(10)$ without E_6 symmetry

beyond the NAHE set

Add $\{\alpha, \beta, \gamma\}$

number of generations is reduced to three

$$SO(10) \longrightarrow SU(3) \times SU(2) \times U(1)_{T_{3R}} \times U(1)_{B-L}$$

$$U(1)_Y = \frac{1}{2}(B - L) + T_{3R} \in SO(10) !$$

$$SO(6)^{1,2,3} \longrightarrow U(1)^{1,2,3} \times U(1)^{1,2,3}$$

The massless spectrum

Three twisted generations

b_1, b_2, b_3

Untwisted Higgs doublets

$h_{11,0,0}$

$\bar{h}_{1-1,0,0}$

$h_{20,1,0}$

$\bar{h}_{20,-1,0}$

$h_{30,0,1}$

$\bar{h}_{30,0,-1}$

Sector $b_1 + b_2 + \alpha + \beta$

$h_{\alpha\beta -\frac{1}{2}, -\frac{1}{2}, 0, 0, 0, 0}$

$\bar{h}_{\alpha\beta \frac{1}{2}, \frac{1}{2}, 0, 0, 0, 0}$

\oplus $SO(10)$ singlets

Sectors $b_j + 2\gamma$ $j = 1, 2, 3$ \longrightarrow hidden matter multiplets

“standard” $SO(10)$ representations

NAHE + { α, β, γ } \rightarrow exotic vector-like matter \rightarrow superheavy

\oplus Quasi-realistic phenomenology

Moduli?

Untwisted moduli \rightarrow shape & size of the internal dimensions

Twisted moduli – > arise from the twisted sectors

models

$$\frac{T^6}{Z_2 \times Z_2}$$

$$T^6 \ : \quad G_{IJ} \quad ; \quad B_{IJ} \quad \quad I, J \ = \ 1, \dots, 6 \ .$$

untwisted moduli: coefficients of exactly marginal operators

moduli fields: massless chiral superfields with flat scalar potential

Scalar couplings of $N = 4$ SUGRA

Up to orbifold projections

$$\frac{T^6}{Z_2 \times Z_2} \implies \frac{SO(6,6)}{SO(6) \times SO(6)} \longrightarrow \left(\frac{SO(2,2)}{SO(2) \times SO(2)} \right)^3$$

\Rightarrow 3 complex structures + 3 Kähler moduli

In all symmetric $Z_2 \times Z_2$ orbifolds

We would like to identify these moduli in the fermionic formalism

In symmetric orbifolds:

$$\text{EMO} \rightarrow \partial X^I \bar{\partial} X^J$$

$$X^I \quad I = 1, \dots, 6 \quad \rightarrow \quad T^6$$

$$S = \frac{1}{8\pi} \int d^2\sigma \ (G_{IJ} \partial X^I \bar{\partial} X^J + B_{IJ} \partial X^I \bar{\partial} X^J)$$

In FFF $\partial X_L^I \rightarrow y^i \omega^I$

$i\partial X_L^I \rightarrow U(1)$ current in the Cartan subalgebra

In the fermionic language

$$i\partial X_L^I \rightarrow J_L^I = y^I \omega^I$$

$$\Rightarrow \partial X^I \bar{\partial} X^J \rightarrow J_L^I(z) \bar{J}_R(\bar{z})$$

→ WS Thirring interactions $(R - \frac{1}{R}) J_L(z) \bar{J}(\bar{z})$

Thirring interactions vanish at free fermionic point with $R = \frac{1}{R}$

To identify the untwisted moduli in the free fermionic models

→ find the operators of the form

$$J_L^I(z) \bar{J}_R^J(\bar{z})$$

that are allowed by the orbifold (fermionic) symmetry group

Bosonization

$$\xi_i = \sqrt{\frac{1}{2}}(y_i + i\omega_i) = e^{iX_i}, \eta_i = \sqrt{\frac{i}{2}}(y_i - i\omega_i) = ie^{-iX_i}$$

similarly for the right-movers

Boundary conditions translate to twists and shifts

$$X^I(z, \bar{z}) = X_L^I(z) + X_R^I(\bar{z})$$

Complex internal coordinates

$$Z_k^\pm = (X^{2k-1} \pm iX^{2k}), \quad \psi_k^\pm = (\chi^{2k-1} \pm i\chi^{2k}) \quad (k = 1, 2, 3)$$

Untwisted moduli

$$S = \int d^2z h_{ij}(X) J_L^i(z) \bar{J}_R^j(\bar{z})$$

$J_L^i \sim y^i \omega^i$ $i = 1, \dots, 6$ are chiral currents of $U(1)_L^6$

$J_R^j \sim \bar{\phi}^j \bar{\phi}^{*j}$ $j = 1, \dots, 22$ are chiral currents of $U(1)_R^{22}$

$h_{ij} \rightarrow$ scalar components of untwisted moduli

some of these operators are projected out in concrete models

\Rightarrow some of the EMO may not be invariant

<u>Models</u>	$\{1, S\}$	$N = 4$	$SO(44)$
		$\frac{SO(6, 22)}{SO(6) \times SO(22)}$	Moduli space
		$\chi^i \otimes \bar{\phi}^a \bar{\phi}^{*a} 0\rangle$	moduli fields
		6×22	scalar fields

$$\begin{aligned}
 Z_2 \times Z_2 & \quad \{ 1, S, \xi_1, \xi_2 \} + \{ b_1, b_2 \} \\
 & \quad SO(12) \times E_8 \times E_8 \quad Z_2 \times Z_2 \\
 & \rightarrow SO(4)^3 \times E_6 \times U(1)^2 \times E_8
 \end{aligned}$$

The Thirring interactions that remain invariant are

$$\begin{aligned}
 J_L^{1,2} \bar{J}_R^{1,2} & ; \quad J_L^{3,4} \bar{J}_R^{3,4} ; \quad J_L^{5,6} \bar{J}_R^{5,6} \\
 y^{1,2} \omega^{1,2} \bar{y}^{1,2} \bar{\omega}^{1,2} & ; \quad y^{3,4} \omega^{3,4} \bar{y}^{3,4} \bar{\omega}^{3,4} ; \quad y^{5,6} \omega^{5,6} \bar{y}^{5,6} \bar{\omega}^{5,6}
 \end{aligned}$$

These moduli are always present in symmetric $Z_2 \times Z_2$ orbifolds

in realistic models

$$\{ 1, S, \xi_1, \xi_2 \} \oplus \{ b_1, b_2 \} \oplus \{ \alpha, \beta, \gamma \}$$

$$N = 4 \quad N = 1$$

$$E_8 \times E_8 \quad Z_2 \times Z_2$$

new feature Asymmetric orbifold

the key focus: boundary conditions of the internal fermions

$$\{ y, \omega \mid \bar{y}, \omega \}$$

WS fermions that have same B.C. in all basis vectors are paired

pairing of LR fermions \rightarrow Ising model \rightarrow symmetric real fermions

pairing of LL & RR fermions \rightarrow complex fermions \rightarrow asymmetric

STRING DERIVED STANDARD-LIKE MODEL (PLB278)

	ψ^μ	χ^{12}	χ^{34}	χ^{56}	$y^{3,\dots,6}$	$\bar{y}^{3,\dots,6}$	$y^{1,2}, \omega^{5,6}$	$\bar{y}^{1,2}, \bar{\omega}^{5,6}$	$\omega^{1,\dots,4}$	$\bar{\omega}^{1,\dots,4}$	$\bar{\psi}^{1,\dots,5}$	$\bar{\eta}^1$	$\bar{\eta}^2$	$\bar{\eta}^3$	$\bar{\phi}^{1,\dots,8}$
1	1	1	1	1	$1,\dots,1$	$1,\dots,1$	$1,\dots,1$	$1,\dots,1$	$1,\dots,1$	$1,\dots,1$	$1,\dots,1$	1	1	1	$1,\dots,1$
S	1	1	1	1	$0,\dots,0$	$0,\dots,0$	$0,\dots,0$	$0,\dots,0$	$0,\dots,0$	$0,\dots,0$	$0,\dots,0$	0	0	0	$0,\dots,0$
b_1	1	1	0	0	$1,\dots,1$	$1,\dots,1$	$0,\dots,0$	$0,\dots,0$	$0,\dots,0$	$0,\dots,0$	$1,\dots,1$	1	0	0	$0,\dots,0$
b_2	1	0	1	0	$0,\dots,0$	$0,\dots,0$	$1,\dots,1$	$1,\dots,1$	$0,\dots,0$	$0,\dots,0$	$1,\dots,1$	0	1	0	$0,\dots,0$
b_3	1	0	0	1	$0,\dots,0$	$0,\dots,0$	$0,\dots,0$	$0,\dots,0$	$1,\dots,1$	$1,\dots,1$	$1,\dots,1$	0	0	1	$0,\dots,0$

	ψ^μ	χ^{12}	χ^{34}	χ^{56}	y^3y^6	$y^4\bar{y}^4$	$y^5\bar{y}^5$	$\bar{y}^3\bar{y}^6$	$y^1\omega^5$	$y^2\bar{y}^2$	$\omega^6\bar{\omega}^6$	$\bar{y}^1\bar{\omega}^5$	$\omega^2\omega^4$	$\omega^1\bar{\omega}^1$	$\omega^3\bar{\omega}^3$	$\bar{\omega}^2\bar{\omega}^4$	$\bar{\psi}^{1,\dots,5}$	$\bar{\eta}^1$	$\bar{\eta}^2$	$\bar{\eta}^3$	
α	0	0	0	0	1	0	0	0	0	0	1	1	0	0	1	1	1 1 1 0 0	0	0	0	1 1
β	0	0	0	0	0	0	1	1	1	0	0	0	0	1	0	1	1 1 1 0 0	0	0	0	1 1
γ	0	0	0	0	0	1	0	1	0	1	0	1	1	0	0	0	$\frac{1}{2} \frac{1}{2} \frac{1}{2} \frac{1}{2} \frac{1}{2}$	$\frac{1}{2}$	$\frac{1}{2}$	$\frac{1}{2}$	$\frac{1}{2} 0 1$

Asymmetric $BC \Rightarrow$ all untwisted moduli are projected out!

all $y_i\omega_i\bar{y}_i\bar{\omega}_i$ are disallowed

can be translated to asymmetric bosonic identifications

$$X_L + X_R \rightarrow X_L - X_R$$

moduli fixed at enhanced symmetry point

Twisted moduli

$$(2,2) \ b_j \oplus b_j + \xi_2 \rightarrow (16 \oplus 10 + 1) + 1 = 27 + 1 \Rightarrow \text{twisted moduli}$$

$$(2,0) \ b_j + b_j + 2\gamma \rightarrow 16_{SO(10)} + 16_{SO(16)}$$

all twisted moduli are projected out

MINIMAL DOUBLET HIGGS CONTENT (EJPC50)

	ψ^μ	χ^{12}	χ^{34}	χ^{56}	y^3y^6	$y^4\bar{y}^4$	$y^5\bar{y}^5$	$\bar{y}^3\bar{y}^6$	$y^1\omega^5$	$y^2\bar{\omega}^2$	$\omega^6\bar{\omega}^6$	$\bar{y}^1\bar{\omega}^5$	$\omega^2\omega^4$	$\omega^1\bar{\omega}^1$	$\omega^3\bar{\omega}^3$	$\bar{\omega}^2\bar{\omega}^4$	$\bar{\psi}^{1,\dots,5}$	$\bar{\eta}^1$	$\bar{\eta}^2$	$\bar{\eta}^3$	$\bar{\phi}$
α	0	0	0	0	1	0	0	1	1	0	0	1	1	0	0	1	1	1	0	0	1 1 0
β	0	0	0	0	0	0	1	1	1	0	0	1	0	0	1	0	1	1	1	0	0 0 1
γ	0	0	0	0	0	1	0	0	0	1	0	0	0	1	0	0	0	$\frac{1}{2}$	$\frac{1}{2}$	$\frac{1}{2}$	0 0 0

(With Elisa Manno and Cristina Timirgaziu)

SYMMETRIC \leftrightarrow ASYMMETRIC

with respect to b_1 & b_2

$h_1, \bar{h}_1, D_1, \bar{D}_1, h_2, \bar{h}_2, D_2, \bar{D}_2$ are projected out

h_3, \bar{h}_3 remain in the spectrum

$\lambda_t Q_3 t_3^c \bar{h}_3$ with $\lambda_t O(1)$

No Phenomenologically viable flat directions

Calculation of Mass Terms

nonvanishing correlators

$$\langle V_1^f V_2^f V_3^b \cdots V_N^b \rangle$$

gauge & string invariant

“anomalous” $U(1)_A$

$$\text{Tr}Q_A \neq 0 \Rightarrow D_A = A + \sum Q_k^A |\langle \phi_k \rangle|^2$$

$$D_j = 0 = \sum Q_k^j |\langle \phi_k \rangle|^2$$

$$\langle W \rangle = \langle \frac{\partial W_N}{\partial \eta_i} \rangle = 0 \quad N = 3, \dots$$

Supersymmetric vacuum $\langle F \rangle = \langle D \rangle = 0$.

nonrenormalizable terms \rightarrow effective renormalizable operators

$$V_1^f V_2^f V_3^b \cdots V_N^b \rightarrow V_1^f V_2^f V_3^b \frac{\langle V_4^b \cdots V_N^b \rangle}{M^{N-3}}$$

Cleaver, Faraggi, Manno, Timirgaziu, PRD 78 (2008) 046009

Classification of F and D flat directions in EMT reduced Higgs model

No D flat direction which is F-flat up to order eight in the superpotential

no stringent flat directions to all orders

Suggesting no supersymmetric flat directions in this model (class of models)

implying no supersymmetric moduli

only remaining perturbative moduli is the dilaton

quasi-realistic model: SLM; 3 gen; $SO(10)$ embed; Higgs & $\lambda_t \sim 1$; ...

vanishing one-loop partition function, perturbatively broken SUSY

Fixed geometrical, twisted and SUSY moduli

Conclusions

Phenomenological string models produce interesting lessons

Spinor–vector duality

relevance of non–standard geometries

Free Fermionic Models \longrightarrow $Z_2 \times Z_2$ orbifold near the self–dual point

Duality & Self–Duality \Leftrightarrow String Vacuum Selection