

Dark Olives (and other Dark Middle Eastern Delights)



- Wilsonian Dark Matter in string derived Z' model
- Time permitting: The equivalence postulate approach to quantum gravity

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HAPPY FESTDAY KEITH!

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Probing the Desert with Ultra-Energetic Neutrinos from the Sun and the Earth

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Abstract

Realistic superstring models generically give rise to exotic matter states, which arise due to the “Wilson-line” breaking of the non-Abelian unifying gauge symmetry. Often such states are protected by a gauge or local discrete symmetry and therefore may be stable or meta-stable. We study the possibility of a flux of high energy neutrinos coming from the sun and the earth due to the annihilation of such exotic string states. We also discuss the expected flux for other heavy stable particles – like the gluino LSP. We comment that the detection of ultra-energetic neutrinos from the sun and the earth imposes model independent constraints on the high energy cutoff, as for example in the recently entertained TeV scale Kaluza-Klein theories. We therefore propose that improved experimental resolution of the energy of the muons in neutrino detectors together with their correlation with neutrinos from the sun and the center of the earth will serve as a probe of the desert in Gravity Unified Theories.

Realistic free fermionic models

'Phenomenology of the Standard Model and Unification'

- Minimal Superstring Standard Model NPB 335 (1990) 347
(with Nanopoulos & Yuan)
- Top quark mass $\sim 175\text{--}180\text{GeV}$ PLB 274 (1992) 47
- Generation mass hierarchy NPB 407 (1993) 57
- CKM mixing NPB 416 (1994) 63 (with Halyo)
- Stringy seesaw mechanism PLB 307 (1993) 311 (with Halyo)
- Gauge coupling unification NPB 457 (1995) 409 (with Dienes)
- Proton stability NPB 428 (1994) 111
- Squark degeneracy NPB 526 (1998) 21 (with Pati)
- Moduli fixing NPB 728 (2005) 83
- Classification & Exophobia 2003 – . . .
(with Nooij, Assel, Christodoulides, Kounnas, Rizos & Sonmez)

Other approaches

Geometrical

Greene, Kirklin, Miron, Ross (1987)
Donagi, Ovrut, Pantev, Waldram (1999)
Blumenhagen, Moster, Reinbacher, Weigand (2006)
Heckman, Vafa (2008)
.....

Orbifolds

Ibanez, Nilles, Quevedo (1987)
Bailin, Love, Thomas (1987)
Kobayashi, Raby, Zhang (2004)
Lebedev, Nilles, Raby, Ramos-Sanchez, Ratz, Vaudrevange, Wingerter (2007)
Blaszczyk, Groot–Nibbelink, Ruehle, Trapletti, Vaudrevange (2010)
.....

Other CFTs

Gepner (1987)
Schellekens, Yankielowicz (1989)
Gato–Rivera, Schellekens (2009)
.....

Orientifolds

Cvetic, Shiu, Uranga (2001)
Ibanez, Marchesano, Rabadan (2001)
Kiristis, Schellekens, Tsulaia (2008)
.....

Low scale Z' in free fermionic models:

- $\frac{3}{2}U(1)_{B-L} - 2U(1)_R \in SO(10) @ 1TeV$ MPL A6 (1991) 61
(with Nanopoulos)
- But $m_t = m_{\nu_\tau}$ & $1TeV Z' \Rightarrow m_{\nu_\tau} \approx 10MeV$ PLB 245 (1990) 435
- $E_6 \rightarrow SO(10) \times U(1)_A \Rightarrow U(1)_A$ is anomalous!
 $\Rightarrow U(1)_A \notin \text{low scale } U(1)_{Z'}$
- 1996-2013, Pati, AEF, Guzzi, Mehta, Athanasopoulos, $U(1) \notin E_6$
- On the other hand(AEF, Viraf Mehta, PRD88 (2013) 025006)
 $\sin^2 \theta_W(M_Z), \alpha_s(M_Z) \Rightarrow U(1)_{Z'} \in E_6$
- Z' string derived model, (with Rizos) NPB 895 (2015) 233

Free Fermionic Construction

Left-Movers: $\psi_{1,2}^\mu$, χ_i , y_i , ω_i ($i = 1, \dots, 6$)

Right-Movers

$$\bar{\phi}_{A=1, \dots, 44} = \left\{ \begin{array}{ll} \bar{y}_i, \bar{\omega}_i & i = 1, \dots, 6 \\ \bar{\eta}_i & i = 1, 2, 3 \\ \bar{\psi}_{1, \dots, 5} \\ \bar{\phi}_{1, \dots, 8} \end{array} \right.$$

Models \longleftrightarrow Basis vectors + one-loop phases

Free Fermionic Models:

$Z_2 \times Z_2$ orbifolds
with discrete Wilson lines

Exotic matter :

In realistic string models

Unifying gauge group \Rightarrow broken by “Wilson lines” .

\Rightarrow non-GUT physical states.

\Rightarrow Meta-stable heavy string relics

\rightarrow Dark Mater

NPB 477 (1996) 65

(with Coriano and Chang)

UHECR candidates

NPB 614 (2001) 233

(with Coriano and Plümacher)

Exotics classified by: $SO(10) \rightarrow$ subgroup $b(\bar{\psi}_{\frac{1}{2}}^{1 \cdots 5})$:

notation $[(SU(3)_C \times U(1)_C); (SU(2)_L \times U(1)_L)](Q_Y, Q_{Z'}, Q_{\text{e.m.}})$

$SO(6) \times SO(4)$ type states

$[(3, \frac{1}{2}); (1, 0)](1/6, 1/2, 1/6)$; $[(\bar{3}, -\frac{1}{2}); (1, 0)](-1/6, -1/2, -1/6)$

$[(1, 0); (2, 0)](0, 0, \pm 1/2)$

$[(1, 0); (1, \pm 1)](\pm 1/2, \mp 1/2, \pm 1/2)$ $[(1, \pm 3/2); (1, 0)](\pm 1/2, \pm 1/2, \pm 1/2)$

$SU(5) \times U(1)$ type states

$[(1, \pm 3/4); (1, \pm 1/2)](\pm 1/2, \pm 1/4, \pm 1/2)$

$SU(3) \times SU(2) \times U(1)^2$ type states

$[(3, \frac{1}{4}); (1, \frac{1}{2})](-1/3, -1/4, -1/3)$; $[(\bar{3}, -\frac{1}{4}); (1, \frac{1}{2})](1/3, 1/4, 1/3)$

$[(1, \pm \frac{3}{4}); (2, \pm \frac{1}{2})](\pm 1/2, \pm 1/4, (1, 0); (0, -1))$

$[(1, \pm \frac{3}{4}); (1, \mp \frac{1}{2})](0, \pm 5/4, 0)$

CCR, SSR, NPB1996.

Classification of fermionic $Z_2 \times Z_2$ orbifolds

Basis vectors:

$$1 = \{\psi^\mu, \chi^{1,\dots,6}, y^{1,\dots,6}, \omega^{1,\dots,6} \mid \bar{y}^{1,\dots,6}, \bar{\omega}^{1,\dots,6}, \bar{\eta}^{1,2,3}, \bar{\psi}^{1,\dots,5}, \bar{\phi}^{1,\dots,8}\}$$

$$S = \{\psi^\mu, \chi^{1,\dots,6}\},$$

$$z_1 = \{\bar{\phi}^{1,\dots,4}\},$$

$$z_2 = \{\bar{\phi}^{5,\dots,8}\},$$

$$e_i = \{y^i, \omega^i \mid \bar{y}^i, \bar{\omega}^i\}, \quad i = 1, \dots, 6, \quad N = 4 \text{ Vacua}$$

$$b_1 = \{\chi^{34}, \chi^{56}, y^{34}, y^{56} \mid \bar{y}^{34}, \bar{y}^{56}, \bar{\eta}^1, \bar{\psi}^{1,\dots,5}\}, \quad N = 4 \rightarrow N = 2$$

$$b_2 = \{\chi^{12}, \chi^{56}, y^{12}, y^{56} \mid \bar{y}^{12}, \bar{y}^{56}, \bar{\eta}^2, \bar{\psi}^{1,\dots,5}\}, \quad N = 2 \rightarrow N = 1$$

$$\alpha = \{\bar{\psi}^{4,5}, \bar{\phi}^{1,2}\} \quad \& \quad SO(10) \rightarrow SO(6) \times SO(4) \times \dots$$

$$\beta = \{\bar{\psi}^{1,\dots,5} \equiv \frac{1}{2}, \dots\} \quad \& \quad SO(10) \rightarrow SU(5) \times U(1) \times \dots$$

Independent phases $c_{[v_i|v_j]}^{[v_i]} = \exp[i\pi(v_i|v_j)]:$ **upper block**

$$\begin{array}{c}
 1 \\
 S \\
 e_1 \\
 e_2 \\
 e_3 \\
 e_4 \\
 e_5 \\
 e_6 \\
 z_1 \\
 z_2 \\
 b_1 \\
 b_2 \\
 \alpha
 \end{array}
 \left(
 \begin{array}{cccccccccccccc}
 1 & S & e_1 & e_2 & e_3 & e_4 & e_5 & e_6 & z_1 & z_2 & b_1 & b_2 & \alpha \\
 -1 & -1 & \pm \\
 & & -1 & -1 & -1 & -1 & -1 & -1 & -1 & -1 & 1 & 1 & -1 \\
 & & & \pm \\
 & & & & \pm \\
 & & & & & \pm \\
 & & & & & & \pm \\
 & & & & & & & \pm & \pm & \pm & \pm & \pm & \pm \\
 & & & & & & & & \pm & \pm & \pm & \pm & \pm \\
 & & & & & & & & & \pm & \pm & \pm & \pm \\
 & & & & & & & & & & \pm & \pm & \pm \\
 & & & & & & & & & & & \pm & \pm \\
 & & & & & & & & & & & -1 & \pm \\
 & & & & & & & & & & & & \pm
 \end{array}
 \right)$$

A priori 66 independent coefficients $\rightarrow 2^{66}$ distinct vacua

The twisted matter spectrum:

$$B_{l_3^1 l_4^1 l_5^1 l_6^1}^1 = S + b_1 + l_3^1 e_3 + l_4^1 e_4 + l_5^1 e_5 + l_6^1 e_6$$

$$B_{l_1^2 l_2^2 l_5^2 l_6^2}^2 = S + b_2 + l_1^2 e_1 + l_2^2 e_2 + l_5^2 e_5 + l_6^2 e_6$$

$$B_{l_1^3 l_2^3 l_3^3 l_4^3}^3 = S + b_3 + l_1^3 e_1 + l_2^3 e_2 + l_3^3 e_3 + l_4^3 e_4 \quad l_i^j = 0, 1$$

sectors $B_{pqrs}^i \rightarrow 16$ or $\overline{16}$ of $SO(10)$ with multiplicity $(1, 0, -1)$

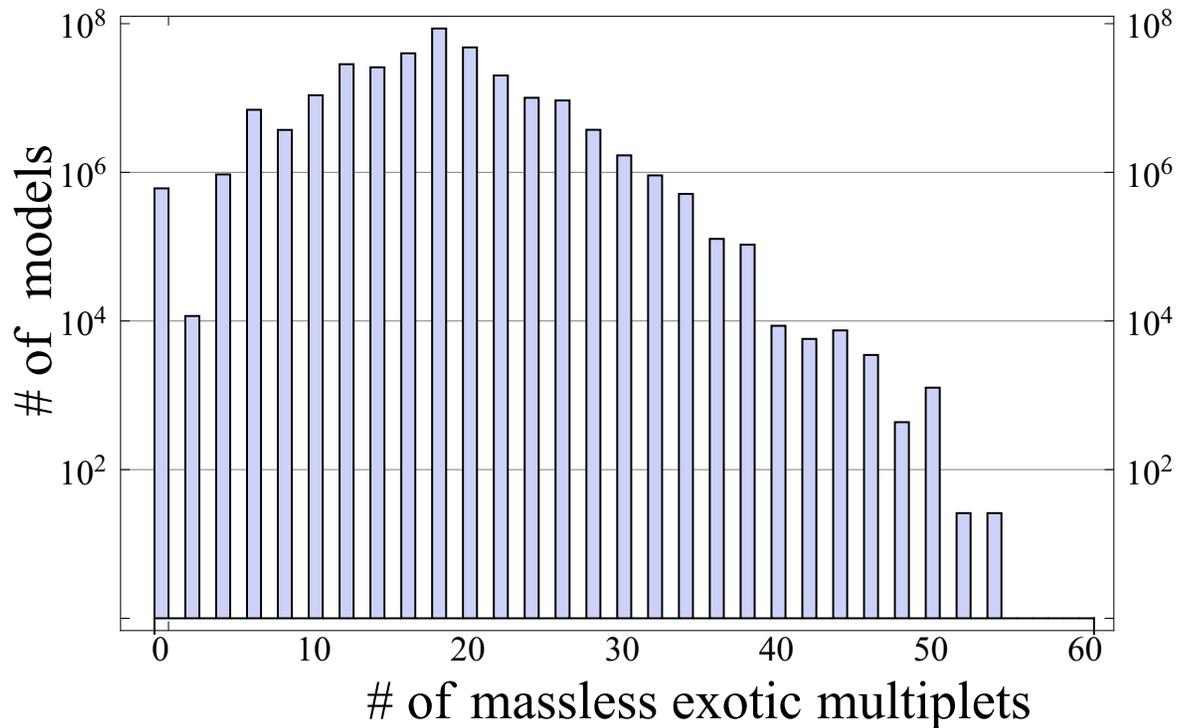
$B_{pqrs}^i + x \rightarrow 10$ of $SO(10)$ with multiplicity $(1, 0)$

$x = \{\bar{\psi}^{1, \dots, 5}, \bar{\eta}^1, \bar{\eta}^2, \bar{\eta}^3\}$ x - map \leftrightarrow spinor-vector map

Algebraic formulas for $S = \sum_{i=1}^3 S_+^{(i)} - S_-^{(i)}$ and $V = \sum_{i=1}^3 V^{(i)}$

Pati–Salam class: with Assel, Christodoulides, Kounnas, Rizos

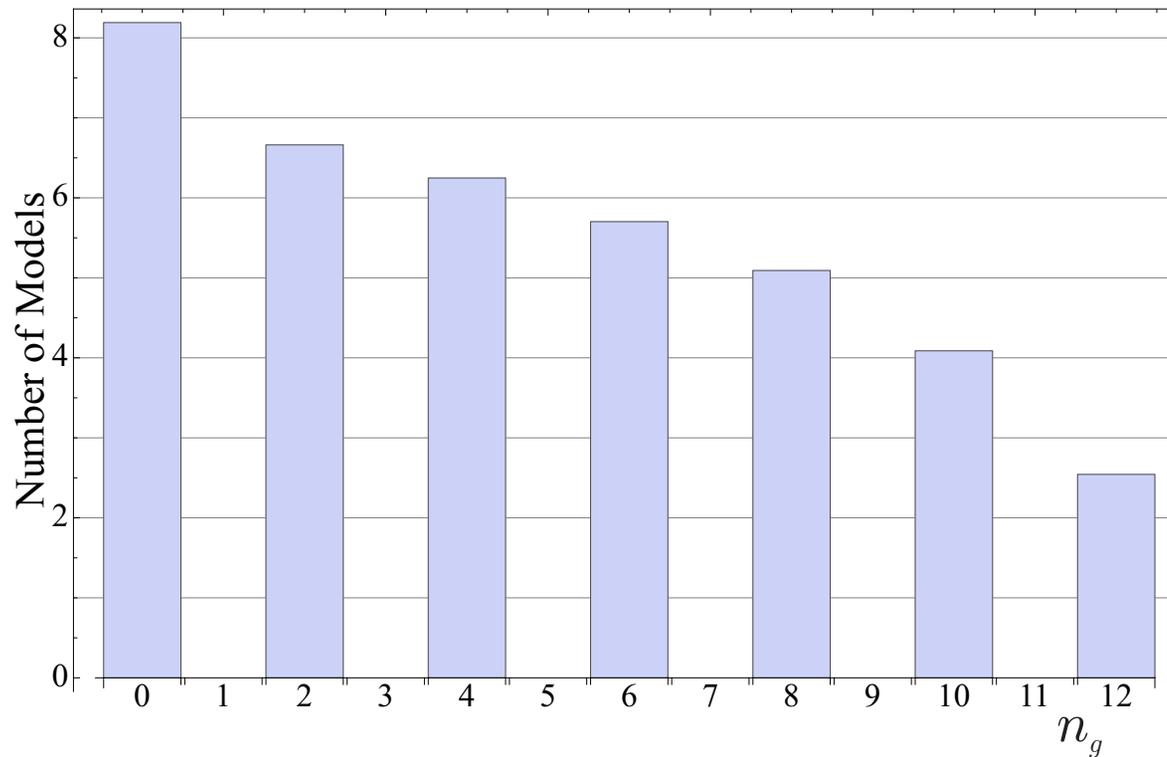
RESULTS: of random search of over 10^{11} vacua



Number of 3-generation models versus total number of exotic multiplets

flipped $SU(5)$ class: with Sonmez, Rizos

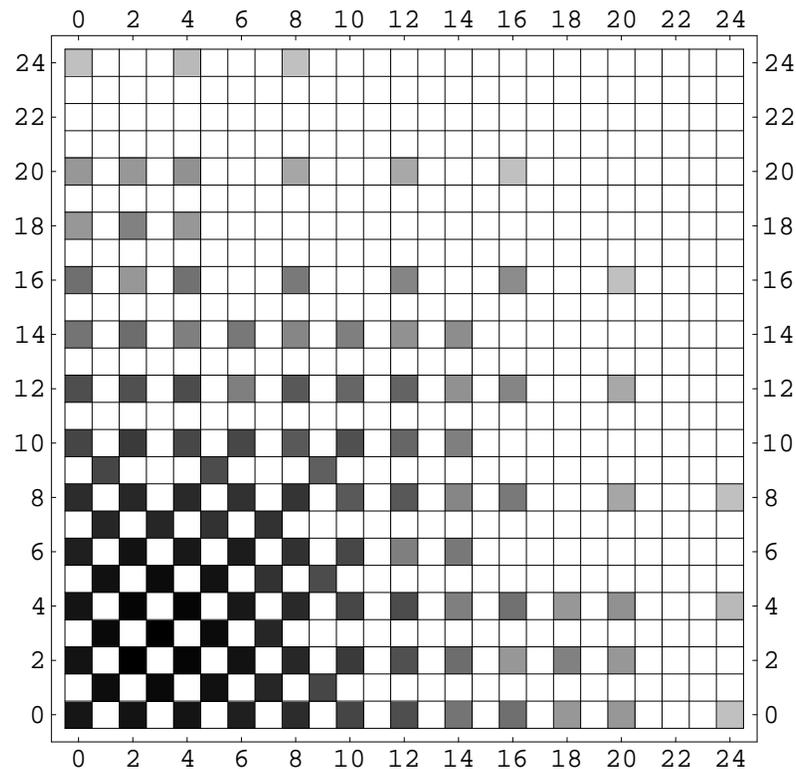
RESULTS: of random search of over 10^{12} vacua



Number of exophobic models versus the number of generations

Spinor–vector duality:

Invariance under exchange of $\#(16 + \overline{16}) < - > \#(10)$



Symmetric under exchange of rows and columns

$$E_6 : \quad 27 = 16 + 10 + 1 \quad \overline{27} = \overline{16} + 10 + 1$$

Self-dual: $\#(16 + \overline{16}) = \#(10)$ without E_6 symmetry

light Z' heterotic-string model $c \begin{bmatrix} v_i \\ v_j \end{bmatrix} = \exp[i\pi(v_i|v_j)]:$

$$(v_i|v_j) = \begin{matrix} & 1 & S & e_1 & e_2 & e_3 & e_4 & e_5 & e_6 & b_1 & b_2 & z_1 & z_2 & \alpha \\ \begin{matrix} 1 \\ S \\ e_1 \\ e_2 \\ e_3 \\ e_4 \\ e_5 \\ e_6 \\ b_1 \\ b_2 \\ z_1 \\ z_2 \\ \alpha \end{matrix} & \begin{pmatrix} 1 & 1 & 1 & 1 & 1 & 1 & 1 & 1 & 1 & 1 & 1 & 1 & 1 & 1 \\ 1 & 1 & 1 & 1 & 1 & 1 & 1 & 1 & 1 & 1 & 1 & 1 & 1 & 1 \\ 1 & 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 \\ 1 & 1 & 0 & 0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 & 1 & 0 \\ 1 & 1 & 0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 & 1 & 0 & 0 \\ 1 & 1 & 0 & 0 & 0 & 0 & 0 & 0 & 1 & 0 & 0 & 0 & 1 & 1 \\ 1 & 1 & 0 & 1 & 0 & 0 & 1 & 0 & 0 & 0 & 0 & 1 & 0 & 0 \\ 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 & 1 & 0 & 0 & 0 \\ 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 & 1 & 0 & 0 & 1 \\ 1 & 1 & 0 & 0 & 0 & 1 & 0 & 1 & 0 & 0 & 0 & 1 & 1 & 0 \\ 1 & 1 & 0 & 1 & 1 & 0 & 1 & 0 & 0 & 0 & 0 & 1 & 1 & 0 \\ 1 & 1 & 1 & 0 & 1 & 0 & 1 & 0 & 1 & 0 & 0 & 1 & 0 & 1 \end{pmatrix} \end{matrix}$$

Observable gauge group: $SO(6) \times SO(4) \times U(1)_{1,2,3}$

$U(1)_\zeta = U(1)_1 + U(1)_2 + U(1)_3$ is anomaly free

| sector | field | $SU(4) \times SU(2)_L \times SU(2)_R$ | $U(1)_1$ | $U(1)_2$ | $U(1)_3$ | $U(1)_\zeta$ |
|---------------------------------|-----------------------------|----------------------------------------|----------|----------|----------|--------------|
| $S + b_1$ | \bar{F}_{1R} | $(\bar{4}, \mathbf{1}, \mathbf{2})$ | 1/2 | 0 | 0 | 1/2 |
| $S + b_1 + e_3 + e_5$ | F_{1R} | $(4, \mathbf{1}, \mathbf{2})$ | 1/2 | 0 | 0 | 1/2 |
| $S + b_2$ | F_{1L} | $(4, \mathbf{2}, \mathbf{1})$ | 0 | 1/2 | 0 | 1/2 |
| $S + b_2 + e_1 + e_2 + e_5$ | F_{2L} | $(4, \mathbf{2}, \mathbf{1})$ | 0 | 1/2 | 0 | 1/2 |
| $S + b_2 + e_1$ | \bar{F}_{2R} | $(\bar{4}, \mathbf{1}, \mathbf{2})$ | 0 | 1/2 | 0 | 1/2 |
| $S + b_2 + e_2 + e_5$ | \bar{F}_{3R} | $(\bar{4}, \mathbf{1}, \mathbf{2})$ | 0 | 1/2 | 0 | 1/2 |
| $S + b_3 + e_1 + e_2$ | F_{3L} | $(4, \mathbf{2}, \mathbf{1})$ | 0 | 0 | 1/2 | 1/2 |
| $S + b_3 + e_2$ | \bar{F}_{4R} | $(\bar{4}, \mathbf{1}, \mathbf{2})$ | 0 | 0 | 1/2 | 1/2 |
| $S + b_3 + x$ | h_1 | $(\mathbf{1}, \mathbf{2}, \mathbf{2})$ | -1/2 | -1/2 | 0 | -1 |
| $S + b_2 + x + e_5$ | h_2 | $(\mathbf{1}, \mathbf{2}, \mathbf{2})$ | -1/2 | 0 | -1/2 | -1 |
| $S + b_2 + x + e_1 + e_2$ | h_3 | $(\mathbf{1}, \mathbf{2}, \mathbf{2})$ | -1/2 | 0 | -1/2 | -1 |
| $S + b_3 + x + e_1$ | D_4 | $(\mathbf{6}, \mathbf{1}, \mathbf{1})$ | -1/2 | -1/2 | 0 | -1 |
| | χ_1^+ | $(\mathbf{1}, \mathbf{1}, \mathbf{1})$ | 1/2 | 1/2 | 1 | +2 |
| | χ_1^- | $(\mathbf{1}, \mathbf{1}, \mathbf{1})$ | 1/2 | 1/2 | -1 | 0 |
| | $\zeta_a, a = 2, 3$ | $(\mathbf{1}, \mathbf{1}, \mathbf{1})$ | 1/2 | -1/2 | 0 | 0 |
| | $\bar{\zeta}_a, a = 2, 3$ | $(\mathbf{1}, \mathbf{1}, \mathbf{1})$ | -1/2 | 1/2 | 0 | 0 |
| $S + b_2 + x + e_1 + e_5$ | D_5 | $(\mathbf{6}, \mathbf{1}, \mathbf{1})$ | -1/2 | 0 | -1/2 | -1 |
| | χ_2^+ | $(\mathbf{1}, \mathbf{1}, \mathbf{1})$ | 1/2 | 1 | 1/2 | +2 |
| | χ_2^- | $(\mathbf{1}, \mathbf{1}, \mathbf{1})$ | 1/2 | -1 | 1/2 | 0 |
| | $\zeta_a, a = 4, 5$ | $(\mathbf{1}, \mathbf{1}, \mathbf{1})$ | 1/2 | 0 | -1/2 | 0 |
| | $\bar{\zeta}_a, a = 4, 5$ | $(\mathbf{1}, \mathbf{1}, \mathbf{1})$ | -1/2 | 0 | 1/2 | 0 |
| $S + b_2 + x + e_2$ | D_6 | $(\mathbf{6}, \mathbf{1}, \mathbf{1})$ | -1/2 | 0 | -1/2 | -1 |
| | χ_3^+ | $(\mathbf{1}, \mathbf{1}, \mathbf{1})$ | 1/2 | 1 | 1/2 | +2 |
| | χ_3^- | $(\mathbf{1}, \mathbf{1}, \mathbf{1})$ | 1/2 | -1 | 1/2 | 0 |
| | $\zeta_a, a = 6, 7$ | $(\mathbf{1}, \mathbf{1}, \mathbf{1})$ | 1/2 | 0 | -1/2 | 0 |
| | $\bar{\zeta}_a, a = 6, 7$ | $(\mathbf{1}, \mathbf{1}, \mathbf{1})$ | -1/2 | 0 | 1/2 | 0 |
| $S + b_1 + x + e_3$ | \bar{D}_6 | $(\mathbf{6}, \mathbf{1}, \mathbf{1})$ | 0 | 1/2 | 1/2 | +1 |
| | $\bar{\chi}_4^+$ | $(\mathbf{1}, \mathbf{1}, \mathbf{1})$ | -1 | -1/2 | -1/2 | -2 |
| | $\bar{\chi}_4^-$ | $(\mathbf{1}, \mathbf{1}, \mathbf{1})$ | 1 | -1/2 | -1/2 | 0 |
| | $\zeta_a, a = 8, 9$ | $(\mathbf{1}, \mathbf{1}, \mathbf{1})$ | 0 | 1/2 | -1/2 | 0 |
| | $\bar{\zeta}_a, a = 8, 9$ | $(\mathbf{1}, \mathbf{1}, \mathbf{1})$ | 0 | -1/2 | 1/2 | 0 |
| $S + b_1 + x + e_5$ | D_7 | $(\mathbf{6}, \mathbf{1}, \mathbf{1})$ | 0 | -1/2 | -1/2 | -1 |
| | χ_5^+ | $(\mathbf{1}, \mathbf{1}, \mathbf{1})$ | 1 | 1/2 | 1/2 | +2 |
| | χ_5^- | $(\mathbf{1}, \mathbf{1}, \mathbf{1})$ | -1 | 1/2 | 1/2 | 0 |
| | $\zeta_a, a = 10, 11$ | $(\mathbf{1}, \mathbf{1}, \mathbf{1})$ | 0 | 1/2 | -1/2 | 0 |
| | $\bar{\zeta}_a, a = 10, 11$ | $(\mathbf{1}, \mathbf{1}, \mathbf{1})$ | 0 | -1/2 | 1/2 | 0 |
| $S + b_3 + x + e_2 + e_3$ | ζ_1 | $(\mathbf{1}, \mathbf{1}, \mathbf{1})$ | 1/2 | -1/2 | 0 | 0 |
| | $\bar{\zeta}_1$ | $(\mathbf{1}, \mathbf{1}, \mathbf{1})$ | -1/2 | 1/2 | 0 | 0 |
| $S + b_1 + x + e_3 + e_4 + e_6$ | ϕ_1 | $(\mathbf{1}, \mathbf{1}, \mathbf{1})$ | 0 | 1/2 | 1/2 | +1 |
| | $\bar{\phi}_1$ | $(\mathbf{1}, \mathbf{1}, \mathbf{1})$ | 0 | -1/2 | -1/2 | -1 |
| $S + b_1 + x + e_4 + e_5 + e_6$ | ϕ_2 | $(\mathbf{1}, \mathbf{1}, \mathbf{1})$ | 0 | 1/2 | 1/2 | +1 |
| | $\bar{\phi}_2$ | $(\mathbf{1}, \mathbf{1}, \mathbf{1})$ | 0 | -1/2 | -1/2 | -1 |

Table 1: Observable twisted matter spectrum and $SU(4) \times SU(2)_L \times SU(2)_R \times U(1)^3$ quantum numbers.

The chiral spectrum is self-dual under spinor-vector duality

maintains the E_6 embedding $\Rightarrow U(1)_\zeta$ is anomaly free

Exophobic, three generations + heavy and light higgs + $\lambda_t \sim 1$

$$U(1)_{Z'} = \frac{1}{5} U(1)_C - \frac{1}{5} U(1)_L - U_\zeta$$

$Z' \Rightarrow$ Additional matter at the Z' breaking scale

Exotic $SO(10)$ singlets with non-standard $U(1)_\zeta$ charges

\Rightarrow Natural Wilsonian dark-matter candidates

$[(1, 0); (1, 0)]_{(0, \pm 1, 0)}$ $SO(10)$ singlets E_6 exotics

Several cases to consider :

1. $M \gg M_{Z'}$ without inflation $\Rightarrow M \leq 10^5$ GeV

2. $M \gg M_{Z'}$ with inflation and $T_R > M_{Z'}$ $\Rightarrow M > T_R \left[25 + \frac{1}{2} \ln \left(\frac{M}{T_R} \right) \right]$

3. $M \ll M_{Z'}$ without inflation $\Rightarrow M < 3$ keV

4. $M \ll M_{Z'}$ with inflation

$$M \begin{cases} > T_R \left[25 + \frac{1}{2} \ln \left(\frac{M^5}{M_{Z'}^4 T_R} \right) \right], & T_R < M \\ < \frac{M_{Z'}^4}{T_R^3} 6.9 \times 10^{-25} \left(\frac{g_*}{200} \right)^{1.5} \frac{1}{N_{Z'} g_{\text{eff}}^2}, & T_R > M \end{cases}$$

The Equivalence Postulate of Quantum Mechanics

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Abstract

The removal of the peculiar degeneration arising in the classical concepts of *rest frame* and *time parameterization* is at the heart of the recently formulated Equivalence Principle (EP). The latter, stating that all physical systems can be connected by a coordinate transformation to the free one with vanishing energy, univocally leads to the Quantum Stationary HJ Equation (QSHJE). This is a third-order non-linear differential equation which provides a trajectory representation of Quantum Mechanics (QM). The trajectories depend on the Planck length through hidden variables which arise as initial conditions. The formulation has manifest p - q duality, a consequence of the involutive nature of the Legendre transformation and of its recently observed relation with second-order linear differential equations. This reflects in an intrinsic ψ^D - ψ duality between linearly independent solutions of the Schrödinger equation. Unlike Bohm's theory, there is a non-trivial action even for bound states and no pilot-wave guide is present. A basic property of the formulation is that no use of any axiomatic interpretation of the wave-function is made. For example, tunnelling is a direct consequence of the quantum potential which differs from the Bohmian one and plays the role of particle's self-energy. Furthermore, the QSHJE is defined only if the ratio ψ^D/ψ is a local homeomorphism of the extended real line into itself. This is an important feature as the $L^2(\mathbb{R})$ condition, which in the Copenhagen formulation is a consequence of the axiomatic interpretation of the wave-function, directly follows as a basic theorem which only uses the geometrical gluing conditions of ψ^D/ψ at $q = \pm\infty$ as implied by the EP. As a result, the EP itself implies a dynamical equation that does not require any further assumption and reproduces both tunnelling and energy quantization. Several features of the formulation show how the Copenhagen interpretation hides the underlying nature of QM. Finally, the non-stationary higher dimensional quantum HJ equation and the relativistic extension are derived.

Basic identity:

Identity

$$\left(\frac{\partial S_0}{\partial q}\right)^2 = \frac{\beta^2}{2} \left(\left\{ e^{\frac{i2S_0}{\beta}}; q \right\} - \{S_0; q\} \right)$$

$$\alpha^2 (\nabla S_0) \cdot (\nabla S_0) = \frac{\Delta(R e^{\alpha S_0})}{R e^{\alpha S_0}} - \frac{\Delta R}{R} - \alpha \left(2 \frac{\nabla R \cdot \nabla S_0}{R} + \Delta S_0 \right),$$

$$\alpha^2 (\partial S) \cdot (\partial S) = \frac{\partial^2(R e^{\alpha S})}{R e^{\alpha S}} - \frac{\partial^2 R}{R} - \alpha \left(2 \frac{\partial R \cdot \partial S}{R} + \partial^2 S \right),$$

$$\alpha^2 (\partial S - eA) \cdot (\partial S - eA) = \frac{D^2(R e^{\alpha S})}{R e^{\alpha S}} - \frac{\partial^2 R}{R} - \frac{\alpha}{R^2} \partial \cdot \left(R^2 (\partial S - eA) \right),$$

$$D^\mu = \partial^\mu - \alpha e A^\mu$$

Extend:

$$\alpha^2(\partial_\mu S)(\partial^\mu S) = \frac{\frac{1}{\sqrt{g}}\partial_\mu\sqrt{g}\partial^\mu(Re^{\alpha S})}{Re^{\alpha S}} - \frac{\frac{1}{\sqrt{g}}\partial_\mu(\sqrt{g}\partial^\mu R)}{R} - \frac{\alpha}{R^2}\frac{1}{\sqrt{g}}\partial_\mu(\sqrt{g}R^2\partial^\mu S),$$

$$\alpha^2 G_{\mu\nu\eta\rho} \frac{\delta S}{\delta g_{\mu\nu}} \frac{\delta S}{\delta g_{\eta\rho}} = \frac{1}{Re^{\alpha S}} G_{\mu\nu\eta\rho} \frac{\delta^2(Re^{\alpha S})}{\delta g_{\mu\nu} \delta g_{\eta\rho}} - G_{\mu\nu\eta\rho} \frac{1}{R} \frac{\delta^2(R)}{\delta g_{\mu\nu} \delta g_{\eta\rho}} - \frac{\alpha}{R^2} G_{\mu\nu\eta\rho} \frac{\delta}{\delta g_{\mu\nu}} \left(R^2 \frac{\delta S}{\delta g_{\eta\rho}} \right),$$

Conclusions

- DATA \longrightarrow UNIFICATION
- STRINGS \longrightarrow GAUGE & GRAVITY UNIFICATION
- Free fermionic models \longrightarrow A Fertile Crescent
- Free fermionic models \longleftrightarrow $Z_2 \times Z_2$ orbifolds
- Question: Does the $Z_2 \times Z_2$ orbifold structure play a role in nature?
- Dark Matter is in the Dark. A low scale Z' will help.
- String Phenomenology \longrightarrow Physics of the third millenium
e.g. Aristarchus to Copernicus

HAPPY

FESTDAY

KEITH!