Higgs–Matter splitting in Heterotic–String Vacua

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- Develop : tools to study string vacua
- dictionaries between different approaches: free fermion $\leftrightarrow$ orbifold

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Plan

1. Higgs–matter splitting in free fermion models
2. $SO(10)$ Classification & spinor–vector duality
3. Higgs–matter splitting in orbifold
4. Conclusions

Standard Model: $(SU321) \oplus (Q \;\; L \;\; U \;\; D \;\; E \;\; N) \quad \oplus \quad (h)$:

$$\longrightarrow \quad \text{Unification} \quad \longrightarrow \quad SU(5)$$

$$\longrightarrow \quad SO(10) \oplus (\, 6 + 2 + \bar{3} + \bar{3} + 1 + 1) = 16 \oplus (5 + \bar{5}) = 10$$

$$\longrightarrow \quad E_6 \to 16 \;\; + \;\; 10 \;\; + \;\; 1 = 27$$

BUT need large representations : $e.g$ 126 in $SO(10)$;

351 in E_6

Heterotic–string theory $E_8 \times E_8 \to E_6 \times U(1)_2 \times E_8$

BUT no large massless representations

What can we learn from phenomenological string models?

Realistic free fermionic models

'Phenomenology of the Standard Model and string unification'

- Top quark mass $\sim$ 175–180GeV — PLB 274 (1992) 47
- Generation mass hierarchy — NPB 407 (1993) 57
- CKM mixing — NPB 416 (1994) 63 (with Halyo)
- Stringy seesaw mechanism — PLB 307 (1993) 311 (with Halyo)
- Gauge coupling unification — NPB 457 (1993) 409 (with Dienes)
- Proton stability — NPB 428 (1994) 111
- Squark degeneracy — NPB 526 (1998) 21 (with Pati)
- Minimal Superstring Standard Model — PLB 455 (1999) 135 (with Cleaver & Nanopoulos)
- Moduli fixing — NPB 728 (2005) 83

Other efforts

Geometrical

Greene, Kirklin, Miron, Ross (1987)
Donagi, Ovrut, Pantev, Waldram (1999)
Blumenhagen, Moster, Reinbacher, Weigand (2006)
Heckman, Vafa (2008)
....

Orbifolds

Ibanez, Nilles, Quevedo (1987)
Bailin, Love, Thomas (1987)
Kobayashi, Raby, Zhang (2004)
Lebedev, Nilles, Raby, Ramos-Sanchez, Ratz, Vaudrevange, Wingerter (2007)
Blaszczyk, Groot–Nibbelink, Ruehle, Trapletti, Vaudrevange (2010)
....

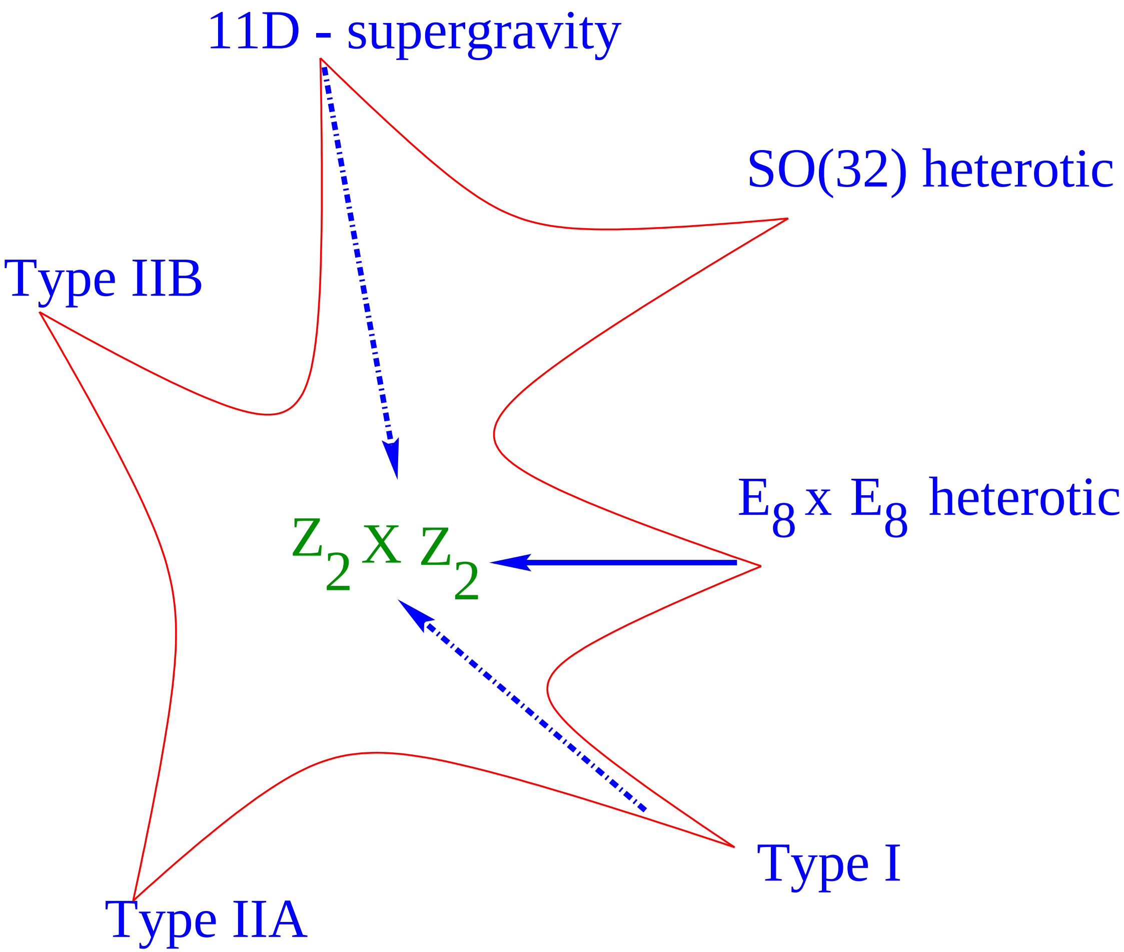
Other CFTs

Gepner (1987)
Schellekens, Yankielowicz (1989)
Gato–Rivera, Schellekens (2009)
....

Orientifolds

Cvetic, Shiu, Uranga (2001)
Ibanez, Marchesano, Rabadan (2001)
Kiristis, Schellekens, Tsulaia (2008)
....

Point, String, Membrane ....



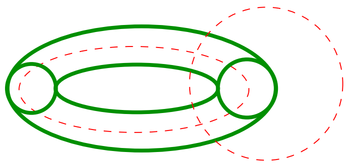
Free Fermionic Construction

Left-Movers: $\psi^{\mu}_{1,2}, \quad \chi_i, \quad y_i, \quad \omega_i \quad (i = 1, \cdots, 6)$

Right-Movers

$$\bar{\phi}_{A=1,\cdots,44} = \begin{cases} \bar{y}_i \;,\; \bar{\omega}_i & i = 1, \cdots, 6 \\ \\ \bar{\eta}_i & i = 1, 2, 3 \\ \bar{\psi}_{1,\cdots,5} & \\ \bar{\phi}_{1,\cdots,8} & \end{cases}$$

$V \longrightarrow V$



$f \longrightarrow -e^{i\pi\alpha(f)} f$

$$Z = \sum_{\substack{all\ spin \\ structures}} c\binom{\vec{\alpha}}{\vec{\beta}} \, Z\binom{\vec{\alpha}}{\vec{\beta}}$$

Models $\longleftrightarrow$ Basis vectors $+$ one–loop phases

Construction of the physical states

$$b_j \quad j = 1, \cdots, N \qquad \rightarrow \qquad \Xi \;=\; \sum_j n_j b_j$$

For $\vec{\alpha} \;=\; (\vec{\alpha}_L; \vec{\alpha}_R) \;\in\; \Xi \quad \Rightarrow \quad \mathsf{H}_{\vec{\alpha}}$

$$\alpha(f) = 1 \;\;\Rightarrow\;\; |\pm\rangle \quad ; \quad \alpha(f) \neq 1 \;\;\Rightarrow\;\; f, f^* \;\;,\;\; \nu_{f,f^*} = \frac{1 \mp \alpha(f)}{2}$$

$$M_L^2 = -\frac{1}{2} + \frac{\vec{\alpha}_L \cdot \vec{\alpha}_L}{8} + N_L = -1 + \frac{\vec{\alpha}_R \cdot \vec{\alpha}_R}{8} + N_R = M_R^2 \;\; (\;\equiv\; 0 \;)$$

<u>GSO projections</u> $\qquad e^{i\pi(\vec{b}_i \cdot \vec{F}_\alpha)} |s\rangle_{\vec{\alpha}} = \delta_\alpha c^* \begin{pmatrix} \vec{\alpha} \\ \vec{b}_i \end{pmatrix} |s\rangle_{\vec{\alpha}}$

$F_\alpha(f) \;\rightarrow\;$ fermion # operator $\;=\; \begin{cases} -1, & |-\rangle \\ 0, & |+\rangle \end{cases} \;=\; \begin{cases} +1, & f \\ -1, & f^* \end{cases}$

$Q(f) = \frac{1}{2}\alpha(f) + F(f) \quad \rightarrow \quad$ U(1) charges

The NAHE set : $\{\, \mathbf{1},\; S\;, \quad b_1\;, \quad b_2\;, \quad b_3 \,\}$

$$N = \quad 4 \quad \to \quad 2 \qquad\qquad 1 \qquad\qquad 1 \quad \text{vacua}$$

$Z_2 \times Z_2$ orbifold compactification

$$\Longrightarrow \quad \text{Gauge group} \quad SO(10) \times SO(6)^{1,2,3} \times E_8$$

beyond the NAHE set Add $\{\alpha, \beta, \gamma\}$

number of generations is reduced to three

$$SO(10) \quad \longrightarrow \quad SU(3) \times SU(2) \times U(1)_{T_{3_R}} \times U(1)_{B-L}$$

$$U(1)_Y \;=\; \tfrac{1}{2}(B-L) \;+\; T_{3_R} \;\in\; SO(10)\;!$$

$$SO(6)^{1,2,3} \quad \longrightarrow \quad U(1)^{1,2,3} \;\times\; U(1)^{1,2,3}$$

Correspondence with $Z_2 \times Z_2$ orbifold PLB 326 (1994) 62

NAHE $\oplus$ $(\xi_2 = \{\bar{\psi}_{1,\cdots,5}, \bar{\eta}^1, \bar{\eta}^2, \bar{\eta}^3\} = 1) \quad \rightarrow \quad \{1, S, \xi_1, \xi_2, b_1, b_2\}$

Gauge group: $SO(4)^3 \times E_6 \times U(1)^2 \times E_8$ and 24 generations.

toroidal compactification

$$g_{ij} = \begin{pmatrix} 2 & -1 & 0 & 0 & 0 & 0 \\ -1 & 2 & -1 & 0 & 0 & 0 \\ 0 & -1 & 2 & -1 & 0 & 0 \\ 0 & 0 & -1 & 2 & -1 & -1 \\ 0 & 0 & 0 & -1 & 2 & 0 \\ 0 & 0 & 0 & -1 & 0 & 2 \end{pmatrix} \qquad b_{ij} = \begin{cases} g_{ij} & i < j \\ 0 & i = j \\ -g_{ij} & i > j \end{cases}$$

$R_i \quad \rightarrow$ the free fermionic point $\rightarrow$ G.G. $SO(12) \times E_8 \times E_8$

mod out by a $Z_2 \times Z_2$ with standard embedding

$\Rightarrow$ Exact correspondence

In the realistic free fermionic models

replace $\xi_2 \equiv x = \{\bar\psi^{1,\cdots,5}, \bar\eta^1, \bar\eta^2, \bar\eta^3\} = 1$

with $2\gamma = \{\bar\psi^{1,\cdots,5}, \bar\eta^1, \bar\eta^2, \bar\eta^3, \bar\phi^{1,\cdots,4}\} = 1$

Then $\{\vec 1, \vec S, \vec\xi_1 = \vec 1 + \vec b_1 + \vec b_2 + \vec b_3, 2\gamma\} \rightarrow$ N=4 SUSY and

$SO(12)\times SO(16)\times SO(16)$

apply $b_1 \times b_2 \rightarrow Z_2 \times Z_2 \rightarrow$ N=1 SUSY and

$SO(4)^3\times SO(10)\times U(1)^3\times SO(16)$

b_1, b_2, b_3 $\Rightarrow (3\times 8)\cdot 16$ of $SO(10)_O$

$b_1+2\gamma$, $b_2+2\gamma$, $b_3+2\gamma$ $\Rightarrow (3\times 8)\cdot 16$ of $SO(16)_H$

Alternatively, $c\binom{\xi_1}{\xi_2} = +1 \rightarrow -1$

$b_1+\xi_1$, $b_2+\xi_1$, $b_3+\xi_1$ $\Rightarrow (3\times 8)\cdot 16$ of $SO(16)_H$

Classification of fermionic $Z_2 \times Z_2$ orbifolds (AEF, Kounnas, Rizos)

Basis vectors: consistent modular blocks 4,8 periodic fermions

$$
\begin{aligned}
1 &= \{\psi^\mu,\ \chi^{1,\dots,6}, y^{1,\dots,6}, \omega^{1,\dots,6} \mid \bar{y}^{1,\dots,6}, \bar{\omega}^{1,\dots,6}, \bar{\eta}^{1,2,3}, \bar{\psi}^{1,\dots,5}, \bar{\phi}^{1,\dots,8}\} \\
S &= \{\psi^\mu, \chi^{1,\dots,6}\}, \\
z_1 &= \{\bar{\phi}^{1,\dots,4}\}, \\
z_2 &= \{\bar{\phi}^{5,\dots,8}\}, \\
e_i &= \{y^i, \omega^i | \bar{y}^i, \bar{\omega}^i\},\ i = 1, \dots, 6,
\end{aligned}
$$

$N = 4$ Vacua

$$
\begin{aligned}
b_1 &= \{\chi^{34}, \chi^{56}, y^{34}, y^{56} | \bar{y}^{34}, \bar{y}^{56}, \bar{\eta}^1, \bar{\psi}^{1,\dots,5}\}, & N = 4 \rightarrow N = 2 \\
b_2 &= \{\chi^{12}, \chi^{56}, y^{12}, y^{56} | \bar{y}^{12}, \bar{y}^{56}, \bar{\eta}^2, \bar{\psi}^{1,\dots,5}\}, & N = 2 \rightarrow N = 1
\end{aligned}
$$

Vector bosons: NS, $z_{1,2}$, $z_1 + z_2$, $x = 1 + s + \sum e_i + z_1 + z_2$

impose: $c\binom{z_1}{z_2} = -1$ & Gauge group $SO(10) \times U(1)^3 \times \text{hidden}$

Independent phases $c\left[{}^{vi}_{v_j}\right] = \exp[i\pi(v_i|v_j)]$: upper block

	1	S	e_1	e_2	e_3	e_4	e_5	e_6	z_1	z_2	b_1	b_2
1	-1	-1	$\pm$	$\pm$	$\pm$	$\pm$	$\pm$	$\pm$	$\pm$	$\pm$	$\pm$	$\pm$
S			-1	-1	-1	-1	-1	-1	-1	-1	1	1
e_1				$\pm$	$\pm$	$\pm$	$\pm$	$\pm$	$\pm$	$\pm$	$\pm$	$\pm$
e_2					$\pm$	$\pm$	$\pm$	$\pm$	$\pm$	$\pm$	$\pm$	$\pm$
e_3						$\pm$	$\pm$	$\pm$	$\pm$	$\pm$	$\pm$	$\pm$
e_4							$\pm$	$\pm$	$\pm$	$\pm$	$\pm$	$\pm$
e_5								$\pm$	$\pm$	$\pm$	$\pm$	$\pm$
e_6									$\pm$	$\pm$	$\pm$	$\pm$
z_1										$\pm$	$\pm$	$\pm$
z_2											$\pm$	$\pm$
b_1												$\pm$
b_2												

Apriori 55 independent coefficients $\rightarrow\ 2^{55}$ distinct vacua

Impose: Gauge group $SO(10) \times U(1)^3 \times SO(8)^2$

$\rightarrow$ 40 independent coefficients

The twisted matter spectrum:

$$B^1_{\ell^1_3\ell^1_4\ell^1_5\ell^1_6} = S + b_1 + \ell^1_3 e_3 + \ell^1_4 e_4 + \ell^1_5 e_5 + \ell^1_6 e_6$$

$$B^2_{\ell^2_1\ell^2_2\ell^2_5\ell^2_6} = S + b_2 + \ell^2_1 e_1 + \ell^2_2 e_2 + \ell^2_5 e_5 + \ell^2_6 e_6$$

$$B^3_{\ell^3_1\ell^3_2\ell^3_3\ell^3_4} = S + b_3 + \ell^3_1 e_1 + \ell^3_2 e_2 + \ell^3_3 e_3 + \ell^3_4 e_4 \qquad l^j_i = 0, 1$$

sectors $B^i_{pqrs} \rightarrow 16$ or $\overline{16}$ of $SO(10)$ with multiplicity $(1, 0, -1)$

$B^i_{pqrs} + x \rightarrow 10$ of $SO(10)$ with multiplicity $(1, 0)$

Counting: for each B^i_{pqrs}:

Projectors:

$$P^{(1)}_{p^1q^1r^1s^1} = \frac{1}{16} \prod \left(1 - c\binom{e_i}{B^{(1)}_{p^1q^1r^1s^1}}\right) \prod \left(1 - c\binom{z_i}{B^{(1)}_{p^1q^1r^1s^1}}\right)$$

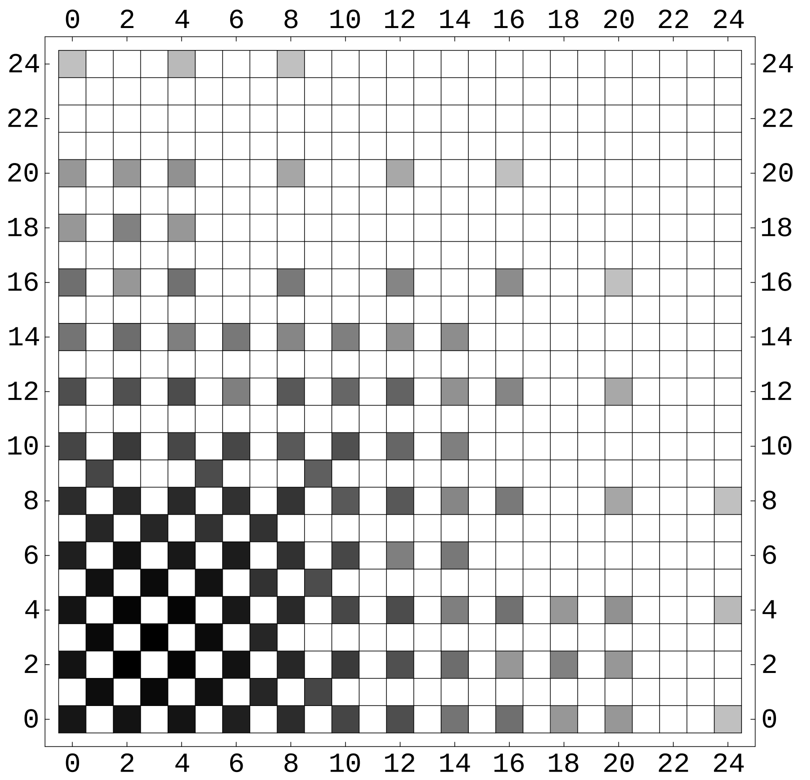
$$S^{(i)}_{\pm} = \sum_{pqrs} \frac{1 \pm X^{(i)}_{p^iq^ir^is^i}}{2} P^{(i)}_{p^iq^ir^is^i}\ , \ i = 1, 2, 3$$

similarly for vectorials

Algebraic formulas for $S = \sum_{i=1}^{3} S^{(i)}_{+} - S^{(i)}_{-}$ and $V = \sum_{i=1}^{3} V^{(i)}$

Spinor–vector duality:

Invariance under exchange of $\#(16+\overline{16}) \; <-> \; \#(10)$



Symmetric under exchange of rows and columns

$$E_6 \; : \qquad 27 = 16 + 10 + 1 \qquad \overline{27} = \overline{16} + 10 + 1$$

Self-dual: $\#(16+\overline{16}) \; = \; \#(10)$ without E_6 symmetry

$\Rightarrow$ Algebraic proof of spinor-vector duality

The number of vectorials and spinorials from a specific

orbifold plane I are interchanged when the ranks of the

associated Y-vectors are interchanged

$$\text{rank}\left[\Delta^{(I)}, Y_{16}^{(I)}\right] \quad \longleftrightarrow \quad \text{rank}\left[\Delta^{(I)}, Y_{10}^{(I)}\right]$$

Induced by GSO phase change

Higgs–matter splitting in Orbifolds: w Carlo Angelantonj, Mirian Tsulaia

Using the level-one $\mathrm{SO}(2n)$ characters

$$O_{2n} = \tfrac{1}{2}\left(\frac{\theta_3^n}{\eta^n} + \frac{\theta_4^n}{\eta^n}\right), \qquad V_{2n} = \tfrac{1}{2}\left(\frac{\theta_3^n}{\eta^n} - \frac{\theta_4^n}{\eta^n}\right),$$

$$S_{2n} = \tfrac{1}{2}\left(\frac{\theta_2^n}{\eta^n} + i^{-n}\frac{\theta_1^n}{\eta^n}\right), \qquad C_{2n} = \tfrac{1}{2}\left(\frac{\theta_2^n}{\eta^n} - i^{-n}\frac{\theta_1^n}{\eta^n}\right).$$

where

$$\theta_3 \equiv Z_f\binom{0}{0} \quad \theta_4 \equiv Z_f\binom{0}{1} \quad \theta_2 \equiv Z_f\binom{1}{0} \quad \theta_1 \equiv Z_f\binom{1}{1}$$

The partition function of the heterotic string on $SO(12)$ lattice:

$$Z_+ = (V_8 - S_8)\left[|O_{12}|^2 + |V_{12}|^2 + |S_{12}|^2 + |C_{12}|^2\right]\left(\bar{O}_{16} + \bar{S}_{16}\right)\left(\bar{O}_{16} + \bar{S}_{16}\right) ,$$

and

$$\begin{aligned} Z_- \, = \, (V_8 - S_8) \Big[& \left(|O_{12}|^2 + |V_{12}|^2\right)\left(\bar{O}_{16}\bar{O}_{16} + \bar{C}_{16}\bar{C}_{16}\right) \\ & + \left(|S_{12}|^2 + |C_{12}|^2\right)\left(\bar{S}_{16}\bar{S}_{16} + \bar{V}_{16}\bar{V}_{16}\right) \\ & + \left(O_{12}\bar{V}_{12} + V_{12}\bar{O}_{12}\right)\left(\bar{S}_{16}\bar{V}_{16} + \bar{V}_{16}\bar{S}_{16}\right) \\ & + \left(S_{12}\bar{C}_{12} + C_{12}\bar{S}_{12}\right)\left(\bar{O}_{16}\bar{C}_{16} + \bar{C}_{16}\bar{O}_{16}\right)\Big] \, . \end{aligned}$$

where $\pm$ refers to

$$c\binom{\xi_1}{\xi_2} \quad = \quad \pm 1$$

connected by : $\qquad Z_- = Z_+ / a \otimes b \, ,$

$$a = (-1)^{F_{\rm L}^{\rm int} + F_\xi^1} \, , \qquad\qquad b = (-1)^{F_{\rm L}^{\rm int} + F_\xi^2} \, .$$

Starting from:

$$Z_+ = (V_8 - S_8) \left(\sum_{m,n} \Lambda_{m,n} \right)^{\otimes 6} (\bar{O}_{16} + \bar{S}_{16}) (\bar{O}_{16} + \bar{S}_{16}) \ ,$$

where as usual, for each circle,

$$p^i_{\rm L,R} = \frac{m_i}{R_i} \pm \frac{n_i R_i}{\alpha'} \,,$$

and

$$\Lambda_{m,n} = \frac{q^{\frac{\alpha'}{4} p^2_{\rm L}} \, \bar{q}^{\frac{\alpha'}{4} p^2_{\rm R}}}{|\eta|^2} \,.$$

apply $\quad Z_2 \times Z_2' : g \times g' = (-1)^{F_{\xi^1}} \delta_1 \times (-1)^{F_{\xi^2}} \delta_1$

with $\quad \delta_1 X^9 = X_9 + \pi R_9 ,$

$$
\begin{aligned}
Z_-^{9d} = (V_8 - S_8) \Big[\quad & \Lambda_{2m,n} && \left(\overline{O}_{16}\overline{O}_{16} + \overline{C}_{16}\overline{C}_{16}\right) \\
+ \; & \Lambda_{2m+1,n} && \left(\overline{S}_{16}\overline{S}_{16} + \overline{V}_{16}\overline{V}_{16}\right) \\
+ \; & \Lambda_{2m,n+\frac{1}{2}} && \left(\overline{S}_{16}\overline{V}_{16} + \overline{V}_{16}\overline{S}_{16}\right) \\
+ \; & \Lambda_{2m+1,n+\frac{1}{2}} && \left(\overline{O}_{16}\overline{C}_{16} + \overline{C}_{16}\overline{O}_{16}\right) \Big] \; .
\end{aligned}
$$

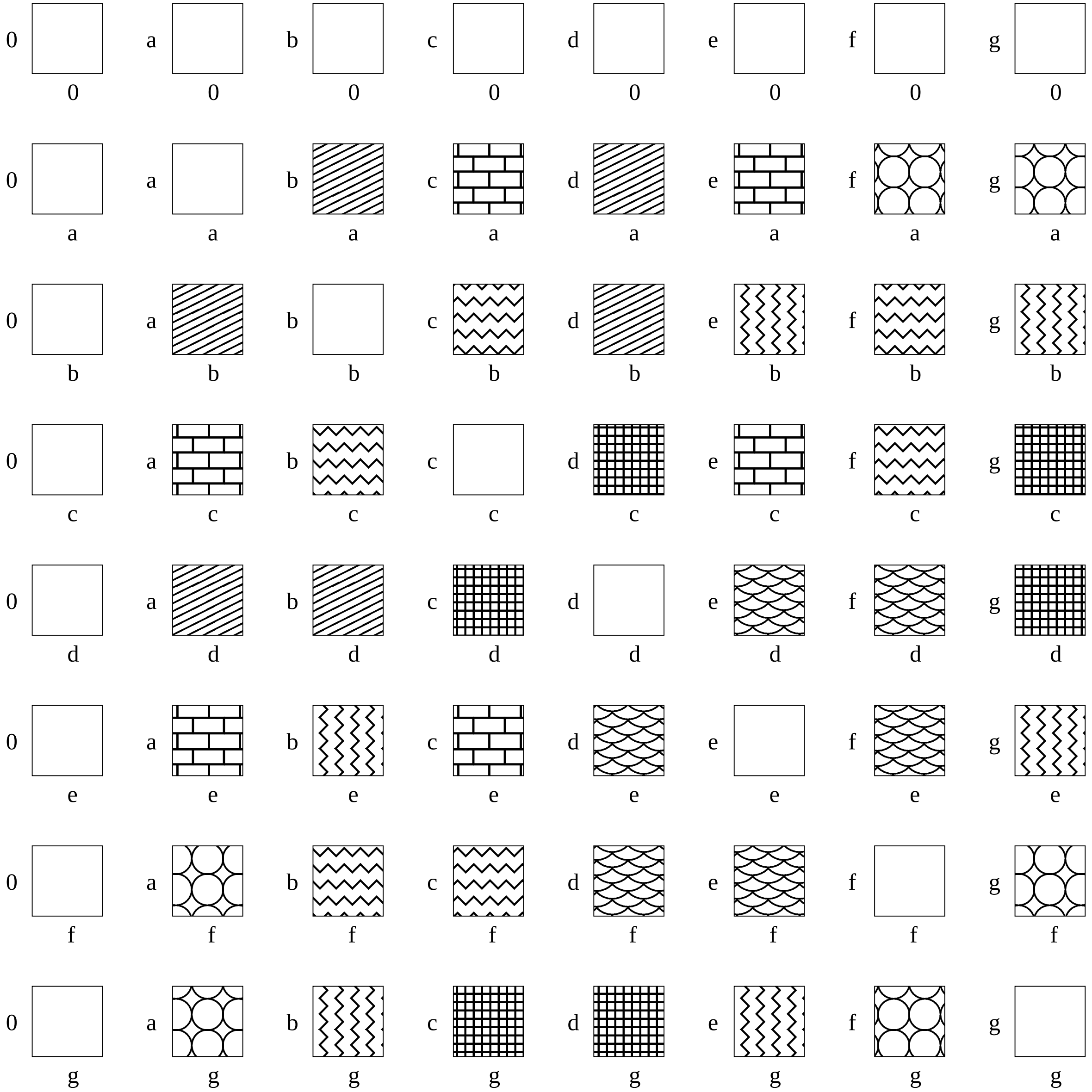
Add Z_2'' : $(x_4, x_5, x_6, x_7, x_8, x_9) \longrightarrow (+x_4, +x_5, -x_6, -x_7, -x_8, -x_9)$

Naively

$$\left(\frac{Z_+}{(Z_2 \times Z_2')} \right) / Z_2''$$

Produces massless vectorials. No massless spinorials. (Elisa Manno, 0908.3164)

$\Longrightarrow$ Analyze $$\left(\frac{Z_+}{(Z_g \times Z_{g'} \times Z_{g''})} \right)$$



eight independent orbits $\rightarrow$ ϵ_i $\quad i = 1, \cdots 7$ discrete torsions

- sector c

$$Q_c\left(\frac{1}{2}\left(\left|\frac{2\eta}{\theta_4}\right|^4+\left|\frac{2\eta}{\theta_3}\right|^4\right)\Lambda_{p,q}\left(\frac{\epsilon_{06}^{-}+\epsilon_{24}^{+}(-1)^m}{2}\right)\Lambda_{m,n}\right)\overline{V}_{12}\overline{S}_4\overline{O}_{16}$$

$$Q_c\left(\frac{1}{2}\left(\left|\frac{2\eta}{\theta_4}\right|^4+\left|\frac{2\eta}{\theta_3}\right|^4\right)\Lambda_{p,q}\left(\frac{\epsilon_{06}^{+}-\epsilon_{24}^{-}(-1)^m}{2}\right)\Lambda_{m,n}\right)\overline{S}_{12}\overline{O}_4\overline{O}_{16}$$

$$\epsilon_1=+1,\ \epsilon_2=-1,\ \epsilon_3=+1,\ \epsilon_4=+1,\ \epsilon_5=-1,\ \epsilon_6=+1,\ \epsilon_7=-1,$$

No gauge enhancement. Spinors massless. Vectors Massive.

$$\epsilon_1=+1,\ \epsilon_2=+1,\ \epsilon_3=+1,\ \epsilon_4=+1,\ \epsilon_5=+1,\ \epsilon_6=-1,\ \epsilon_7=+1,$$

No gauge enhancement. Spinors massive. Vectors Massless.

Spinor–Vector duality map $\{\epsilon_2,\epsilon_5,\epsilon_6,\epsilon_7\}\rightarrow-\{\epsilon_2,\epsilon_5,\epsilon_6,\epsilon_7\}$

Conclusions

Phenomenological string models produce interesting lessons

Higgs–matter splitting – A Cheshire Cat's Grin

Spinor–vector duality

All string vacua are connected

String point of view: Organisation of low energy spectrum $\rightarrow$ secondary

Consistency *i.e. number of d.o.f.* $\rightarrow$ primary