

NOVEL SYMMETRIES FROM HETEROTIC STRING MODELS

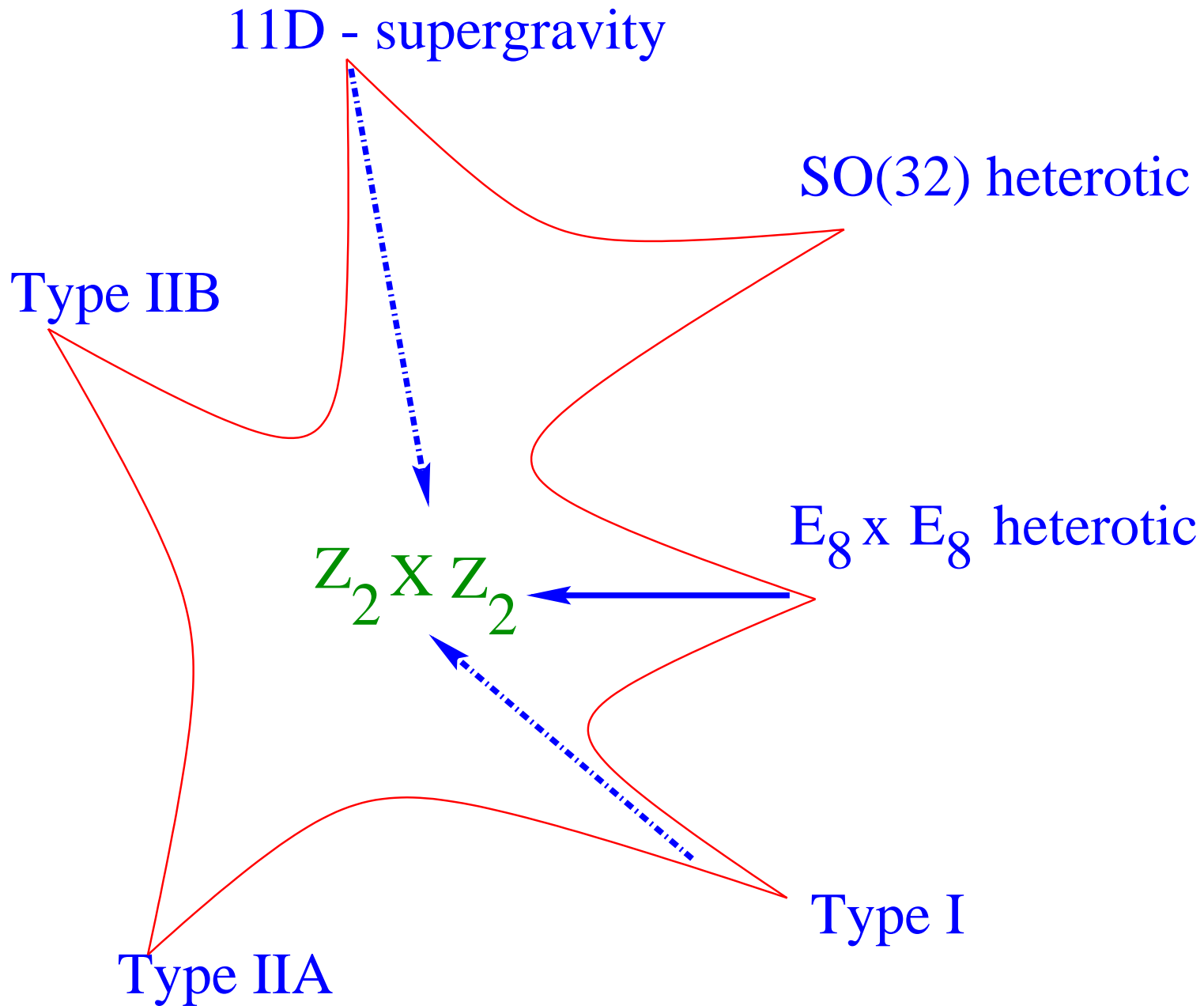
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- With: Costas Kounnas; John Rizos; hep-th/0611251.
- Claudio Coriano; Marco Guzzi; hep-ph/0704.1256.
- Related work: Elisa Manno; Cristina Timirgaziu.

String Phenomenology 2007, Frascati, 4-8 June 2007

Point, String, Membrane



Realistic free fermionic models

'Phenomenology of the Standard Model and string unification'

- Top quark mass $\sim 175\text{--}180\text{GeV}$ PLB 274 (1992) 47
- Generation mass hierarchy NPB 407 (1993) 57
- CKM mixing NPB 416 (1994) 63 (with Halyo)
- Stringy seesaw mechanism PLB 307 (1993) 311 (with Halyo)
- Gauge coupling unification NPB 457 (1993) 409 (with Dienes)
- Proton stability NPB 428 (1994) 111
- Squark degeneracy NPB 526 (1998) 21 (with Pati)
- Minimal Superstring Standard Model PLB 455 (1999) 135
(with Cleaver & Nanopoulos)
- Moduli fixing NPB 728 (2005) 83

Free Fermionic Construction

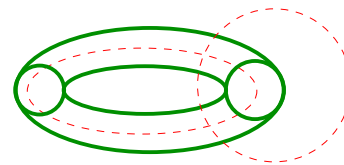
Left-Movers: $\psi_{1,2}^\mu$, χ_i , y_i , ω_i ($i = 1, \dots, 6$)

Right-Movers

$$\bar{\phi}_{A=1, \dots, 44} = \left\{ \begin{array}{ll} \bar{y}_i, \bar{\omega}_i & i = 1, \dots, 6 \\ \bar{\eta}_i & i = 1, 2, 3 \\ \bar{\psi}_{1, \dots, 5} \\ \bar{\phi}_{1, \dots, 8} \end{array} \right.$$

$$f \longrightarrow -e^{i\pi\alpha(f)} f$$

$$V \longrightarrow V$$



$$Z = \sum_{\text{all spin structures}} c\left(\begin{smallmatrix} \vec{\alpha} \\ \vec{\beta} \end{smallmatrix}\right) Z\left(\begin{smallmatrix} \vec{\alpha} \\ \vec{\beta} \end{smallmatrix}\right)$$

Models \longleftrightarrow Basis vectors + one-loop phases

Basis vectors: consistent modular blocks 4,8 periodic fermions

$$1 = \{\psi^\mu, \chi^{1,\dots,6}, y^{1,\dots,6}, \omega^{1,\dots,6} \mid \bar{y}^{1,\dots,6}, \bar{\omega}^{1,\dots,6}, \bar{\eta}^{1,2,3}, \bar{\psi}^{1,\dots,5}, \bar{\phi}^{1,\dots,8}\}$$

$$S = \{\psi^\mu, \chi^{1,\dots,6}\},$$

$$z_1 = \{\bar{\phi}^{1,\dots,4}\},$$

$$z_2 = \{\bar{\phi}^{5,\dots,8}\},$$

$$e_i = \{y^i, \omega^i \mid \bar{y}^i, \bar{\omega}^i\}, \quad i = 1, \dots, 6, \quad N = 4 \text{ Vacua}$$

$$b_1 = \{\chi^{34}, \chi^{56}, y^{34}, y^{56} \mid \bar{y}^{34}, \bar{y}^{56}, \bar{\eta}^1, \bar{\psi}^{1,\dots,5}\}, \quad N = 4 \rightarrow N = 2$$

$$b_2 = \{\chi^{12}, \chi^{56}, y^{12}, y^{56} \mid \bar{y}^{12}, \bar{y}^{56}, \bar{\eta}^2, \bar{\psi}^{1,\dots,5}\}, \quad N = 2 \rightarrow N = 1$$

Vector bosons: NS, $z_{1,2}$, $z_1 + z_2$, $x = 1 + s + \sum e_i + z_1 + z_2$

impose: only NS vector bosons survive GSO projections

\implies Gauge group $SO(10) \times U(1)^3 \times SO(8) \times SO(8)$

Independent phases $c_{\begin{smallmatrix} v_i \\ v_j \end{smallmatrix}} = \exp[i\pi(v_i|v_j)]:$ **upper block**

$$\begin{array}{c} \\ 1 \\ S \\ e_1 \\ e_2 \\ e_3 \\ e_4 \\ e_5 \\ e_6 \\ z_1 \\ z_2 \\ b_1 \\ b_2 \end{array} \begin{pmatrix} 1 & S & e_1 & e_2 & e_3 & e_4 & e_5 & e_6 & z_1 & z_2 & b_1 & b_2 \\ -1 & -1 & \pm & \pm & \pm & \pm & \pm & \pm & \pm & \pm & \pm & \pm \\ & -1 & -1 & -1 & -1 & -1 & -1 & -1 & -1 & -1 & 1 & 1 \\ & & \pm & \pm & \pm & \pm & \pm & \pm & \pm & \pm & \pm & \pm \\ & & & \pm & \pm & \pm & \pm & \pm & \pm & \pm & \pm & \pm \\ & & & & \pm & \pm & \pm & \pm & \pm & \pm & \pm & \pm \\ & & & & & \pm & \pm & \pm & \pm & \pm & \pm & \pm \\ & & & & & & \pm & \pm & \pm & \pm & \pm & \pm \\ & & & & & & & \pm & \pm & \pm & \pm & \pm \\ & & & & & & & & \pm & \pm & \pm & \pm \\ & & & & & & & & & \pm & \pm & \pm \\ & & & & & & & & & & \pm & \pm \\ & & & & & & & & & & & \pm \end{pmatrix}$$

Apriori 55 independent coefficients $\rightarrow 2^{55}$ distinct vacua

Impose: Gauge group $SO(10) \times U(1)^3 \times SO(8)^2$

\rightarrow 40 independent coefficients

The twisted matter spectrum:

$$B_{l_3^1 l_4^1 l_5^1 l_6^1}^1 = S + b_1 + l_3^1 e_3 + l_4^1 e_4 + l_5^1 e_5 + l_6^1 e_6$$

$$B_{l_1^2 l_2^2 l_5^2 l_6^2}^2 = S + b_2 + l_1^2 e_1 + l_2^2 e_2 + l_5^2 e_5 + l_6^2 e_6$$

$$B_{l_1^3 l_2^3 l_3^3 l_4^3}^3 = S + b_3 + l_1^3 e_1 + l_2^3 e_2 + l_3^3 e_3 + l_4^3 e_4 \quad l_i^j = 0, 1$$

sectors $B_{pqrs}^i \rightarrow 16$ or $\overline{16}$ of $SO(10)$ with multiplicity $(1, 0, -1)$

$B_{pqrs}^i + x \rightarrow 10$ of $SO(10)$ with multiplicity $(1, 0)$

Counting: for each B_{pqrs}^i :

Projectors:

$$P_{p^1 q^1 r^1 s^1}^{(1)} = \frac{1}{16} \prod \left(1 - c \left[\begin{matrix} e_i \\ B_{p^1 q^1 r^1 s^1}^{(1)} \end{matrix} \right] \right) \prod \left(1 - c \left[\begin{matrix} z_i \\ B_{p^1 q^1 r^1 s^1}^{(1)} \end{matrix} \right] \right)$$

$$S_{\pm}^{(i)} = \sum_{pqrs} \frac{1 \pm X^{(i)}}{2} P_{p^i q^i r^i s^i}^{(i)}, \quad i = 1, 2, 3$$

similarly for vectorials

Algebraic formulas for $S = \sum_{i=1}^3 S_+^{(i)} - S_-^{(i)}$ and $V = \sum_{i=1}^3 V^{(i)}$

Spinor–vector duality:

Duality under exchange of spinors and vectors.

First Plane			Second plane			Third Plane			# of models
s	\bar{s}	v	s	\bar{s}	v	s	\bar{s}	v	
2	0	0	0	0	0	0	0	0	1325963712
0	2	0	0	0	0	0	0	0	1340075584
1	1	0	0	0	0	0	0	0	3718991872
0	0	2	0	0	0	0	0	0	6385031168

of models with $\#(16 + \overline{16}) = \#$ of models with $\#(10)$

For every model with $\#(16 + \overline{16})$ & $\#(10)$

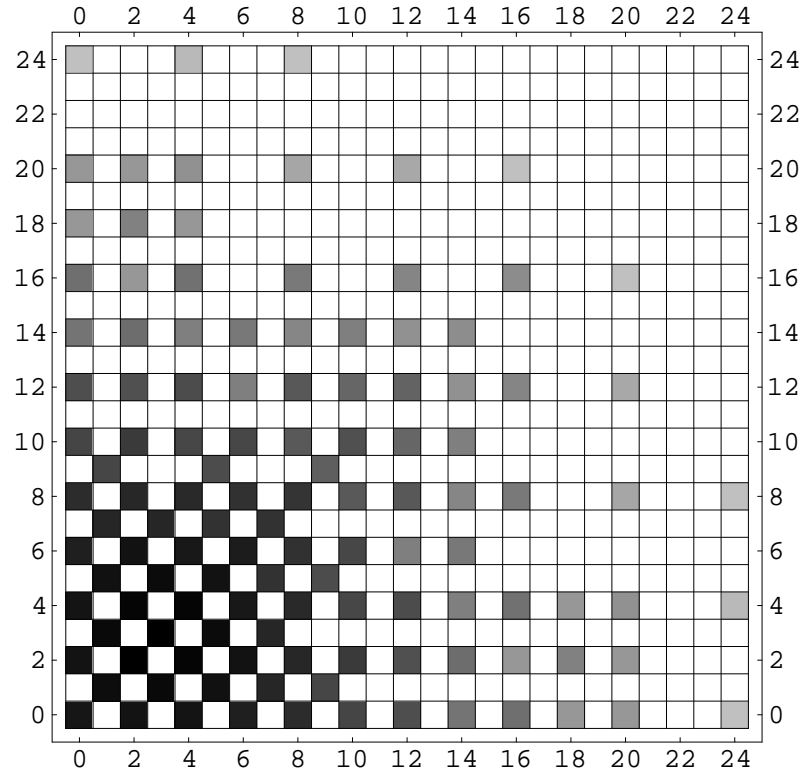
There exist another model in which they are interchanged

Reflects discrete exchange of phases

	0	1	2	3	4	5	6	7	8
0	14424168320	0	19155093504	0	17251226688	0	5722036224	0	1663598208
1	0	35042893824	0	54063267840	0	24984354816	0	3050569728	0
2	19155093504	0	138128891904	0	80400635904	0	22541905920	0	2593253376
3	0	54063267840	0	128713392128	0	43913576448	0	3064725504	0
4	17251226688	0	80400635904	0	78871289088	0	11554105344	0	2246205312
5	0	24984354816	0	43913576448	0	21663891456	0	856424448	0
6	5722036224	0	22541905920	0	11554105344	0	8043915264	0	937728000
7	0	3050569728	0	3064725504	0	856424448	0	866942976	0
8	1663598208	0	2593253376	0	2246205312	0	937728000	0	703000320
9	0	113541120	0	0	0	67829760	0	0	0
10	135948288	0	406695936	0	107403264	0	104902656	0	17467392
11	0	0	0	0	0	0	0	0	0
12	50867584	0	42448896	0	65853312	0	387072	0	18590208
13	0	0	0	0	0	0	0	0	0
14	1210368	0	2420736	0	387072	0	774144	0	202752
15	0	0	0	0	0	0	0	0	0
16	1854336	0	36864	0	1514688	0	0	0	714816
17	0	0	0	0	0	0	0	0	0
18	36864	0	313344	0	36864	0	0	0	0
19	0	0	0	0	0	0	0	0	0
20	33984	0	36864	0	62784	0	0	0	7680
21	0	0	0	0	0	0	0	0	0
22	0	0	0	0	0	0	0	0	0
23	0	0	0	0	0	0	0	0	0
24	576	0	0	0	1152	0	0	0	576

Spinor–vector duality:

Invariance under exchange of $\#(16 + \overline{16}) \leftrightarrow \#(10)$



Symmetric under exchange of rows and columns

Self-dual: $\#(16 + \overline{16}) = \#(10)$ without E_6 symmetry

Introducing the notation $c \begin{bmatrix} a_i \\ a_j \end{bmatrix} = e^{i\pi(a_i|a_j)}$, $(a_i | a_j) = 0, 1$

the projectors can be written as system of equations

$$\Delta^{(I)} U_{16}^{(I)} = Y_{16}^{(I)} \quad , \quad \Delta^{(I)} U_{10}^{(I)} = Y_{10}^{(I)} \quad , \quad I = 1, 2, 3$$

where the unknowns are the fixed point labels

$$U_{16}^{(I)} = \begin{bmatrix} p_{16}^I \\ q_{16}^I \\ r_{16}^I \\ s_{16}^I \end{bmatrix} \quad , \quad U_{10}^{(I)} = \begin{bmatrix} p_{10}^I \\ q_{10}^I \\ r_{10}^I \\ s_{10}^I \end{bmatrix}$$

and

$$\Delta^{(1)} = \begin{bmatrix} (e_1 | e_3) & (e_1 | e_4) & (e_1 | e_5) & (e_1 | e_6) \\ (e_2 | e_3) & (e_2 | e_4) & (e_2 | e_5) & (e_2 | e_6) \\ (z_1 | e_3) & (z_1 | e_4) & (z_1 | e_5) & (z_1 | e_6) \\ (z_2 | e_3) & (z_2 | e_4) & (z_2 | e_5) & (z_2 | e_6) \end{bmatrix} \quad ; \quad \Delta^{(2)} \dots$$

$$Y_{16}^{(1)} = \begin{bmatrix} (e_1 | b_1) \\ (e_2 | b_1) \\ (z_1 | b_1) \\ (z_2 | b_2) \end{bmatrix}, \quad Y_{16}^{(2)} = \dots$$

$$Y_{10}^{(1)} = \begin{bmatrix} (e_1 | b_1 + x) \\ (e_2 | b_1 + x) \\ (z_1 | b_1 + x) \\ (z_2 | b_2 + x) \end{bmatrix}, \quad Y_{10}^{(2)} = \dots$$

The number of solutions per plane

$$S^{(I)} = \begin{cases} 2^{4-\text{rank}(\Delta^{(I)})} & \text{rank}(\Delta^{(I)}) = \text{rank} \begin{bmatrix} \Delta^{(I)} & Y_{16}^{(I)} \end{bmatrix} \\ 0 & \text{rank}(\Delta^{(I)}) < \text{rank} \begin{bmatrix} \Delta^{(I)} & Y_{16}^{(I)} \end{bmatrix} \end{cases}$$

$$V^{(I)} = \begin{cases} 2^{4-\text{rank}(\Delta^{(I)})} & \text{rank}(\Delta^{(I)}) = \text{rank} \begin{bmatrix} \Delta^{(I)} & Y_{10}^{(I)} \end{bmatrix} \\ 0 & \text{rank}(\Delta^{(I)}) < \text{rank} \begin{bmatrix} \Delta^{(I)} & Y_{10}^{(I)} \end{bmatrix} \end{cases}$$

rank $(\Delta^{(I)})$	rank $\Delta^{(I)}, Y_{16}^{(I)}$	rank $\Delta^{(I)}, Y_{10}^{(I)}$	# of Spinorials	# of vectorials
4	4	4	1	1
3	4	4	0	0
	3	4	2	0
	4	3	0	2
	3	3	2	2
2	3	3	0	0
	2	3	4	0
	3	2	0	4
	3	3	4	4
1	2	2	0	0
	1	2	8	0
	2	1	0	8
	1	1	8	8
0	1	1	0	0
	0	1	16	0
	1	0	0	16
	0	0	16	16

Table 1: Total number of $SO(10)$ spinorial and vectorial representations in a given orbifold plane $I = 1, 2, 3$ for all possible ranks of the projection matrices $(\Delta^{(I)})$, $[\Delta^{(I)}, Y_{16}^{(I)}]$, and $[\Delta^{(I)}, Y_{10}^{(I)}]$.

⇒ Algebraic proof of spinor-vector duality

The number of vectorials and spinorials from a specific orbifold plane I are interchanged when the ranks of the associated Y -vectors are interchanged

$$\text{rank} \left[\Delta^{(I)}, Y_{16}^{(I)} \right] \longleftrightarrow \text{rank} \left[\Delta^{(I)}, Y_{10}^{(I)} \right]$$

Induced by GSO phase change

Spinor–vector duality in $N = 2$ vacua: (with Kounnas and Rizos; in progress)

First Plane			Second plane			Third Plane			# of models
s	\bar{s}	v	s	\bar{s}	v	s	\bar{s}	v	
2	0	0	0	0	0	0	0	0	1325963712
0	2	0	0	0	0	0	0	0	1340075584
1	1	0	0	0	0	0	0	0	3718991872
0	0	2	0	0	0	0	0	0	6385031168

Duality exists plane by plane \implies Exists at $N = 2$ level

Untwisted gauge group: $SO(12) \times SO(4) \times SO(8) \times SO(8)$

Exchange: $\#(32 + 32') \longleftrightarrow \#(12)$

Novel Basis

$$S = \{\psi^\mu, \chi^{1,\dots,6}\},$$

$$z_1 = \{\bar{\phi}^{1,\dots,4}\},$$

$$z_2 = \{\bar{\phi}^{5,\dots,8}\},$$

$$z_3 = \{\bar{\psi}^{1,\dots,4}\},$$

$$z_4 = \{\bar{\eta}^{0,\dots,3}\},$$

$$e_i = \{y^i, \omega^i | \bar{y}^i, \bar{\omega}^i\}, \quad i = 1, \dots, 6,$$

$$\bar{\eta}^0 \equiv \bar{\psi}^5$$

$N = 4$ Vacua

$$1 = S + \sum e_i + z_1 + z_2 + z_3 + z_4$$

$$b_1 = \{\chi^{34}, \chi^{56}, y^{34}, y^{56} | \bar{y}^{34}, \bar{y}^{56}, \bar{\eta}^1, \bar{\psi}^{1,\dots,5}\},$$

$$N = 4 \rightarrow N = 2$$

Vector bosons: NS, $z_{1,2,3,4}$, $z_i + z_j$

$$\text{NS} \rightarrow SO(8)^4$$

$SO(12)$ -GUT \rightarrow from enhancement

Duality picture is facilitated

Spinor \longleftrightarrow Vector map $\longrightarrow B \longleftrightarrow B + z_4$

$SO(12)$ enhancement $\longrightarrow B \longleftrightarrow B + z_3$

A convenient basis to study dualities; modular properties

GUT structure is obscured

Proton stability and superstring Z' (with Coriano and Guzzi)

Standard Model: $(SU_{321}) \oplus (Q L U D E N) \oplus (h)$:

Effective renormalizable QFT below a cutoff

baryon and lepton numbers protected by accidental global symmetries

Non-renormalizable operators suppressed by cutoff M

B & L numbers violating operators

Dimension six: $QQQL \frac{1}{M^2} \Rightarrow M \sim 10^{16} \text{GeV}$

supersymmetry:

Dimension four: $\eta_1 QLD \ \& \ \eta_2 UDD \Rightarrow (\eta_1 \cdot \eta_2 \leq 10^{-24})$

Dimension five: $\lambda QQQQL \frac{1}{M} \Rightarrow \left(\frac{\lambda}{M}\right) \leq 10^{-26}$

Appealing Proposition: Low scale gauged $U(1)$ symmetry

Additional Facts

Standard Model: \longrightarrow Unification \longleftrightarrow $SO(10)$

Dimension four: $16^4 \Rightarrow \eta'_1 UDD \frac{\langle N \rangle}{M} + \eta'_2 QLD \frac{\langle N \rangle}{M}$

+ ... dimension five ; dimension six

left-handed neutrino masses: $M_{\nu_L} \approx M_{Up} \left(\frac{M_{\text{weak}}}{M_{\langle N \rangle}} \right)^2$

Fermion masses: $\lambda_{ij}^{\text{Up}} Q^i U^j \bar{h} ; \lambda_{ij}^{\text{Down}} Q^i D^j h ; \lambda_{ij}^{\text{CLepton}} L^i E^j h ;$

Flavour universality

Freedom from anomalies

What can we learn from string constructions?

Fermionic $Z_2 \times Z_2$ orbifolds

Models \longrightarrow set of boundary condition basis vectors

The NAHE set $\{1, S, b_1, b_2, b_3\}$

$\longrightarrow Z_2 \times Z_2$ orbifold compactification

Gauge Group : $SO(10) \times SO(6)^{1,2,3} \times E_8$

beyond the NAHE set Add $\{\alpha, \beta, \gamma\}$

number of generations is reduced to three

$SO(10) \longrightarrow$ subgroup

e.g. $SU(3) \times SU(2) \times U(1)_{T_{3R}} \times U(1)_{B-L}$

$U(1)_Y = \frac{1}{2}(B - L) + T_{3R} \in SO(10) !$

$SO(6)^{1,2,3} \longrightarrow U(1)^{1,2,3} \times U(1)^{1,2,3}$

The massless spectrum

Three twisted generations

$$b_1, \quad b_2, \quad b_3$$

Untwisted Higgs doublets

$$\begin{array}{ll}
 h_{1,0,0} & \bar{h}_{1,-1,0,0} \\
 h_{2,0,1,0} & \bar{h}_{2,0,-1,0} \\
 h_{3,0,0,1} & \bar{h}_{3,0,0,-1}
 \end{array}$$

Sector $b_1 + b_2 + \alpha + \beta$

$$\begin{array}{ll}
 h_{\alpha\beta} & \bar{h}_{\alpha\beta} \\
 -\frac{1}{2}, -\frac{1}{2}, 0, 0, 0, 0 & \frac{1}{2}, \frac{1}{2}, 0, 0, 0, 0
 \end{array}$$

\oplus $SO(10)$ singlets

Sectors $b_j + 2\gamma \quad j = 1, 2, 3 \quad \longrightarrow \quad$ hidden matter multiplets

“standard” $SO(10)$ representations

NAHE + $\{ \alpha, \beta, \gamma \} \longrightarrow$ exotic vector-like matter \longrightarrow superheavy

\oplus Quasi-realistic phenomenology

Extra $U(1)$'s beyond the Standard Model

e.g. PLB 278 (1992) 131 → seven extra $U(1)$'s

$$U(1)_{Z'} = \frac{B-L}{2} - \frac{2}{3}T_{3R}$$

The $U(1)$ combinations

$$U_A = \frac{1}{\sqrt{15}}(2(U_1 + U_2 + U_3) - (U_4 + U_5 + U_6))$$

$$U_\chi = \frac{1}{\sqrt{15}}(U_1 + U_2 + U_3 + 2U_4 + 2U_5 + 2U_6)$$

$$U_{12} = \frac{1}{\sqrt{2}}(U_1 - U_2) \quad , \quad U_\psi = \frac{1}{\sqrt{6}}(U_1 + U_2 - 2U_3),$$

$$U_{45} = \frac{1}{\sqrt{2}}(U_4 - U_5) \quad , \quad U_\zeta = \frac{1}{\sqrt{6}}(U_4 + U_5 - 2U_6)$$

Pati, PLB388 (1996) 532; $U(1)_\psi$ in conjunction with $U(1)_{B-L}$ or $U(1)_\chi$

provides adequate protection as well as allowing light neutrino masses

PLB499 (2001) 147; the extra $U(1)$ s are broken at high scale

Patterns of $SO(10)$ symmetry breaking

The $SO(10) \rightarrow$ subgroup $b(\bar{\psi}_{\frac{1}{2}}^{1\dots 5})$:

1. $b\{\bar{\psi}_{\frac{1}{2}}^{1\dots 5} \bar{\eta}^1 \bar{\eta}^2 \bar{\eta}^3\} = \left\{ \frac{111111}{222222} \frac{111}{222} \right\} \Rightarrow SU(5) \times U(1) \quad U(1) \quad U(1) \quad U(1)$
2. $b\{\bar{\psi}_{\frac{1}{2}}^{1\dots 5} \bar{\eta}^1 \bar{\eta}^2 \bar{\eta}^3\} = \{ 11100 \quad 000 \} \Rightarrow SO(6) \times SO(4) \quad U(1) \quad U(1) \quad U(1)$

$$(1. + 2.) \Rightarrow SO(10) \rightarrow SU(3)_C \times SU(2)_L \times U(1)_C \times U(1)_L$$

$$SO(10) \rightarrow SU(3)_C \times SU(2)_L \times SU(2)_R \times U(1)_{B-L}$$

2. $b\{\bar{\psi}_{\frac{1}{2}}^{1\dots 5} \bar{\eta}^1 \bar{\eta}^2 \bar{\eta}^3\} = \{ 11100000 \} \Rightarrow SO(6) \times SO(4) \quad U(1) \quad U(1) \quad U(1)$
3. $b\{\bar{\psi}_{\frac{1}{2}}^{1\dots 5} \bar{\eta}^1 \bar{\eta}^2 \bar{\eta}^3\} = \left\{ \frac{111}{222} 00 \frac{111}{222} \right\} \Rightarrow$
 $SU(3)_C \times U(1)_C \times SU(2)_L \times SU(2)_R \quad U(1) \quad U(1) \quad U(1)$

$U(1)$ matter charges

in cases 1. 2.

$$\implies Q_{U(1)_j}(16 = \{Q, L, U, D, E, N\}) = +\frac{1}{2}$$

\implies the $U(1)_{1,2,3}$ are anomalous

In the LRS model of case 3.

$$\implies Q_{U(1)_j}(Q_L, L_L) = -\frac{1}{2}$$

$$Q_{U(1)_j}(Q_R = \{U, D\}, L_R = \{E, N\}) = +\frac{1}{2}$$

\implies the $U(1)_{1,2,3}$ are anomaly free

The $U(1)$ combination $U(1) = U(1)_1 + U(1)_2 + U(1)_3$ is :

- a. family universal
- b. anomaly free

The Baryon number violating terms :

$$Q_L Q_L Q_L L_L \rightarrow QQQ L$$

$$Q_R Q_R Q_R L_R \rightarrow \{UDDN, UUDE\}$$

are forbidden

The Lepton number violating terms :

$$Q_L Q_R L_L L_R \rightarrow QDLN$$

$$L_L L_L L_R L_R \rightarrow LLEN$$

are allowed

The fermion mass terms:

$$Q_L Q_R h \quad \text{and} \quad L_L L_R h \quad \text{and} \quad NN\bar{N}_H\bar{N}_H .$$

are allowed

STRING DERIVED LEFT-RIGHT SYMMETRIC MODEL

	ψ^μ	χ^{12}	χ^{34}	χ^{56}	$y^{3,\dots,6}$	$\bar{y}^{3,\dots,6}$	$y^{1,2}, \omega^{5,6}$	$\bar{y}^{1,2}, \bar{\omega}^{5,6}$	$\omega^{1,\dots,4}$	$\bar{\omega}^{1,\dots,4}$	$\bar{\psi}^{1,\dots,5}$	$\bar{\eta}^1$	$\bar{\eta}^2$	$\bar{\eta}^3$	$\bar{\phi}^{1,\dots,8}$
1	1	1	1	1	1,...,1	1,...,1	1,...,1	1,...,1	1,...,1	1,...,1	1,...,1	1	1	1	1,...,1
<i>S</i>	1	1	1	1	0,...,0	0,...,0	0,...,0	0,...,0	0,...,0	0,...,0	0,...,0	0	0	0	0,...,0
<i>b</i> ₁	1	1	0	0	1,...,1	1,...,1	0,...,0	0,...,0	0,...,0	0,...,0	1,...,1	1	0	0	0,...,0
<i>b</i> ₂	1	0	1	0	0,...,0	0,...,0	1,...,1	1,...,1	0,...,0	0,...,0	1,...,1	0	1	0	0,...,0
<i>b</i> ₃	1	0	0	1	0,...,0	0,...,0	0,...,0	0,...,0	1,...,1	1,...,1	1,...,1	0	0	1	0,...,0

	ψ^μ	χ^{12}	χ^{34}	χ^{56}	$y^3 y^6$	$y^4 \bar{y}^4$	$y^5 \bar{y}^5$	$\bar{y}^3 \bar{y}^6$	$y^1 \omega^5$	$y^2 \bar{y}^2$	$\omega^6 \bar{\omega}^6$	$\bar{y}^1 \bar{\omega}^5$	$\omega^2 \omega^4$	$\omega^1 \bar{\omega}^1$	$\omega^3 \bar{\omega}^3$	$\bar{\omega}^2 \bar{\omega}^4$	$\bar{\psi}^{1,\dots,5}$	$\bar{\eta}^1$	$\bar{\eta}^2$	$\bar{\eta}^3$	$\bar{\phi}^{1,\dots,8}$
α	0	0	0	0	1	0	0	0	0	0	1	1	0	0	1	1	1 1 1 0 0	0	0	0	1 1 1 1 0 0 0 0
β	0	0	0	0	0	0	1	1	1	0	0	0	0	1	0	1	1 1 1 0 0	0	0	0	1 1 0 0 1 1 0 0
γ	0	0	0	0	0	1	0	1	0	1	0	1	1	0	0	0	$\frac{1}{2} \frac{1}{2} \frac{1}{2} 0 0$	$\frac{1}{2}$	$\frac{1}{2}$	$\frac{1}{2}$	1 $\frac{1}{2} \frac{1}{2} \frac{1}{2} \frac{1}{2} \frac{1}{2} \frac{1}{2} 0$

Gerald Cleaver, AEF and Christopher Savage, PRD 63:066001,2001.

3 generations;

3 untwisted Higgs bi-doublets;

Fermion mass terms arise from $N = 3$ and $N = 5$ superpotential terms

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Conclusions

Free fermionic models $\longleftrightarrow Z_2 \times Z_2$ orbifolds

A novel Global structure: \longleftrightarrow spinor–vector duality

A novel $U(1)$ symmetry \longleftrightarrow proton lifeguard

FSU5, PS and SLM models descend from an $N = 4$ vacuum with
 $SO(12) \times E_8 \times E_8$ or $SO(12) \times SO(16) \times SO(16)$ gauge symmetry

LRS models descend from an $N = 4$ vacuum with
 $SO(16) \times E_7 \times E_7$ gauge symmetry

Phenomenology: Low scale Z' with Lepton number violation