D–term uplift in non-supersymmetric heterotic–string vacua

- Non–supersymmetric heterotic–string vacua
- D–term uplift in non–supersymmetric vacua
- Asymmetric   free fermion – orbifold   and string moduli

AEF, V Matyas, B Percival, EPJC80 (2020) 337; NPB961 (2020) 115231;
PRD104 (2021) 046002; PRD106 (2022) 026011
A Diaz Avalos, AEF, V Matyas, B Percival, arXiv:2302:10075; 2306:16878
AEF, NPB728 (2005) 83
AEF, S Groot Nibbelink, B Percival, arXiv:2306.16443

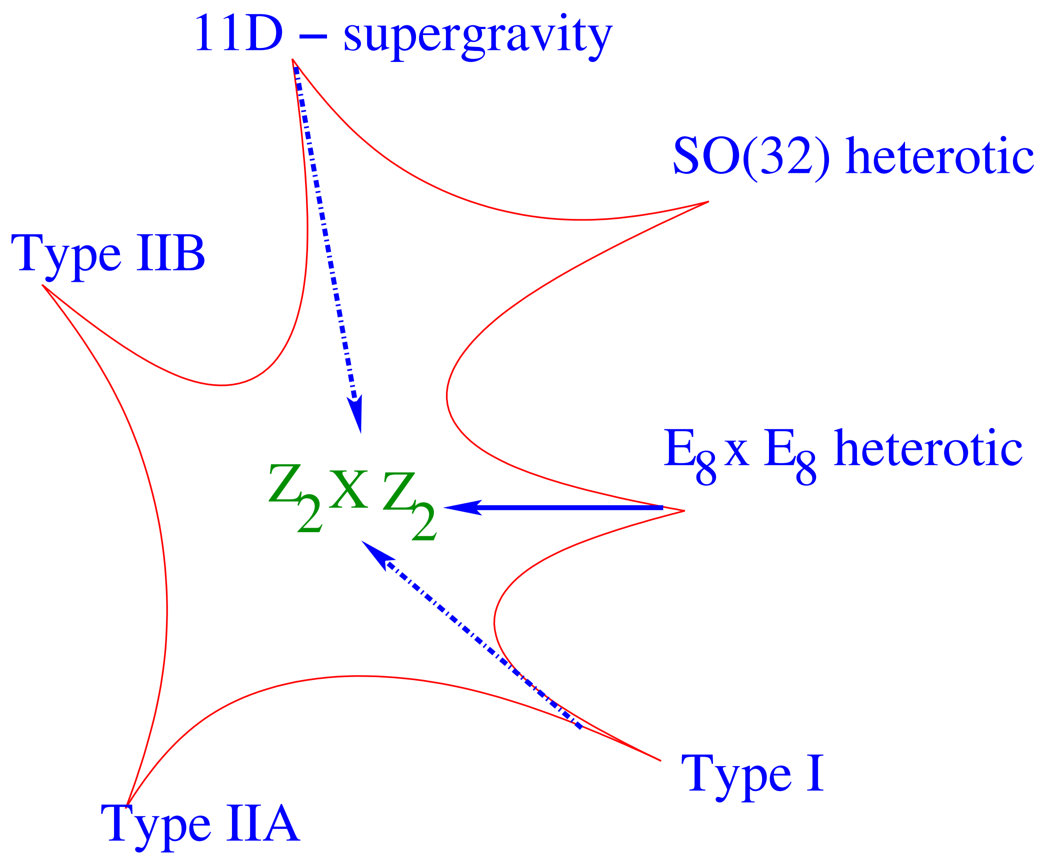
SUSY 2023, Southampton University, 11 July 2023

Fermionic $Z_2 \times Z_2$ orbifolds

'Phenomenology of the Standard Model and Unification'

- Minimal Superstring Standard Model — NPB 335 (1990) 347 (with Nanopoulos & Yuan)
- Top quark mass $\sim$ 175–180GeV — PLB 274 (1992) 47
- Generation mass hierarchy — NPB 407 (1993) 57
- CKM mixing — NPB 416 (1994) 63 (with Halyo)
- Stringy seesaw mechanism — PLB 307 (1993) 311 (with Halyo)
- Gauge coupling unification — NPB 457 (1995) 409 (with Dienes)
- Proton stability — NPB 428 (1994) 111
- Squark degeneracy — NPB 526 (1998) 21 (with Pati)
- Moduli fixing — NPB 728 (2005) 83
- Classification — 2003 – $\cdots$ (with Kounnas, Rizos & ... Percival, Matyas)

Point, String, Membrane ....



+ ... SO(16)xSO(16), E8, SO(16)xE8 + ...

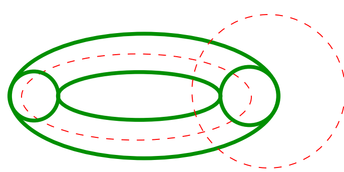
... Abel, Basile, Dienes, Kaidi, Itoyama ...

Fermionic Construction

Left-Movers: $\psi^{\mu=1,2}\ ,\quad \chi_i,\quad y_i,\ \ \omega_i\quad (i=1,\cdots,6)$

Right-Movers

$$\bar{\phi}_{A=1,\cdots,44} = \begin{cases} \bar{y}_i\ ,\ \bar{\omega}_i & i=1,\cdots,6 \\ & \\ \bar{\eta}_i & U(1)_i \quad i=1,2,3 \\ \bar{\psi}_{1,\cdots,5} & SO(10) \\ \bar{\phi}_{1,\cdots,8} & SO(16) \end{cases}$$



$$V \longrightarrow V \qquad\qquad f \longrightarrow -e^{i\pi\alpha(f)}f$$

$$Z = \sum_{\substack{all\ spin\\ structures}} c\binom{\vec{\alpha}}{\vec{\beta}}\ Z\binom{\vec{\alpha}}{\vec{\beta}}$$

Models $\longleftrightarrow$ Basis vectors $+$ one–loop phases

Classification of fermionic $Z_2 \times Z_2$ orbifolds

Basis vectors:

$$1 = \{\psi^\mu,\ \chi^{1,\dots,6}, y^{1,\dots,6}, \omega^{1,\dots,6} \mid \bar{y}^{1,\dots,6}, \bar{\omega}^{1,\dots,6}, \bar{\eta}^{1,2,3}, \bar{\psi}^{1,\dots,5}, \bar{\phi}^{1,\dots,8}\}$$

$$S = \{\psi^\mu, \chi^{1,\dots,6}\},$$

$$z_1 = \{\bar{\phi}^{1,\dots,4}\},$$

$$z_2 = \{\bar{\phi}^{5,\dots,8}\},$$

$$e_i = \{y^i, \omega^i | \bar{y}^i, \bar{\omega}^i\},\ i = 1, \dots, 6,$$ $N = 4$ Vacua

$$b_1 = \{\chi^{34}, \chi^{56}, y^{34}, y^{56} | \bar{y}^{34}, \bar{y}^{56}, \bar{\eta}^1, \bar{\psi}^{1,\dots,5}\},$$ $N = 4 \rightarrow N = 2$

$$b_2 = \{\chi^{12}, \chi^{56}, y^{12}, y^{56} | \bar{y}^{12}, \bar{y}^{56}, \bar{\eta}^2, \bar{\psi}^{1,\dots,5}\},$$ $N = 2 \rightarrow N = 1$

Vector bosons: NS, $z_{1,2}$, $z_1 + z_2$, $\quad X = 1 + s + \sum e_i + z_1 + z_2$

impose: $c\begin{bmatrix} z_1 \\ z_2 \end{bmatrix} = -1$ & Gauge group $SO(10) \times U(1)^3 \times \text{hidden}$

Independent phases $c\left[{vi \atop v_j}\right] = \exp[i\pi(v_i|v_j)]$: upper block

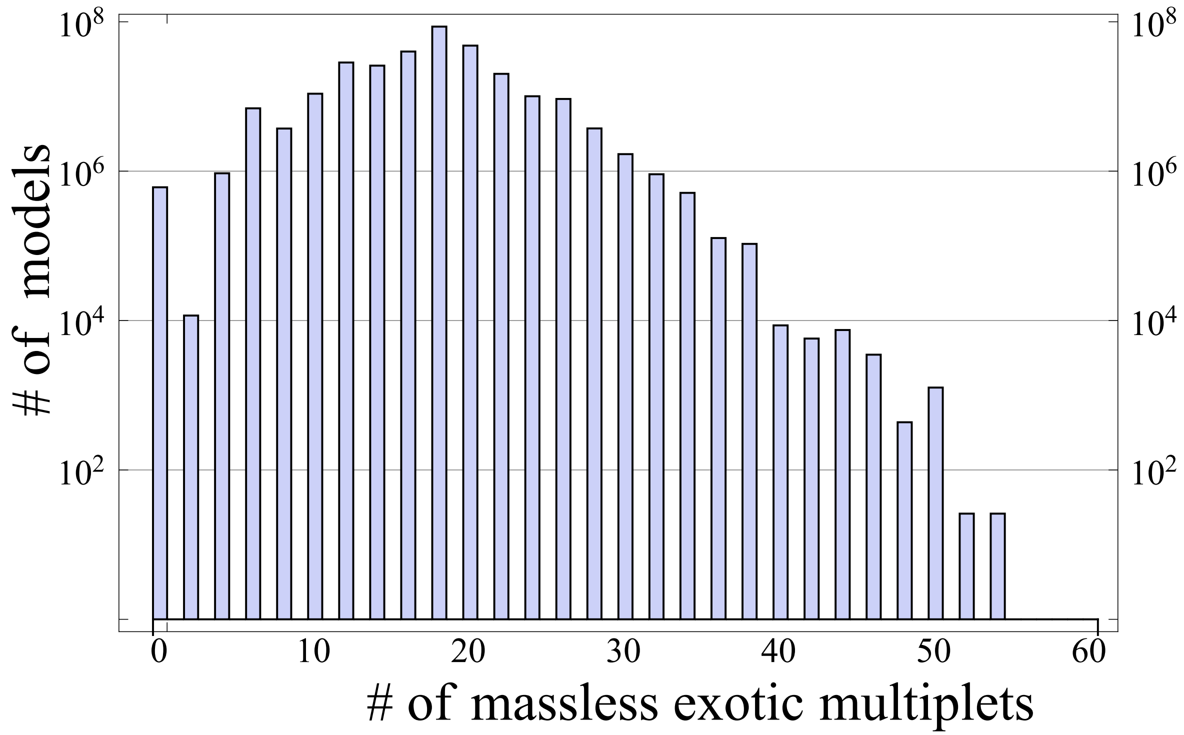
$$
\begin{array}{c}
 \\ 1 \\ S \\ e_1 \\ e_2 \\ e_3 \\ e_4 \\ e_5 \\ e_6 \\ z_1 \\ z_2 \\ b_1 \\ b_2
\end{array}
\begin{array}{c}
\begin{array}{cccccccccccc}
1 & S & e_1 & e_2 & e_3 & e_4 & e_5 & e_6 & z_1 & z_2 & b_1 & b_2
\end{array} \\
\left(\begin{array}{cccccccccccc}
-1 & -1 & \pm & \pm & \pm & \pm & \pm & \pm & \pm & \pm & \pm & \pm \\
 & & -1 & -1 & -1 & -1 & -1 & -1 & -1 & -1 & 1 & 1 \\
 & & & \pm & \pm & \pm & \pm & \pm & \pm & \pm & \pm & \pm \\
 & & & & \pm & \pm & \pm & \pm & \pm & \pm & \pm & \pm \\
 & & & & & \pm & \pm & \pm & \pm & \pm & \pm & \pm \\
 & & & & & & \pm & \pm & \pm & \pm & \pm & \pm \\
 & & & & & & & \pm & \pm & \pm & \pm & \pm \\
 & & & & & & & & \pm & \pm & \pm & \pm \\
 & & & & & & & & & \pm & \pm & \pm \\
 & & & & & & & & & & \pm & \pm \\
 & & & & & & & & & & & \pm \\
 & & & & & & & & & & & -1
\end{array}\right)
\end{array}
$$

A priori 55 independent coefficients $\rightarrow$ 2^{55} distinct vacua

PLB2021, Percival *et al* $\rightarrow$ Satisfiability Modulo Theories $\longrightarrow$ $t \times 10^{-3}$

Pati–Salam class: with Assel, Christodoulides, Kounnas, Rizos

RESULTS: of random search of over 10^{11} vacua



Number of 3-generation models versus total number of exotic multiplets

NON–SUSY String Phenomenology:

Starting with: $$Z^{+}_{10d} = (V_8 - S_8)\left(\overline{O}_{16} + \overline{S}_{16}\right)\left(\overline{O}_{16} + \overline{S}_{16}\right),$$

using the level–one $SO(2n)$ characters

$$O_{2n} = \tfrac{1}{2}\left(\frac{\theta_3^n}{\eta^n} + \frac{\theta_4^n}{\eta^n}\right), \qquad V_{2n} = \tfrac{1}{2}\left(\frac{\theta_3^n}{\eta^n} - \frac{\theta_4^n}{\eta^n}\right),$$

$$S_{2n} = \tfrac{1}{2}\left(\frac{\theta_2^n}{\eta^n} + i^{-n}\frac{\theta_1^n}{\eta^n}\right), \qquad C_{2n} = \tfrac{1}{2}\left(\frac{\theta_2^n}{\eta^n} - i^{-n}\frac{\theta_1^n}{\eta^n}\right).$$

where

$$\theta_3 \equiv Z_f\binom{0}{0} \qquad \theta_4 \equiv Z_f\binom{0}{1} \qquad \theta_2 \equiv Z_f\binom{1}{0} \qquad \theta_1 \equiv Z_f\binom{1}{1}$$

Apply $g = (-1)^{F + F_{z_1} + F_{z_2}}$

$$\begin{aligned} Z^{-}_{10d} = & \left[\, V_8\left(\overline{O}_{16}\overline{O}_{16} + \overline{S}_{16}\overline{S}_{16}\right) - S_8\left(\overline{O}_{16}\overline{S}_{16} + \overline{S}_{16}\overline{O}_{16}\right)\right. \\ + & \left. \underline{O_8\left(\overline{C}_{16}\overline{V}_{16} + \overline{V}_{16}\overline{C}_{16}\right)} - C_8\left(\overline{C}_{16}\overline{C}_{16} + \overline{V}_{16}\overline{V}_{16}\right)\right]. \end{aligned}$$

In fermionic language: $\{ \mathbf{1} , z_1 , z_2 \}$

where $z_1 = \{\bar\psi^{1,\cdots,5}, \bar\eta^{1,2,3}\}$; $z_2 = \{\bar\phi^{1,\cdots,8}\}$ $\Rightarrow$ $S = \mathbf{1} + z_1 + z_2$

$c\binom{z_1}{z_2} = +1 \implies E_8 \times E_8$; $c\binom{z_1}{z_2} = -1 \implies SO(16) \times SO(16)$

Tachyon free non-SUSY string phenomenology

Alternatively: Apply $g = (-1)^{F+F_{z_1}}$

$$Z^-_{10d} = (V_8\overline{O}_{16} - S_8\overline{S}_{16} + \underline{O_8\overline{V}_{16}} - C_8\overline{C}_{16})\,(\overline{O}_{16} + \overline{S}_{16}) \,,$$

$O_8\overline{V}_{16}\overline{O}_{16} \implies$ tachyonic 10D vacuum

In fermionic language: $\{ \mathbf{1} , z_2 \}$ $\implies$ No S

In both cases $\longrightarrow$ tachyon free 4D GSO configurations

Tachyon free models: $S \longleftrightarrow \tilde{S}$–map $\longleftarrow$ "modular map"

Modified NAHE $\longleftrightarrow$ $\overline{\mathrm{NAHE}}$

	ψ^μ	χ^{12}	χ^{34}	χ^{56}	$y^{3,\ldots,6}$	$\bar{y}^{3,\ldots,6}$	$y^{1,2},\omega^{5,6}$	$\bar{y}^{1,2},\bar{\omega}^{5,6}$	$\omega^{1,\ldots,4}$	$\bar{\omega}^{1,\ldots,4}$	$\bar{\psi}^{1,\ldots,5}$	$\bar{\eta}^1$	$\bar{\eta}^2$	$\bar{\eta}^3$	$\bar{\phi}^{1,\ldots,8}$
1	1	1	1	1	1,...,1	1,...,1	1,...,1	1,...,1	1,...,1	1,...,1	1,...,1	1	1	1	1,1,1,1,1,1,1,1
$\tilde{S}$	1	1	1	1	0,...,0	0,...,0	0,...,0	0,...,0	0,...,0	0,...,0	0,...,0	0	0	0	1,1,1,1,0,0,0,0
b_1	1	1	0	0	1,...,1	1,...,1	0,...,0	0,...,0	0,...,0	0,...,0	1,...,1	1	0	0	0,0,0,0,0,0,0,0
b_2	1	0	1	0	0,...,0	0,...,0	1,...,1	1,...,1	0,...,0	0,...,0	1,...,1	0	1	0	0,0,0,0,0,0,0,0
b_3	1	0	0	1	0,...,0	0,...,0	0,...,0	0,...,0	1,...,1	1,...,1	1,...,1	0	0	1	0,0,0,0,0,0,0,0

Beyond the $\overline{\mathrm{NAHE}}$–set

	ψ^μ	χ^{12}	χ^{34}	χ^{56}	y^3y^6	$y^4\bar{y}^4$	$y^5\bar{y}^5$	$\bar{y}^3\bar{y}^6$	$y^1\omega^5$	$y^2\bar{y}^2$	$\omega^6\bar{\omega}^6$	$\bar{y}^1\bar{\omega}^5$	$\omega^2\omega^4$	$\omega^1\bar{\omega}^1$	$\omega^3\bar{\omega}^3$	$\bar{\omega}^2\bar{\omega}^4$	$\bar{\psi}^{1,\ldots,5}$	$\bar{\eta}^1$	$\bar{\eta}^2$	$\bar{\eta}^3$	$\bar{\phi}^{1,}$
α	0	0	0	0	1	0	0	1	0	0	1	1	0	0	1	1	1 1 1 0 0	1	0	0	0 0 0 0
β	0	0	0	0	0	0	1	1	1	0	0	1	0	1	0	1	1 1 1 0 0	0	1	0	1 1 0 0
γ	0	0	0	0	0	1	0	0	0	1	0	0	1	0	0	0	$\frac{1}{2}$ $\frac{1}{2}$ $\frac{1}{2}$ $\frac{1}{2}$ $\frac{1}{2}$	$\frac{1}{2}$	$\frac{1}{2}$	$\frac{1}{2}$	0 0 $\frac{1}{2}$ $\frac{1}{2}$

Up to the $S \longleftrightarrow \tilde{S}$–map

Same model as published with

with Cleaver, Manno and Timirgaziu in PRD78 (2008) 046009

Stable non–SUSY heterotic–string vacuum?

Classification of tachyon free models

Basis vectors:

$$1 = \{\psi^\mu,\ \chi^{1,\dots,6}, y^{1,\dots,6}, \omega^{1,\dots,6} \mid \bar{y}^{1,\dots,6}, \bar{\omega}^{1,\dots,6}, \bar{\eta}^{1,2,3}, \bar{\psi}^{1,\dots,5}, \bar{\phi}^{1,\dots,8}\}$$

$$S = \{\psi^\mu, \chi^{1,\dots,6}\}$$ and $$\tilde{S} = \{\psi^\mu, \chi^{1,\dots,6} \mid \bar{\phi}^{3,\cdots,6}\},$$

$$z_1 = \{\bar{\phi}^{1,\dots,4}\},$$

$$z_2 = \{\bar{\phi}^{5,\dots,8}\},$$

$$e_i = \{y^i, \omega^i | \bar{y}^i, \bar{\omega}^i\},\ i = 1, \dots, 6,$$

$$b_1 = \{\psi^{12}, \chi^{12}, y^{34}, y^{56} | \bar{y}^{34}, \bar{y}^{56}, \bar{\eta}^1, \bar{\psi}^{1,\dots,5}\}, \qquad N = 4 \to N = 2$$

$$b_2 = \{\psi^{12}, \chi^{34}, y^{12}, y^{56} | \bar{y}^{12}, \bar{y}^{56}, \bar{\eta}^2, \bar{\psi}^{1,\dots,5}\}, \qquad N = 2 \to N = 1$$

$$\alpha = \{\bar{\psi}^{4,5}, \bar{\phi}^{1,2}\}$$

with Viktor Matyas and Ben Percival, NPB 961 (2020) 115231;

PRD 104 (2021) 04600; PRD 106 (2022) 026011

Partition functions and the cosmological constant

Full Partition Function for Free Fermionic models:

$$Z_{ToT} \quad = \quad \int_{\mathfrak{F}} \frac{d^2\tau}{\tau_2^2} Z_B \, Z_F \quad \equiv \quad \Lambda$$

Integral over the inequivalent tori

- Fermionic contribution:

$$Z_F = \sum_{Sp.Str.} c\binom{\alpha}{\beta} \prod_f Z\begin{bmatrix}\alpha(f)\\ \beta(f)\end{bmatrix}$$

$$Z\begin{bmatrix}1\\1\end{bmatrix} = \sqrt{\frac{\theta_1}{\eta}}, \qquad Z\begin{bmatrix}1\\0\end{bmatrix} = \sqrt{\frac{\theta_2}{\eta}}, \qquad Z\begin{bmatrix}0\\0\end{bmatrix} = \sqrt{\frac{\theta_3}{\eta}}, \qquad Z\begin{bmatrix}0\\1\end{bmatrix} = \sqrt{\frac{\theta_4}{\eta}},$$

- Bosonic : $Z_B = \frac{1}{\tau_2}\frac{1}{\eta^2\bar{\eta}^2}$ from spacetime Bosons.

Evaluated using $q \equiv e^{2\pi i\tau}$ expansion

$$Z = \sum_{n.m} a_{mn} \int_{\mathcal{F}} \frac{d^2\tau}{\tau_2^3} q^m \bar{q}^n$$

$$\begin{cases} d\tau_1 & \longrightarrow \textit{analytic} \\ d\tau_2 & \longrightarrow \textit{numeric} \end{cases}$$

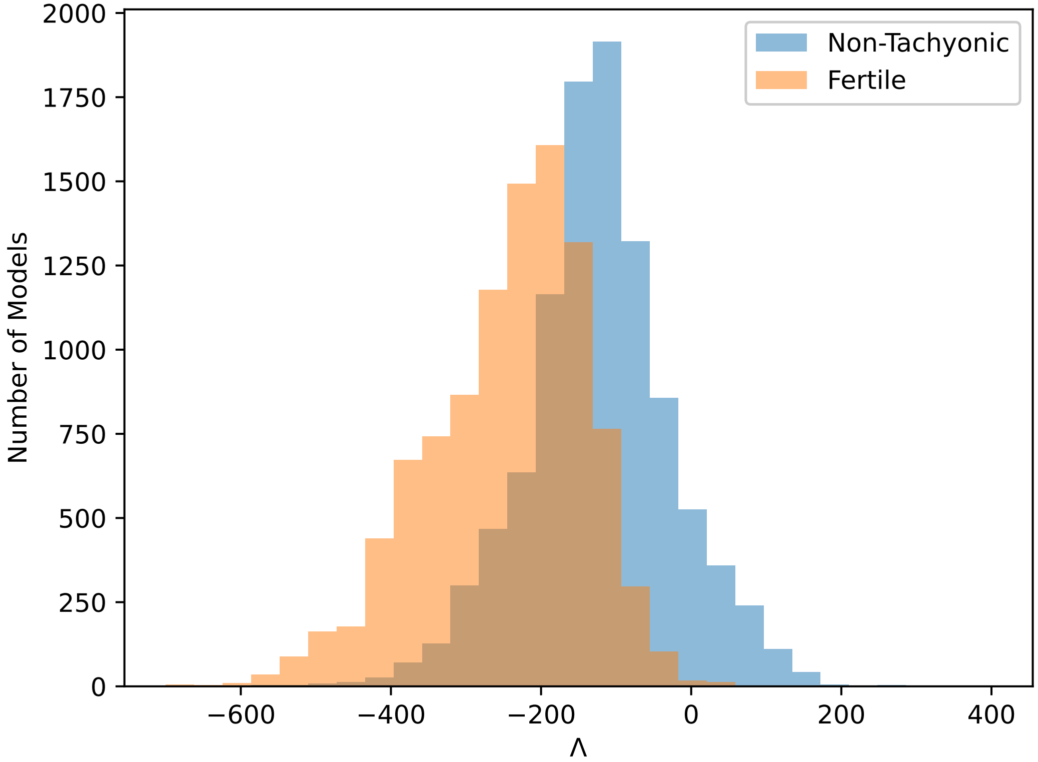
$q-$ expansion of Z

$$I_{mn} = \begin{cases} \infty & \text{if} \quad m+n<0 \ \wedge \ m-n \notin \mathbb{Z}\backslash\{0\} \\ \text{Finite} & \text{Otherwise.} \end{cases}$$

- On-Shell Tachyons cause divergence
- Off-Shell Tachyons allowed (necessary)

Modular invariance $\longrightarrow m-n \in \mathbb{Z}$.

Distribution of Λ



Away from the free fermionic point:

$$Z = \int \frac{d^2\tau}{\tau_2^2} \frac{\tau_2^{-1}}{\eta^{12}\bar\eta^{24}} \frac{1}{2^3} \left(\sum (-)^{a+b+ab} \vartheta \begin{bmatrix} a \\ b \end{bmatrix} \vartheta \begin{bmatrix} a+h_1 \\ b+g_1 \end{bmatrix} \vartheta \begin{bmatrix} a+h_2 \\ b+g_2 \end{bmatrix} \vartheta \begin{bmatrix} a+h_3 \\ b+g_3 \end{bmatrix} \right)_{\psi^\mu,\chi}$$

$$\times \left(\frac{1}{2} \sum_{\epsilon,\xi} \bar\vartheta \begin{bmatrix} \epsilon \\ \xi \end{bmatrix}^5 \bar\vartheta \begin{bmatrix} \epsilon+h_1 \\ \xi+g_1 \end{bmatrix} \bar\vartheta \begin{bmatrix} \epsilon+h_2 \\ \xi+g_2 \end{bmatrix} \bar\vartheta \begin{bmatrix} \epsilon+h_3 \\ \xi+g_3 \end{bmatrix} \right)_{\bar\psi^{1\ldots5},\bar\eta^{1,2,3}}$$

$$\times \left(\frac{1}{2} \sum_{H_1,G_1} \frac{1}{2} \sum_{H_2,G_2} (-)^{H_1G_1+H_2G_2} \bar\vartheta \begin{bmatrix} \epsilon+H_1 \\ \xi+G_1 \end{bmatrix}^4 \bar\vartheta \begin{bmatrix} \epsilon+H_2 \\ \xi+G_2 \end{bmatrix}^4 \right)_{\bar\phi^{1\ldots8}}$$

$$\times \left(\sum_{s_i,t_i} \Gamma_{6,6} \begin{bmatrix} h_i|s_i \\ g_i|t_i \end{bmatrix} \right)_{(y\omega\bar y\bar\omega)^{1\ldots6}} \times e^{i\pi\Phi(\gamma,\delta,s_i,t_i,\epsilon,\xi,h_i,g_i,H_1,G_1,H_2,G_2)}$$

$$\Gamma_{1,1}[^h_g] = \frac{R}{\sqrt{\tau_2}} \sum_{\tilde m,n} \exp\left[-\frac{\pi R^2}{\tau_2} \left| (2\tilde m + g) + (2n+h)\,\tau \right|^2 \right]$$

Fayet–Iliopoulos D–term uplift (w Diaz-Avalos, Matyas, Percival)

"anomalous" $U(1)_A$ $\Rightarrow$ FI–term in $N=1$ supersymmetric vacua

$$\text{Tr}Q_A \neq 0 \Rightarrow \quad D_A = 0 = A + \sum Q_k^A |\langle\phi_k\rangle|^2$$

$$D_j = 0 = \qquad \sum Q_k^j |\langle\phi_k\rangle|^2$$

$$\langle W\rangle = \langle \frac{\partial W_N}{\partial \eta_i}\rangle = 0 \;\; N = 3\cdots\cdot$$

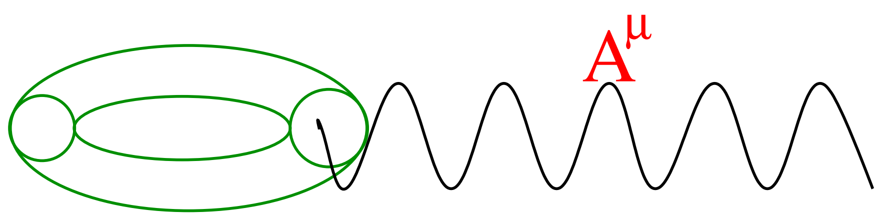
nonvanishing correlators

$$\langle V_1^f V_2^f V_3^b \cdots \cdot V_N^b\rangle$$

gauge & string invariant

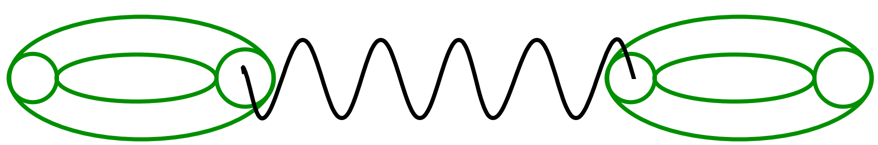
Supersymmetric vacuum $\langle F\rangle = \langle D\rangle = 0.$

An anomalous $U(1)_A$ $\longleftrightarrow$ Recurring feature in $N=0$ string vacua



The calculation of the "FI–term" follows through

$$V=\frac{1}{2}g_s^2\zeta^2$$



SUSY broken by the choice of the phase Φ:

- explicit breaking $e.g.\ c\binom{S}{z_1} = +1$

  gravitino is projected out with mass $M \sim M_S$

- Scherk-Schwarz mechanism $\rightarrow$ Supersymmetry is broken spontaneously

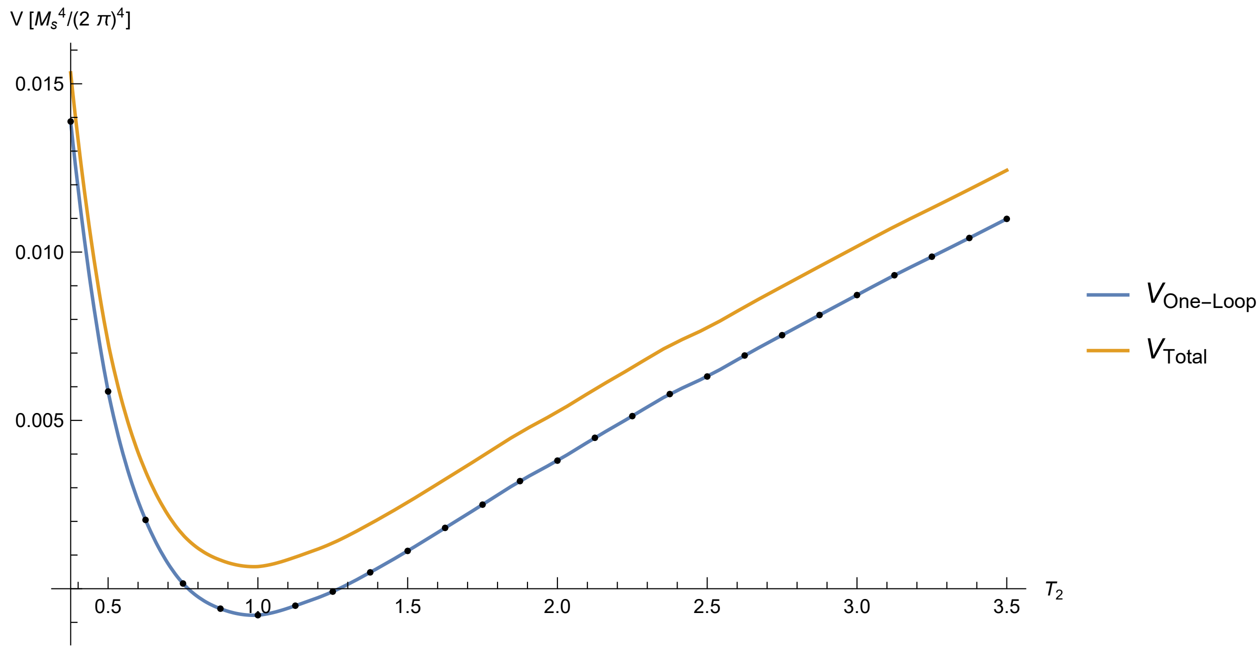
  $\Phi(X^5 + 2\pi R) = \mathrm{e}^{iQ}\Phi(X^5)$ Q fermion number

  $g = (-1)^F \delta \qquad \delta \rightarrow X = X + \pi R \qquad\qquad \longleftrightarrow \quad c\binom{S}{e_i} = +1$

$$M_{\frac{3}{2}} \sim \frac{1}{R}$$

Supersymmetry is restored in the $R \rightarrow \infty$ limit

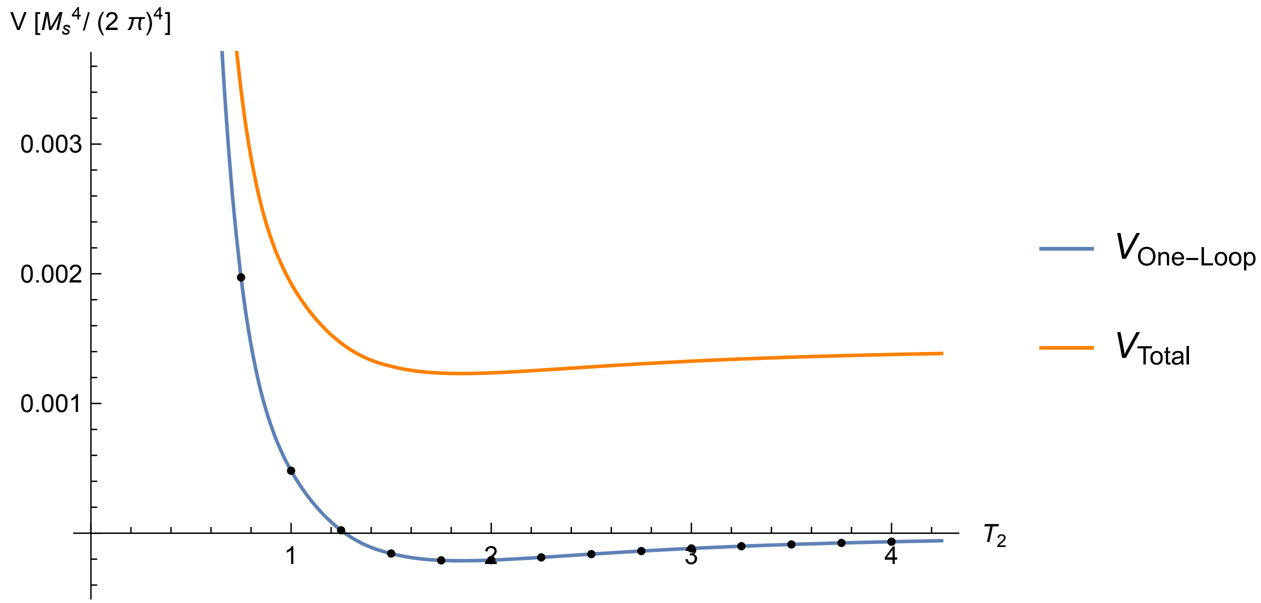
Uplift with explicit breaking $(g_s \sim 1)$



$$\Lambda = \quad -0.000785598\mathcal{M}^4$$

$$V_D = \quad 0.00144365\mathcal{M}^4$$

Uplift with Scherk–Schwarz breaking



$$\Lambda = -0.000215338\mathcal{M}^4$$

$$V_D = 0.00144365\mathcal{M}^4$$

Moduli $\rightarrow$ WS Thirring interactions $(R-\frac{1}{R})J_L^i(z)\bar{J}_R^j(\bar{z}) = (R-\frac{1}{R})y^i\omega^i\bar{y}^j\bar{\omega}^j$

To identify the untwisted moduli in the free fermionic models

$\rightarrow$ find the operators of the form

$$J_L^I(z)\bar{J}_R^J(\bar{z}) \quad \equiv \quad y^I\omega^I\bar{y}^J\bar{\omega}^J$$

that are allowed by the orbifold (fermionic) symmetry group

$$Z_2 \times Z_2 \qquad \{\,1\,,\ S\,,\ z_1\,,\ z_2\,\} \ + \ \{\,b_1\,,\ b_2\,\}$$

$$\rightarrow \quad SO(4)^3 \times E_6 \times U(1)^2 \times E_8$$

The Thirring interactions that remain invariant are

$$J_L^{1,2}\bar{J}_R^{1,2} \quad ; \quad J_L^{3,4}\bar{J}_R^{3,4} \quad ; \quad J_L^{5,6}\bar{J}_R^{5,6}$$

$$y^{1,2}\omega^{1,2}\bar{y}^{1,2}\bar{\omega}^{1,2} \quad ; \quad y^{3,4}\omega^{3,4}\bar{y}^{3,4}\bar{\omega}^{3,4} \quad ; \quad y^{5,6}\omega^{5,6}\bar{y}^{5,6}\bar{\omega}^{5,6}$$

These moduli are always present in symmetric $Z_2 \times Z_2$ orbifolds in realistic models

$$\{\,1\,,\,S\,,\,z_1\,,\,z_2\,\}\;\oplus\;\{\,b_1\,,\,b_2\,\}\;\oplus\;\{\,\alpha\,,\,\beta\,,\,\gamma\}$$

$$N=4 \qquad\qquad N=1$$

$$E_8\times E_8 \qquad\qquad Z_2\times Z_2$$

new feature Asymmetric orbifold $y^i\omega^i\bar{y}^i\bar{\omega}^i \;\rightarrow\; -\,y^i\omega^i\bar{y}^i\bar{\omega}^i$

the key focus: boundary conditions of the internal fermions

$$\{\;y\;,\;\omega\;\mid\;\bar{y}\;,\;\bar{\omega}\;\}$$

WS fermions that have same B.C. in all basis vectors are paired

pairing of LR fermions $\rightarrow$ Ising model $\rightarrow$ symmetric real fermions

pairing of LL & RR fermions $\rightarrow$ complex fermions $\rightarrow$ asymmetric

STRING DERIVED STANDARD-LIKE MODEL (PLB278)

	ψ^μ	χ^{12}	χ^{34}	χ^{56}	$y^{3,...,6}$	$\bar{y}^{3,...,6}$	$y^{1,2},\omega^{5,6}$	$\bar{y}^{1,2},\bar{\omega}^{5,6}$	$\omega^{1,...,4}$	$\bar{\omega}^{1,...,4}$	$\bar{\psi}^{1,...,5}$	$\bar{\eta}^1$	$\bar{\eta}^2$	$\bar{\eta}^3$	$\bar{\phi}^{1,...,8}$
1	1	1	1	1	1,...,1	1,...,1	1,...,1	1,...,1	1,...,1	1,...,1	1,...,1	1	1	1	1,...,1
S	1	1	1	1	0,...,0	0,...,0	0,...,0	0,...,0	0,...,0	0,...,0	0,...,0	0	0	0	0,...,0
b_1	1	1	0	0	1,...,1	1,...,1	0,...,0	0,...,0	0,...,0	0,...,0	1,...,1	1	0	0	0,...,0
b_2	1	0	1	0	0,...,0	0,...,0	1,...,1	1,...,1	0,...,0	0,...,0	1,...,1	0	1	0	0,...,0
b_3	1	0	0	1	0,...,0	0,...,0	0,...,0	0,...,0	1,...,1	1,...,1	1,...,1	0	0	1	0,...,0

	ψ^μ	χ^{12}	χ^{34}	χ^{56}	y^3y^6	$y^4\bar{y}^4$	$y^5\bar{y}^5$	$\bar{y}^3\bar{y}^6$	$y^1\omega^5$	$y^2\bar{y}^2$	$\omega^6\bar{\omega}^6$	$\bar{y}^1\bar{\omega}^5$	$\omega^2\omega^4$	$\omega^1\bar{\omega}^1$	$\omega^3\bar{\omega}^3$	$\bar{\omega}^2\bar{\omega}^4$	$\bar{\psi}^{1,...,5}$	$\bar{\eta}^1$	$\bar{\eta}^2$	$\bar{\eta}^3$	$\bar{\phi}$
α	0	0	0	0	1	0	0	0	0	0	1	1	0	0	1	1	1 1 1 0 0	0	0	0	1 1 1
β	0	0	0	0	0	0	1	1	1	0	0	0	0	1	0	1	1 1 1 0 0	0	0	0	1 1 1
γ	0	0	0	0	0	1	0	1	0	1	0	1	1	0	0	0	$\frac{1}{2}$ $\frac{1}{2}$ $\frac{1}{2}$ $\frac{1}{2}$ $\frac{1}{2}$	$\frac{1}{2}$	$\frac{1}{2}$	$\frac{1}{2}$	$\frac{1}{2}$ 0 1

Asymmetric $BC \Rightarrow$ all untwisted moduli are projected out!

all $y_i\omega_i\bar{y}_i\bar{\omega}_i$ are disallowed

can be translated to asymmetric bosonic identifications

$$X_L + X_R \rightarrow X_L - X_R$$

moduli fixed at enhanced symmetry point

Free Fermionic Webs of Heterotic T–folds :

- w Groot–Nibellink & Benjamin Percival, arXiv:2606.16443,

  free fermion $\longleftrightarrow$ asymmetric orbifolds via bosonisation

- the space of possible bosonisations via permutations.

  symmetric vs asymmetric is in the eye of the bosonisation

- enhanced T–duality group $\longleftrightarrow$ generalised T–folds

- intrinsically asymmetric $\longleftrightarrow$ all geometrical moduli are projected out

- moduli are projected in groups of four: 0, 4, 8, 12

Conclusions

- DATA $\longrightarrow$ UNIFICATION $\longleftrightarrow$ HiggStructure?
- Non–SUSY string phenomenology $\cdots\cdots\cdots$

  Role of the "Fayet–Iliopoulos" term in non–SUSY string vacua

  D–term up lift $\rightarrow$ Toward string dynamics and vacuum selection

  Role of non–geometric backgrounds $\longleftrightarrow$ Moduli Fixing

  Symmetric $\longleftrightarrow$ Asymmetric

  Classical geometry quantum geometry

- String Phenomenology $\longrightarrow$ Physics of the third millennium

  $e.g.$ Aristarchus to Galileo