Moduli fixing and Other Manias in Heterotic String Vacua

Progress Report:

- Moduli fixing NPB 728 (2005) 83; EPJC 50 (2007) 701; PRD 78 (2008) 046009; arXiv:1105.0447
- Spinor–Vector duality, NPB 848 (2011) 332

With: Elisa Manno, Gerald Cleaver, Jared Greenwald, Cristina Timiragaziu
Costas Konnas, John Rizos
Yiannis Florakis, Thomas Mohaupt, Mirian Tsulaia

StringVac 2011, Busan, 5-7 September 2011

DATA $\longrightarrow$ STANDARD MODEL

$SU(3) \times SU(2) \times U(1)_Y \longrightarrow SU(5) \longrightarrow SO(10)$

$$\left[\begin{pmatrix} \nu \\ e \end{pmatrix} + D^c_L \right] + \left[U^c_L + \begin{pmatrix} u \\ d \end{pmatrix} + E^c_L \right] + N^c_L$$

$$\bar{5} \quad + \quad 10 \quad 1 \qquad \overline{} \; 16$$

STANDARD MODEL -> UNIFICATION

ADDITIONAL EVIDENCE:

Logarithmic running, proton longevity, neutrino masses

PRIMARY GUIDES:

3 generations

SO(10) embedding

Realistic free fermionic models

'Phenomenology of the Standard Model and string unification'

- Top quark mass $\sim$ 175–180GeV — PLB 274 (1992) 47
- Generation mass hierarchy — NPB 407 (1993) 57
- CKM mixing — NPB 416 (1994) 63 (with Halyo)
- Stringy seesaw mechanism — PLB 307 (1993) 311 (with Halyo)
- Gauge coupling unification — NPB 457 (1995) 409 (with Dienes)
- Proton stability — NPB 428 (1994) 111
- Squark degeneracy — NPB 526 (1998) 21 (with Pati)
- Minimal Superstring Standard Model — PLB 455 (1999) 135 (with Cleaver & Nanopoulos)
- Moduli fixing — NPB 728 (2005) 83
- Exophobia — PLB 683 (2010) 306 (with Assel, Christodoulides, Kounnas & Rizos)

Other approaches

Geometrical

Greene, Kirklin, Miron, Ross (1987)
Donagi, Ovrut, Pantev, Waldram (1999)
Blumenhagen, Moster, Reinbacher, Weigand (2006)
Heckman, Vafa (2008)
....

Orbifolds

Ibanez, Nilles, Quevedo (1987)
Bailin, Love, Thomas (1987)
Kobayashi, Raby, Zhang (2004)
Lebedev, Nilles, Raby, Ramos-Sanchez, Ratz, Vaudrevange, Wingerter (2007)
Blaszczyk, Groot–Nibbelink, Ruehle, Trapletti, Vaudrevange (2010)
....

Other CFTs

Gepner (1987)
Schellekens, Yankielowicz (1989)
Gato–Rivera, Schellekens (2009)
....

Orientifolds

Cvetic, Shiu, Uranga (2001)
Ibanez, Marchesano, Rabadan (2001)
Kiristis, Schellekens, Tsulaia (2008)
....

Elements of string unification:

Classically $\quad g^{\alpha\beta} \longrightarrow \eta^{\alpha\beta}$

Quantum $\quad D = 26$ (Bosonic) $\quad D = 10$ (Fermionic)

Heterotic–string $\quad D_L = 10 \quad D_R = 26$

REAL WORLD $\quad D = 4$

$\Rightarrow$ Bosonic $\rightarrow \quad 4_{L+R} + 22_L + 22_R$

$\Rightarrow$ Fermionic $\rightarrow \quad 4_{L+R} + 6_L + 6_R$

$\Rightarrow$ Heterotic–string $\rightarrow \quad 4_{L+R} + (6_L + 6_R) + 16_R$

6D IM $\quad$ 16D w $R_J = \sqrt{2}$

Moduli $\rightarrow$ size & shape of internal 6D manifold

Additionally. Twisted moduli. SUSY moduli.

How are the moduli fixed?

Realistic string vacua $\longleftrightarrow$ phenomenological guide?

Free Fermionic Construction

Left-Movers: $\psi^{\mu}_{1,2}, \quad \chi_i, \quad y_i, \quad \omega_i \quad (i = 1, \cdots, 6)$

Right-Movers

$$\bar{\phi}_{A=1,\cdots,44} = \begin{cases} \bar{y}_i \ , \ \bar{\omega}_i & i = 1, \cdots, 6 \\ & \\ \bar{\eta}_i & i = 1, 2, 3 \\ \bar{\psi}_{1,\cdots,5} & \\ \bar{\phi}_{1,\cdots,8} & \end{cases}$$

$$V \longrightarrow V \qquad\qquad f \longrightarrow -e^{i\pi\alpha(f)} f$$

$$Z = \sum_{\substack{all\ spin \\ structures}} c\binom{\vec{\alpha}}{\vec{\beta}} Z\binom{\vec{\alpha}}{\vec{\beta}}$$

Models $\longleftrightarrow$ Basis vectors $+$ one–loop phases

The NAHE set:

$$1 \;=\; \{\psi^\mu,\ \chi^{1,\dots,6}, y^{1,\dots,6}, \omega^{1,\dots,6} \mid \bar{y}^{1,\dots,6}, \bar{\omega}^{1,\dots,6}, \bar{\eta}^{1,2,3}, \bar{\psi}^{1,\dots,5}, \bar{\phi}^{1,\dots,8}\}$$

$$S \;=\; \{\psi^\mu, \chi^{1,\dots,6}\}, \qquad N=4 \text{ Vacua}$$

$$b_1 \;=\; \{\chi^{34}, \chi^{56}, y^{34}, y^{56} | \bar{y}^{34}, \bar{y}^{56}, \bar{\eta}^1, \bar{\psi}^{1,\dots,5}\}, \qquad N=4 \rightarrow N=2$$

$$b_2 \;=\; \{\chi^{12}, \chi^{56}, y^{12}, \omega^{56} | \bar{y}^{12}, \bar{\omega}^{56}, \bar{\eta}^2, \bar{\psi}^{1,\dots,5}\}, \qquad N=2 \rightarrow N=1$$

$$b_3 \;=\; \{\chi^{12}, \chi^{34}, \omega^{12}, \omega^{34} | \bar{\omega}^{12}, \bar{\omega}^{34}, \bar{\eta}^3, \bar{\psi}^{1,\dots,5}\}, \qquad N=2 \rightarrow N=1$$

$Z_2 \times Z_2$ orbifold compactification

$\Longrightarrow$ Gauge group $SO(10) \times SO(6)^{1,2,3} \times E_8$

Correspondence with $Z_2 \times Z_2$ orbifold

PLB 326 (1994) 62

NAHE $\oplus$ $(\xi_2 = \{\bar{\psi}_{1,\cdots,5}, \bar{\eta}^1, \bar{\eta}^2, \bar{\eta}^3\} = 1)$ $\rightarrow$ $\{1, S, \xi_1, \xi_2, b_1, b_2\}$

Gauge group: $SO(4)^3 \times E_6 \times U(1)^2 \times E_8$ and 24 generations.

toroidal compactification $(6_L + 6_R)$ g_{ij}, b_{ij}

$$g_{ij} = \begin{pmatrix} 2 & -1 & 0 & 0 & 0 & 0 \\ -1 & 2 & -1 & 0 & 0 & 0 \\ 0 & -1 & 2 & -1 & 0 & 0 \\ 0 & 0 & -1 & 2 & -1 & -1 \\ 0 & 0 & 0 & -1 & 2 & 0 \\ 0 & 0 & 0 & -1 & 0 & 2 \end{pmatrix} \qquad b_{ij} = \begin{cases} g_{ij} & i < j \\ 0 & i = j \\ -g_{ij} & i > j \end{cases}$$

R_i $\rightarrow$ the free fermionic point $\rightarrow$G.G. $SO(12) \times E_8 \times E_8$

mod out by a $Z_2 \times Z_2$ with standard embedding

$\Rightarrow SO(4)^3 \times E_6 \times U(1)^2 \times E_8$ with 24 generations

Exact correspondence

In the realistic free fermionic models

replace $X = \{\bar{\psi}^{1,\cdots,5}, \bar{\eta}^1, \bar{\eta}^2, \bar{\eta}^3\} = 1$

with $2\gamma = \{\bar{\psi}^{1,\cdots,5}, \bar{\eta}^1, \bar{\eta}^2, \bar{\eta}^3, \bar{\phi}^{1,\cdots,4}\} = 1$

Then $\{\vec{1}, \vec{S}, \vec{\xi}_1 = \vec{1} + \vec{b}_1 + \vec{b}_2 + \vec{b}_3, 2\gamma\} \rightarrow$ N=4 SUSY and

$$SO(12) \times SO(16) \times SO(16)$$

apply $b_1 \times b_2 \rightarrow Z_2 \times Z_2 \rightarrow$ N=1 SUSY and

$$SO(4)^3 \times SO(10) \times U(1)^3 \times SO(16)$$

b_1, b_2, b_3 $\Rightarrow (3 \times 8) \cdot 16$ of $SO(10)_O$

$b_1 + 2\gamma$, $b_2 + 2\gamma$, $b_3 + 2\gamma$ $\Rightarrow (3 \times 8) \cdot 16$ of $SO(16)_H$

This will be important for the twisted moduli.

beyond the NAHE set   Add $\{\alpha, \beta, \gamma\}$

number of generations is reduced to three

$$SO(10) \longrightarrow SU(3) \times SU(2) \times U(1)_{T_{3_R}} \times U(1)_{B-L}$$

$$U(1)_Y = \frac{1}{2}(B-L) + T_{3_R} \in SO(10) \, !$$

$$SO(6)^{1,2,3} \longrightarrow U(1)^{1,2,3} \times U(1)^{1,2,3}$$

Moduli?

Untwisted moduli $->$ shape & size of the internal dimensions

Twisted moduli $->$ arise from the twisted sectors

models

$$\frac{T^6}{Z_2 \times Z_2}$$

$$T^6 \ : \qquad G_{IJ} \qquad ; \qquad B_{IJ} \qquad\qquad I, J \ = \ 1, \cdots, 6 \ .$$

untwisted moduli: coefficients of exactly marginal operators

moduli fields: massless chiral superfields with flat scalar potential

Scalar couplings of $N = 4$ SUGRA

$$\frac{SO(6,6)}{SO(6) \times SO(6)} \qquad \times \qquad \frac{SU(1,1)}{U(1)}$$

internal manifold   dilaton

Moduli?

Untwisted moduli $->$ shape & size of the internal dimensions

Twisted moduli $->$ arise from the twisted sectors

models

$$\frac{T^6}{Z_2 \times Z_2}$$

$$T^6 \ : \quad G_{IJ} \quad ; \quad B_{IJ} \qquad I, J \ = \ 1, \cdots, 6 \ .$$

untwisted moduli: coefficients of exactly marginal operators

moduli fields: massless chiral superfields with flat scalar potential

Scalar couplings of $N = 4$ SUGRA

$$\frac{SO(6,6)}{SO(6) \times SO(6)} \quad \times \quad \frac{SU(1,1)}{U(1)}$$

internal manifold dilaton

Up to orbifold projections

$$\frac{T^6}{Z_2 \times Z_2} \Longrightarrow \frac{SO(6,6)}{SO(6)\times SO(6)} \longrightarrow \left(\frac{SO(2,2)}{SO(2)\times SO(2)}\right)^3$$

$\Rightarrow$ 3 complex structures $+$ 3 Kähler moduli

In all symmetric $Z_2 \times Z_2$ orbifolds

We would like to identify these moduli in the fermionic formalism

In symmetric orbifolds:

$$\text{EMO} \rightarrow \ \partial X^I \bar{\partial} X^J$$

$$X^I \quad I = 1, \cdots, 6 \ \rightarrow \ T^6$$

$$S = \frac{1}{8\pi}\int d^2\sigma \ \left(G_{IJ}\, \partial X^I \bar{\partial} X^J + B_{IJ}\, \partial X^I \bar{\partial} X^J\right)$$

In FFF $\partial X_L^I \; \rightarrow \; y^i \omega^I$

$i\partial X_L^I \; \rightarrow \; U(1)$ current in the Cartan subalgebra

In the fermionic language

$$i\partial X_L^I \; \rightarrow \; J_L^I = y^I \omega^I$$

$$\Rightarrow \;\; \partial X^I \bar{\partial} X^J \rightarrow J_L^I(z) \bar{J}_R(\bar{z})$$

$\rightarrow$ WS Thirring interactions $\quad (R - \frac{1}{R}) J_L(z) \bar{J}(\bar{z})$

Thirring interactions vanish at free fermionic point with $R = \frac{1}{R}$

To identify the untwisted moduli in the free fermionic models

$\rightarrow$ find the operators of the form

$$J_L^I(z) \bar{J}_R^J(\bar{z})$$

that are allowed by the orbifold (fermionic) symmetry group

Bosonization

$$\xi_i = \sqrt{\frac{1}{2}}(y_i + i\omega_i) = e^{iX_i}, \eta_i = \sqrt{\frac{i}{2}}(y_i - i\omega_i) = ie^{-iX_i}$$

simlarly for the right-movers

Boundary conditions translate to twists and shifts

$$X^I(z,\bar{z}) = X^I_L(z) + X^I_R(\bar{z})$$

Complex internal coordinates

$$Z^{\pm}_k = (X^{2k-1} \pm iX^{2k}), \quad \psi^{\pm}_k = (\chi^{2k-1} \pm i\chi^{2k}) \quad (k = 1, 2, 3)$$

Untwisted moduli

$$S = \int d^2 z h_{ij}(X) J_L^i(z) \bar{J}_R^j(\bar{z})$$

$J_L^i \sim y^i \omega^i$ $\quad$ $i = 1, \cdots, 6$ are chiral currents of $U(1)_L^6$

$J_R^j \sim \bar{\phi}^j \bar{\phi}^{*j}$ $\quad$ $j = 1, \cdots, 22$ are chiral currents of $U(1)_R^{22}$

$h_{ij} \rightarrow$ scalar components of untwisted moduli

some of these operators are projected out in concrete models

$\Rightarrow$ some of the EMO may not be invariant

Models $\{\mathbf{1}, S\}$ $N=4$ $SO(44)$

$$\frac{SO(6,22)}{SO(6)\times SO(22)}$$ Moduli space

$$\chi^i \otimes \bar{\phi}^a \bar{\phi}^{*^a}|0\rangle$$ moduli fields

$$6 \ \times \ 22$$ scalar fields

$Z_2 \times Z_2$ $\{ \, 1 \, , \ S \ , \ \xi_1 \ , \ \xi_2 \, \} \ + \ \{ \, b_1 \ , \ b_2 \, \}$

$$SO(12)\times E_8 \times E_8 \qquad Z_2 \times Z_2$$

$$\rightarrow \quad SO(4)^3 \times E_6 \times U(1)^2 \times E_8$$

The Thirring interactions that remain invariant are

$$J_L^{1,2} \bar{J}_R^{1,2} \quad ; \quad J_L^{3,4} \bar{J}_R^{3,4} \quad ; \quad J_L^{5,6} \bar{J}_R^{5,6}$$

$$y^{1,2} \omega^{1,2} \bar{y}^{1,2} \bar{\omega}^{1,2} \quad ; \quad y^{3,4} \omega^{3,4} \bar{y}^{3,4} \bar{\omega}^{3,4} \quad ; \quad y^{5,6} \omega^{5,6} \bar{y}^{5,6} \bar{\omega}^{5,6}$$

These moduli are always present in symmetric $Z_2 \times Z_2$ orbifolds in realistic models

$$\{\, 1 \,,\, S \,,\, \xi_1 \,,\, \xi_2 \,\} \;\oplus\; \{\, b_1 \,,\, b_2 \,\} \;\oplus\; \{\, \alpha \,,\, \beta \,,\, \gamma \}$$

$$N = 4 \qquad\qquad\qquad N = 1$$

$$E_8 \times E_8 \qquad\qquad\qquad Z_2 \times Z_2$$

new feature Asymmetric orbifold

the key focus: boundary conditions of the internal fermions

$$\{\, y \,,\, \omega \;\mid\; \bar{y} \,,\, \bar{\omega} \,\}$$

WS fermions that have same B.C. in all basis vectors are paired

pairing of LR fermions $\longrightarrow$ Ising model $\longrightarrow$ symmetric real fermions

pairing of LL & RR fermions $\longrightarrow$ complex fermions $\longrightarrow$ asymmetric

STRING DERIVED STANDARD-LIKE MODEL (PLB278)

	ψ^μ	χ^{12}	χ^{34}	χ^{56}	$y^{3,...,6}$	$\bar{y}^{3,...,6}$	$y^{1,2},\omega^{5,6}$	$\bar{y}^{1,2},\bar{\omega}^{5,6}$	$\omega^{1,...,4}$	$\bar{\omega}^{1,...,4}$	$\bar{\psi}^{1,...,5}$	$\bar{\eta}^1$	$\bar{\eta}^2$	$\bar{\eta}^3$	$\bar{\phi}^{1,...,8}$
1	1	1	1	1	1,...,1	1,...,1	1,...,1	1,...,1	1,...,1	1,...,1	1,...,1	1	1	1	1,...,1
S	1	1	1	1	0,...,0	0,...,0	0,...,0	0,...,0	0,...,0	0,...,0	0,...,0	0	0	0	0,...,0
b_1	1	1	0	0	1,...,1	1,...,1	0,...,0	0,...,0	0,...,0	0,...,0	1,...,1	1	0	0	0,...,0
b_2	1	0	1	0	0,...,0	0,...,0	1,...,1	1,...,1	0,...,0	0,...,0	1,...,1	0	1	0	0,...,0
b_3	1	0	0	1	0,...,0	0,...,0	0,...,0	0,...,0	1,...,1	1,...,1	1,...,1	0	0	1	0,...,0

	ψ^μ	χ^{12}	χ^{34}	χ^{56}	y^3y^6	$y^4\bar{y}^4$	$y^5\bar{y}^5$	$\bar{y}^3\bar{y}^6$	$y^1\omega^5$	$y^2\bar{y}^2$	$\omega^6\bar{\omega}^6$	$\bar{y}^1\bar{\omega}^5$	$\omega^2\omega^4$	$\omega^1\bar{\omega}^1$	$\omega^3\bar{\omega}^3$	$\bar{\omega}^2\bar{\omega}^4$	$\bar{\psi}^{1,...,5}$	$\bar{\eta}^1$	$\bar{\eta}^2$	$\bar{\eta}^3$	
α	0	0	0	0	1	0	0	0	0	0	1	1	0	0	1	1	1 1 1 0 0	0	0	0	1 1
β	0	0	0	0	0	0	1	1	1	0	0	0	0	1	0	1	1 1 1 0 0	0	0	0	1 1
γ	0	0	0	0	0	1	0	1	0	1	0	1	1	0	0	0	$\frac{1}{2}$ $\frac{1}{2}$ $\frac{1}{2}$ $\frac{1}{2}$ $\frac{1}{2}$	$\frac{1}{2}$	$\frac{1}{2}$	$\frac{1}{2}$	$\frac{1}{2}$ 0 1

Asymmetric BC $\Rightarrow$ all untwisted moduli are projected out!

all $y_i\omega_i\bar{y}_i\bar{\omega}_i$ are disallowed

can be translated to asymmetric bosonic identifications

$$X_L + X_R \rightarrow X_L - X_R$$

moduli fixed at enhanced symmetry point

Twisted moduli

(2,2) $b_j \oplus b_j + \xi_2 \quad \longrightarrow \quad (16 \oplus 10 + 1) + 1 = 27 + 1 \Rightarrow$ twisted moduli

(2,0) $b_j + b_j + 2\gamma \quad \longrightarrow 16_{SO(10)} + 16_{SO(16)}$

all twisted moduli are projected out

MINIMAL DOUBLET HIGGS CONTENT (EJPC50)

	ψ^μ	χ^{12}	χ^{34}	χ^{56}	y^3y^6	$y^4\bar{y}^4$	$y^5\bar{y}^5$	$\bar{y}^3\bar{y}^6$	$y^1\omega^5$	$y^2\bar{y}^2$	$\omega^6\bar{\omega}^6$	$\bar{y}^1\bar{\omega}^5$	$\omega^2\omega^4$	$\omega^1\bar{\omega}^1$	$\omega^3\bar{\omega}^3$	$\bar{\omega}^2\bar{\omega}^4$	$\bar{\psi}^{1,...,5}$	$\bar{\eta}^1$	$\bar{\eta}^2$	$\bar{\eta}^3$	$\bar{\phi}$
α	0	0	0	0	1	0	0	1	0	0	1	1	0	0	1	1	1 1 1 0 0	1	0	0	1 1 0
β	0	0	0	0	0	0	1	1	1	0	0	1	0	1	0	1	1 1 1 0 0	0	1	0	0 0 1
γ	0	0	0	0	0	1	0	0	0	1	0	0	1	0	0	0	$\frac{1}{2}$ $\frac{1}{2}$ $\frac{1}{2}$ $\frac{1}{2}$ $\frac{1}{2}$	$\frac{1}{2}$	$\frac{1}{2}$	$\frac{1}{2}$	0 0 0

(With Elisa Manno and Cristina Timirgaziu)

SYMMETRIC $\leftrightarrow$ ASYMMETRIC

with respect to b_1 & b_2

$h_1, \bar{h}_1, D_1, \bar{D}_1$, $h_2, \bar{h}_2, D_2, \bar{D}_2$ are projected out

$h_3, \bar{h}_3$ remain in the spectrum

$$\lambda_t Q_3 t_3^c \bar{h}_3 \quad \text{with} \quad \lambda_t \; O(1)$$

No Phenomenologically viable flat directions

Calculation of Mass Terms

nonvanishing correlators

$$\langle V_1^f V_2^f V_3^b \cdot \cdots V_N^b \rangle$$

gauge & string invariant

"anomalous" $U(1)_A$

$$\mathrm{Tr} Q_A \neq 0 \Rightarrow \quad D_A = 0 = A + \sum Q_k^A |\langle \phi_k \rangle|^2$$

$$D_j \ = 0 = \qquad \sum Q_k^j |\langle \phi_k \rangle|^2$$

$$\langle W \rangle = \langle \frac{\partial W_N}{\partial \eta_i} \rangle = 0 \quad N = 3 \cdots\cdot$$

Supersymmetric vacuum $\langle F \rangle = \langle D \rangle = 0.$

nonrenormalizable terms $\longrightarrow$ effective renormalizable operators

$$V_1^f V_2^f V_3^b \cdot \cdots V_N^b \ \rightarrow \ V_1^f V_2^f V_3^b \frac{\langle V_4^b \cdots V_N^b \rangle}{M^{N-3}}$$

Cleaver, Faraggi, Manno, Timirgaziu, PRD 78 (2008) 046009

Classification of F and D flat directions in FMT reduced Higgs model

No D flat direction which is F-flat up to order eight in the superpotential

no stringent flat directions to all orders

Suggesting no supersymmetric flat directions in this model (class of models)

implying no supersymmetric moduli

only remaining perturbative moduli is the dilaton

quasi–realistic model: SLM; 3 gen; $SO(10)$ embed; Higgs & $\lambda_t \sim 1$; ...

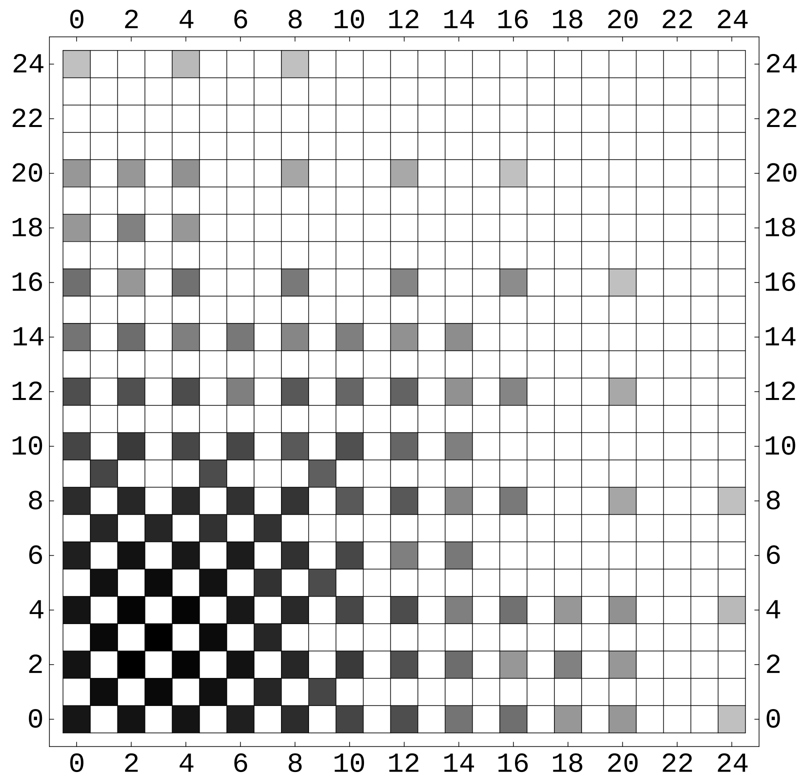
vanishing one–loop partition function, perturbatively broken SUSY

Fixed geometrical, twisted and SUSY moduli

Cleaver $etal$, $SO(10)$ and FSU5 analysis $->$ stringent flat directions

Spinor–vector duality:

Invariance under exchange of $\#(16+\overline{16})$ $<->$ $\#(10)$



Symmetric under exchange of rows and columns

$$E_6 \quad : \qquad 27 = 16 + 10 + 1 \qquad \overline{27} = \overline{16} + 10 + 1$$

Self-dual: $\#(16+\overline{16}) \; = \; \#(10)$ without E_6 symmetry

Spinor–Vector duality in Orbifolds:

Using the level-one $\mathrm{SO}(2n)$ characters

$$O_{2n} = \tfrac{1}{2}\left(\frac{\theta_3^n}{\eta^n} + \frac{\theta_4^n}{\eta^n}\right) , \qquad V_{2n} = \tfrac{1}{2}\left(\frac{\theta_3^n}{\eta^n} - \frac{\theta_4^n}{\eta^n}\right) ,$$

$$S_{2n} = \tfrac{1}{2}\left(\frac{\theta_2^n}{\eta^n} + i^{-n}\frac{\theta_1^n}{\eta^n}\right) , \qquad C_{2n} = \tfrac{1}{2}\left(\frac{\theta_2^n}{\eta^n} - i^{-n}\frac{\theta_1^n}{\eta^n}\right) .$$

where

$$\theta_3 \equiv Z_f\binom{0}{0} \quad \theta_4 \equiv Z_f\binom{0}{1}$$

$$\theta_2 \equiv Z_f\binom{1}{0} \quad \theta_1 \equiv Z_f\binom{1}{1}$$

Starting from:

$$Z_+ = (V_8 - S_8) \left(\sum_{m,n} \Lambda_{m,n} \right)^{\otimes 6} (\bar{O}_{16} + \bar{S}_{16}) \, (\bar{O}_{16} + \bar{S}_{16}) \ ,$$

where as usual, for each circle,

$$p^i_{\rm L,R} = \frac{m_i}{R_i} \pm \frac{n_i R_i}{\alpha'} \,,$$

and

$$\Lambda_{m,n} = \frac{q^{\frac{\alpha'}{4} p^2_{\rm L}} \, \bar{q}^{\frac{\alpha'}{4} p^2_{\rm R}}}{|\eta|^2} \,.$$

apply $Z_2 \times Z_2' : g \times g'$

$$g : (-1)^{(F_1+F_2)}\delta$$

$$F_{1,2} : (\,\overline{O}_{16}^{1,2},\ \overline{V}_{16}^{1,2},\ \overline{S}_{16}^{1,2},\ \overline{C}_{16}^{1,2}\,) \longrightarrow (\,\overline{O}_{16}^{1,2},\ \overline{V}_{16}^{1,2},\ -\overline{S}_{16}^{1,2},\ -\overline{C}_{16}^{1,2}\,)$$

with $\delta X_9 = X_9 + \pi R_9$,

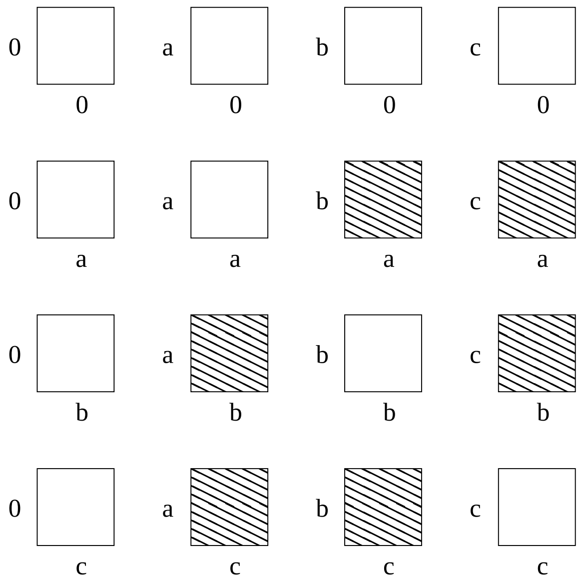
$$\delta : \Lambda_{m,n}^{9} \longrightarrow (-1)^m \Lambda_{m,n}^{9}$$

$$g' : (x_4, x_5, x_6, x_7, x_8, x_9) \longrightarrow (-x_4, -x_5, -x_6, -x_7, +x_8, +x_9)$$

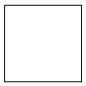
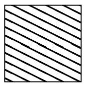
Note: A single space twisting Z_2' $\Rightarrow$ $N = 4 \to N = 2$

$E_7 \to SO(12) \times SU(2)$

$\Rightarrow$ Analyze $\quad Z = \left(\frac{Z_+}{Z_g \times Z_{g'}}\right) = \left[\frac{(1+g)}{2}\frac{(1+g')}{2}\right] Z_+$



a = g ; b = g' ; c = gg'

P.F. = ( + ε ) = $\Lambda_{m,n}$ • ( ) + $\Lambda_{m,n+1/2}$ • ( )

ε = ± 1

massless massive

• sector b

$$\Lambda_{p,q}\left\{\frac{1}{2}\left(\left|\frac{2\eta}{\theta_4}\right|^4+\left|\frac{2\eta}{\theta_3}\right|^4\right)\left[P_\epsilon^+ Q_s \overline{V}_{12}\overline{C}_4\overline{O}_{16}+P_\epsilon^- Q_s \overline{S}_{12}\overline{O}_4\overline{O}_{16}\right]+\frac{1}{2}\left(\left|\frac{2\eta}{\theta_4}\right|^4-\left|\frac{2\eta}{\theta_3}\right|^4\right)\left[P_\epsilon^+ Q_s \overline{O}_{12}\overline{S}_4\overline{O}_{16}\right]\right\} + \text{massive}$$

where

$$P_\epsilon^+ = \left(\frac{1+\epsilon(-1)^m}{2}\right)\Lambda_{m,n} \qquad P_\epsilon^- = \left(\frac{1-\epsilon(-1)^m}{2}\right)\Lambda_{m}$$

$$\epsilon = +1 \Rightarrow P_\epsilon^+ = \Lambda_{2m,n} \qquad P_\epsilon^- = \Lambda_{2m+1,n}$$

$$\epsilon = -1 \Rightarrow P_\epsilon^+ = \Lambda_{2m+1,n} \qquad P_\epsilon^- = \Lambda_{2m,n}$$

and $12\cdot 2 + 4\cdot 2 = 32$

Further :

- The spinor–vector duality in this model is realised in terms of a continuous interpolation between two discrete Wilson lines.

- The spinor-vector duality is realised in terms of a spectral flow operator that operates in the bosonic side of the heterotic string. In the case of enhanced E_6 symmetry, the spectral flow operator acts as an internal E_6 generator. When E_6 is broken the spectral flow operator induces the spinor–vector duality map.

Conclusions

Moduli fixing $\longrightarrow$ Model dependence.

spinor–vector duality $\rightarrow$ Physics & Geometry