EXOTICA

- Wilsonian Dark Matter in string derived Z' model
- Time permitting: Other Exotica: SVD & the Moonshine; QG from FP

With: John Rizos, NPB 895 (2015) 233; EPJC 76 (2016) 170;

Luigi Delle Rose, Carlo Marzo, John Rizos, arXiv:1704:02579.

String Phenomenology 2017, Blacksburg, Virginia, 3 July 2017

PHENOMENA

DATA $\longrightarrow$ STANDARD MODEL

EWX −> PERTUBATIVE

STANDARD MODEL −> UNIFICATION

EVIDENCE: 16 of SO(10), Log running, proton stability, neutrino masses

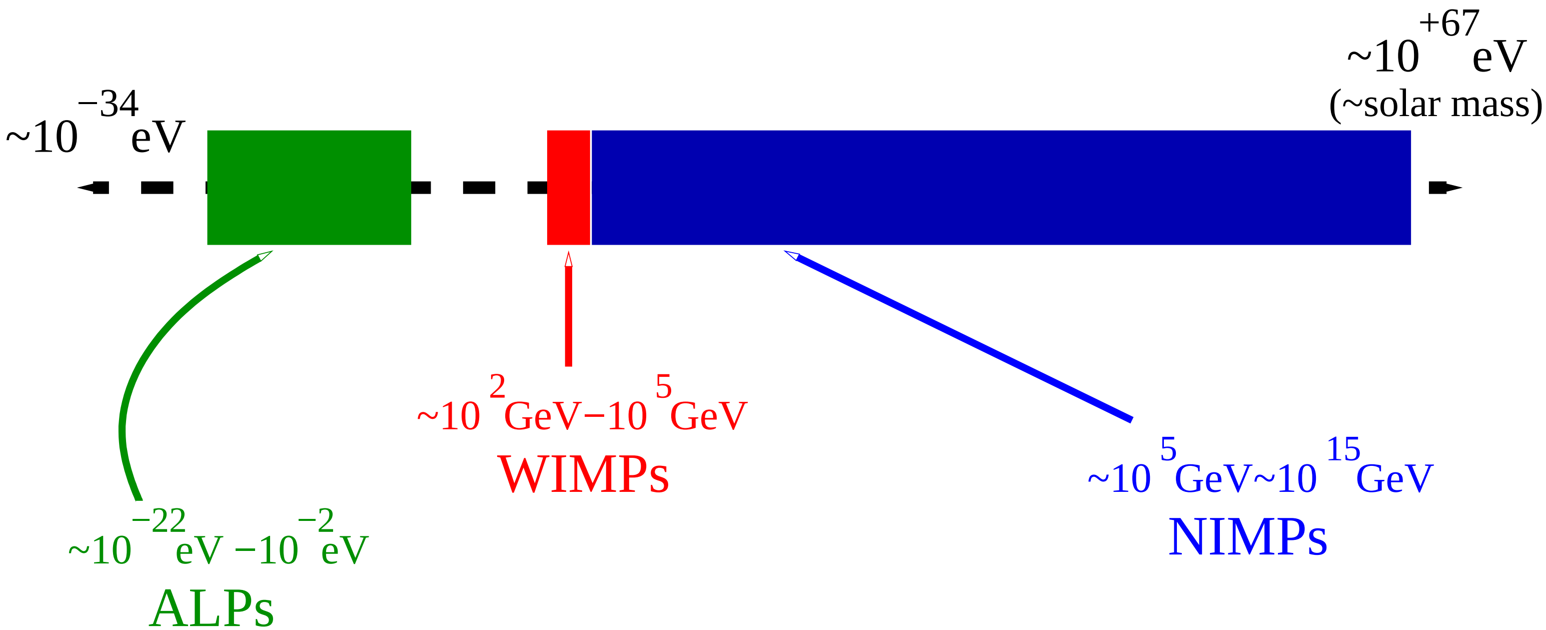
+ GRAVITY < −− > STRINGS

PRIMARY GUIDES:

3 generations
SO(10) embedding

THE DARK MATTER LANDSCAPE *(in mass)*

Energy budget: 70% dark energy
25% dark matter
5% visible matter



Realistic free fermionic models

'Phenomenology of the Standard Model and Unification'

- Minimal Superstring Standard Model NPB 335 (1990) 347 (with Nanopoulos & Yuan)
- Top quark mass $\sim$ 175–180GeV PLB 274 (1992) 47
- Generation mass hierarchy NPB 407 (1993) 57
- CKM mixing NPB 416 (1994) 63 (with Halyo)
- Stringy seesaw mechanism PLB 307 (1993) 311 (with Halyo)
- Gauge coupling unification NPB 457 (1995) 409 (with Dienes)
- Proton stability NPB 428 (1994) 111
- Squark degeneracy NPB 526 (1998) 21 (with Pati)
- Moduli fixing NPB 728 (2005) 83
- Classification & Exophobia 2003 – $\cdots$ (with Nooij, Assel, Christodoulides, Kounnas, Rizos & Sonmez)

Other approaches

Geometrical

Greene, Kirklin, Miron, Ross (1987)
Donagi, Ovrut, Pantev, Waldram (1999)
Blumenhagen, Moster, Reinbacher, Weigand (2006)
Heckman, Vafa (2008)
....

Orbifolds

Ibanez, Nilles, Quevedo (1987)
Bailin, Love, Thomas (1987)
Kobayashi, Raby, Zhang (2004)
Lebedev, Nilles, Raby, Ramos-Sanchez, Ratz, Vaudrevange, Wingerter (2007)
Blaszczyk, Groot–Nibbelink, Ruehle, Trapletti, Vaudrevange (2010)
....

Other CFTs

Gepner (1987)
Schellekens, Yankielowicz (1989)
Gato–Rivera, Schellekens (2009)
....

Orientifolds

Cvetic, Shiu, Uranga (2001)
Ibanez, Marchesano, Rabadan (2001)
Kiristis, Schellekens, Tsulaia (2008)
....

Exotic matter :

In realistic string models

Unifying gauge group $\Rightarrow$ broken by "Wilson lines".

$\Rightarrow$ non–GUT physical states.

$\Rightarrow$ Meta-stable heavy string relics

$\longrightarrow$ Dark Mater NPB 477 (1996) 65

(with Coriano and Chang)

UHECR candidates NPB 614 (2001) 233

(with Coriano and Plümacher)

Free Fermionic Construction

Left-Movers: $\psi^{\mu}_{1,2}, \quad \chi_i, \quad y_i, \quad \omega_i \quad (i = 1, \cdots, 6)$

Right-Movers

$$\bar{\phi}_{A=1,\cdots,44} = \begin{cases} \bar{y}_i \;,\; \bar{\omega}_i & i = 1, \cdots, 6 \\ \bar{\eta}_i & i = 1, 2, 3 \\ \bar{\psi}_{1,\cdots,5} & \\ \bar{\phi}_{1,\cdots,8} & \end{cases}$$

Models $\longleftrightarrow$ Basis vectors $+$ one–loop phases

Free Fermionic Models: $Z_2 \times Z_2$ orbifolds with discrete Wilson lines

Exotics classified by: $SO(10) \to \text{subgroup} \quad b(\bar\psi^{1\cdots5}_{\frac{1}{2}})$:

notation $[(SU(3)_C \times U(1)_C);(SU(2)_L \times U(1)_L)]_{(\ Q_Y\ ,\ Q_{Z'}\ ,\ Q_{\rm e.m.}\ )}$

$SO(6) \times SO(4)$ type states

$[(3,\frac{1}{2});(1,0)]_{(\ 1/6\ ,\ 1/2\ ,\ 1/6\ )}$; $[(\bar3,-\frac{1}{2});(1,0)]_{(\ -1/6\ ,\ -1/2\ ,\ -1/6)}$

$[(1,0);(2,0)]_{(\ 0\ ,\ 0\ ,\ \pm1/2\ )}$

$[(1,0);(1,\pm1)]_{(\ \pm1/2\ ,\ \mp1/2\ ,\ \pm1/2\ )}$ $[(1,\pm3/2);(1,0)]_{(\ \pm1/2\ ,\ \pm1/2\ ,\ \pm1/2\ )}$

$SU(5) \times U(1)$ type states

$[(1,\pm3/4);(1,\pm1/2)]_{(\ \pm1/2\ ,\ \pm1/4\ ,\ \pm1/2\ )}$

$SU(3) \times SU(2) \times U(1)^2$ type states

$[(3,\frac{1}{4});(1,\frac{1}{2})]_{(\ -1/3\ ,\ -1/4\ ,\ -1/3\ )}$; $[(\bar3,-\frac{1}{4});(1,\frac{1}{2})]_{(\ 1/3\ ,\ 1/4\ ,\ 1/3\ )}$

$[(1,\pm\frac{3}{4});(2,\pm\frac{1}{2})]_{(\ \pm1/2\ ,\ \pm1/4\ ,\ (1,0)\ ;\ (0,-1)\ )}$

$[(1,\pm\frac{3}{4});(1,\mp\frac{1}{2})]_{(\ 0\ ,\ \pm5/4\ ,\ 0\ )}$

CCR, SSR, NPB1996.

Low scale Z' in free fermionic models:

- $\frac{3}{2}U(1)_{B-L} - 2U(1)_R \in SO(10)$ @ $1TeV$ MPL A6 (1991) 61

  (with Nanopoulos)

- But $m_t = m_{\nu_\tau}$ & $1TeV \; Z' \Rightarrow \; m_{\nu_\tau} \approx 10MeV$ PLB 245 (1990) 435

  $\underline{E_6 \rightarrow SO(10) \times U(1)_A} \quad \Longrightarrow \quad U(1)_A$ is anomalous!

  $\Longrightarrow \quad U(1)_A \quad \notin$ low scale $U(1)_{Z'}$

- 1996-2013, Pati, AEF, Guzzi, Mehta, Athanasopoulos, $U(1) \notin E_6$

- On the other hand ....(AEF, Viraf Mehta, PRD88 (2013) 025006)

$$\sin^2\theta_W(M_Z) \; , \; \alpha_s(M_Z) \quad \Longrightarrow \quad U(1)_{Z'} \; \in \; E_6$$

- Z' string derived model, (with Rizos) NPB 895 (2015) 233

Classification of fermionic $Z_2 \times Z_2$ orbifolds

Basis vectors:

$$1 = \{\psi^\mu,\ \chi^{1,\dots,6}, y^{1,\dots,6}, \omega^{1,\dots,6} \mid \bar{y}^{1,\dots,6}, \bar{\omega}^{1,\dots,6}, \bar{\eta}^{1,2,3}, \bar{\psi}^{1,\dots,5}, \bar{\phi}^{1,\dots,8}\}$$

$$S = \{\psi^\mu, \chi^{1,\dots,6}\},$$

$$z_1 = \{\bar{\phi}^{1,\dots,4}\},$$

$$z_2 = \{\bar{\phi}^{5,\dots,8}\},$$

$$e_i = \{y^i, \omega^i | \bar{y}^i, \bar{\omega}^i\},\ i = 1, \dots, 6,$$ $N = 4$ Vacua

$$b_1 = \{\chi^{34}, \chi^{56}, y^{34}, y^{56} | \bar{y}^{34}, \bar{y}^{56}, \bar{\eta}^1, \bar{\psi}^{1,\dots,5}\},\qquad N = 4 \rightarrow N = 2$$

$$b_2 = \{\chi^{12}, \chi^{56}, y^{12}, y^{56} | \bar{y}^{12}, \bar{y}^{56}, \bar{\eta}^2, \bar{\psi}^{1,\dots,5}\},\qquad N = 2 \rightarrow N = 1$$

$$\alpha = \{\bar{\psi}^{4,5}, \bar{\phi}^{1,2}\} \quad \& \quad SO(10) \rightarrow SO(6) \times SO(4) \times \cdots$$

$$\beta = \{\bar{\psi}^{1,\cdots,5} \equiv \tfrac{1}{2}, \cdots\} \quad \& \quad SO(10) \rightarrow SU(5) \times U(1) \times \cdots$$

Independent phases $c\binom{v_i}{v_j} = \exp[i\pi(v_i|v_j)]$: upper block

$$
\begin{array}{c}
\\ 1 \\ S \\ e_1 \\ e_2 \\ e_3 \\ e_4 \\ e_5 \\ e_6 \\ z_1 \\ z_2 \\ b_1 \\ b_2 \\ \alpha
\end{array}
\begin{array}{c}
\begin{array}{ccccccccccccc}
1 & S & e_1 & e_2 & e_3 & e_4 & e_5 & e_6 & z_1 & z_2 & b_1 & b_2 & \alpha
\end{array} \\
\left(\begin{array}{ccccccccccccc}
-1 & -1 & \pm & \pm & \pm & \pm & \pm & \pm & \pm & \pm & \pm & \pm & \pm \\
 & & -1 & -1 & -1 & -1 & -1 & -1 & -1 & -1 & 1 & 1 & -1 \\
 & & & \pm & \pm & \pm & \pm & \pm & \pm & \pm & \pm & \pm & \pm \\
 & & & & \pm & \pm & \pm & \pm & \pm & \pm & \pm & \pm & \pm \\
 & & & & & \pm & \pm & \pm & \pm & \pm & \pm & \pm & \pm \\
 & & & & & & \pm & \pm & \pm & \pm & \pm & \pm & \pm \\
 & & & & & & & \pm & \pm & \pm & \pm & \pm & \pm \\
 & & & & & & & & \pm & \pm & \pm & \pm & \pm \\
 & & & & & & & & & \pm & \pm & \pm & \pm \\
 & & & & & & & & & & \pm & \pm & \pm \\
 & & & & & & & & & & & \pm & \pm \\
 & & & & & & & & & & & -1 & \pm \\
 & & & & & & & & & & & &
\end{array}\right)
\end{array}
$$

A priori 66 independent coefficients $\rightarrow$ 2^{66} distinct vacua

The twisted matter spectrum:

$$B^1_{\ell^1_3\ell^1_4\ell^1_5\ell^1_6} = S + b_1 + \ell^1_3 e_3 + \ell^1_4 e_4 + \ell^1_5 e_5 + \ell^1_6 e_6$$

$$B^2_{\ell^2_1\ell^2_2\ell^2_5\ell^2_6} = S + b_2 + \ell^2_1 e_1 + \ell^2_2 e_2 + \ell^2_5 e_5 + \ell^2_6 e_6$$

$$B^3_{\ell^3_1\ell^3_2\ell^3_3\ell^3_4} = S + b_3 + \ell^3_1 e_1 + \ell^3_2 e_2 + \ell^3_3 e_3 + \ell^3_4 e_4 \qquad l^j_i = 0,1$$

sectors B^i_{pqrs} $\rightarrow$ 16 or $\overline{16}$ of $SO(10)$ with multiplicity $(1, 0, -1)$

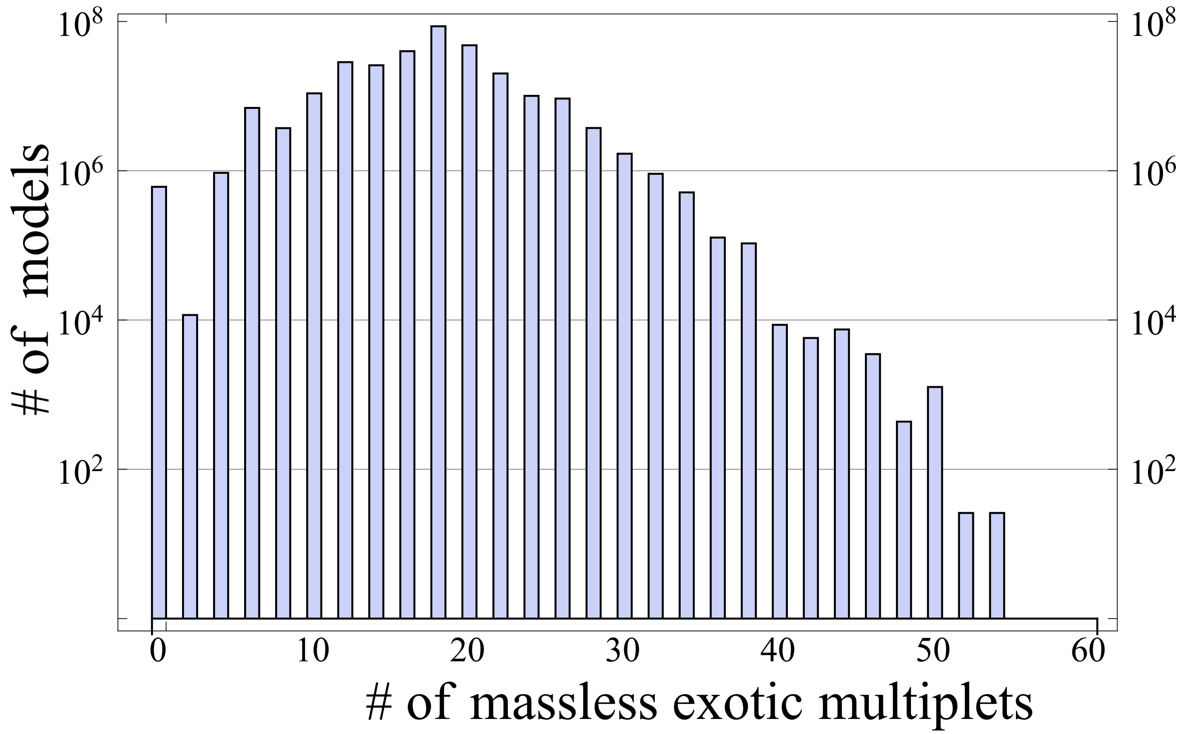
$B^i_{pqrs} + x$ $\rightarrow$ 10 of $SO(10)$ with multiplicity $(1, 0)$

$x = \{\bar{\psi}^{1,\cdots,5}, \bar{\eta}^1, \bar{\eta}^2, \bar{\eta}^3\}$ $\qquad x -$ map $\leftrightarrow$ spinor–vector map

Algebraic formulas for $S = \sum_{i=1}^{3} S_+^{(i)} - S_-^{(i)}$ and $V = \sum_{i=1}^{3} V^{(i)}$

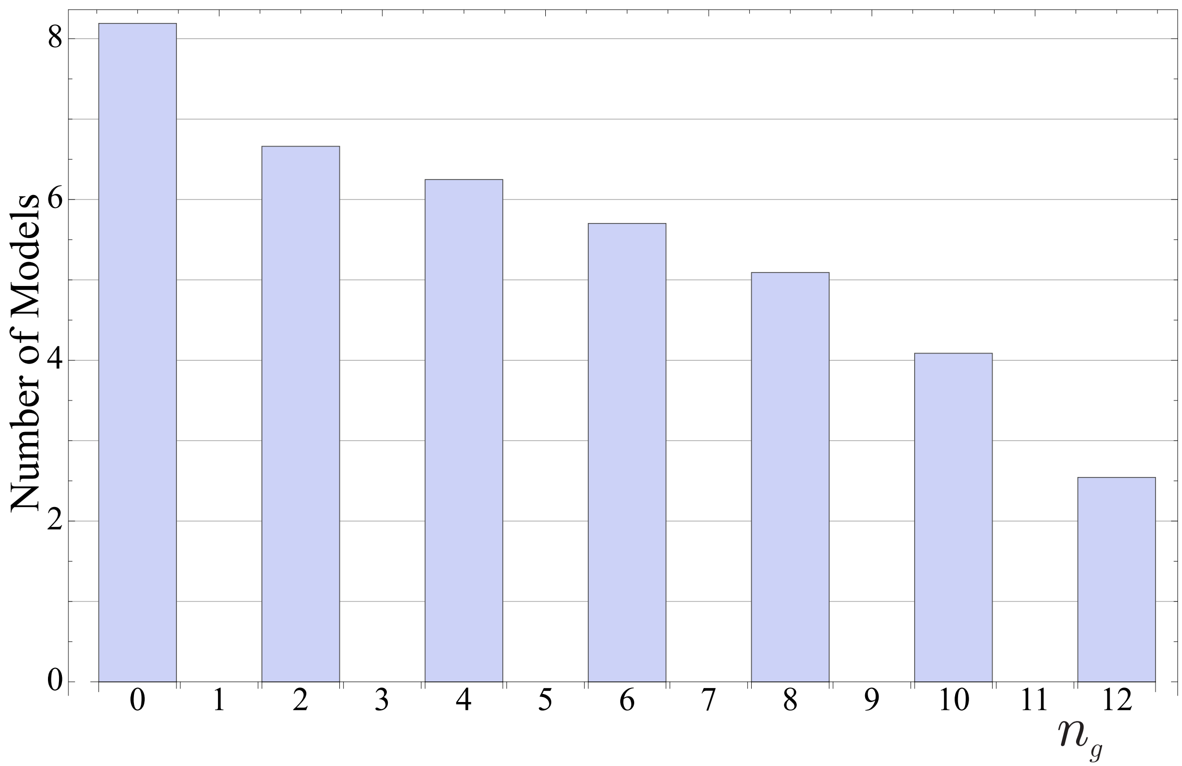
Pati–Salam class: with Assel, Christodoulides, Kounnas, Rizos

RESULTS: of random search of over 10^{11} vacua



Number of 3-generation models versus total number of exotic multiplets flipped $SU(5)$ class: with Sonmez, Rizos

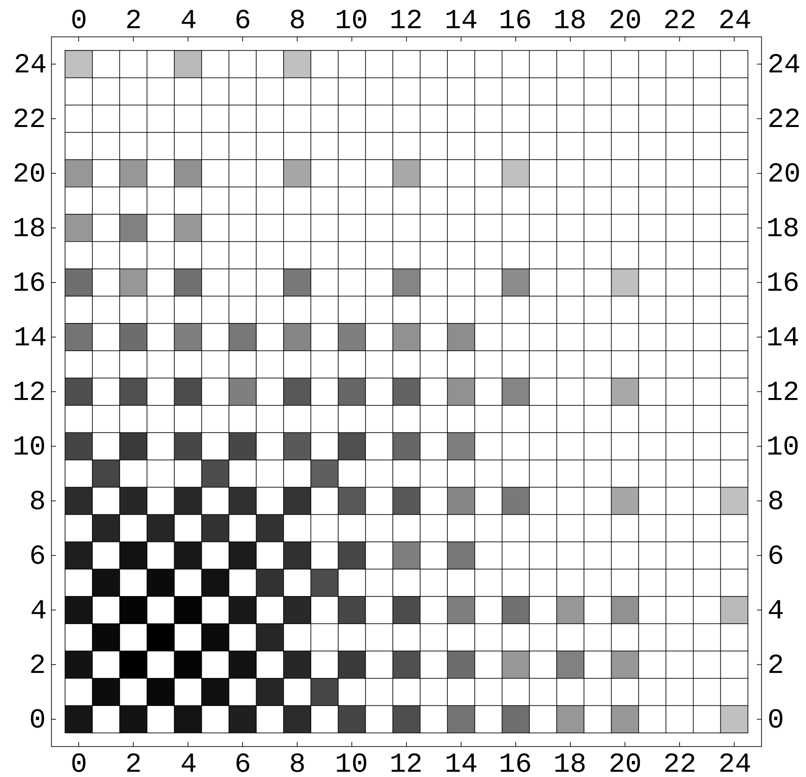
RESULTS: of random search of over 10^{12} vacua



Number of exophobic models versus the number of generations

Spinor–vector duality:

Invariance under exchange of $\#(16+\overline{16})$ $<->$ $\#(10)$



Symmetric under exchange of rows and columns

$$E_6 \quad : \qquad 27 = 16 + 10 + 1 \qquad \overline{27} = \overline{16} + 10 + 1$$

Self-dual: $\#(16+\overline{16}) \;=\; \#(10)$ without E_6 symmetry

light Z' heterotic–string model $c\left[{v_i \atop v_j}\right] = \exp[i\pi(v_i|v_j)]$:

$$
(v_i|v_j) \;=\; \begin{array}{c} \\ 1\\ S\\ e_1\\ e_2\\ e_3\\ e_4\\ e_5\\ e_6\\ b_1\\ b_2\\ z_1\\ z_2\\ \alpha \end{array}
\begin{array}{c} \begin{array}{ccccccccccccc} 1 & S & e_1 & e_2 & e_3 & e_4 & e_5 & e_6 & b_1 & b_2 & z_1 & z_2 & \alpha \end{array} \\
\left(\begin{array}{ccccccccccccc}
1 & 1 & 1 & 1 & 1 & 1 & 1 & 1 & 1 & 1 & 1 & 1 & 1\\
1 & 1 & 1 & 1 & 1 & 1 & 1 & 1 & 1 & 1 & 1 & 1 & 1\\
1 & 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1\\
1 & 1 & 0 & 0 & 0 & 0 & 0 & 1 & 0 & 0 & 0 & 1 & 0\\
1 & 1 & 0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 & 1 & 1\\
1 & 1 & 0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 & 1 & 0 & 0\\
1 & 1 & 0 & 0 & 0 & 0 & 0 & 1 & 0 & 0 & 0 & 1 & 1\\
1 & 1 & 0 & 1 & 0 & 0 & 1 & 0 & 0 & 0 & 1 & 0 & 0\\
1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 & 1 & 0 & 0 & 0\\
1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 & 1 & 0 & 0 & 1\\
1 & 1 & 0 & 0 & 0 & 1 & 0 & 1 & 0 & 0 & 1 & 1 & 0\\
1 & 1 & 0 & 1 & 1 & 0 & 1 & 0 & 0 & 0 & 1 & 1 & 0\\
1 & 1 & 1 & 0 & 1 & 0 & 1 & 0 & 1 & 0 & 1 & 0 & 1
\end{array}\right) \end{array}
$$

Observable gauge group: $SO(6) \times SO(4) \times U(1)_{1,2,3}$

$U(1)_\zeta = U(1)_1 + U(1)_2 + U(1)_3$ is anomaly free

sector	field	$SU(4)\times SU(2)_L\times SU(2)_R$	$U(1)_1$	$U(1)_2$	$U(1)_3$	$U(1)_\zeta$
$S+b_1$	$\bar{F}_{1R}$	$(\bar{\mathbf{4}},\mathbf{1},\mathbf{2})$	1/2	0	0	1/2
$S+b_1+e_3+e_5$	F_{1R}	$(\mathbf{4},\mathbf{1},\mathbf{2})$	1/2	0	0	1/2
$S+b_2$	F_{1L}	$(\mathbf{4},\mathbf{2},\mathbf{1})$	0	1/2	0	1/2
$S+b_2+e_1+e_2+e_5$	F_{2L}	$(\mathbf{4},\mathbf{2},\mathbf{1})$	0	1/2	0	1/2
$S+b_2+e_1$	$\bar{F}_{2R}$	$(\bar{\mathbf{4}},\mathbf{1},\mathbf{2})$	0	1/2	0	1/2
$S+b_2+e_2+e_5$	$\bar{F}_{3R}$	$(\bar{\mathbf{4}},\mathbf{1},\mathbf{2})$	0	1/2	0	1/2
$S+b_3+e_1+e_2$	F_{3L}	$(\mathbf{4},\mathbf{2},\mathbf{1})$	0	0	1/2	1/2
$S+b_3+e_2$	$\bar{F}_{4R}$	$(\bar{\mathbf{4}},\mathbf{1},\mathbf{2})$	0	0	1/2	1/2
$S+b_3+x$	h_1	$(\mathbf{1},\mathbf{2},\mathbf{2})$	−1/2	−1/2	0	−1
$S+b_2+x+e_5$	h_2	$(\mathbf{1},\mathbf{2},\mathbf{2})$	−1/2	0	−1/2	−1
$S+b_2+x+e_1+e_2$	h_3	$(\mathbf{1},\mathbf{2},\mathbf{2})$	−1/2	0	−1/2	−1
$S+b_3+x+e_1$	D_4	$(\mathbf{6},\mathbf{1},\mathbf{1})$	−1/2	−1/2	0	−1
	χ_1^+	$(\mathbf{1},\mathbf{1},\mathbf{1})$	1/2	1/2	1	+2
	χ_1^-	$(\mathbf{1},\mathbf{1},\mathbf{1})$	1/2	1/2	−1	0
	$\zeta_a, a=2,3$	$(\mathbf{1},\mathbf{1},\mathbf{1})$	1/2	−1/2	0	0
	$\bar{\zeta}_a, a=2,3$	$(\mathbf{1},\mathbf{1},\mathbf{1})$	−1/2	1/2	0	0
$S+b_2+x+e_1+e_5$	D_5	$(\mathbf{6},\mathbf{1},\mathbf{1})$	−1/2	0	−1/2	−1
	χ_2^+	$(\mathbf{1},\mathbf{1},\mathbf{1})$	1/2	1	1/2	+2
	χ_2^-	$(\mathbf{1},\mathbf{1},\mathbf{1})$	1/2	−1	1/2	0
	$\zeta_a, a=4,5$	$(\mathbf{1},\mathbf{1},\mathbf{1})$	1/2	0	−1/2	0
	$\bar{\zeta}_a, a=4,5$	$(\mathbf{1},\mathbf{1},\mathbf{1})$	−1/2	0	1/2	0
$S+b_2+x+e_2$	D_6	$(\mathbf{6},\mathbf{1},\mathbf{1})$	−1/2	0	−1/2	−1
	χ_3^+	$(\mathbf{1},\mathbf{1},\mathbf{1})$	1/2	1	1/2	+2
	χ_3^-	$(\mathbf{1},\mathbf{1},\mathbf{1})$	1/2	−1	1/2	0
	$\zeta_a, a=6,7$	$(\mathbf{1},\mathbf{1},\mathbf{1})$	1/2	0	−1/2	0
	$\bar{\zeta}_a, a=6,7$	$(\mathbf{1},\mathbf{1},\mathbf{1})$	−1/2	0	1/2	0
$S+b_1+x+e_3$	$\bar{D}_6$	$(\mathbf{6},\mathbf{1},\mathbf{1})$	0	1/2	1/2	+1
	$\bar{\chi}_4^+$	$(\mathbf{1},\mathbf{1},\mathbf{1})$	−1	−1/2	−1/2	−2
	$\bar{\chi}_4^-$	$(\mathbf{1},\mathbf{1},\mathbf{1})$	1	−1/2	−1/2	0
	$\zeta_a, a=8,9$	$(\mathbf{1},\mathbf{1},\mathbf{1})$	0	1/2	−1/2	0
	$\bar{\zeta}_a, a=8,9$	$(\mathbf{1},\mathbf{1},\mathbf{1})$	0	−1/2	1/2	0
$S+b_1+x+e_5$	D_7	$(\mathbf{6},\mathbf{1},\mathbf{1})$	0	−1/2	−1/2	−1
	χ_5^+	$(\mathbf{1},\mathbf{1},\mathbf{1})$	1	1/2	1/2	+2
	χ_5^-	$(\mathbf{1},\mathbf{1},\mathbf{1})$	−1	1/2	1/2	0
	$\zeta_a, a=10,11$	$(\mathbf{1},\mathbf{1},\mathbf{1})$	0	1/2	−1/2	0
	$\bar{\zeta}_a, a=10,11$	$(\mathbf{1},\mathbf{1},\mathbf{1})$	0	−1/2	1/2	0
$S+b_3+x+e_2+e_3$	ζ_1	$(\mathbf{1},\mathbf{1},\mathbf{1})$	1/2	−1/2	0	0
	$\bar{\zeta}_1$	$(\mathbf{1},\mathbf{1},\mathbf{1})$	−1/2	1/2	0	0
$S+b_1+x+e_3+e_4+e_6$	ϕ_1	$(\mathbf{1},\mathbf{1},\mathbf{1})$	0	1/2	1/2	+1
	$\bar{\phi}_1$	$(\mathbf{1},\mathbf{1},\mathbf{1})$	0	−1/2	−1/2	−1
$S+b_1+x+e_4+e_5+e_6$	ϕ_2	$(\mathbf{1},\mathbf{1},\mathbf{1})$	0	1/2	1/2	+1
	$\bar{\phi}_2$	$(\mathbf{1},\mathbf{1},\mathbf{1})$	0	−1/2	−1/2	−1

Table 1: *Observable twisted matter spectrum and* $SU(4)\times SU(2)_L\times SU(2)_R\times U(1)^3$ *quantum numbers.*

The chiral spectrum is self–dual under spinor–vector duality

maintains the E_6 embedding $\quad \Rightarrow \quad U(1)_\zeta$ is anomaly free

Exophobic, three generations + heavy and light higgs + $\lambda_t \sim 1$

$$U(1)_{Z'} \;=\; \tfrac{1}{5}\,U(1)_C \;-\; \tfrac{1}{5}\,U(1)_L - U_\zeta$$

$Z' \Rightarrow$ Additional matter at the Z' breaking scale

Exotic $SO(10)$ singlets with non–standard $U(1)_\zeta$ charges

$\Rightarrow$ Natural Wilsonian dark–matter candidates

"Wilsonian" singlet dark matter   Delle Rose, AEF, Marzo, Rizos, 1704.02579

$[(1,0);(1,0)]_{(\ 0\ ,\ \pm 1\ ,\ 0\ )}$ $SO(10)$ singlets E_6 exotics

Several cases to consider :

1. $M >> M_{Z'}$ without inflation $\Rightarrow M \leq 10^5$ GeV

2. $M >> M_{Z'}$ with inflation and $T_R > M_{Z'}$ $\Rightarrow M > T_R \left[25 + \frac{1}{2}\ln\left(\frac{M}{T_R}\right)\right.$

3. $M << M_{Z'}$ without inflation $\Rightarrow M < 3$ keV

4. $M << M_{Z'}$ with inflation

$$M \begin{cases} > \ T_R \left[25 + \frac{1}{2}\ln\left(\frac{M^5}{M_{Z'}^4 T_R}\right)\right] , & T_R < M \\ < \ \frac{M_{Z'}^4}{T_R^3} 6.9 \times 10^{-25} \left(\frac{g_*}{200}\right)^{1.5} \frac{1}{N_{Z'} g_{\text{eff}}^2}, & T_R > M \end{cases}$$

Novel Basis

$$\begin{aligned} S &= \{\psi^\mu, \chi^{1,\dots,6}\}, \\ z_1 &= \{\bar\phi^{1,\dots,4}\}, \\ z_2 &= \{\bar\phi^{5,\dots,8}\}, \\ z_3 &= \{\bar\psi^{1,\dots,4}\}, \\ z_4 &= \{\bar\eta^{0,\dots,3}\}, \\ e_i &= \{y^i, \omega^i | \bar y^i, \bar\omega^i\}, \ i = 1, \dots, 6, \end{aligned}$$

$\bar\eta^0 \equiv \bar\psi^5$

$N = 4$ Vacua

$$1 = S + \sum e_i + z_1 + z_2 + z_3 + z_4$$

$$b_1 = \{\chi^{34}, \chi^{56}, y^{34}, y^{56} | \bar y^{34}, \bar y^{56}, \bar\eta^2, \bar\eta^3\},$$

$N = 4 \longrightarrow N = 2$

Vector bosons: NS, $z_{1,2,3,4}$, $z_i + z_j$

NS $\leftrightarrow$ $SO(8)^4 \rightarrow SO(8) \times SO(4)^2 \times SO(8)^2$

$SO(12)$–GUT $\rightarrow$ from enhancement

Duality picture is facilitated

Spinor $\longleftrightarrow$ Vector map $\longrightarrow$ $B \longleftrightarrow B + z_4$

$SO(12)$ enhancement $\longrightarrow$ $B \longleftrightarrow B + z_3$

A convenient basis to study dualities; modular properties

GUT structure is obscured

Compactifications to two dimensions

Left-Movers: $\chi_i, \quad y_i, \quad \omega_i \quad (i = 1, \cdots, 8)$

Right-Movers

$$\bar{\phi}_{A=1,\cdots,48} = \begin{cases} \bar{y}_i \ , \ \bar{\omega}_i & i = 1, \cdots, 8 \\ \\ \bar{\eta}_i & i = 0, 1, 2, 3 \\ \bar{\psi}_{1,\cdots,4} & \\ \bar{\phi}_{1,\cdots,8} & \end{cases}$$

Novel Basis

$$
\begin{aligned}
1 &= \{\chi^{1,\dots,8}, y^{1,\dots,8}, \omega^{1,\dots,8} | \bar{y}^{1,\dots,8}, \bar{\omega}^{1,\dots,8}, \bar{\eta}^{0,\dots,3}, \bar{\psi}^{1,\dots,4} \bar{\phi}^{1,\dots,8}\}, \\
H_L &= \{\chi^{1,\dots,8}, y^{1,\dots,8}, \chi^{1,\dots,8}\}, \\
z_1 &= \{\bar{\phi}^{1,\dots,4}\}, \\
z_2 &= \{\bar{\phi}^{5,\dots,8}\}, \\
z_3 &= \{\bar{\psi}^{1,\dots,4}\}, \\
z_4 &= \{\bar{\eta}^{0,\dots,3}\}, \\
z_5 &= \{\bar{\rho}_i = \bar{y}^i + i\bar{\omega}^i\}, \ i = 1, \dots, 4,
\end{aligned}
$$

$$
z_6 = \mathbf{1} + H_L + z_1 + z_2 + z_3 + z_4 + z_5 = \{\bar{\rho}^{5,6,7,8}\}.
$$

massless bosons: NS, $z_{1,2,3,4,5,6}$, $z_i + z_j \ \ i, j = 1, \dots, 6, \ i \neq j$

24 dimensional lattices $\rightarrow$ from enhancements

$c\binom{z_1}{H_L}$	$c\binom{z_2}{H_L}$	$c\binom{z_3}{H_L}$	$c\binom{z_4}{H_L}$	$c\binom{z_1}{z_2}$	$c\binom{z_1}{z_3}$	$c\binom{z_1}{z_4}$	$c\binom{z_2}{z_3}$	$c\binom{z_2}{z_4}$	$c\binom{z_3}{z_4}$	Gauge group G
+	+	+	+	+	+	+	+	+	+	$E_8 \times SO(32)$
+	+	+	+	+	+	+	+	+	−	$SO(16) \times SO(16) \times SO(16)$
+	+	+	+	+	+	+	+	−	−	$SO(16) \times SO(8) \times SO(8) \times SO(16)$
+	+	+	+	+	+	+	−	−	−	$E_8 \times SO(24) \times SO(8)$
+	+	+	+	−	+	+	−	+	−	$SO(16) \times SO(8) \times SO(8) \times SO(8) \times SO(8)$
+	+	+	+	−	−	−	+	+	+	$SO(24) \times SO(16) \times SO(8)$
+	+	+	+	+	−	−	−	−	+	$E_8 \times SO(16) \times SO(16)$
−	+	+	+	+	+	+	+	+	+	$SO(24) \times SO(24)$
+	+	+	+	−	−	−	−	−	−	$SO(32) \times SO(16)$
−	−	+	+	−	+	+	+	+	+	$E_8 \times E_8 \times SO(16)$
−	−	−	−	+	+	+	+	+	+	$SO(48)$
−	−	−	+	+	+	+	+	+	+	$SO(40) \times SO(8)$
−	−	−	−	−	+	+	+	+	−	$E_8 \times E_8 \times E_8$

Table 2: The configuration of the symmetry group with six basis vectors.

From a subset with six basis vector

A subset of enhanced lattices

Additionally obtain representation of 24 dimensional Niemeier lattices

in terms of free fermion data (with Panos Athanasopoulos arXiv:1610.04898)

Motivation General Relativity: Covariance & Equivalence Principle

$\longrightarrow$ fundamental geometrical principle

Quantum Mechanics: No Such Principle

Axiomatic formulation ... $\mathrm{P} \sim |\Psi|^2$

However Quantum $+$ Gravity Theory

not known

Main effort: quantize GR; quantize space–time: $e.g.$ superstring theory

The main successes of string theory:

1) Viable perturbative approach to quantum gravity

2) Unification of gravity, gauge & matter structures

$i.e.$ construction of phenomenologically realistic models

$\longrightarrow$ relevant for experimental observation

Quantum gravity from fundamental principles

Basic identity:

Schwarzian Identity

$$\left(\frac{\partial S_0}{\partial q}\right)^2 = \frac{\beta^2}{2}\left(\{\mathrm{e}^{\frac{i2S_0}{\beta}}; q\} - \{S_0; q\}\right)$$

$$\alpha^2(\nabla S_0)\cdot(\nabla S_0) = \frac{\Delta(Re^{\alpha S_0})}{Re^{\alpha S_0}} - \frac{\Delta R}{R} - \alpha\left(2\frac{\nabla R\cdot\nabla S_0}{R} + \Delta S_0\right),$$

$$\alpha^2(\partial S)\cdot(\partial S) = \frac{\partial^2(Re^{\alpha S})}{Re^{\alpha S}} - \frac{\partial^2 R}{R} - \alpha\left(2\frac{\partial R\cdot\partial S}{R} + \partial^2 S\right),$$

$$\alpha^2(\partial S - eA)\cdot(\partial S - eA) = \frac{D^2(Re^{\alpha S})}{Re^{\alpha S}} - \frac{\partial^2 R}{R} - \frac{\alpha}{R^2}\partial\cdot\left(R^2(\partial S - eA)\right),$$

$$D^\mu \;=\; \partial^\mu - \alpha e A^\mu$$

Extend:

$$\alpha^2(\partial_\mu S)(\partial^\mu S) = \frac{\frac{1}{\sqrt{g}}\partial_\mu\sqrt{g}\partial^\mu\left(R\mathrm{e}^{\alpha S}\right)}{R\mathrm{e}^{\alpha S}} - \frac{\frac{1}{\sqrt{g}}\partial_\mu\left(\sqrt{g}\partial^\mu R\right)}{R} - \frac{\alpha}{R^2}\frac{1}{\sqrt{g}}\partial_\mu\left(\sqrt{g}R^2\partial^\mu S\right),$$

$$\alpha^2 G_{\mu\nu\eta\rho}\frac{\delta S}{\delta g_{\mu\nu}}\frac{\delta S}{\delta g_{\eta\rho}} = \frac{1}{R\mathrm{e}^{\alpha S}}G_{\mu\nu\eta\rho}\frac{\delta^2\left(R\mathrm{e}^{\alpha S}\right)}{\delta g_{\mu\nu}\delta g_{\eta\rho}} - G_{\mu\nu\eta\rho}\frac{1}{R}\frac{\delta^2(R)}{\delta g_{\mu\nu}\delta g_{\eta\rho}} - \frac{\alpha}{R^2}G_{\mu\nu\eta\rho}\frac{\delta}{\delta g_{\mu\nu}}\left(R^2\frac{\delta S}{\delta g_{\eta\rho}}\right),$$

Conclusions

- DATA $\longrightarrow$ UNIFICATION
- STRINGS $\longrightarrow$ GAUGE & GRAVITY UNIFICATION
- Free fermionic models $\longrightarrow$ A Fertile Crescent
- Free fermionic models $\longleftrightarrow$ $Z_2 \times Z_2$ orbifolds
- Dark Matter is in the Dark. A low scale Z' will help.
- Quantum gravity from basic principles
- String Phenomenology $\longrightarrow$ Physics of the third millenium

  $e.g.$ Aristarchus to Copernicus