Exophobic Quasi–Realistic Heterotic–String Vacua

Alon E. Faraggi

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- Recent results from free fermion heterotic–string model building

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STANDARD MODEL $\longrightarrow$ UNIFICATION

STRONG WEAK ELECTROMAGNETIC $\longrightarrow$ SO(10)

$$\bar{5} = \binom{5}{4} = \frac{5!}{4!1!}$$

$$+$$

$$10 = \binom{5}{2}$$

$$+$$

$$1 = \binom{5}{0}$$

$$\overline{16}$$

Additional evidence: Log running , τ_p , m_ν

Guides: 3 Generations & SO(10) embedding

Point, String, Membrane ....

11D - supergravity

SO(32) heterotic

Type IIB

$Z_2 \times Z_2$

$E_8 \times E_8$ heterotic

Type I

Type IIA
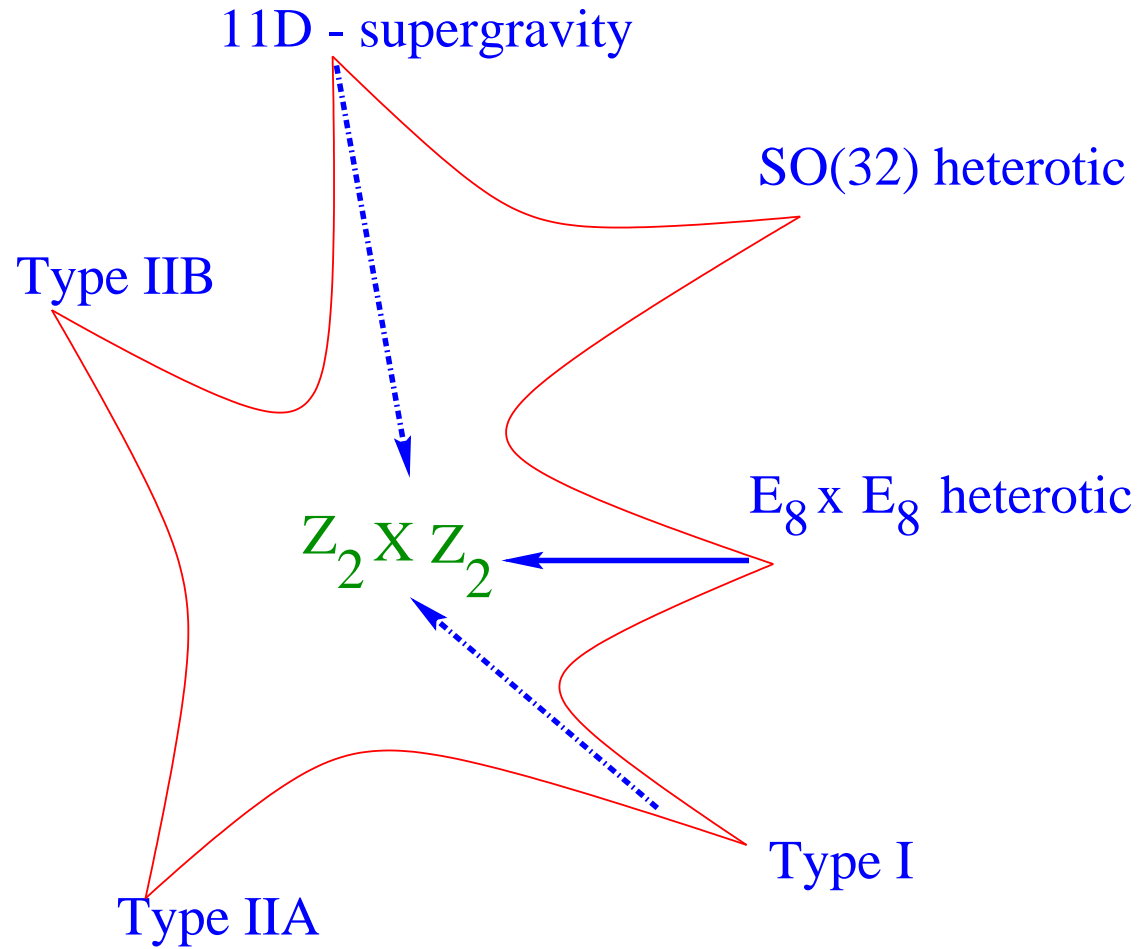
Realistic free fermionic models

'Phenomenology of the Standard Model and string unification'

- Top quark mass $\sim$ 175–180GeV   PLB 274 (1992) 47

- Generation mass hierarchy   NPB 407 (1993) 57

- CKM mixing   NPB 416 (1994) 63 (with Halyo)

- Stringy seesaw mechanism   PLB 307 (1993) 311 (with Halyo)

- Gauge coupling unification   NPB 457 (1993) 409 (with Dienes)

- Proton stability   NPB 428 (1994) 111

- Squark degeneracy   NPB 526 (1998) 21 (with Pati)

- Minimal Superstring Standard Model   PLB 455 (1999) 135

        (with Cleaver & Nanopoulos)

- Moduli fixing   NPB 728 (2005) 83

Free Fermionic Construction

Left-Movers: $\quad \psi^{\mu}_{1,2}, \quad \chi_i, \quad y_i, \ \omega_i \quad (i = 1, \cdots, 6)$

Right-Movers

$$\bar{\phi}_{A=1,\cdots,44} = \begin{cases} \bar{y}_i \ , \ \bar{\omega}_i \quad i = 1, \cdots, 6 \\[2em] \bar{\eta}_i \qquad\quad i = 1, 2, 3 \\[1em] \bar{\psi}_{1,\cdots,5} \\[1em] \bar{\phi}_{1,\cdots,8} \end{cases}$$
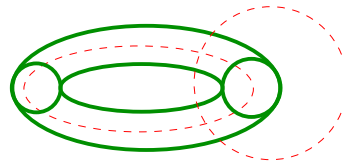
$V \longrightarrow V$ $f \longrightarrow -e^{i\pi\alpha(f)}f$

$$Z \ = \ \sum_{\substack{all \ spin \\ structures}} \ c\!\begin{pmatrix} \vec{\alpha} \\ \vec{\beta} \end{pmatrix} \ Z\!\begin{pmatrix} \vec{\alpha} \\ \vec{\beta} \end{pmatrix}$$

Models $\longleftrightarrow$ Basis vectors $\ + \ $ one–loop phases

Model building – Construction of the physical states

$$b_j \quad j = 1, \cdots, N \quad \rightarrow \quad \Xi = \sum_j n_j b_j$$

$$\text{For} \quad \vec{\alpha} = (\vec{\alpha}_L; \vec{\alpha}_R) \in \Xi \quad \Rightarrow \quad \mathsf{H}_{\vec{\alpha}}$$

$$\alpha(f) = 1 \quad \Rightarrow \quad |\pm\rangle \quad ; \quad \alpha(f) \neq 1 \quad \Rightarrow \quad f, f^* \ , \ \nu_{f,f^*} = \frac{1 \mp \alpha(f)}{2}$$

$$M_L^2 = -\tfrac{1}{2} + \frac{\vec{\alpha}_L \cdot \vec{\alpha}_L}{8} + N_L = -1 + \frac{\vec{\alpha}_R \cdot \vec{\alpha}_R}{8} + N_R = M_R^2 \ (\equiv \ 0)$$

$$\underline{\text{GSO projections}} \qquad e^{i\pi(\vec{b}_i \cdot \vec{F}_\alpha)} |s\rangle_{\vec{\alpha}} = \delta_\alpha c^* \begin{pmatrix} \vec{\alpha} \\ \vec{b}_i \end{pmatrix} |s\rangle_{\vec{\alpha}}$$

$$F_\alpha(f) \rightarrow \text{fermion} \ \# \ \text{operator} \ = \ \begin{cases} -1, & |-\rangle \\ 0, & |+\rangle \end{cases} = \begin{cases} +1, \\ -1, \end{cases}$$

$$Q(f) = \tfrac{1}{2}\alpha(f) + F(f) \quad \rightarrow \quad \mathsf{U(1)} \ \text{charges}$$

Calculation of Mass Terms

nonvanishing correlators

$$\langle V_1^f V_2^f V_3^b \cdot \cdots \cdot V_N^b \rangle$$

gauge & string invariant

"anomalous" $U(1)_A$

$$\text{Tr} Q_A \neq 0 \Rightarrow \quad D_A = 0 = A + \sum Q_k^A |\langle \phi_k \rangle|^2$$

$$D_j = 0 = \quad \sum Q_k^j |\langle \phi_k \rangle|^2$$

$$\langle W \rangle = \langle \frac{\partial W_N}{\partial \eta_i} \rangle = 0 \quad N = 3 \cdot \cdots$$

Supersymmetric vacuum $\langle F \rangle = \langle D \rangle = 0.$

nonrenormalizable terms $\rightarrow$ effective renormalizable operators

$$V_1^f V_2^f V_3^b \cdot \cdots \cdot V_N^b \;\rightarrow\; V_1^f V_2^f V_3^b \frac{\langle V_4^b \cdots V_N^b \rangle}{M^{N-3}}$$

<u>The NAHE set</u> : $\{ \mathbf{1}, S, \quad b_1, \quad b_2, \quad b_3 \}$

$$N = \quad 4 \quad \rightarrow \quad 2 \qquad 1 \qquad 1 \quad \text{vacua}$$

$Z_2 \times Z_2$ orbifold compactification

$$\implies \quad \text{Gauge group} \quad SO(10) \times SO(6)^{1,2,3} \times E_8$$

<u>beyond the NAHE set</u> Add $\{\alpha, \beta, \gamma\}$

number of generations is reduced to three

$$SO(10) \quad \longrightarrow \quad SU(3) \times SU(2) \times U(1)_{T_{3_R}} \times U(1)_{B-L}$$

$$U(1)_Y = \tfrac{1}{2}(B-L) + T_{3_R} \in SO(10) \, !$$

$$SO(6)^{1,2,3} \quad \longrightarrow \quad U(1)^{1,2,3} \times U(1)^{1,2,3}$$

The massless spectrum

Three twisted generations $\qquad$ $b_1, \quad b_2, \quad b_3$

	$h1_{1,0,0}$	$\bar{h}1_{-1,0,0}$
Untwisted Higgs doublets	$h2_{0,1,0}$	$\bar{h}2_{0,-1,0}$
	$h3_{0,0,1}$	$\bar{h}3_{0,0,-1}$

Sector $\quad b_1 + b_2 + \alpha + \beta$ $\qquad$ $h_{\alpha\beta_{-\frac{1}{2},-\frac{1}{2},0,0,0,0}}$ $\qquad$ $\bar{h}_{\alpha\beta_{\frac{1}{2},\frac{1}{2},0,0,0,0}}$

$$\oplus \ SO(10) \text{ singlets}$$

Sectors $\quad b_j + 2\gamma \ \ j = 1,2,3 \quad \longrightarrow \quad$ hidden matter multiplets

"standard" $SO(10)$ representations

NAHE $+ \ \{ \ \alpha \ , \ \beta \ , \ \gamma \ \} \ \rightarrow \quad$ exotic vector–like matter $\quad \rightarrow \quad$ superheavy

$$\oplus \ \text{Quasi-realistic phenomenology}$$

Fermion mass hierarchy

Fermion mass terms

$$cg f_i f_j h \left(\frac{\langle \phi \rangle}{M} \right)^{N-3}$$

c - calculable coefficients g - gauge coupling

$$f_i, \, f_j \;\; \in \;\; b_j \;\;\; j \;=\; 1, 2, 3$$

$$h \rightarrow \text{light Higgs multiplets}$$

$$M \;\sim\; 10^{18} \; GeV$$

$$\langle \phi \rangle \;\; \text{generalized VEVs, several sources}$$

Up/Down–type Yukawa Selection Mechanism

	ψ^μ	χ^{12}	χ^{34}	χ^{56}	y^3y^6	$y^4\bar{y}^4$	$y^5\bar{y}^5$	$\bar{y}^3\bar{y}^6$	$y^1\omega^5$	$y^2\bar{y}^2$	$\omega^6\bar{\omega}^6$	$\bar{y}^1\bar{\omega}^5$	$\omega^2\omega^4$	$\omega^1\bar{\omega}^1$	$\omega^3\bar{\omega}^3$	$\bar{\omega}^2\bar{\omega}^4$	$\bar{\psi}^{1,...,5}$					$\bar{\eta}^1$	$\bar{\eta}^2$	$\bar{\eta}^3$	$\bar{\phi}$		
α	0	0	0	0	1	0	0	0	0	0	1	1	0	0	1	1	1	1	1	0	0	0	0	0	1	1	1
β	0	0	0	0	0	0	1	1	1	0	0	0	0	1	0	1	1	1	1	0	0	0	0	0	1	1	1
γ	0	0	0	0	0	1	0	1	0	1	0	1	1	0	0	0	$\tfrac{1}{2}$ $\tfrac{1}{2}$ $\tfrac{1}{2}$ $\tfrac{1}{2}$ $\tfrac{1}{2}$					$\tfrac{1}{2}$	$\tfrac{1}{2}$	$\tfrac{1}{2}$	0	0	0

At the cubic level $\rightarrow$ γ selects trilevel Yukawa coupling:

$$\Delta_j = \left|\gamma(U(1)_{L_{j+3}}) - \gamma(U(1)_{R_{j+3}})\right| = 0, 1 \quad j = 1, 2, 3$$

$$\Delta_j = 1 \Rightarrow u_j Q_j \bar{h}_j \qquad\qquad \Delta_j = 0 \Rightarrow d_j Q_j h_j \ ; \ e_j L_j h_j$$

$b_1 \ : \ y_3 y_6 \quad y_4 \bar{y}_4 \quad y_5 \bar{y}_5 \quad \bar{y}_3 \bar{y}_6 \qquad\qquad y_3 y_6 \quad y_4 \bar{y}_4 \quad y_5 \bar{y}_5 \quad \bar{y}_3 \bar{y}_6$

$\gamma \ : \quad 1 \qquad 0 \qquad 0 \qquad 0 \qquad\qquad\quad 1 \qquad 0 \qquad 0 \qquad 1$

$$\Delta_1 = 1 \qquad\qquad\qquad\qquad\qquad \Delta_1 = 0$$

$$\underline{\lambda_j t_j^c Q_j \bar{h}_j}$$

Top quark mass prediction

$$\text{only} \quad \lambda_t = \langle t^c Q_t \bar{h}_1 \rangle = \sqrt{2}g \neq 0 \quad \text{at} \quad N = 3$$

$$W_4 \longrightarrow b^c Q_b h_{\alpha\beta} \Phi_1 + \tau^c L_\tau h_{\alpha\beta} \Phi_1$$

$$\Longrightarrow \quad \lambda_b = \left(c_b \frac{\langle \phi \rangle}{M} \right) \qquad \lambda_\tau = \left(c_\tau \frac{\langle \phi \rangle}{M} \right)$$

$$\longrightarrow \quad \lambda_b = \lambda_\tau = 0.35 g^3 \sim \frac{1}{8} \lambda_t$$

Evolve $\quad \lambda_t \,, \quad \lambda_b \quad$ to low energies

$$m_t = \lambda_t v_1 = \lambda_t \frac{v_0}{\sqrt{2}} \sin \beta \qquad\qquad m_b = \lambda_b v_2 = \lambda_b \frac{v_0}{\sqrt{2}} \cos \beta$$

where $\quad v_0 = \dfrac{2 m_W}{g_2(M_Z)} = 246 \text{GeV} \qquad$ and $\quad (v_1^2 + v_2^2) = \dfrac{v_0^2}{2}$

$$m_t = \lambda_t(m_t) \frac{v_0}{\sqrt{2}} \frac{\tan \beta}{(1 + \tan^2 \beta)^{\frac{1}{2}}} \Longrightarrow \quad m_t \sim 175 \text{GeV} \quad \text{PLB274(1992)47}$$

Hierarchical top–bottom mass relation in a superstring derived standard-like model

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I propose a mechanism in a class of superstring standard-like models which explains the mass hierarchy between the top and bottom quarks. At the trilinear level of the superpotential only the top quark gets a nonvanishing mass term while the bottom quarks and tau lepton mass terms are obtained from nonrenormalizable terms. I construct a model which realized this mechanism. In this model the bottom quark and tau lepton Yukawa couplings are obtained from quartic order terms. I show that $\lambda_b = \lambda_\tau \sim \frac{1}{2}\lambda_t$ at the unification scale. A naive estimate yields $m_t \sim 175$–180 GeV.

One of the unresolved puzzles of the standard model is the mass splitting between the top quark and the lighter quarks and leptons. Especially difficult to understand within the context of the standard model is the big splitting in the heaviest generation. Experimental limits [1] indicate the top mass to be above 80 GeV, while the bottom and tau lepton masses are found at 5 GeV and 1.78 GeV respectively. Possible extensions to the standard model are grand unified theories. Although the main prediction of GUTs, proton decay, has not yet been observed, calculations of $\sin^2\theta_w$ and of the mass ratio m_b/m_τ support their validity. Recent calculations seem to support supersymmetric GUTs versus nonsupersymmetric ones [2]. In spite of the success of SUSY GUTs in confronting LEP data [2], an understanding of the mass splitting between the top quark and the lighter quarks and leptons is still lacking. The next level in which such an understanding may be developed is in the context of superstring theory [3].

Find anomaly free solution

$$
M_d \sim \begin{pmatrix} \epsilon & \dfrac{V_2 \bar{V}_3 \Phi_{\alpha\beta}}{M^3} & 0 \\[2ex] \dfrac{V_2 \bar{V}_3 \Phi_{\alpha\beta} \xi_1}{M^4} & \dfrac{\bar{\Phi}_2^- \xi_1}{M^2} & 0 \\[2ex] 0 & 0 & \dfrac{\Phi_1^+ \xi_2}{M^2} \end{pmatrix} v_2,
$$

$$
\epsilon < 10^{-8} \qquad \frac{V_2 \bar{V}_3 \Phi_{\alpha\beta}}{M^3} = \frac{\sqrt{5}\, g^6}{64\, \pi^3} \approx 2 - 3 \times 10^{-4}.
$$

$$
\Rightarrow \quad |V| \sim \begin{pmatrix} 0.98 & 0.2 & 0 \\ 0.2 & 0.98 & 0 \\ 0 & 0 & 1 \end{pmatrix}
$$

Three generation mixing $\longrightarrow$ NPB 416 (1994) 63 $\qquad |J| \sim 10^{-6}$

Classification of fermionic $Z_2 \times Z_2$ orbifolds   (AEF, Kounnas, Rizos)

Basis vectors: consistent modular blocks 4,8 periodic fermions

$$1 = \{\psi^\mu, \; \chi^{1,\dots,6}, y^{1,\dots,6}, \omega^{1,\dots,6} \mid \bar{y}^{1,\dots,6}, \bar{\omega}^{1,\dots,6}, \bar{\eta}^{1,2,3}, \bar{\psi}^{1,\dots,5}, \bar{\phi}^{1,\dots,8}\}$$

$$S = \{\psi^\mu, \chi^{1,\dots,6}\},$$

$$z_1 = \{\bar{\phi}^{1,\dots,4}\},$$

$$z_2 = \{\bar{\phi}^{5,\dots,8}\},$$

$$e_i = \{y^i, \omega^i \mid \bar{y}^i, \bar{\omega}^i\}, \; i = 1, \dots, 6, \qquad\qquad N = 4 \text{ Vacua}$$

$$b_1 = \{\chi^{34}, \chi^{56}, y^{34}, y^{56} \mid \bar{y}^{34}, \bar{y}^{56}, \bar{\eta}^1, \bar{\psi}^{1,\dots,5}\}, \qquad N = 4 \to N = 2$$

$$b_2 = \{\chi^{12}, \chi^{56}, y^{12}, y^{56} \mid \bar{y}^{12}, \bar{y}^{56}, \bar{\eta}^2, \bar{\psi}^{1,\dots,5}\}, \qquad N = 2 \to N = 1$$

Vector bosons: NS, $z_{1,2}$, $z_1 + z_2$, $x = 1 + s + \sum e_i + z_1 + z_2$

impose: Gauge group $SO(10) \times U(1)^3 \times$ hidden

	1	S	e_1	e_2	e_3	e_4	e_5	e_6	z_1	z_2	b_1	b_2
1	−1	−1	±	±	±	±	±	±	±	±	±	±
S			−1	−1	−1	−1	−1	−1	−1	−1	1	1
e_1			±	±	±	±	±	±	±	±	±	±
e_2				±	±	±	±	±	±	±	±	±
e_3					±	±	±	±	±	±	±	±
e_4						±	±	±	±	±	±	±
e_5							±	±	±	±	±	±
e_6								±	±	±	±	±
z_1									±	±	±	±
z_2										±	±	±
b_1											±	±
b_2												±

Apriori 55 independent coefficients $\rightarrow$ 2^{55} distinct vacua

Impose: Gauge group $SO(10) \times U(1)^3 \times SO(8)^2$

$\rightarrow$ 40 independent coefficients

The twisted matter spectrum:

$$B^1_{\ell^1_3 \ell^1_4 \ell^1_5 \ell^1_6} = S + b_1 + \ell^1_3 e_3 + \ell^1_4 e_4 + \ell^1_5 e_5 + \ell^1_6 e_6$$

$$B^2_{\ell^2_1 \ell^2_2 \ell^2_5 \ell^2_6} = S + b_2 + \ell^2_1 e_1 + \ell^2_2 e_2 + \ell^2_5 e_5 + \ell^2_6 e_6$$

$$B^3_{\ell^3_1 \ell^3_2 \ell^3_3 \ell^3_4} = S + b_3 + \ell^3_1 e_1 + \ell^3_2 e_2 + \ell^3_3 e_3 + \ell^3_4 e_4 \qquad l^j_i = 0, 1$$

sectors $B^i_{pqrs} \quad \rightarrow \quad$ 16 or $\overline{16}$ of $SO(10)$ with multiplicity $(1, 0, -1)$

$\qquad B^i_{pqrs} + x \quad \rightarrow \quad$ 10 $\quad$ of $SO(10)$ with multiplicity $(1, 0)$

Counting: for each B^i_{pqrs}:

Projectors:

$$P^{(1)}_{p^1 q^1 r^1 s^1} = \frac{1}{16} \prod \left(1 - c\left({}^{e_i}_{B^{(1)}_{p^1 q^1 r^1 s^1}} \right) \right) \prod \left(1 - c\left({}^{z_i}_{B^{(1)}_{p^1 q^1 r^1 s^1}} \right) \right)$$

$$S^{(i)}_{\pm} = \sum_{pqrs} \frac{1 \pm X^{(i)}_{p^i q^i r^i s^i}}{2} P^{(i)}_{p^i q^i r^i s^i} \, , \; i = 1, 2, 3$$

similarly for vectorials

Algebraic formulas for $\quad S = \sum^3_{i=1} S^{(i)}_+ - S^{(i)}_- \quad$ and $\quad V = \sum^3_{i=1} V^{(i)}$

Introducing the notation $\qquad c\binom{a_i}{a_j} = e^{i\pi(a_i|a_j)}$, $(a_i | a_j) = 0, 1$

the projectors can be written as system of equations

$$\Delta^{(I)} U_{16}^{(I)} = Y_{16}^{(I)} \quad , \quad \Delta^{(I)} U_{10}^{(I)} = Y_{10}^{(I)} \,, \; I = 1, 2, 3$$

where the unknowns are the fixed point labels

$$U_{16}^{(I)} = \begin{bmatrix} p_{16}^I \\ q_{16}^I \\ r_{16}^I \\ s_{16}^I \end{bmatrix} \quad , \qquad U_{10}^{(I)} = \begin{bmatrix} p_{10}^I \\ q_{10}^I \\ r_{10}^I \\ s_{10}^I \end{bmatrix}$$

and

$$\Delta^{(1)} = \begin{bmatrix} (e_1 | e_3) & (e_1 | e_4) & (e_1 | e_5) & (e_1 | e_6) \\ (e_2 | e_3) & (e_2 | e_4) & (e_2 | e_5) & (e_2 | e_6) \\ (z_1 | e_3) & (z_1 | e_4) & (z_1 | e_5) & (z_1 | e_6) \\ (z_2 | e_3) & (z_2 | e_4) & (z_2 | e_5) & (z_2 | e_6) \end{bmatrix} \quad ; \quad \Delta^{(2)} ...$$

$$Y_{16}^{(1)} = \begin{bmatrix} (e_1 \,|\, b_1) \\ (e_2 \,|\, b_1) \\ (z_1 \,|\, b_1) \\ (z_2 \,|\, b_2) \end{bmatrix} \quad , \quad Y_{16}^{(2)} = \ldots$$

$$Y_{10}^{(1)} = \begin{bmatrix} (e_1 \,|\, b_1 + x) \\ (e_2 \,|\, b_1 + x) \\ (z_1 \,|\, b_1 + x) \\ (z_2 \,|\, b_2 + x) \end{bmatrix} \quad , \quad Y_{10}^{(2)} = \ldots$$

The number of solutions per plane

$$
S^{(I)} = \begin{cases} 2^{4-\mathrm{rank}\left(\Delta^{(I)}\right)} & \mathrm{rank}\left(\Delta^{(I)}\right) = \mathrm{rank}\left[\Delta^{(I)}, Y_{16}^{(I)}\right] \\ 0 & \mathrm{rank}\left(\Delta^{(I)}\right) < \mathrm{rank}\left[\Delta^{(I)}, Y_{16}^{(I)}\right] \end{cases}
$$

$$
V^{(I)} = \begin{cases} 2^{4-\mathrm{rank}\left(\Delta^{(I)}\right)} & \mathrm{rank}\left(\Delta^{(I)}\right) = \mathrm{rank}\left[\Delta^{(I)}, Y_{10}^{(I)}\right] \\ 0 & \mathrm{rank}\left(\Delta^{(I)}\right) < \mathrm{rank}\left[\Delta^{(I)}, Y_{10}^{(I)}\right] \end{cases}
$$

rank $\left(\Delta^{(I)}\right)$	rank $\left[\Delta^{(I)}, Y_{16}^{(I)}\right]$	rank $\left[\Delta^{(I)}, Y_{10}^{(I)}\right]$	# of Spinorials	# of vectorials
4	4	4	1	1
3	4	4	0	0
	3	4	2	0
	4	3	0	2
	3	3	2	2
2	3	3	0	0
	2	3	4	0
	3	2	0	4
	3	3	4	4
1	2	2	0	0
	1	2	8	0
	2	1	0	8
	1	1	8	8
0	1	1	0	0
	0	1	16	0
	1	0	0	16
	0	0	16	16

Table 1: *Total number of $SO(10)$ spinorial and vectorial representations in a given orbifold plane $I = 1, 2, 3$ for all possible ranks of the projection matrices $\left(\Delta^{(I)}\right)$, $\left[\Delta^{(I)}, Y_{16}^{(I)}\right]$, and $\left[\Delta^{(I)}, Y_{10}^{(I)}\right]$.*
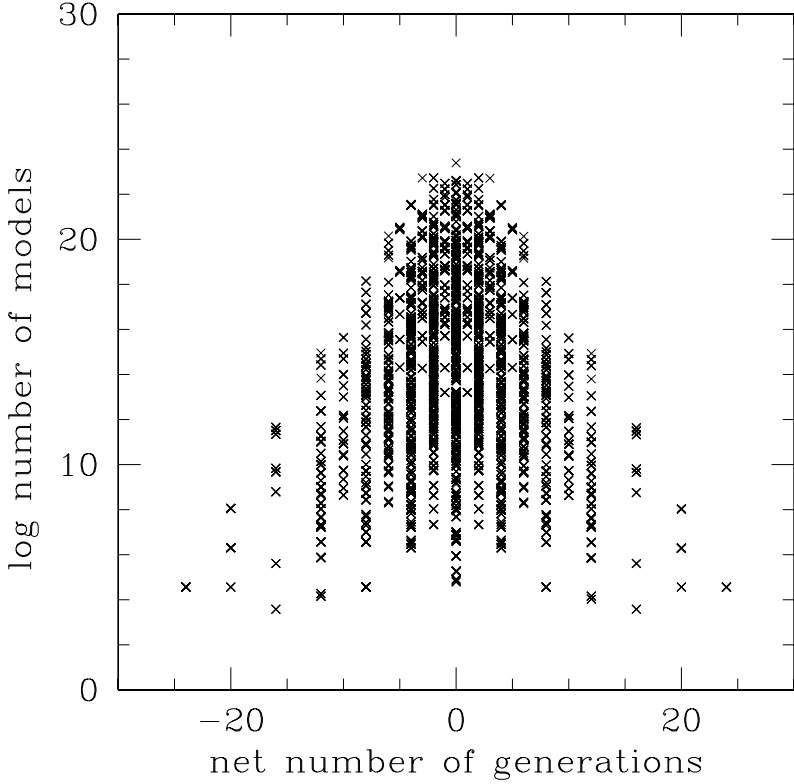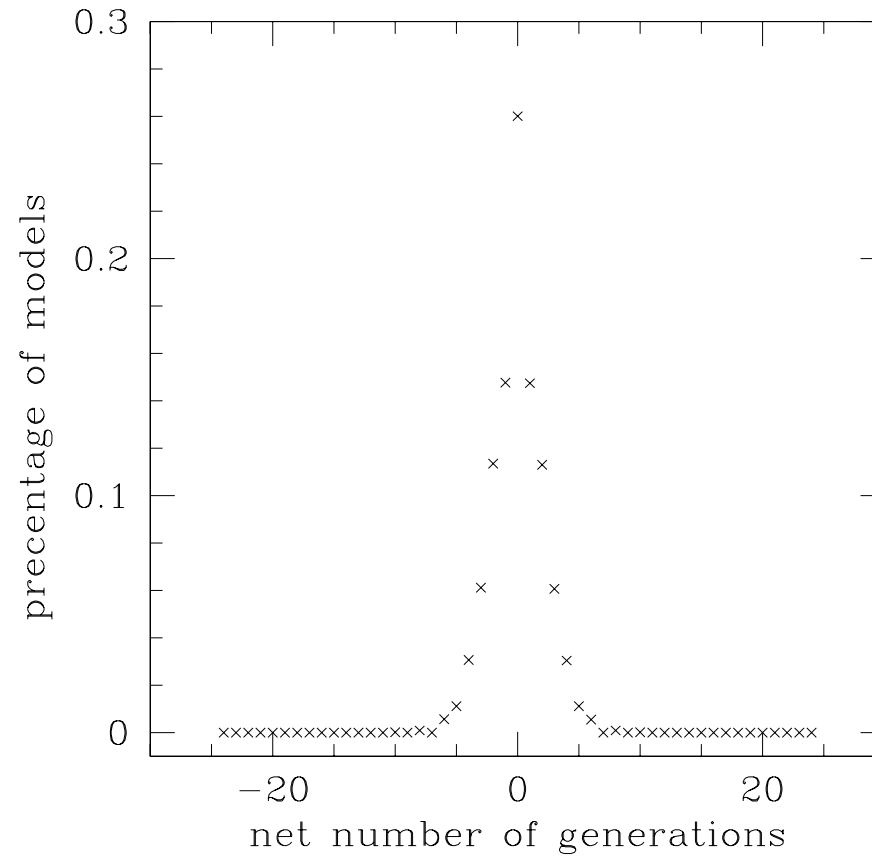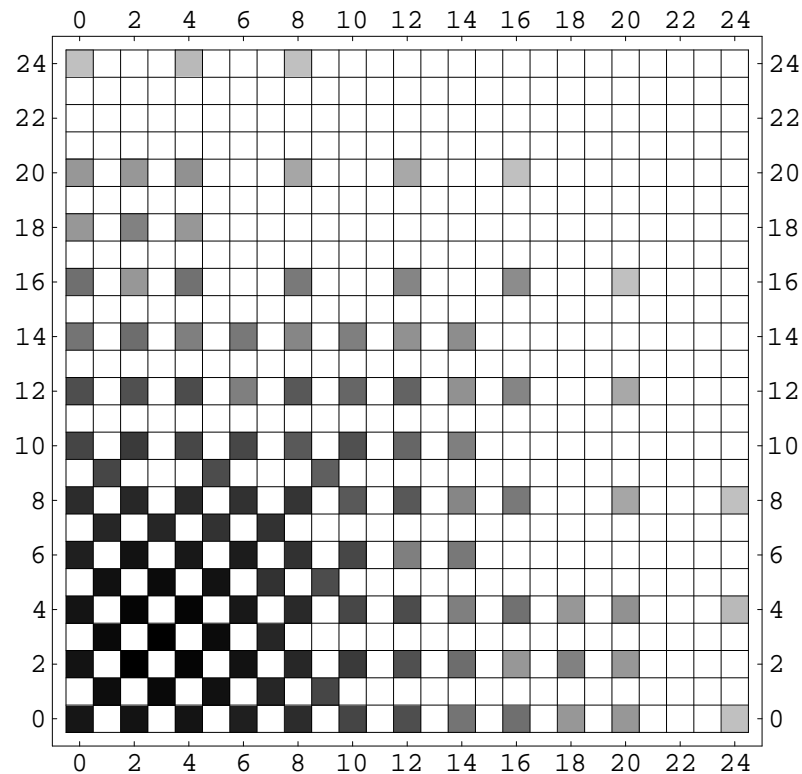
7×10^9 models $\sim$ 15% with 3 gen   FKRI

Spinor–vector duality:

Invariance under exchange of $\#(16 + \overline{16}) \; <-> \;\; \#(10)$

Symmetric under exchange of rows and columns

$$E_6 : \qquad 27 = 16 + 10 + 1 \qquad \overline{27} = \overline{16} + 10 + 1$$

Self-dual: $\#(16 + \overline{16}) \;=\;\; \#(10)$ without E_6 symmetry

$\Rightarrow$ Algebraic proof of spinor-vector duality

The number of vectorials and spinorials from a specific

orbifold plane I are interchanged when the ranks of the

associated Y-vectors are interchanged

$$\mathrm{rank}\left[\Delta^{(I)}, Y_{16}^{(I)}\right] \quad \longleftrightarrow \quad \mathrm{rank}\left[\Delta^{(I)}, Y_{10}^{(I)}\right]$$

Induced by GSO phase change

<u>Exotics :</u>   (in FNY SLM–model, NPB 335 (1990) 347)

$$W_2 = \frac{1}{\sqrt{2}}\{ \ H_1 H_2 \phi_4 + H_3 H_4 \bar{\phi}_4 + H_5 H_6 \bar{\phi}_4 + (H_7 H_8 + H_9 H_{10})\phi'_4 +$$

$$(H_{11} + H_{12})(H_{13} + H_{14})\bar{\phi}'_4 + V_{41} V_{42} \bar{\phi}_4 + V_{43} V_{44} \bar{\phi}_4 +$$

$$V_{45} V_{46} \phi_4 + (V_{47} V_{48} + V_{49} V_{50})\bar{\phi}'_4 + V_{51} V_{52} \phi'_4\}$$

$\langle \ \bar{\phi}_4, \bar{\phi}'_4, \phi_4, \phi'_4 \ \rangle \rightarrow$ massive exotic states at N=3 (PRD46 (1993) 3204)

CFN $\rightarrow$ Classification of flat directions   (PLB 455 (1999) 135)

Example: $\{\phi_{12}, \phi_{23}, \bar{\phi}_{56}, \phi_4, \phi'_4, \bar{\phi}_4, \bar{\phi}'_4, H_{15}, H_{30}, H_{31}, H_{38}\}$

All Standard Model charged states beyond MSSM $\rightarrow$ $\approx M_{\text{string}}$

MINIMAL STANDARD HETEROTIC STRING MODEL

Beyond $SO(10)$ Classification: Pati–Salam Heterotic–String :

$$SO(10) \to SU(4) \times SU(2)_L \times SU(2)_R \to SU(3) \times SU(2) \times U(1)$$

single additional vector $\qquad v_{13} = \alpha = \{\bar{\psi}^{45}\bar{\phi}^{1,2}\}$

12 new GSO phases $c[\alpha, v_j], j = 1, \ldots, 12.$

$$\mathbf{16} = F_L(\mathbf{4}, \mathbf{2}, \mathbf{1}) + F_R(\bar{\mathbf{4}}, \mathbf{1}, \mathbf{2}) \to (Q_L + L_L) + (Q_R + L_R)$$

$$\mathbf{10} = D(\mathbf{6}, \mathbf{1}, \mathbf{1}) + h(\mathbf{1}, \mathbf{2}, \mathbf{2}) \to (D_3 + \bar{D}_3) + (h^d + h^u)$$

PS Symmetry broken by $\langle \nu_H^c \rangle, \quad \langle \nu_H \rangle$

$$\bar{H}(\bar{\mathbf{4}}, \mathbf{1}, \mathbf{2}) \to u_H^c(\bar{\mathbf{3}}, \mathbf{1}, +\tfrac{2}{3}) + d_H^c(\bar{\mathbf{3}}, \mathbf{1}, -\tfrac{1}{3}) + \langle \nu_H^c(\mathbf{1}, \mathbf{1}, 0) \rangle + e_H^c(\mathbf{1}, \mathbf{1}, -1)$$

$$H(\mathbf{4}, \mathbf{1}, \mathbf{2}) \to u_H(\mathbf{3}, \mathbf{1}, -\tfrac{2}{3}) + d_H(\mathbf{3}, \mathbf{1}, +\tfrac{1}{3}) + \langle \nu_H(\mathbf{1}, \mathbf{1}, 0) \rangle + e_H(\mathbf{1}, \mathbf{1}, +1)$$

Models to date contain additional fractionally charged matter

ALGEBRAIC ANALYSIS OF ALL SECTORS

Search strategy

A model is characterized by 9 integers $\left(n_g, k_L, k_R, n_6, n_h, n_4, n_{\bar{4}}, n_{2L}, n_{2R}\right)$

$n_{4L} - n_{\bar{4}R} = n_{\bar{4}L} - n_{4R} = n_g = \#$ of generations

$n_{\bar{4}L} = k_L = \#$ of non chiral left pairs

$n_{4R} = k_R = \#$ of non chiral right pairs

$n_6 = \#$ of $(\mathbf{6}, \mathbf{1}, \mathbf{1})$

$n_h = \#$ of $(\mathbf{1}, \mathbf{2}, \mathbf{2})$

$n_4 = \#$ of $(\mathbf{4}, \mathbf{1}, \mathbf{1})$ (exotic)

$n_{\bar{4}} = \#$ of $(\bar{\mathbf{4}}, \mathbf{1}, \mathbf{1})$ (exotic)

$n_{2L} = \#$ of $(\mathbf{1}, \mathbf{2}, \mathbf{1})$ (exotic)

$n_{2R} = \#$ of $(\mathbf{1}, \mathbf{1}, \mathbf{2})$ (exotic)

51 independent GSO phases, $\implies$ $2^{51} \sim 2 \times 10^{15}$ models.

RESULTS: of random search of over 10^{11} of 5×10^{15} vacua
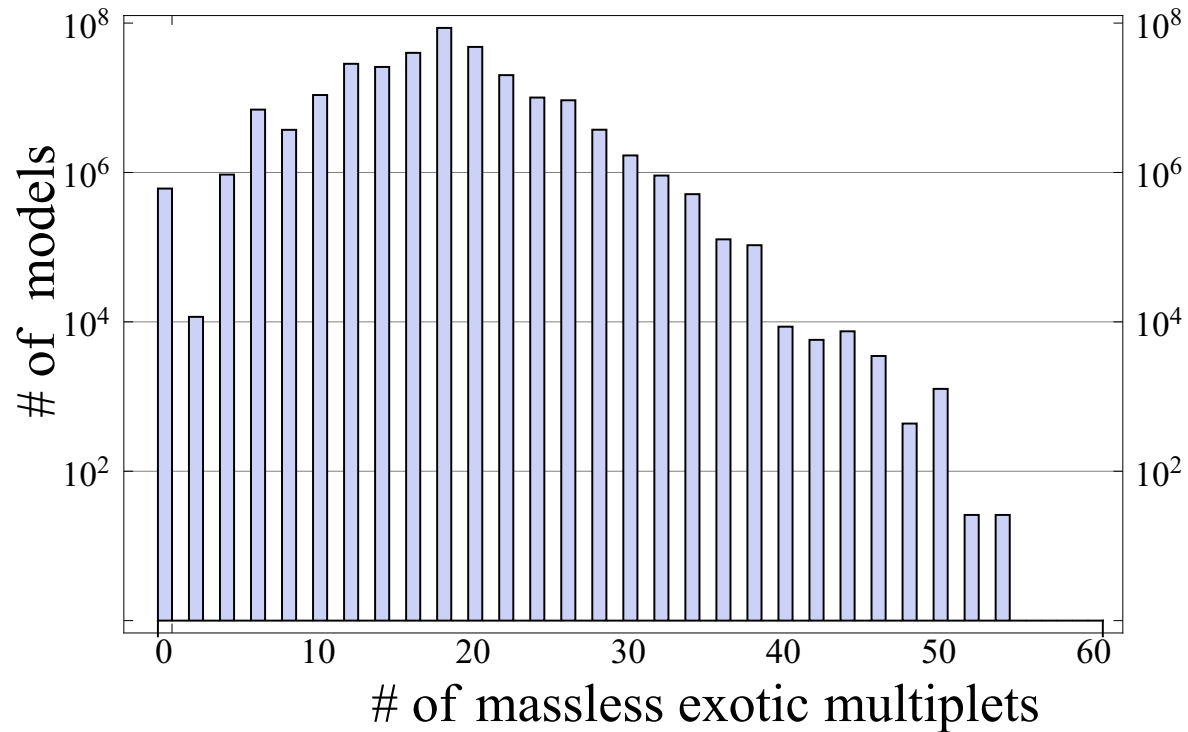
Number of 3-generation models versus total number of exotic multiplets

Pati–Salam models statistics with respect to phenomenological constraints

constraint	# of models	probability	# of models
None	100000000000	1	2.25×10^{15}
+ No gauge group enhancements.	78977078333	7.90×10^{-1}	1.78×10^{15}
+ Complete families	22497003372	2.25×10^{-1}	5.07×10^{14}
+ 3 generations	298140621	2.98×10^{-3}	6.71×10^{12}
+ PS breaking Higgs	23694017	2.37×10^{-4}	5.34×10^{11}
+ SM breaking Higgs	19191088	1.92×10^{-4}	4.32×10^{11}
+ No massless exotics	121669	1.22×10^{-6}	2.74×10^{9}

Constraints in second column act additionally.

Correspondence with $Z_2 \times Z_2$ orbifold

NAHE $\oplus$ $(\xi_2 = \{\bar{\psi}_{1,\ldots,5}, \bar{\eta}^1, \bar{\eta}^2, \bar{\eta}^3\} = 1)$ $\rightarrow$ $\{1, S, \xi_1, \xi_2, b_1, b_2\}$

Gauge group: $SO(4)^3 \times E_6 \times U(1)^2 \times E_8$ and 24 generations.

toroidal compactification

$$g_{ij} = \begin{pmatrix} 2 & -1 & 0 & 0 & 0 & 0 \\ -1 & 2 & -1 & 0 & 0 & 0 \\ 0 & -1 & 2 & -1 & 0 & 0 \\ 0 & 0 & -1 & 2 & -1 & -1 \\ 0 & 0 & 0 & -1 & 2 & 0 \\ 0 & 0 & 0 & -1 & 0 & 2 \end{pmatrix}$$

$$b_{ij} = \begin{cases} g_{ij} & i < \\ 0 & i = \\ -g_{ij} & i > \end{cases}$$

R_i $\rightarrow$ the free fermionic point $\rightarrow$ G.G. $SO(12) \times E_8 \times E_8$

mod out by a $Z_2 \times Z_2$ with standard embedding

$\Rightarrow$ Exact correspondence

In the realistic free fermionic models

replace $\xi_2 \equiv x = \{\bar{\psi}^{1,\cdots,5}, \bar{\eta}^1, \bar{\eta}^2, \bar{\eta}^3\} = 1$

with $\qquad 2\gamma = \{\bar{\psi}^{1,\cdots,5}, \bar{\eta}^1, \bar{\eta}^2, \bar{\eta}^3, \bar{\phi}^{1,\cdots,4}\} = 1$

Then $\{\vec{1}, \vec{S}, \vec{\xi}_1 = \vec{1} + \vec{b}_1 + \vec{b}_2 + \vec{b}_3, 2\gamma\} \rightarrow$ N=4 SUSY and

$$SO(12) \times SO(16) \times SO(16)$$

apply $b_1 \times b_2 \quad \rightarrow \quad Z_2 \times Z_2 \quad \rightarrow$ N=1 SUSY and

$$SO(4)^3 \times SO(10) \times U(1)^3 \times SO(16)$$

$\quad b_1, \qquad b_2, \qquad b_3 \qquad\qquad\qquad \Rightarrow (3 \times 8) \cdot 16$ of $SO(10)_O$

$b_1 + 2\gamma, \, b_2 + 2\gamma, \, b_3 + 2\gamma \qquad\qquad \Rightarrow (3 \times 8) \cdot 16$ of $SO(16)_H$

$$\text{Alternatively,} \quad c\binom{\xi_1}{\xi_2} = +1 \quad \rightarrow \quad -1$$

Using the level-one $\mathrm{SO}(2n)$ characters

$$O_{2n} = \tfrac{1}{2} \left(\frac{\theta_3^n}{\eta^n} + \frac{\theta_4^n}{\eta^n} \right) , \qquad V_{2n} = \tfrac{1}{2} \left(\frac{\theta_3^n}{\eta^n} - \frac{\theta_4^n}{\eta^n} \right) ,$$

$$S_{2n} = \tfrac{1}{2} \left(\frac{\theta_2^n}{\eta^n} + i^{-n} \frac{\theta_1^n}{\eta^n} \right) , \qquad C_{2n} = \tfrac{1}{2} \left(\frac{\theta_2^n}{\eta^n} - i^{-n} \frac{\theta_1^n}{\eta^n} \right) .$$

where

$$\theta_3 \equiv Z_f \begin{pmatrix} 0 \\ 0 \end{pmatrix} \quad \theta_4 \equiv Z_f \begin{pmatrix} 0 \\ 1 \end{pmatrix} \quad \theta_2 \equiv Z_f \begin{pmatrix} 1 \\ 0 \end{pmatrix} \quad \theta_1 \equiv Z_f \begin{pmatrix} 1 \\ 1 \end{pmatrix}$$

The partition function of the heterotic string on $SO(12)$ lattice:

$$Z_+ = (V_8 - S_8) \left[|O_{12}|^2 + |V_{12}|^2 + |S_{12}|^2 + |C_{12}|^2 \right] (\bar{O}_{16} + \bar{S}_{16}) (\bar{O}_{16} + \bar{S}_{16}) ,$$

and

$$
\begin{aligned}
Z_- = (V_8 - S_8) \Big[& \left(|O_{12}|^2 + |V_{12}|^2 \right) (\bar{O}_{16}\bar{O}_{16} + \bar{C}_{16}\bar{C}_{16}) \\
+ & \left(|S_{12}|^2 + |C_{12}|^2 \right) (\bar{S}_{16}\bar{S}_{16} + \bar{V}_{16}\bar{V}_{16}) \\
+ & \left(O_{12}\bar{V}_{12} + V_{12}\bar{O}_{12} \right) (\bar{S}_{16}\bar{V}_{16} + \bar{V}_{16}\bar{S}_{16}) \\
+ & \left(S_{12}\bar{C}_{12} + C_{12}\bar{S}_{12} \right) (\bar{O}_{16}\bar{C}_{16} + \bar{C}_{16}\bar{O}_{16}) \Big] .
\end{aligned}
$$

where $\pm$ refers to

$$c \begin{pmatrix} \xi_1 \\ \xi_2 \end{pmatrix} = \pm 1$$

connected by :
$$Z_- = Z_+ / a \otimes b ,$$

$$a = (-1)^{F_{\mathrm{L}}^{\mathrm{int}} + F_\xi^1} , \qquad\qquad b = (-1)^{F_{\mathrm{L}}^{\mathrm{int}} + F_\xi^2} .$$

$$Z_+ = (V_8 - S_8) \left(\sum_{m,n} \Lambda_{m,n} \right)^{\otimes 6} \left(\bar{O}_{16} + \bar{S}_{16} \right) \left(\bar{O}_{16} + \bar{S}_{16} \right) ,$$

where as usual, for each circle,

$$p_{\mathrm{L,R}}^i = \frac{m_i}{R_i} \pm \frac{n_i R_i}{\alpha'} ,$$

and

$$\Lambda_{m,n} = \frac{q^{\frac{\alpha'}{4} p_{\mathrm{L}}^2} \, \bar{q}^{\frac{\alpha'}{4} p_{\mathrm{R}}^2}}{|\eta|^2} .$$

apply $\quad Z_2 \times Z_2' : g \times g' = (-1)^{F_{\xi 1}}\delta_1 \times (-1)^{F_{\xi 2}}\delta_1$

with $\quad \delta_1 X^9 = X_9 + \pi R_9$,

$$Z_-^{9d} = (V_8 - S_8) \, [\quad \Lambda_{2m,n} \quad \left(\overline{O}_{16}\overline{O}_{16} + \overline{C}_{16}\overline{C}_{16}\right)$$
$$+ \, \Lambda_{2m+1,n} \quad \left(\overline{S}_{16}\overline{S}_{16} + \overline{V}_{16}\overline{V}_{16}\right)$$
$$+ \, \Lambda_{2m,n+\frac{1}{2}} \quad \left(\overline{S}_{16}\overline{V}_{16} + \overline{V}_{16}\overline{S}_{16}\right)$$
$$+ \, \Lambda_{2m+1,n+\frac{1}{2}} \left(\overline{O}_{16}\overline{C}_{16} + \overline{C}_{16}\overline{O}_{16}\right)] \quad .$$

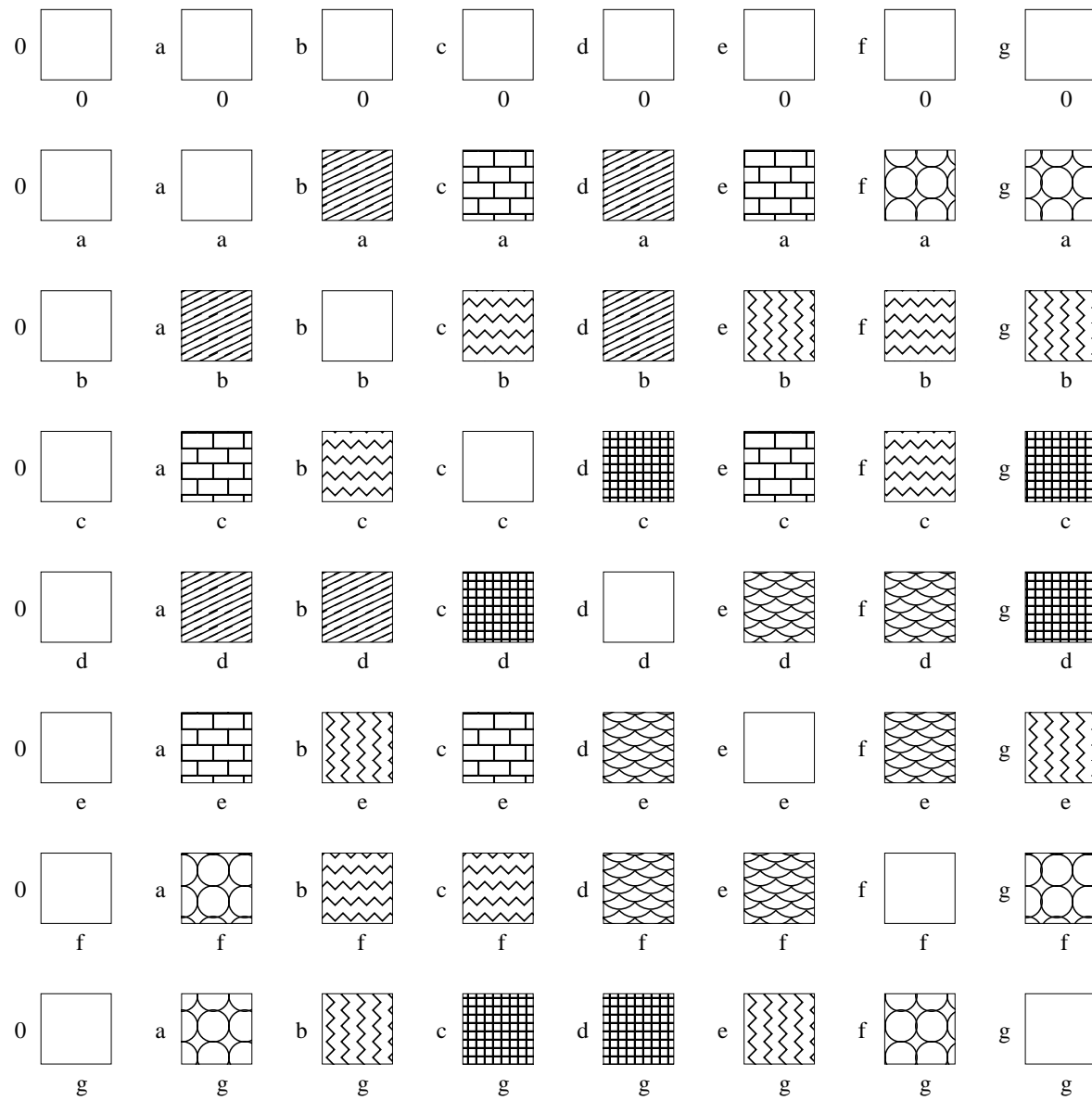Add Z_2'' : $(x_4, x_5, x_6, x_7, x_8, x_9) \longrightarrow (+x_4, +x_5, -x_6, -x_7, -x_8, -x_9)$

Naively

$$\left(\frac{Z_+}{(Z_2 \times Z_2')} \right) / Z_2''$$

Produces massless vectorials. No massless spinorials. (Elisa Manno, 0908.3164

$$\implies \quad \text{Analyze} \quad \left(\frac{Z_+}{(Z_g \times Z_{g'} \times Z_{g''})} \right)$$

eight independent orbits $\rightarrow$ ϵ_i $i = 1, \cdots 7$ discrete torsions

- <u>sector c</u>

$$Q_c \left(\frac{1}{2} \left(\left| \frac{2\eta}{\theta_4} \right|^4 + \left| \frac{2\eta}{\theta_3} \right|^4 \right) \Lambda_{p,q} \left(\frac{\epsilon_{06}^- + \epsilon_{24}^+ (-1)^m}{2} \right) \Lambda_{m,n} \right) \overline{V}_{12} \overline{S}_4 \overline{O}_{16}$$

$$Q_c \left(\frac{1}{2} \left(\left| \frac{2\eta}{\theta_4} \right|^4 + \left| \frac{2\eta}{\theta_3} \right|^4 \right) \Lambda_{p,q} \left(\frac{\epsilon_{06}^+ - \epsilon_{24}^- (-1)^m}{2} \right) \Lambda_{m,n} \right) \overline{S}_{12} \overline{O}_4 \overline{O}_{16}$$

$\epsilon_1 = +1, \ \epsilon_2 = -1, \ \epsilon_3 = +1, \ \epsilon_4 = +1, \ \epsilon_5 = -1, \ \epsilon_6 = +1, \ \epsilon_7 = -1,$
No gauge enhancement. Spinors massless. Vectors Massive.

$\epsilon_1 = +1, \ \epsilon_2 = +1, \ \epsilon_3 = +1, \ \epsilon_4 = +1, \ \epsilon_5 = +1, \ \epsilon_6 = -1, \ \epsilon_7 = +1,$
No gauge enhancement. Spinors massive. Vectors Massless.

Spinor–Vector duality map $\quad \{\epsilon_2, \epsilon_5, \epsilon_6, \epsilon_7\} \rightarrow - \{\epsilon_2, \epsilon_5, \epsilon_6, \epsilon_7\}$

Conclusions

Phenomenological string models produce interesting lessons

relevance of non–standard geometries

Spinor–vector duality

Exophobia

Free Fermionic Models $\longrightarrow$ $Z_2 \times Z_2$ orbifold near the self–dual point

Duality & Self–Duality $\Leftrightarrow$ String Vacuum Selection

Proton stability and superstring Z' (with Coriano and Guzzi)

Standard Model: $(SU321) \oplus (Q\ L\ U\ D\ E\ N) \oplus (h)$:

Effective renormalizable QFT below a cutoff

baryon and lepton numbers protected by accidental global symmetries

Non-renormalizable operators suppressed by cutoff M

B & L numbers violating operators

Dimension six: $QQQL\dfrac{1}{M^2}$ $\Rightarrow$ $M \sim 10^{16} GeV$

supersymmetry:

Dimension four: $\eta_1 QLD$ & $\eta_2 UDD$ $\Rightarrow$ $(\eta_1 \cdot \eta_2 \leq 10^{-24})$

Dimension five: $\lambda QQQL\dfrac{1}{M}$ $\Rightarrow$ $(\frac{\lambda}{M}) \leq 10^{-26}$

<u>Appealing Proposition:</u>   Low scale gauged $U(1)$ symmetry

<u>Additional Facts</u>

Standard Model: $\longrightarrow$ Unification $\longleftrightarrow$ $SO(10)$

Dimension four: $16^4 \;\Rightarrow\; \eta_1' UDD\frac{\langle N\rangle}{M} \;+\; \eta_2' QLD\frac{\langle N\rangle}{M}$

$+$ ... dimension five ; dimension six

left–handed neutrino masses: $M_{\nu_L} \approx M_{\text{Up}} \left(\frac{M_{\text{weak}}}{M_{\langle N\rangle}}\right)^2$

Fermion masses: $\lambda_{ij}^{\text{Up}} Q^i U^j \bar{h}$; $\lambda_{ij}^{\text{Down}} Q^i D^j h$; $\lambda_{ij}^{\text{CLepton}} L^i E^j h$;

Flavour universality

Freedom from anomalies

What can we learn from string constructions?

Extra $U(1)$'s beyond the Standard Model

e.g. PLB 278 (1992) 131 $\rightarrow$ seven extra $U(1)$'s

$$U(1)_{Z'} = \frac{B-L}{2} - \frac{2}{3}T3_R$$

The $U(1)$ combinations

$$U_A = \frac{1}{\sqrt{15}}(2(U_1 + U_2 + U_3) - (U_4 + U_5 + U_6))$$

$$U_\chi = \frac{1}{\sqrt{15}}(U_1 + U_2 + U_3 + 2U_4 + 2U_5 + 2U_6)$$

$$U_{12} = \frac{1}{\sqrt{2}}(U_1 - U_2) \quad , \quad U_\psi = \frac{1}{\sqrt{6}}(U_1 + U_2 - 2U_3),$$

$$U_{45} = \frac{1}{\sqrt{2}}(U_4 - U_5) \quad , \quad U_\zeta = \frac{1}{\sqrt{6}}(U_4 + U_5 - 2U_6)$$

Pati, PLB388 (1996) 532; $U(1)_\psi$ in conjuction with $U(1)_{B-L}$ or $U(1)_\chi$

provides adequate protection as well as allowing light neutrino masses

PLB499 (2001) 147; the extra $U(1)$s are broken at high scale

Patterns of $SO(10)$ symmetry breaking

The $SO(10) \rightarrow$ subgroup $\quad b(\bar{\psi}_{\frac{1}{2}}^{1\cdots5})$:

1. $b\{\bar{\psi}_{\frac{1}{2}}^{1\cdots5} \ \bar{\eta}^1 \ \bar{\eta}^2 \ \bar{\eta}^3\} = \{\frac{1\,1\,1\,1\,1\,{\color{red}1}\,{\color{blue}1}\,{\color{green}1}}{2\,2\,2\,2\,2\,{\color{red}2}\,{\color{blue}2}\,{\color{green}2}}\} \Rightarrow \ SU(5) \times U(1) \ \ {\color{red}U(1)} \ {\color{blue}U(1)} \ {\color{green}U(1)}$

2. $b\{\bar{\psi}_{\frac{1}{2}}^{1\cdots5} \ \bar{\eta}^1 \ \bar{\eta}^2 \ \bar{\eta}^3\} = \{\ 1\,1\,1\,0\,0 \ {\color{red}0}\,{\color{blue}0}\,{\color{green}0} \ \} \Rightarrow SO(6) \times SO(4) \ {\color{red}U(1)} \ {\color{blue}U(1)} \ {\color{green}U(1)}$

$(1. + 2.) \ \Rightarrow \ SO(10) \ \rightarrow SU(3)_C \times SU(2)_L \times U(1)_C \times U(1)_L$

$SO(10) \rightarrow SU(3)_C \times SU(2)_L \times SU(2)_R \times U(1)_{B-L}$

2. $b\{\bar{\psi}_{\frac{1}{2}}^{1\cdots5}\bar{\eta}^1 \ \bar{\eta}^2 \ \bar{\eta}^3\} = \{\ 1\,1\,1\,0\,0{\color{red}0}{\color{blue}0}{\color{green}0} \ \} \Rightarrow SO(6) \times SO(4) \ {\color{red}U(1)} \ {\color{blue}U(1)} \ {\color{green}U(1)}$

3. $b\{\bar{\psi}_{\frac{1}{2}}^{1\cdots5} \ \bar{\eta}^1 \ \bar{\eta}^2 \ \bar{\eta}^3\} = \{\frac{1\,1\,1}{2\,2\,2}0\,0\frac{{\color{red}1}\,{\color{blue}1}\,{\color{green}1}}{{\color{red}2}\,{\color{blue}2}\,{\color{green}2}}\} \Rightarrow$

$SU(\ 3\ )_C \times U(1)_C \times SU(2)_L \times SU(2)_R \ {\color{red}U(1)} \ {\color{blue}U(1)} \ {\color{green}U(1)}$

$U(1)$ matter charges

in cases 1. 2.

$$\Longrightarrow \qquad Q_{U(1)_j}(16 = \{Q, L, U, D, E, N\}) \;=\; +\tfrac{1}{2}$$

$\Longrightarrow$ the $U(1)_{1,2,3}$ are anomalous

In the LRS model of case 3.

$$\Longrightarrow \qquad Q_{U(1)_j}(Q_L, L_L) \qquad\qquad\qquad\qquad =\; -\tfrac{1}{2}$$

$$Q_{U(1)_j}(Q_R = \{U, D\}, L_R = \{E, N\}) \;=\; +\tfrac{1}{2}$$

$\Longrightarrow$ the $U(1)_{1,2,3}$ are anomaly free

The $U(1)$ combination $U(1) = U(1)_1 + U(1)_2 + U(1)_3$ is :

  a. family universal

  b. anomaly free

The Baryon number violating terms :

$$Q_L Q_L Q_L L_L \rightarrow QQQL$$
$$Q_R Q_R Q_R L_R \rightarrow \{UDDN, UUDE\}$$

are forbidden

The Lepton number violating terms :

$$Q_L Q_R L_L L_R \rightarrow QDLN$$
$$L_L L_L L_R L_R \rightarrow LLEN$$

are allowed

The fermion mass terms:

$$Q_L Q_R h \qquad \text{and} \qquad L_L L_R h \qquad \text{and} \qquad NN\bar{N}_H \bar{N}_H \, .$$

are allowed

STRING DERIVED LEFT-RIGHT SYMMETRIC MODEL

	ψ^μ	χ^{12}	χ^{34}	χ^{56}	$y^{3,...,6}$	$\bar{y}^{3,...,6}$	$y^{1,2},\omega^{5,6}$	$\bar{y}^{1,2},\bar{\omega}^{5,6}$	$\omega^{1,...,4}$	$\bar{\omega}^{1,...,4}$	$\bar{\psi}^{1,...,5}$	$\bar{\eta}^1$	$\bar{\eta}^2$	$\bar{\eta}^3$	$\bar{\phi}^{1,...,8}$
1	1	1	1	1	1,...,1	1,...,1	1,...,1	1,...,1	1,...,1	1,...,1	1,...,1	1	1	1	1,...,1
S	1	1	1	1	0,...,0	0,...,0	0,...,0	0,...,0	0,...,0	0,...,0	0,...,0	0	0	0	0,...,0
b_1	1	1	0	0	1,...,1	1,...,1	0,...,0	0,...,0	0,...,0	0,...,0	1,...,1	1	0	0	0,...,0
b_2	1	0	1	0	0,...,0	0,...,0	1,...,1	1,...,1	0,...,0	0,...,0	1,...,1	0	1	0	0,...,0
b_3	1	0	0	1	0,...,0	0,...,0	0,...,0	0,...,0	1,...,1	1,...,1	1,...,1	0	0	1	0,...,0

	ψ^μ	χ^{12}	χ^{34}	χ^{56}	y^3y^6	$y^4\bar{y}^4$	$y^5\bar{y}^5$	$\bar{y}^3\bar{y}^6$	$y^1\omega^5$	$y^2\bar{y}^2$	$\omega^6\bar{\omega}^6$	$\bar{y}^1\bar{\omega}^5$	$\omega^2\omega^4$	$\omega^1\bar{\omega}^1$	$\omega^3\bar{\omega}^3$	$\bar{\omega}^2\bar{\omega}^4$	$\bar{\psi}^{1,...,5}$					$\bar{\eta}^1$	$\bar{\eta}^2$	$\bar{\eta}^3$	$\bar{\phi}^{1,...}$
α	0	0	0	0	1	0	0	0	0	0	1	1	0	0	1	1	1 1 1 0 0					0	0	0	1 1 1 1
β	0	0	0	0	0	0	1	1	1	0	0	0	0	1	0	1	1 1 1 0 0					0	0	0	1 1 0 0
γ	0	0	0	0	0	1	0	1	0	1	0	1	1	0	0	0	$\frac{1}{2}\frac{1}{2}\frac{1}{2}$ 0 0					$\frac{1}{2}$	$\frac{1}{2}$	$\frac{1}{2}$	1 $\frac{1}{2}\frac{1}{2}\frac{1}{2}$

Gerald Cleaver, AEF and Christopher Savage, PRD 63:066001,2001.

3 generations;

3 untwisted Higgs bi–doublets;

Fermion mass terms arise from $N = 3$ and $N = 5$ superpotential terms

Moduli?

Untwisted moduli $->$ shape & size of the internal dimensions

Twisted moduli $->$ arise from the twisted sectors

models

$$\frac{T^6}{Z_2 \times Z_2}$$

$$T^6 : \quad G_{IJ} \quad ; \quad B_{IJ} \qquad I, J = 1, \cdots, 6 .$$

untwisted moduli: coefficients of exactly marginal operators

moduli fields: massless chiral superfields with flat scalar potential

Scalar couplings of $N = 4$ SUGRA

$$\frac{SO(6,6)}{SO(6) \times SO(6)} \quad \times \quad \frac{SU(1,1)}{U(1)}$$

internal manifold $\qquad\qquad$ dilaton

Up to orbifold projections

$$\frac{T^6}{Z_2 \times Z_2} \implies \frac{SO(6,6)}{SO(6) \times SO(6)} \longrightarrow \left(\frac{SO(2,2)}{SO(2) \times SO(2)}\right)^3$$

$\Rightarrow$ 3 complex structures $+$ 3 Kähler moduli

In all symmetric $Z_2 \times Z_2$ orbifolds

We would like to identify these moduli in the fermionic formalism

In symmetric orbifolds:

$$\text{EMO} \rightarrow \partial X^I \bar{\partial} X^J$$

$$X^I \quad I = 1, \cdots, 6 \quad \rightarrow \quad T^6$$

$$S = \frac{1}{8\pi} \int d^2\sigma \ (G_{IJ} \, \partial X^I \bar{\partial} X^J \ + \ B_{IJ} \, \partial X^I \bar{\partial} X^J)$$

In FFF $\partial X_L^I \rightarrow y^i \omega^I$

$\qquad i\partial X_L^I \rightarrow U(1)$ current in the Cartan subalgebra

In the fermionic language

$$i\partial X_L^I \rightarrow J_L^I = y^I \omega^I$$

$$\Rightarrow \partial X^I \bar{\partial} X^J \rightarrow J_L^I(z)\bar{J}_R(\bar{z})$$

$\rightarrow$ WS Thirring interactions $\quad (R - \frac{1}{R})J_L(z)\bar{J}(\bar{z})$

Thirring interactions vanish at free fermionic point with $R = \frac{1}{R}$

To identify the untwisted moduli in the free fermionic models

$\rightarrow$ find the operators of the form

$$J_L^I(z)\bar{J}_R^J(\bar{z})$$

that are allowed by the orbifold (fermionic) symmetry group

Bosonization

$$\xi_i = \sqrt{\frac{1}{2}}(y_i + i\omega_i) = e^{iX_i}, \eta_i = \sqrt{\frac{i}{2}}(y_i - i\omega_i) = ie^{-iX_i}$$

simlarly for the right-movers

Boundary conditions translate to twists and shifts

$$X^I(z, \bar{z}) = X_L^I(z) + X_R^I(\bar{z})$$

Complex internal coordinates

$$Z_k^\pm = (X^{2k-1} \pm iX^{2k}), \quad \psi_k^\pm = (\chi^{2k-1} \pm i\chi^{2k}) \quad (k = 1, 2, 3)$$

Untwisted moduli

$$S = \int d^2z\, h_{ij}(X) J_L^i(z) \bar{J}_R^j(\bar{z})$$

$J_L^i \sim y^i \omega^i$ $i = 1, \cdots, 6$ are chiral currents of $U(1)_L^6$

$J_R^j \sim \bar{\phi}^j \bar{\phi}^{*j}$ $j = 1, \cdots, 22$ are chiral currents of $U(1)_R^{22}$

$h_{ij} \longrightarrow$ scalar components of untwisted moduli

     some of these operators are projected out in concrete models

         $\Rightarrow$ some of the EMO may not be invariant

<u>Models</u> $\{\mathbf{1}, S\}$ $N = 4$ $SO(44)$

$$\frac{SO(6,22)}{SO(6) \times SO(22)}$$ Moduli space

$$\chi^i \otimes \bar{\phi}^a \bar{\phi}^{*a} |0\rangle$$ moduli fields

6×22 scalar fields

$Z_2 \times Z_2$ $\{\, 1 \,,\, S \,,\, \xi_1 \,,\, \xi_2 \,\} \;+\; \{\, b_1 \,,\, b_2 \,\}$

$SO(12) \times E_8 \times E_8$ $Z_2 \times Z_2$

$\rightarrow \quad SO(4)^3 \times E_6 \times U(1)^2 \times E_8$

The Thirring interactions that remain invariant are

$$J_L^{1,2} \bar{J}_R^{1,2} \qquad ; \qquad J_L^{3,4} \bar{J}_R^{3,4} \qquad ; \qquad J_L^{5,6} \bar{J}_R^{5,6}$$

$$y^{1,2}\omega^{1,2}\bar{y}^{1,2}\bar{\omega}^{1,2} \quad ; \quad y^{3,4}\omega^{3,4}\bar{y}^{3,4}\bar{\omega}^{3,4} \quad ; \quad y^{5,6}\omega^{5,6}\bar{y}^{5,6}\bar{\omega}^{5,6}$$

These moduli are always present in symmetric $Z_2 \times Z_2$ orbifolds

in realistic models

$$\{\, 1 \,,\, S \,,\, \xi_1 \,,\, \xi_2 \,\} \quad \oplus \quad \{\, b_1 \,,\, b_2 \,\} \quad \oplus \quad \{\, \alpha \,,\, \beta \,,\, \gamma \}$$

$$N = 4 \qquad\qquad N = 1$$

$$E_8 \times E_8 \qquad\qquad Z_2 \times Z_2$$

new feature   Asymmetric orbifold

the key focus: boundary conditions of the internal fermions

$$\{\, y \,,\, \omega \,\mid\, \bar{y} \,,\, \omega \,\}$$

WS fermions that have same B.C. in all basis vectors are paired

pairing of LR fermions $\rightarrow$ Ising model $\rightarrow$ symmetric real fermions

pairing of LL & RR fermions $\rightarrow$ complex fermions $\rightarrow$ asymmetric

STRING DERIVED STANDARD-LIKE MODEL (PLB278)

	ψ^μ	χ^{12}	χ^{34}	χ^{56}	$y^{3,...,6}$	$\bar{y}^{3,...,6}$	$y^{1,2},\omega^{5,6}$	$\bar{y}^{1,2},\bar{\omega}^{5,6}$	$\omega^{1,...,4}$	$\bar{\omega}^{1,...,4}$	$\bar{\psi}^{1,...,5}$	$\bar{\eta}^1$	$\bar{\eta}^2$	$\bar{\eta}^3$	$\bar{\phi}^{1,...,8}$
1	1	1	1	1	1,...,1	1,...,1	1,...,1	1,...,1	1,...,1	1,...,1	1,...,1	1	1	1	1,...,1
S	1	1	1	1	0,...,0	0,...,0	0,...,0	0,...,0	0,...,0	0,...,0	0,...,0	0	0	0	0,...,0
b_1	1	1	0	0	1,...,1	1,...,1	0,...,0	0,...,0	0,...,0	0,...,0	1,...,1	1	0	0	0,...,0
b_2	1	0	1	0	0,...,0	0,...,0	1,...,1	1,...,1	0,...,0	0,...,0	1,...,1	0	1	0	0,...,0
b_3	1	0	0	1	0,...,0	0,...,0	0,...,0	0,...,0	1,...,1	1,...,1	1,...,1	0	0	1	0,...,0

	ψ^μ	χ^{12}	χ^{34}	χ^{56}	y^3y^6	$y^4\bar{y}^4$	$y^5\bar{y}^5$	$\bar{y}^3\bar{y}^6$	$y^1\omega^5$	$y^2\bar{y}^2$	$\omega^6\bar{\omega}^6$	$\bar{y}^1\bar{\omega}^5$	$\omega^2\omega^4$	$\omega^1\bar{\omega}^1$	$\omega^3\bar{\omega}^3$	$\bar{\omega}^2\bar{\omega}^4$	$\bar{\psi}^{1,...,5}$					$\bar{\eta}^1$	$\bar{\eta}^2$	$\bar{\eta}^3$		
α	0	0	0	0	1	0	0	0	0	0	1	1	0	0	1	1	1	1	1	0	0	0	0	0	1	1
β	0	0	0	0	0	0	1	1	1	0	0	0	0	1	0	1	1	1	1	0	0	0	0	0	1	1
γ	0	0	0	0	0	1	0	1	0	1	0	1	1	0	0	0	$\frac{1}{2}$	$\frac{1}{2}$	$\frac{1}{2}$	$\frac{1}{2}$	$\frac{1}{2}$	$\frac{1}{2}$	$\frac{1}{2}$	$\frac{1}{2}$	$\frac{1}{2}$ 0	1

Asymmetric $BC \Rightarrow$ all untwisted moduli are projected out!

all $y_i\omega_i\bar{y}_i\bar{\omega}_i$ are disallowed

can be translated to asymmetric bosonic identifications

$$X_L + X_R \rightarrow X_L - X_R$$

moduli fixed at enhanced symmetry point

Twisted moduli

$(2,2)$ $b_j \oplus b_j + \xi_2 \quad \rightarrow \quad (16 \oplus 10 + 1) + 1 = 27 + 1 \Rightarrow$ twisted moduli

$(2,0)$ $b_j + b_j + 2\gamma \quad \rightarrow 16_{SO(10)} + 16_{SO(16)}$

all twisted moduli are projected out

A STRINGY DOUBLET–TRIPLET SPLITTING MECHANISM

NAHE $\rightarrow$ $\chi_j \bar{\psi}^{1,\cdots,5} \bar{\eta}_j + c.c. \rightarrow (5 + \bar{5})_j = 10_j$ of $SO(10)$

$\alpha \quad \rightarrow \quad SO(10) \quad \rightarrow \quad SO(6) \times SO(4)$

$y_3 \bar{y}_3 \quad y_4 \bar{y}_4 \quad y_5 \bar{y}_5 \quad y_6 \bar{y}_6$ $\qquad\qquad$ $y_3 y_6 \quad y_4 \bar{y}_4 \quad y_5 \bar{y}_5 \quad \bar{y}_3 \bar{y}_6$

$\quad 1 \qquad 0 \qquad 0 \qquad 1$ $\qquad\qquad$ $1 \qquad 0 \qquad 0 \qquad 0$

$$\Delta_j = |\alpha_L(T_2^j) - \alpha_R(T_2^j)|$$

$\Delta_j = 0 \rightarrow D_j, \bar{D}_j$ $\qquad\qquad$ $\Delta_j = 1 \rightarrow h_j, \bar{h}_j$

$\Delta_{1,2,3} = 1 \Rightarrow h_j \ \bar{h}_j \qquad j = 1, 2, 3$

A superstring solution to the GUT hierarchy problem

MINIMAL DOUBLET HIGGS CONTENT
(w Manno & Timirgaziu (SLM); Christodoulides (FSU5) in progress)

	ψ^μ	χ^{12}	χ^{34}	χ^{56}	y^3y^6	$y^4\bar{y}^4$	$y^5\bar{y}^5$	$\bar{y}^3\bar{y}^6$	$y^1\omega^5$	$y^2\bar{y}^2$	$\omega^6\bar{\omega}^6$	$\bar{y}^1\bar{\omega}^5$	$\omega^2\omega^4$	$\omega^1\bar{\omega}^1$	$\omega^3\bar{\omega}^3$	$\bar{\omega}^2\bar{\omega}^4$	$\bar{\psi}^{1,\dots,5}$	$\bar{\eta}^1$	$\bar{\eta}^2$	$\bar{\eta}^3$	$\bar{\phi}$
α	0	0	0	0	1	0	0	1	0	0	1	1	0	0	1	1	1 1 1 0 0	1	0	0	1 1 0
β	0	0	0	0	0	0	1	1	1	0	0	1	0	1	0	1	1 1 1 0 0	0	1	0	0 0 1
γ	0	0	0	0	0	1	0	0	0	1	0	0	1	0	0	0	$\frac{1}{2}\frac{1}{2}\frac{1}{2}\frac{1}{2}\frac{1}{2}$	$\frac{1}{2}$	$\frac{1}{2}$	$\frac{1}{2}$	0 0 0

SYMMETRIC $\leftrightarrow$ ASYMMETRIC

with respect to b_1 & b_2

$h_1,\ \bar{h}_1,\ D_1,\ \bar{D}_1\ ,\ \ h_2,\ \bar{h}_2,\ D_2,\ \bar{D}_2$ are projected out

$h_3,\ \bar{h}_3$ \qquad remain in the spectrum

$\lambda_t Q_3 t_3^c \bar{h}_3\ $ with $\ \lambda_t\ O(1)$

No Phenomenologically viable flat directions

Cleaver, Faraggi, Manno, Timirgaziu, PRD 78 (2008) 046009

Classification of F and D flat directions in EMT reduced Higgs model

No D flat direction which is F-flat up to order eight in the superpotential

no stringent flat directions to all orders

Suggesting no supersymmetric flat directions in this model (class of models)

implying no supersymmetric moduli

only remaining perturbative moduli is the dilaton

quasi–realistic model: SLM; 3 gen; $SO(10)$ embed; Higgs & $\lambda_t \sim 1$; ...

vanishing one–loop partition function

Fixed geometrical, twisted and SUSY moduli

$Z_2 \times Z_2$ orbifolds

torus: One complex parameter $Z = Z + n\, e_1 + m\, e_2$

$T^2 \times T^2 \times T^2 \longrightarrow$ Three complex coordinates z_1 , z_2 and z_3

Z_2 orbifold : $Z = -Z + \sum_i m_i\, e_i \longrightarrow$ 4 fixed points

$$Z = \{\ 0\ ,\ 1/2\, e_1\ ,\ 1/2\, e_2\ ,\ 1/2\, (\, e_1 + e_2\,)\ \}$$

$\dfrac{T^2 \times T^2 \times T^2}{Z_2 \times Z_2}$

$\alpha : (\ z1\,,\, z2\,,\, z3\ )\ \to\ (-z1\ ,\ -z2\ ,\ +z3\ )\ \to\ 16$

$\beta : (\ z1\,,\, z2\,,\, z3\ )\ \to\ (+\, z1\ ,\ -z2\ ,\ -z3\ )\ \to\ 16$

$\alpha\beta : (\ z1\,,\, z2\,,\, z3\ )\ \to\ (-z1\ ,\ +z2\ ,\ -z3\ )\ \to\ \dfrac{16}{48}$

$\downarrow$

$\gamma : (\ z1,\, z2,\, z3\ )\ \to\ (z1+1/2,\, z2+1/2,\, z3+1/2) \longrightarrow 24$

Question:

$$\frac{T^6}{Z_2 \times Z_2} \quad \longrightarrow \quad 48 \ \text{ fixed points} \qquad\qquad \frac{SO(12)}{Z_2 \times Z_2} \quad \longrightarrow \quad 24 \ \text{ fixed points}$$

$$Z_2 \ \text{ shift } \ : 48 \ \longleftrightarrow \ 24$$

Is this the same model? In general, no.

shift that reproduces the $SO(12)$ lattice at the free fermionic point?

$$A_1 \ : \ X_{\mathrm{L,R}} \to X_{\mathrm{L,R}} + \tfrac{1}{2}\pi R \,,$$

$$A_2 \ : \ X_{\mathrm{L,R}} \to X_{\mathrm{L,R}} + \tfrac{1}{2}\left(\pi R \pm \frac{\pi \alpha'}{R}\right) \,,$$

$$A_3 \ : \ X_{\mathrm{L,R}} \to X_{\mathrm{L,R}} \pm \tfrac{1}{2}\frac{\pi \alpha'}{R} \,.$$

Using the level-one $\mathrm{SO}(2n)$ characters

$$O_{2n} = \tfrac{1}{2}\left(\frac{\vartheta_3^n}{\eta^n} + \frac{\vartheta_4^n}{\eta^n}\right) \,, \qquad\qquad V_{2n} = \tfrac{1}{2}\left(\frac{\vartheta_3^n}{\eta^n} - \frac{\vartheta_4^n}{\eta^n}\right) \,,$$

$$S_{2n} = \tfrac{1}{2}\left(\frac{\vartheta_2^n}{\eta^n} + i^{-n}\frac{\vartheta_1^n}{\eta^n}\right) \,, \qquad C_{2n} = \tfrac{1}{2}\left(\frac{\vartheta_2^n}{\eta^n} - i^{-n}\frac{\vartheta_1^n}{\eta^n}\right) \,.$$

Starting from:

$$Z_+ = (V_8 - S_8) \left(\sum_{m,n} \Lambda_{m,n} \right)^{\otimes 6} \left(\bar{O}_{16} + \bar{S}_{16} \right) \left(\bar{O}_{16} + \bar{S}_{16} \right) ,$$

where as usual, for each circle,

$$p^i_{\mathrm{L,R}} = \frac{m_i}{R_i} \pm \frac{n_i R_i}{\alpha'} ,$$

and

$$\Lambda_{m,n} = \frac{q^{\frac{\alpha'}{4} p_{\mathrm{L}}^2} \, \bar{q}^{\frac{\alpha'}{4} p_{\mathrm{R}}^2}}{|\eta|^2} .$$

Add shifts : (A_1, A_1, A_1) , (A_3, A_3, A_3)

$$\left(48 \to 24 \quad \text{yes} \right)$$
$$\left(SO(12)? \quad \text{no} \right)$$

$$g \ : \ \left(A_2, A_2, 0\right),$$

$$h \ : \ \left(0, A_2, A_2\right),$$

where each A_2 acts on a complex coordinate

$$\left(48 \rightarrow 24 \ \ \text{yes}\right)$$

$$\left(SO(12)? \ \ \text{yes}\right)$$

$$R = \sqrt{\alpha'}$$

T1 – COMPACTIFICATION
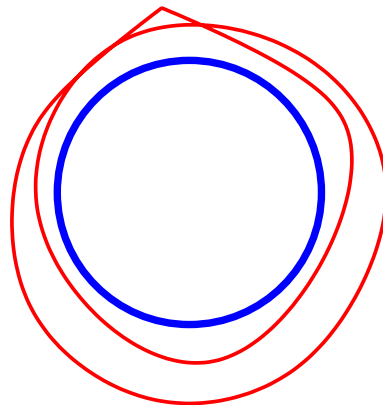
$$X \quad \longrightarrow \quad X \sim X + 2\,\pi\,R\,m$$
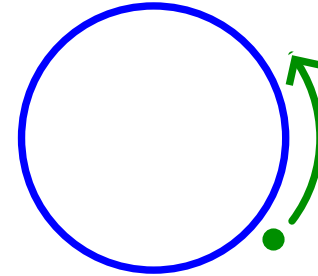
Point particle

$$\Psi \sim \text{Exp}(i\,P\,X) \quad \Rightarrow P = \frac{m}{R}$$

String

$$P_{L,R} = \frac{m}{R} \pm \frac{n\,R}{\alpha'}$$

T – DUALITY

$$\text{mass}^2 = \left(\frac{n}{R}\right)^2 + \left(\frac{m\,R}{\alpha'}\right)^2$$

Invariant under

$$\frac{1}{R} \longleftrightarrow \frac{R}{\alpha'} \qquad \text{with} \qquad m \longleftrightarrow n$$

An exact symmetry in string perturbation theory!

Self–dual point $R = \dfrac{\alpha'}{R}$ = free fermionic point