

“Modular maps” and BSM phenomenology



- ... – ... Spacetime supersymmetry ...
- 2006 – ... Spinor–vector duality ... on resolved orbifolds
- 1990 – ... String derived Z' ... collider phenomenology
- 2019 – ... non–SUSY string phenomenology from 10D tachyonic vacua

AEF, S Groot–Nibbelink, M Hurtado Heredia, PRD2021; NPB2021

AEF, J Rizos, NPB895 (2015) 233; work in progress with M Guzzi

AEF, B Percival, V Matyas, EPJC80 (2020) 337; NPB961 (2020) 115231;
PLB 814 (2021) 136080; 2010.06637; PRD2021.

String Phenomenology 2021, Zoom – Northeastern University, 16 July 2021

WHY?

DATA \rightarrow STANDARD MODEL \leftrightarrow HIGGS!

EWX \rightarrow PERTUBATIVE

STANDARD MODEL \rightarrow UNIFICATION

EVIDENCE: 16 of SO(10), Log running, proton stability, neutrino masses

+ GRAVITY $\langle \text{---} \rangle$ STRINGS

PRIMARY GUIDES:

3 generations

SO(10) embedding

Higgs : Fundamental? Composite? SM? Multi? SSC?!

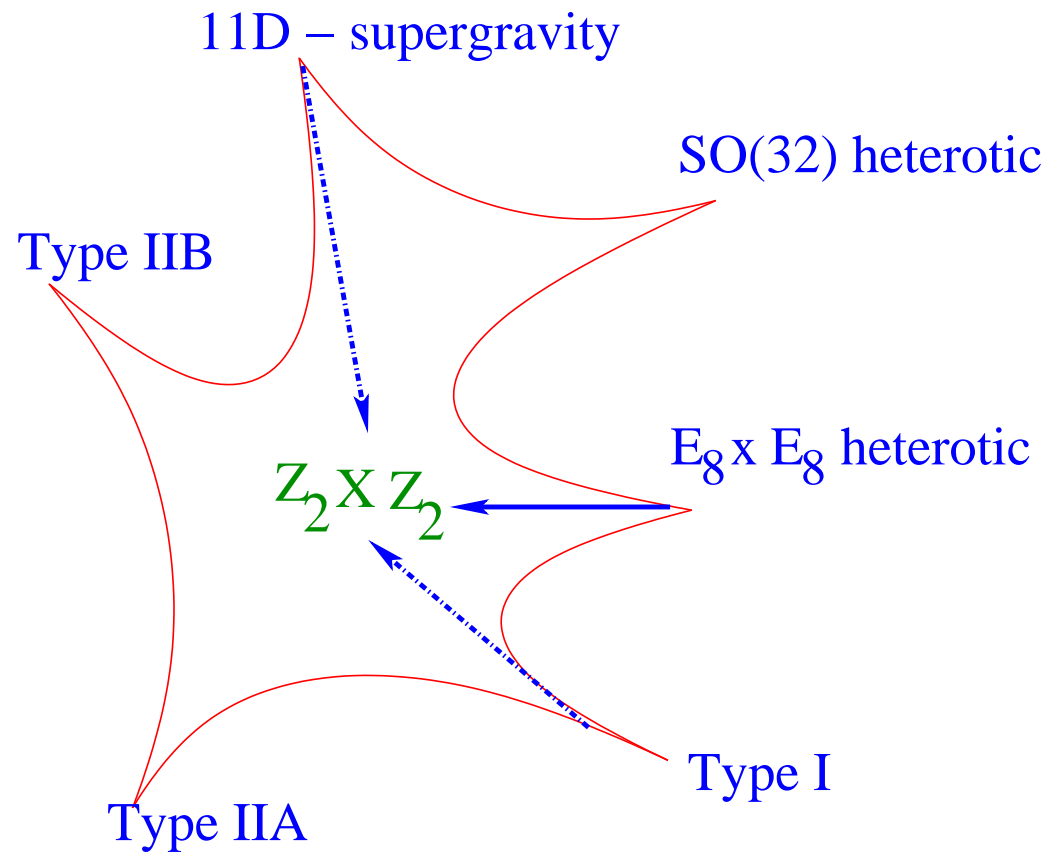
Fermionic $Z_2 \times Z_2$ orbifolds

'Phenomenology of the Standard Model and Unification'

- Minimal Superstring Standard Model NPB 335 (1990) 347
(with Nanopoulos & Yuan)
- Top quark mass $\sim 175\text{--}180\text{GeV}$ PLB 274 (1992) 47
- Generation mass hierarchy NPB 407 (1993) 57
- CKM mixing NPB 416 (1994) 63 (with Halyo)
- Stringy seesaw mechanism PLB 307 (1993) 311 (with Halyo)
- Gauge coupling unification NPB 457 (1995) 409 (with Dienes)
- Proton stability NPB 428 (1994) 111
- Squark degeneracy NPB 526 (1998) 21 (with Pati)
- Moduli fixing NPB 728 (2005) 83
- Classification 2003 – . . .

(with Kounnas, Rizos & ... Percival, Matyas)

Point, String, Membrane



+ ... $SO(16) \times SO(16)$, E_8 , $SO(16) \times E_8$ + ...

... Abel, Basile, Dienes, Kaidi, Itoyama ...

Fermionic Construction

Left-Movers: $\psi^{\mu=1,2}$, χ_i , y_i , ω_i ($i = 1, \dots, 6$)

Right-Movers

$$\bar{\phi}_{A=1, \dots, 44} = \left\{ \begin{array}{ll} \bar{y}_i, \bar{\omega}_i & i = 1, \dots, 6 \\ \bar{\eta}_i & U(1)_i \quad i = 1, 2, 3 \\ \bar{\psi}_{1, \dots, 5} & SO(10) \\ \bar{\phi}_{1, \dots, 8} & SO(16) \end{array} \right.$$

$$V \longrightarrow V \quad \begin{array}{c} \text{Diagram of a genus-2 surface (torus with two handles) with green and red dashed lines} \end{array} \quad f \longrightarrow -e^{i\pi\alpha(f)} f$$

$$Z = \sum_{\text{all spin structures}} c\left(\begin{array}{c} \vec{\alpha} \\ \vec{\beta} \end{array}\right) Z\left(\begin{array}{c} \vec{\alpha} \\ \vec{\beta} \end{array}\right)$$

Models \longleftrightarrow Basis vectors + one-loop phases

Basis vectors:

$$1 = \{\psi^\mu, \chi^{1,\dots,6}, y^{1,\dots,6}, \omega^{1,\dots,6} \mid \bar{y}^{1,\dots,6}, \bar{\omega}^{1,\dots,6}, \bar{\eta}^{1,2,3}, \bar{\psi}^{1,\dots,5}, \bar{\phi}^{1,\dots,8}\}$$

$$S = \{\psi^\mu, \chi^{1,\dots,6}\}, \quad S + \Xi \longrightarrow \text{“modular map”}$$

$$z_1 = \{\bar{\phi}^{1,\dots,4}\},$$

$$z_2 = \{\bar{\phi}^{5,\dots,8}\},$$

$$e_i = \{y^i, \omega^i \mid \bar{y}^i, \bar{\omega}^i\}, \quad i = 1, \dots, 6, \quad N = 4 \text{ Vacua}$$

$$b_1 = \{\chi^{34}, \chi^{56}, y^{34}, y^{56} \mid \bar{y}^{34}, \bar{y}^{56}, \bar{\eta}^1, \bar{\psi}^{1,\dots,5}\}, \quad N = 4 \rightarrow N = 2$$

$$b_2 = \{\chi^{12}, \chi^{56}, y^{12}, y^{56} \mid \bar{y}^{12}, \bar{y}^{56}, \bar{\eta}^2, \bar{\psi}^{1,\dots,5}\}, \quad N = 2 \rightarrow N = 1$$

$$\alpha = \{\bar{\psi}^{4,5}, \bar{\phi}^{1,2}\} \quad \& \quad SO(10) \rightarrow SO(6) \times SO(4) \times \dots$$

$$\beta = \{\bar{\psi}^{1,\dots,5} \equiv \frac{1}{2}, \dots\} \quad \& \quad SO(10) \rightarrow SU(5) \times U(1) \times \dots$$

Independent phases $c \begin{bmatrix} v_i \\ v_j \end{bmatrix} = \exp[i\pi(v_i|v_j)]:$ **upper block**

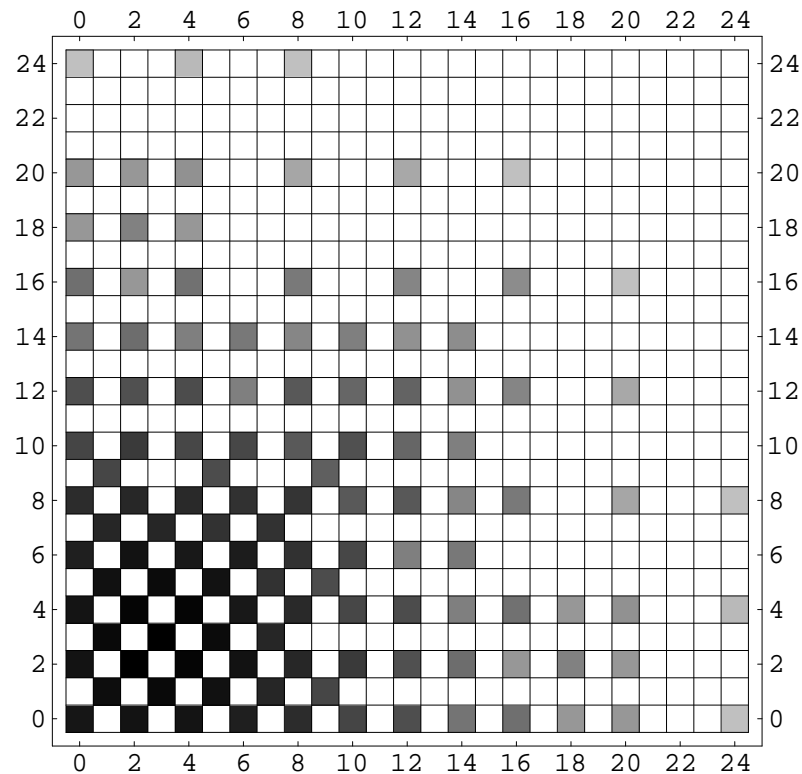
$$\begin{array}{c}
 1 \\
 S \\
 e_1 \\
 e_2 \\
 e_3 \\
 e_4 \\
 e_5 \\
 e_6 \\
 z_1 \\
 z_2 \\
 b_1 \\
 b_2 \\
 \alpha
 \end{array}
 \begin{pmatrix}
 1 & S & e_1 & e_2 & e_3 & e_4 & e_5 & e_6 & z_1 & z_2 & b_1 & b_2 & \alpha \\
 -1 & -1 & \pm & \pm & \pm & \pm & \pm & \pm & \pm & \pm & \pm & \pm & \pm \\
 & & -1 & -1 & -1 & -1 & -1 & -1 & -1 & -1 & 1 & 1 & -1 \\
 & & & \pm & \pm & \pm & \pm & \pm & \pm & \pm & \pm & \pm & \pm \\
 & & & & \pm & \pm & \pm & \pm & \pm & \pm & \pm & \pm & \pm \\
 & & & & & \pm & \pm & \pm & \pm & \pm & \pm & \pm & \pm \\
 & & & & & & \pm & \pm & \pm & \pm & \pm & \pm & \pm \\
 & & & & & & & \pm & \pm & \pm & \pm & \pm & \pm \\
 & & & & & & & & \pm & \pm & \pm & \pm & \pm \\
 & & & & & & & & & \pm & \pm & \pm & \pm \\
 & & & & & & & & & & \pm & \pm & \pm \\
 & & & & & & & & & & & \pm & \pm \\
 & & & & & & & & & & & -1 & \pm \\
 & & & & & & & & & & & & \pm
 \end{pmatrix}$$

A priori 66 independent coefficients $\rightarrow 2^{66}$ distinct vacua

PLB2021, Percival *et al* \rightarrow Satisfiability Modulo Theories $\rightarrow t \times 10^{-3}$

Spinor–vector duality:

Invariance under exchange of $\#(16 + \overline{16}) < - > \#(10)$



Symmetric under exchange of rows and columns

$$E_6 : \quad 27 = 16 + 10 + 1 \quad \overline{27} = \overline{16} + 10 + 1$$

Self-dual: $\#(16 + \overline{16}) = \#(10)$ without E_6 symmetry

Spinor–Vector duality in Orbifolds:

Starting from: $Z_+ = (V_8 - S_8) \left(\sum_{m,n} \Lambda_{m,n} \right)^{\otimes 6} E_8 \times E_8,$

apply $Z_2 \times Z'_2 : g \times g'$

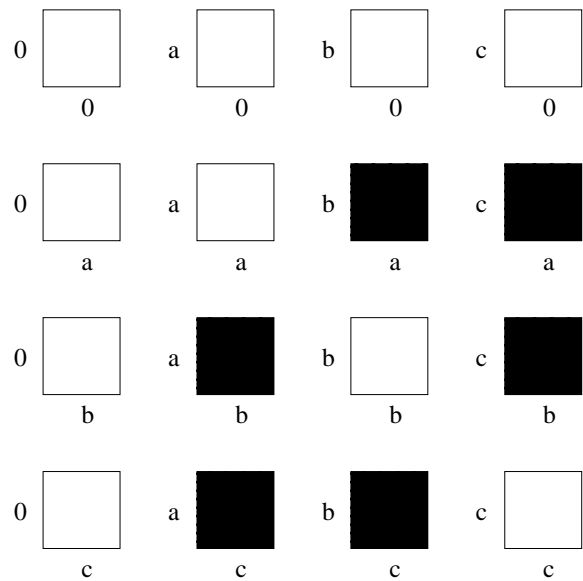
$$g : (0^7, 1|1, 0^7) \rightarrow \text{Wilson line} \rightarrow E_8 \times E_8 \rightarrow SO(16) \times SO(16)$$

$$g' : (x_4, x_5, x_6, x_7, x_8, x_9) \longrightarrow (-x_4, -x_5, -x_6, -x_7, +x_8, +x_9)$$

Note: A single space twisting $Z'_2 \Rightarrow N = 4 \rightarrow N = 2$

$$E_7 \rightarrow SO(12) \times SU(2)$$

⇒ Analyze $Z = \left(\frac{Z_+}{Z_g \times Z_{g'}} \right) = \left[\frac{(1+g)(1+g')}{2 \cdot 2} \right] Z_+$



$a = g$; $b = g'$; $c = gg'$

$P.F. = (\text{white square} + \varepsilon \text{ black square}) = \Lambda_{m,n} \bullet () + \Lambda_{m,n+1/2} \bullet ()$

$\varepsilon = \pm 1$

massless

massive

• sector b

$$\Lambda_{p,q} \left\{ \frac{1}{2} \left(\left| \frac{2\eta}{\theta_4} \right|^4 + \left| \frac{2\eta}{\theta_3} \right|^4 \right) [P_\epsilon^+ Q_s \bar{V}_{12} \bar{C}_4 \bar{O}_{16} + P_\epsilon^- Q_s \bar{S}_{12} \bar{O}_4 \bar{O}_{16}] + \frac{1}{2} \left(\left| \frac{2\eta}{\theta_4} \right|^4 - \left| \frac{2\eta}{\theta_3} \right|^4 \right) [P_\epsilon^+ Q_s \bar{O}_{12} \bar{S}_4 \bar{O}_{16}] \right\} + \text{massive}$$

where

$$P_\epsilon^+ = \left(\frac{1 + \epsilon(-1)^m}{2} \right) \Lambda_{m,n} \quad P_\epsilon^- = \left(\frac{1 - \epsilon(-1)^m}{2} \right) \Lambda_{m,n}$$

$$\epsilon = +1 \Rightarrow P_\epsilon^+ = \Lambda_{2m,n} \quad P_\epsilon^- = \Lambda_{2m+1,n}$$

$$\epsilon = -1 \Rightarrow P_\epsilon^+ = \Lambda_{2m+1,n} \quad P_\epsilon^- = \Lambda_{2m,n}$$

and $12 \cdot 2 + 4 \cdot 2 = 32$

Spinor-Vector duality on Calabi–Yau threefolds with vector bundles :

- From the “Land” to the “Swamp” w Groot–Nibellink & Hurtado-Heredia, arXiv:2103.13442, spinor–vector duality on a resolved orbifold. The role of the discrete torsion in the effective field theory limit
- Vafa–Witten 1994, the role of a discrete torsion in the $Z_2 \times Z_2$ orbifold in mirror symmetry
- In similar spirit \rightarrow the imprint of the worldsheet modular properties in the effective field theory limit

Novel Basis

$$S = \{\psi^\mu, \chi^{1,\dots,6}\},$$

$$z_1 = \{\bar{\phi}^{1,\dots,4}\},$$

$$z_2 = \{\bar{\phi}^{5,\dots,8}\},$$

$$z_3 = \{\bar{\psi}^{1,\dots,4}\},$$

$$z_4 = \{\bar{\eta}^{0,\dots,3}\},$$

$$e_i = \{y^i, \omega^i | \bar{y}^i, \bar{\omega}^i\}, \quad i = 1, \dots, 6,$$

$$\bar{\eta}^0 \equiv \bar{\psi}^5$$

$N = 4$ Vacua

$$1 = S + \sum e_i + z_1 + z_2 + z_3 + z_4$$

$$b_1 = \{\chi^{34}, \chi^{56}, y^{34}, y^{56} | \bar{y}^{34}, \bar{y}^{56}, \bar{\eta}^2, \bar{\eta}^3\},$$

$$N = 4 \rightarrow N = 2$$

Vector bosons: NS, $z_{1,2,3,4}$, $z_i + z_j$

$$\text{NS} \leftrightarrow SO(8)^4 \rightarrow SO(8) \times SO(4) \times SO(4) \times SO(8)^2$$

$SO(12)$ -GUT \rightarrow from enhancement

Duality picture is facilitated

$$SO(12) \text{ enhancement} \quad \longrightarrow \quad B \quad \longleftrightarrow \quad B + z_3$$

$$\text{Spinor} \quad \longleftrightarrow \quad \text{Vector map} \quad \longrightarrow \quad B \quad \longleftrightarrow \quad B + z_4$$

$$z_4 \quad \longrightarrow \quad \text{right-moving spectral flow operator} \quad \longrightarrow \quad \text{“modular map”}$$

The picture extends to compactifications with Interacting Internal CFT

(P. Athanasopoulos, AEF, D. Gepner, PLB 735 (2014) 357)

Low scale Z' in heterotic-string models:

- $E_6 \rightarrow SM \times U(1)_A \times U(1)_B \implies$ anomalous $U(1)_A$; seesaw $U(1)_B$
 $\implies U(1)_A ; U(1)_B \notin$ low scale $U(1)_{Z'}$

- 1996-2013, Pati, AEF, Guzzi, Mehta, Athanasopoulos, $U(1) \notin E_6$

- On the other hand(AEF, Viraf Mehta, PRD88 (2013) 025006)

$$\sin^2 \theta_W(M_Z) , \alpha_s(M_Z) \implies U(1)_{Z'} \in E_6$$

- Z' string derived model, (with Rizos) NPB 895 (2015) 233

Self-dual under SVD; no E_6 enhancement \implies Anomaly free $U(1)_A \in E_6$.

Z' model at low scales Heavy Higgs $\langle \mathcal{N} \rangle \sim M_{\text{String}} \rightarrow$ high seesaw \rightarrow Z'

Field	$SU(3)_C$	$\times SU(2)_L$	$U(1)_Y$	$U(1)_{Z'}$
Q_L^i	3	2	$+\frac{1}{6}$	$-\frac{2}{5}$
u_L^i	$\bar{3}$	1	$-\frac{2}{3}$	$-\frac{2}{5}$
d_L^i	$\bar{3}$	1	$+\frac{1}{3}$	$-\frac{4}{5}$
e_L^i	1	1	+1	$-\frac{2}{5}$
L_L^i	1	2	$-\frac{1}{2}$	$-\frac{4}{5}$
D^i	3	1	$-\frac{1}{3}$	$+\frac{4}{5}$
\bar{D}^i	$\bar{3}$	1	$+\frac{1}{3}$	$+\frac{6}{5}$
H^i	1	2	$-\frac{1}{2}$	$+\frac{6}{5}$
\bar{H}^i	1	2	$+\frac{1}{2}$	$+\frac{4}{5}$
S^i	1	1	0	-2
h	1	2	$-\frac{1}{2}$	$-\frac{4}{5}$
\bar{h}	1	2	$+\frac{1}{2}$	$+\frac{4}{5}$
ϕ	1	1	0	-1
$\bar{\phi}$	1	1	0	+1
ζ^i	1	1	0	0

Additional matter states at $U(1)_{Z'}$ breaking scale

Relevant for: *e.g.* Sterile neutrinos; $g_\mu - 2$; Lepton universality; ...

NON-SUSY String Phenomenology:

10D in fermionic language: $\{ \mathbf{1}, z_1, z_2 \}$

where $z_1 = \{ \bar{\psi}^{1, \dots, 5}, \bar{\eta}^{1, 2, 3} \}$; $z_2 = \{ \bar{\phi}^{1, \dots, 8} \} \Rightarrow S = \mathbf{1} + z_1 + z_2$

$c \begin{pmatrix} z_1 \\ z_2 \end{pmatrix} = +1 \implies E_8 \times E_8$; $c \begin{pmatrix} z_1 \\ z_2 \end{pmatrix} = -1 \implies SO(16) \times SO(16)$

tachyon-free non-SUSY 10D string vacuum

Alternatively:

$\{ \mathbf{1}, z_2 \} \implies \text{No } S \iff SO(16) \times E_8$

tachyonic 10D string vacuum

In both cases \longrightarrow tachyon free 4D GSO configurations

Tachyon free models: $S \longleftrightarrow \tilde{S}$ -map \longleftarrow “modular map”

Modified NAHE \longleftrightarrow $\overline{\text{NAHE}}$

	ψ^μ	χ^{12}	χ^{34}	χ^{56}	$y^{3,\dots,6}$	$\bar{y}^{3,\dots,6}$	$y^{1,2}, \omega^{5,6}$	$\bar{y}^{1,2}, \bar{\omega}^{5,6}$	$\omega^{1,\dots,4}$	$\bar{\omega}^{1,\dots,4}$	$\bar{\psi}^{1,\dots,5}$	$\bar{\eta}^1$	$\bar{\eta}^2$	$\bar{\eta}^3$	$\bar{\phi}^{1,\dots,8}$
1	1	1	1	1	1, ..., 1	1, ..., 1	1, ..., 1	1, ..., 1	1, ..., 1	1, ..., 1	1, ..., 1	1	1	1	1, 1, 1, 1, 1, 1, 1, 1
\tilde{S}	1	1	1	1	0, ..., 0	0, ..., 0	0, ..., 0	0, ..., 0	0, ..., 0	0, ..., 0	0, ..., 0	0	0	0	1, 1, 1, 1, 0, 0, 0, 0
b_1	1	1	0	0	1, ..., 1	1, ..., 1	0, ..., 0	0, ..., 0	0, ..., 0	0, ..., 0	1, ..., 1	1	0	0	0, 0, 0, 0, 0, 0, 0, 0
b_2	1	0	1	0	0, ..., 0	0, ..., 0	1, ..., 1	1, ..., 1	0, ..., 0	0, ..., 0	1, ..., 1	0	1	0	0, 0, 0, 0, 0, 0, 0, 0
b_3	1	0	0	1	0, ..., 0	0, ..., 0	0, ..., 0	0, ..., 0	1, ..., 1	1, ..., 1	1, ..., 1	0	0	1	0, 0, 0, 0, 0, 0, 0, 0

Beyond the $\overline{\text{NAHE}}$ -set

	ψ^μ	χ^{12}	χ^{34}	χ^{56}	$y^3 y^6$	$y^4 \bar{y}^4$	$y^5 \bar{y}^5$	$\bar{y}^3 \bar{y}^6$	$y^1 \omega^5$	$y^2 \bar{y}^2$	$\omega^6 \bar{\omega}^6$	$\bar{y}^1 \bar{\omega}^5$	$\omega^2 \omega^4$	$\omega^1 \bar{\omega}^1$	$\omega^3 \bar{\omega}^3$	$\bar{\omega}^2 \bar{\omega}^4$	$\bar{\psi}^{1,\dots,5}$	$\bar{\eta}^1$	$\bar{\eta}^2$	$\bar{\eta}^3$	$\bar{\phi}$
α	0	0	0	0	1	0	0	1	0	0	1	1	0	0	1	1	1 1 1 0 0	1	0	0	0 0 0
β	0	0	0	0	0	0	1	1	1	0	0	1	0	1	0	1	1 1 1 0 0	0	1	0	1 1 0
γ	0	0	0	0	0	1	0	0	0	1	0	0	1	0	0	0	$\frac{1}{2} \frac{1}{2} \frac{1}{2} \frac{1}{2} \frac{1}{2}$	$\frac{1}{2}$	$\frac{1}{2}$	$\frac{1}{2}$	0 0 $\frac{1}{2}$

Up to the $S \longleftrightarrow \tilde{S}$ -map

Same model as published with

with Cleaver, Manno and Timirgazi in PRD78 (2008) 046009

Stable non-SUSY heterotic-string vacuum?

Classification of tachyon free models

Basis vectors:

$$1 = \{\psi^\mu, \chi^{1,\dots,6}, y^{1,\dots,6}, \omega^{1,\dots,6} \mid \bar{y}^{1,\dots,6}, \bar{\omega}^{1,\dots,6}, \bar{\eta}^{1,2,3}, \bar{\psi}^{1,\dots,5}, \bar{\phi}^{1,\dots,8}\}$$

$$\tilde{S} = \{\psi^\mu, \chi^{1,\dots,6} \mid \bar{\phi}^{3,\dots,6}\}, \quad \longleftarrow \quad \text{"modular map"}$$

$$z_1 = \{\bar{\phi}^{1,\dots,4}\},$$

$$z_2 = \{\bar{\phi}^{5,\dots,8}\},$$

$$e_i = \{y^i, \omega^i \mid \bar{y}^i, \bar{\omega}^i\}, \quad i = 1, \dots, 6, \quad N = 4 \text{ Vacua}$$

$$b_1 = \{\psi^{12}, \chi^{12}, y^{34}, y^{56} \mid \bar{y}^{34}, \bar{y}^{56}, \bar{\eta}^1, \bar{\psi}^{1,\dots,5}\}, \quad N = 4 \rightarrow N = 2$$

$$b_2 = \{\psi^{12}, \chi^{34}, y^{12}, y^{56} \mid \bar{y}^{12}, \bar{y}^{56}, \bar{\eta}^2, \bar{\psi}^{1,\dots,5}\}, \quad N = 2 \rightarrow N = 1$$

$$\alpha = \{\bar{\psi}^{4,5}, \bar{\phi}^{1,2}\}$$

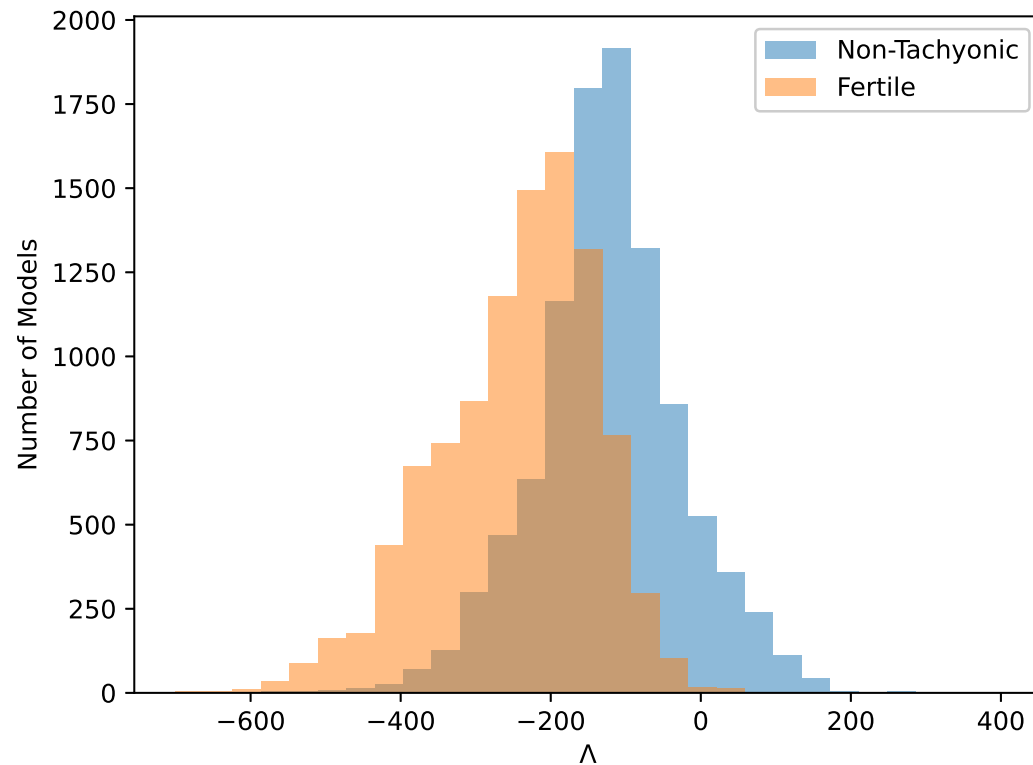
with Viktor Matyas and Ben Percival, NPB 961 (2020) 115231; 2011.04113

Some interesting results

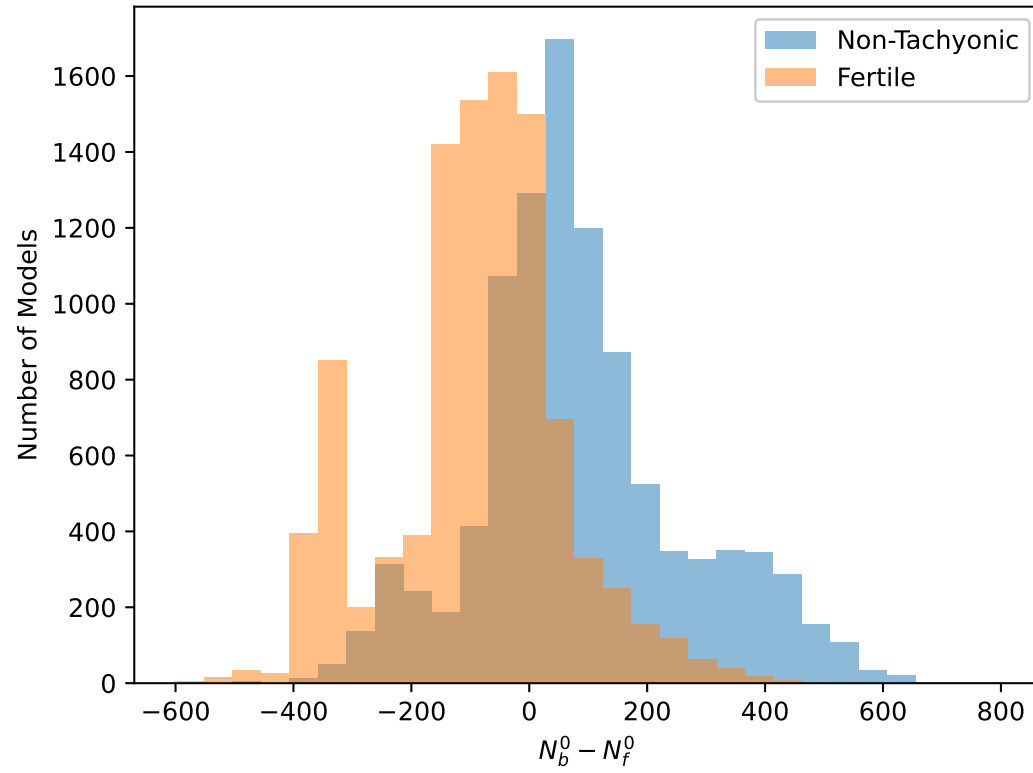
viable \tilde{S} -models: only $SM \times U(1)_{Z'}$.

No heavy Higgs to break FSU5 or PS symmetry; $SM \times U(1)_{Z'} \rightarrow Z'$ exotics

Distribution of Λ



Distribution of a_{00}



“Modular maps” in two dimensions

Left-Movers: $\chi_i, y_i, \omega_i \quad (i = 1, \dots, 8)$

Right-Movers

$$\bar{\phi}_{A=1, \dots, 48} = \left\{ \begin{array}{ll} \bar{y}_i, \bar{\omega}_i & i = 1, \dots, 8 \\ \bar{\eta}_i & i = 0, 1, 2, 3 \\ \bar{\psi}_{1, \dots, 4} \\ \bar{\phi}_{1, \dots, 8} \end{array} \right.$$

Novel Basis

$$1 = \{\chi^{1,\dots,8}, y^{1,\dots,8}, \omega^{1,\dots,8} | \bar{y}^{1,\dots,8}, \bar{\omega}^{1,\dots,8}, \bar{\eta}^{0,\dots,3}, \bar{\psi}^{1,\dots,4} \bar{\phi}^{1,\dots,8}\},$$

$$H_L = \{\chi^{1,\dots,8}, y^{1,\dots,8}, \chi^{1,\dots,8}\},$$

$$z_1 = \{\bar{\phi}^{1,\dots,4}\},$$

$$z_2 = \{\bar{\phi}^{5,\dots,8}\},$$

$$z_3 = \{\bar{\psi}^{1,\dots,4}\},$$

$$z_4 = \{\bar{\eta}^{0,\dots,3}\},$$

$$z_5 = \{\bar{\rho}_i = \bar{y}^i + i\bar{\omega}^i\}, \quad i = 1, \dots, 4,$$

$$z_6 = \mathbf{1} + H_L + z_1 + z_2 + z_3 + z_4 + z_5 = \{\bar{\rho}^{5,6,7,8}\}.$$

massless bosons: NS, $z_{1,2,3,4,5,6}$, $z_i + z_j \quad i, j = 1, \dots, 6, \quad i \neq j$

24 dimensional lattices \rightarrow from enhancements

$c_{H_L}^{(z_1)}$	$c_{H_L}^{(z_2)}$	$c_{H_L}^{(z_3)}$	$c_{H_L}^{(z_4)}$	$c_{z_2}^{(z_1)}$	$c_{z_3}^{(z_1)}$	$c_{z_4}^{(z_1)}$	$c_{z_3}^{(z_2)}$	$c_{z_4}^{(z_2)}$	$c_{z_4}^{(z_3)}$	Gauge group G
+	+	+	+	+	+	+	+	+	+	$E_8 \times SO(32)$
+	+	+	+	+	+	+	+	+	-	$SO(16) \times SO(16) \times SO(16)$
+	+	+	+	+	+	+	+	-	-	$SO(16) \times SO(8) \times SO(8) \times SO(16)$
+	+	+	+	+	+	+	-	-	-	$E_8 \times SO(24) \times SO(8)$
+	+	+	+	-	+	+	-	+	-	$SO(16) \times SO(8) \times SO(8) \times SO(8) \times SO(8)$
+	+	+	+	-	-	-	+	+	+	$SO(24) \times SO(16) \times SO(8)$
+	+	+	+	+	-	-	-	-	+	$E_8 \times SO(16) \times SO(16)$
-	+	+	+	+	+	+	+	+	+	$SO(24) \times SO(24)$
+	+	+	+	-	-	-	-	-	-	$SO(32) \times SO(16)$
-	-	+	+	-	+	+	+	+	+	$E_8 \times E_8 \times SO(16)$
-	-	-	-	+	+	+	+	+	+	$SO(48)$
-	-	-	+	+	+	+	+	+	+	$SO(40) \times SO(8)$
-	-	-	-	-	+	+	+	+	-	$E_8 \times E_8 \times E_8$

Table 1: The configuration of the symmetry group with six basis vectors.

From a subset with six basis vector

A subset of enhanced lattices

Additionally obtain representation of 24 dimensional Niemeier lattices

in terms of free fermion data (with Panos Athanasopoulos arXiv:1610.04898)

Conclusions

- DATA \longrightarrow UNIFICATION \longleftrightarrow HiggsStructure?
- STRINGS THEORY \longrightarrow GAUGE & GRAVITY UNIFICATION
- “Modular maps” \longrightarrow Worldsheet symmetry structure
- \longrightarrow Moduli spaces of (2,0) string compactifications
 \longrightarrow from the “land” to the “swamp”
- SUSY/Non-SUSY string phenomenology
- String derived Z' at LHCb
- String Phenomenology \longrightarrow Physics of the third millennium
e.g. Aristarchus to Galileo