ASPECTS OF FREE FERMIONIC HETEROTIC-STRING MODELS

Alon E. Faraggi

- AEF, 1990's (top quark mass, CKM, MSSM w Cleaver and Nanopoulos)
- AEF, C. Kounnas, J. Rizos, 2003-2009
- AEF, PLB2002, work in progress with Angelantonj, Tsulaia

Galileo Galilei Institute, Firenze, 26 May 2009

DATA $\longrightarrow$ STANDARD MODEL

$SU(3) \times SU(2) \times U(1)_Y \longrightarrow SU(5) \longrightarrow SO(10)$

$$\left[\begin{pmatrix} \nu \\ e \end{pmatrix} + D^c_L \right] + \left[U^c_L + \begin{pmatrix} u \\ d \end{pmatrix} + E^c_L \right] + N^c_L$$

$$\bar{5} \quad + \quad 10 \quad\quad 1 \quad\quad \overline{} \; 16$$

STANDARD MODEL -> UNIFICATION

ADDITIONAL EVIDENCE:

Logarithmic running, proton longevity, neutrino masses

PRIMARY GUIDES:

3 generations

SO(10) embedding

Realistic free fermionic models

'Phenomenology of the Standard Model and string unification'

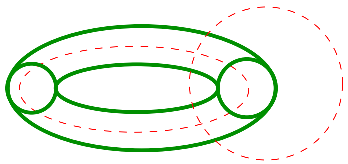
- Top quark mass $\sim$ 175–180GeV — PLB 274 (1992) 47
- Generation mass hierarchy — NPB 407 (1993) 57
- CKM mixing — NPB 416 (1994) 63 (with Halyo)
- Stringy seesaw mechanism — PLB 307 (1993) 311 (with Halyo)
- Gauge coupling unification — NPB 457 (1993) 409 (with Dienes)
- Proton stability — NPB 428 (1994) 111
- Squark degeneracy — NPB 526 (1998) 21 (with Pati)
- Minimal Superstring Standard Model — PLB 455 (1999) 135 (with Cleaver & Nanopoulos)
- Moduli fixing — NPB 728 (2005) 83

Free Fermionic Construction

Left-Movers: $\psi^{\mu}_{1,2}, \quad \chi_i, \quad y_i, \quad \omega_i \quad (i = 1, \cdots, 6)$

Right-Movers

$$\bar{\phi}_{A=1,\cdots,44} = \begin{cases} \bar{y}_i \ , \ \bar{\omega}_i & i = 1, \cdots, 6 \\ \\ \bar{\eta}_i & i = 1, 2, 3 \\ \bar{\psi}_{1,\cdots,5} & \\ \bar{\phi}_{1,\cdots,8} & \end{cases}$$



$$f \longrightarrow -e^{i\pi\alpha(f)} f$$

$$V \longrightarrow V$$

$$Z = \sum_{\substack{all \ spin \\ structures}} c\binom{\vec{\alpha}}{\vec{\beta}} Z\binom{\vec{\alpha}}{\vec{\beta}}$$

Models $\longleftrightarrow$ Basis vectors $+$ one–loop phases

Model building – Construction of the physical states

$$b_j \quad j = 1, \cdots, N \qquad \rightarrow \qquad \Xi \; = \; \sum_j n_j b_j$$

For $\vec{\alpha} \; = \; (\vec{\alpha}_L ; \vec{\alpha}_R) \quad \in \quad \Xi \qquad \Rightarrow \qquad \mathrm{H}_{\vec{\alpha}}$

$$\alpha(f) = 1 \quad \Rightarrow \quad |\pm\rangle \qquad ; \qquad \alpha(f) \neq 1 \quad \Rightarrow \quad f, f^* \quad , \quad \nu_{f,f^*} = \frac{1 \mp \alpha(f)}{2}$$

$$M_L^2 = -\frac{1}{2} + \frac{\vec{\alpha}_L \cdot \vec{\alpha}_L}{8} + N_L = -1 + \frac{\vec{\alpha}_R \cdot \vec{\alpha}_R}{8} + N_R = M_R^2 \;\; (\;\; \equiv \;\; 0 \;\;)$$

GSO projections $\qquad e^{i\pi(\vec{b}_i \cdot \vec{F}_\alpha)} |s\rangle_{\vec{\alpha}} = \delta_\alpha c^* \begin{pmatrix} \vec{\alpha} \\ \vec{b}_i \end{pmatrix} |s\rangle_{\vec{\alpha}}$

$Q(f) = \frac{1}{2}\alpha(f) + F(f) \quad \rightarrow \quad$ U(1) charges

Calculation of Mass Terms

nonvanishing correlators

$$\langle V_1^f V_2^f V_3^b \cdot \cdots V_N^b \rangle$$

gauge & string invariant

"anomalous" $U(1)_A$

$$\text{Tr} Q_A \neq 0 \Rightarrow \quad D_A = 0 = A + \sum Q_k^A |\langle \phi_k \rangle|^2$$

$$D_j \; = 0 = \qquad \sum Q_k^j |\langle \phi_k \rangle|^2$$

$$\langle W \rangle = \langle \frac{\partial W_N}{\partial \eta_i} \rangle = 0 \quad N = 3 \cdots\cdot$$

Supersymmetric vacuum $\langle F \rangle = \langle D \rangle = 0.$

nonrenormalizable terms $\to$ effective renormalizable operators

$$V_1^f V_2^f V_3^b \cdot \cdots V_N^b \;\to\; V_1^f V_2^f V_3^b \frac{\langle V_4^b \cdots V_N^b \rangle}{M^{N-3}}$$

The NAHE set:

$$1 = \{\psi^\mu, \ \chi^{1,\dots,6}, y^{1,\dots,6}, \omega^{1,\dots,6} \mid \bar{y}^{1,\dots,6}, \bar{\omega}^{1,\dots,6}, \bar{\eta}^{1,2,3}, \bar{\psi}^{1,\dots,5}, \bar{\phi}^{1,\dots,8}\}$$

$$S = \{\psi^\mu, \chi^{1,\dots,6}\}, \qquad N = 4 \text{ Vacua}$$

$$b_1 = \{\chi^{34}, \chi^{56}, y^{34}, y^{56} | \bar{y}^{34}, \bar{y}^{56}, \bar{\eta}^1, \bar{\psi}^{1,\dots,5}\}, \qquad N = 4 \to N = 2$$

$$b_2 = \{\chi^{12}, \chi^{56}, y^{12}, \omega^{56} | \bar{y}^{12}, \bar{\omega}^{56}, \bar{\eta}^2, \bar{\psi}^{1,\dots,5}\}, \qquad N = 2 \to N = 1$$

$$b_3 = \{\chi^{12}, \chi^{34}, \omega^{12}, \omega^{34} | \bar{\omega}^{12}, \bar{\omega}^{34}, \bar{\eta}^3, \bar{\psi}^{1,\dots,5}\}, \qquad N = 2 \to N = 1$$

$Z_2 \times Z_2$ orbifold compactification

$\Longrightarrow$ Gauge group $SO(10) \times SO(6)^{1,2,3} \times E_8$

beyond the NAHE set Add $\{\alpha, \beta, \gamma\}$

	ψ^μ	χ^{12}	χ^{34}	χ^{56}	y^3y^6	$y^4\bar{y}^4$	$y^5\bar{y}^5$	$\bar{y}^3\bar{y}^6$	$y^1\omega^5$	$y^2\bar{y}^2$	$\omega^6\bar{\omega}^6$	$\bar{y}^1\bar{\omega}^5$	$\omega^2\omega^4$	$\omega^1\bar{\omega}^1$	$\omega^3\bar{\omega}^3$	$\bar{\omega}^2\bar{\omega}^4$	$\bar{\psi}^{1,\ldots,5}$	$\bar{\eta}^1$	$\bar{\eta}^2$	$\bar{\eta}^3$	$\bar{\phi}^{1,\ldots,8}$
α	0	0	0	0	1	0	0	1	0	0	1	1	0	0	1	1	1 1 1 0 0	1	0	0	1 1 0 0 0 0 0 0
β	0	0	0	0	0	0	1	1	1	0	0	1	0	1	0	1	1 1 1 0 0	0	1	0	0 0 1 1 0 0 0 0
γ	0	0	0	0	0	1	0	0	0	1	0	0	1	0	0	0	$\frac{1}{2}$ $\frac{1}{2}$ $\frac{1}{2}$ $\frac{1}{2}$ $\frac{1}{2}$	$\frac{1}{2}$	$\frac{1}{2}$	$\frac{1}{2}$	0 0 0 $\frac{1}{2}$ $\frac{1}{2}$ $\frac{1}{2}$ $\frac{1}{2}$ 0

number of generations is reduced to three

$$SO(10) \longrightarrow SU(3) \times SU(2) \times U(1)_{T_{3_R}} \times U(1)_{B-L}$$

$$U(1)_Y = \frac{1}{2}(B-L) + T_{3_R} \in SO(10) \; !$$

$$SO(6)^{1,2,3} \longrightarrow U(1)^{1,2,3} \times U(1)^{1,2,3}$$

The massless spectrum

Three twisted generations $b_1, \quad b_2, \quad b_3$

Untwisted Higgs doublets

$h_{1_{1,0,0}} \qquad \bar{h}_{1_{-1,0,0}}$

$h_{2_{0,1,0}} \qquad \bar{h}_{2_{0,-1,0}}$

$h_{3_{0,0,1}} \qquad \bar{h}_{3_{0,0,-1}}$

Sector $b_1 + b_2 + \alpha + \beta$ $\quad h_{\alpha\beta_{-\frac{1}{2},-\frac{1}{2},0,0,0,0}} \qquad \bar{h}_{\alpha\beta_{\frac{1}{2},\frac{1}{2},0,0,0,0}}$

$\oplus$ $SO(10)$ singlets

Sectors $b_j + 2\gamma \quad j = 1, 2, 3 \quad \longrightarrow$ hidden matter multiplets

"standard" $SO(10)$ representations

NAHE $+ \{ \alpha , \beta , \gamma \} \rightarrow$ exotic vector–like matter $\rightarrow$ superheavy

$\oplus$ Quasi-realistic phenomenology

Fermion mass hierarchy

Fermion mass terms

$$c g f_i f_j h \left(\frac{\langle \phi \rangle}{M} \right)^{N-3}$$

c - calculable coefficients g - gauge coupling

$$f_i, f_j \;\in\; b_j \quad j \;=\; 1, 2, 3$$

$h \rightarrow$ light Higgs multiplets

$$M \;\sim\; 10^{18}\, GeV$$

$\langle \phi \rangle$ generalized VEVs, several sources

Top quark mass prediction

only $\lambda_t = \langle t^c Q_t \bar{h}_1 \rangle = \sqrt{2} g \neq 0$ at $N = 3$

$$W_4 \longrightarrow b^c Q_b h_{\alpha\beta} \Phi_1 + \tau^c L_\tau h_{\alpha\beta} \Phi_1$$

$$\Longrightarrow \quad \lambda_b = \left(c_b \frac{\langle \phi \rangle}{M} \right) \qquad \lambda_\tau = \left(c_\tau \frac{\langle \phi \rangle}{M} \right)$$

$$\longrightarrow \quad \lambda_b = \lambda_\tau = 0.35 g^3 \sim \frac{1}{8} \lambda_t$$

Evolve λ_t , λ_b to low energies

$$m_t = \lambda_t v_1 = \lambda_t \frac{v_0}{\sqrt{2}} \sin\beta \qquad m_b = \lambda_b v_2 = \lambda_b \frac{v_0}{\sqrt{2}} \cos\beta$$

where $v_0 = \dfrac{2 m_W}{g_2(M_Z)} = 246 \mathrm{GeV}$ and $(v_1^2 + v_2^2) = \dfrac{v_0^2}{2}$

$$m_t = \lambda_t(m_t) \frac{v_0}{\sqrt{2}} \frac{\tan\beta}{(1 + \tan^2\beta)^{\frac{1}{2}}} \Longrightarrow \quad m_t \sim 175 \mathrm{GeV} \quad \mathrm{PLB274(1992)47}$$

Cabibbo mixing

PLB 307 (1993) 305

Find anomaly free solution

$$M_d \sim \begin{pmatrix} \epsilon & \frac{V_2\bar{V}_3\Phi_{\alpha\beta}}{M^3} & 0 \\ \frac{V_2\bar{V}_3\Phi_{\alpha\beta}\xi_1}{M^4} & \frac{\bar{\Phi}_2^-\xi_1}{M^2} & 0 \\ 0 & 0 & \frac{\Phi_1^+\xi_2}{M^2} \end{pmatrix} v_2,$$

$$\frac{V_2\bar{V}_3\Phi_{\alpha\beta}}{M^3} = \frac{\sqrt{5}\ g^6}{64\ \pi^3} \approx 2-3\times 10^{-4}.$$

$\epsilon < 10^{-8}$. Fix ξ_1, ξ_2, $\bar{\Phi}_2^-$ to fit m_b, m_s

$$\Rightarrow \quad |V| \sim \begin{pmatrix} 0.98 & 0.2 & 0 \\ 0.2 & 0.98 & 0 \\ 0 & 0 & 1 \end{pmatrix}$$

Three generation mixing $\longrightarrow$ NPB 416 (1994) 63

$$|J| \sim 10^{-6}$$

Correspondence with $Z_2 \times Z_2$ orbifold PLB 326 (1994) 62

NAHE $\oplus$ $(\xi_2 = \{\bar{\psi}_{1,\cdots,5}, \bar{\eta}^1, \bar{\eta}^2, \bar{\eta}^3\} = 1) \;\; \rightarrow \;\; \{1, S, \xi_1, \xi_2, b_1, b_2\}$

Gauge group: $SO(4)^3 \times E_6 \times U(1)^2 \times E_8$ and 24 generations.

toroidal compactification $(6_L + 6_R)$ g_{ij}, b_{ij}

$$g_{ij} = \begin{pmatrix} 2 & -1 & 0 & 0 & 0 & 0 \\ -1 & 2 & -1 & 0 & 0 & 0 \\ 0 & -1 & 2 & -1 & 0 & 0 \\ 0 & 0 & -1 & 2 & -1 & -1 \\ 0 & 0 & 0 & -1 & 2 & 0 \\ 0 & 0 & 0 & -1 & 0 & 2 \end{pmatrix} \qquad b_{ij} = \begin{cases} g_{ij} & \text{i ¡ j} \\ 0 & \text{i = j} \\ -g_{ij} & \text{i ¿ j} \end{cases}$$

$R_i \;\; \rightarrow$ the free fermionic point $\rightarrow$G.G. $SO(12) \times E_8 \times E_8$

mod out by a $Z_2 \times Z_2$ with standard embedding

$\Rightarrow SO(4)^3 \times E_6 \times U(1)^2 \times E_8$ with 24 generations

Exact correspondence

In the realistic free fermionic models

replace $X = \{\bar\psi^{1,\cdots,5}, \bar\eta^1, \bar\eta^2, \bar\eta^3\} = 1$

with $2\gamma = \{\bar\psi^{1,\cdots,5}, \bar\eta^1, \bar\eta^2, \bar\eta^3, \bar\phi^{1,\cdots,4}\} = 1$

Then $\{\vec{1}, \vec{S}, \vec{\xi}_1 = \vec{1} + \vec{b}_1 + \vec{b}_2 + \vec{b}_3, 2\gamma\} \rightarrow$ N=4 SUSY and

$$SO(12) \times SO(16) \times SO(16)$$

apply $b_1 \times b_2 \rightarrow Z_2 \times Z_2 \rightarrow$ N=1 SUSY and

$$SO(4)^3 \times SO(10) \times U(1)^3 \times SO(16)$$

b_1, b_2, b_3 $\Rightarrow (3 \times 8) \cdot 16$ of $SO(10)_O$

$b_1 + 2\gamma$, $b_2 + 2\gamma$, $b_3 + 2\gamma$ $\Rightarrow (3 \times 8) \cdot 16$ of $SO(16)_H$

Alternatively, $c\binom{\xi_1}{\xi_2} = +1 \rightarrow -1$

$Z_2 \times Z_2$ orbifolds

A torus One complex parameter $Z = Z + n\, e_1 + m\, e_2$

$T^2 \times T^2 \times T^2 \longrightarrow$ Three complex coordinates z_1 , z_2 and z_3

Z_2 orbifold : $Z = -Z + \sum_i m_i e_i \longrightarrow$ 4 fixed points

$$Z = \{\ 0\ ,\ 1/2\, e_1\ ,\ 1/2\, e_2\ ,\ 1/2\,(e_1 + e_2)\ \}$$

$$\frac{T^2 \times T^2 \times T^2}{Z_2 \times Z_2}$$

$\alpha : (z1, z2, z3) \to (-z1, -z2, +z3) \to 16$

$\beta : (z1, z2, z3) \to (+z1, -z2, -z3) \to 16$

$\alpha\beta : (z1, z2, z3) \to (-z1, +z2, -z3) \to \underline{16}$

48

↓

$\gamma : (z1, z2, z3) \to (z1+1/2, z2+1/2, z3+1/2) \longrightarrow$ 24

Classification of fermionic $Z_2 \times Z_2$ orbifolds (FKNR, FKR)

Basis vectors: consistent modular blocks 4,8 periodic fermions

$$1 = \{\psi^\mu, \ \chi^{1,\dots,6}, y^{1,\dots,6}, \omega^{1,\dots,6} \mid \bar{y}^{1,\dots,6}, \bar{\omega}^{1,\dots,6}, \bar{\eta}^{1,2,3}, \bar{\psi}^{1,\dots,5}, \bar{\phi}^{1,\dots,8}\}$$

$$S = \{\psi^\mu, \chi^{1,\dots,6}\},$$

$$z_1 = \{\bar{\phi}^{1,\dots,4}\},$$

$$z_2 = \{\bar{\phi}^{5,\dots,8}\},$$

$$e_i = \{y^i, \omega^i | \bar{y}^i, \bar{\omega}^i\}, \ i = 1, \dots, 6, \qquad N = 4 \text{ Vacua}$$

$$b_1 = \{\chi^{34}, \chi^{56}, y^{34}, y^{56} | \bar{y}^{34}, \bar{y}^{56}, \bar{\eta}^1, \bar{\psi}^{1,\dots,5}\}, \qquad N = 4 \to N = 2$$

$$b_2 = \{\chi^{12}, \chi^{56}, y^{12}, y^{56} | \bar{y}^{12}, \bar{y}^{56}, \bar{\eta}^2, \bar{\psi}^{1,\dots,5}\}, \qquad N = 2 \to N = 1$$

Vector bosons: NS, $z_{1,2}$, $z_1 + z_2$, $x = 1 + s + \sum e_i + z_1 + z_2$

impose: Gauge group $SO(10) \times U(1)^3 \times \text{hidden}$

Independent phases $c\left[\begin{matrix} v_i \\ v_j \end{matrix}\right] = \exp[i\pi(v_i|v_j)]$: upper block

$$
\begin{array}{c|cccccccccccc}
 & 1 & S & e_1 & e_2 & e_3 & e_4 & e_5 & e_6 & z_1 & z_2 & b_1 & b_2 \\
\hline
1 & -1 & -1 & \pm & \pm & \pm & \pm & \pm & \pm & \pm & \pm & \pm & \pm \\
S & & & -1 & -1 & -1 & -1 & -1 & -1 & -1 & -1 & 1 & 1 \\
e_1 & & & & \pm & \pm & \pm & \pm & \pm & \pm & \pm & \pm & \pm \\
e_2 & & & & & \pm & \pm & \pm & \pm & \pm & \pm & \pm & \pm \\
e_3 & & & & & & \pm & \pm & \pm & \pm & \pm & \pm & \pm \\
e_4 & & & & & & & \pm & \pm & \pm & \pm & \pm & \pm \\
e_5 & & & & & & & & \pm & \pm & \pm & \pm & \pm \\
e_6 & & & & & & & & & \pm & \pm & \pm & \pm \\
z_1 & & & & & & & & & & \pm & \pm & \pm \\
z_2 & & & & & & & & & & & \pm & \pm \\
b_1 & & & & & & & & & & & & \pm \\
b_2 & & & & & & & & & & & &
\end{array}
$$

Apriori 55 independent coefficients $\rightarrow$ 2^{55} distinct vacua

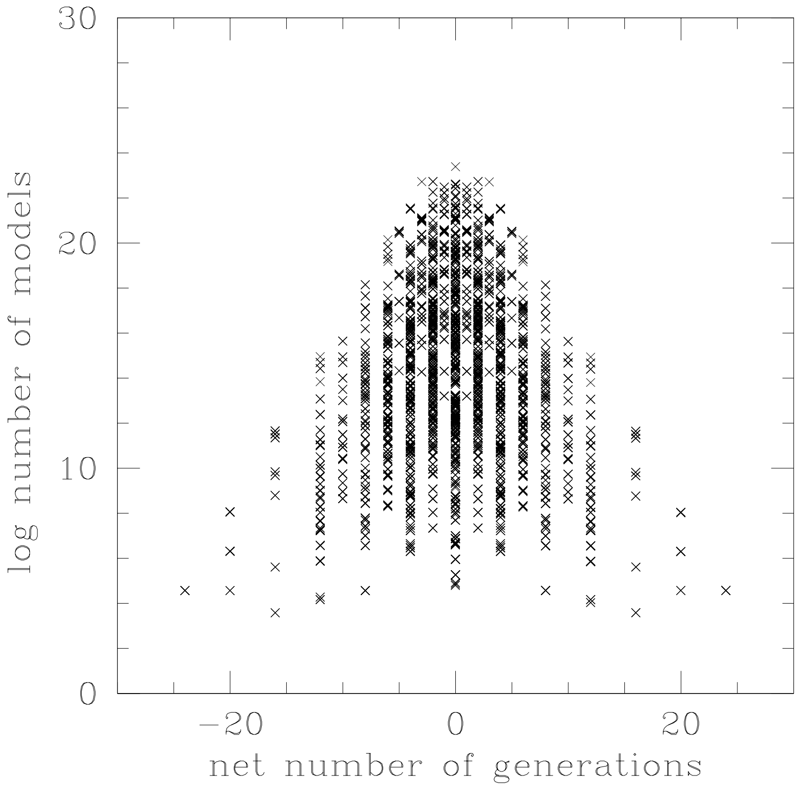
Impose: Gauge group $SO(10) \times U(1)^3 \times SO(8)^2$

$\rightarrow$ 40 independent coefficients

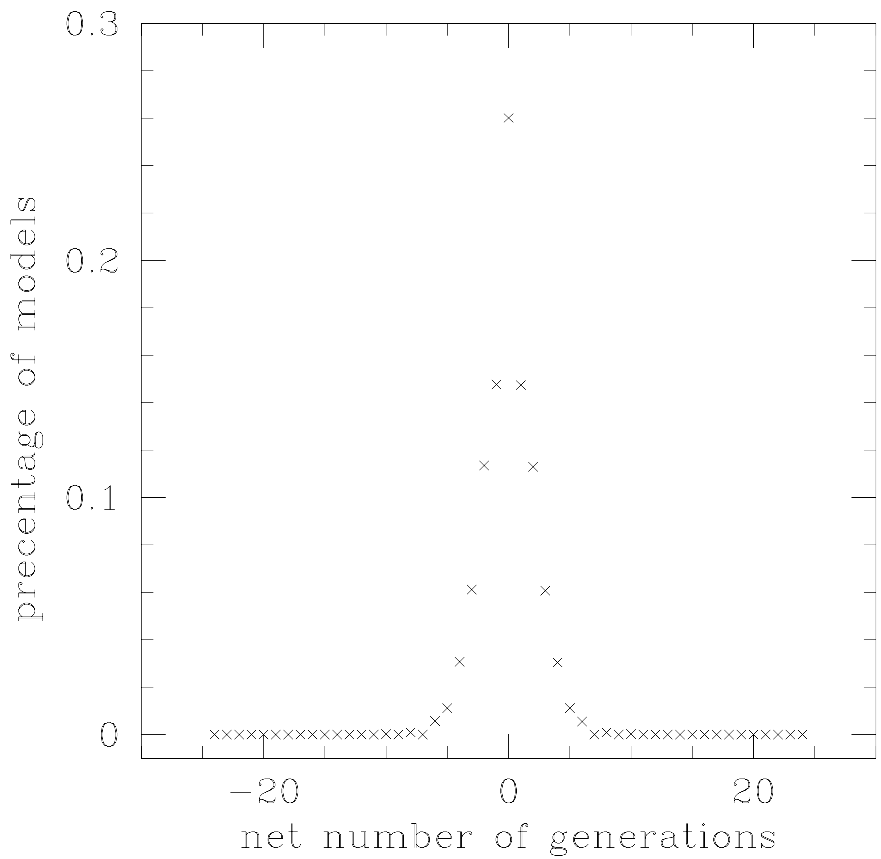
RESULTS:

FKR I: Random sampling of phases. $SO(10) \times U(1)^3 \times \text{hidden}$

FKR II: Complete classification. $SO(10) \times U(1)^3 \times SO(8)^2$



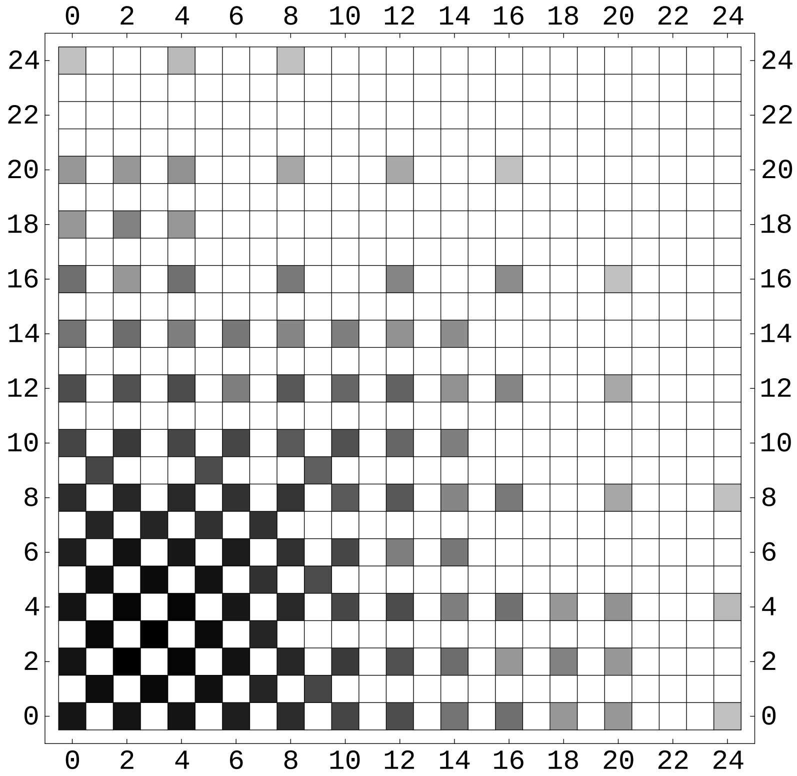
RESULTS ARE SIMILAR



7×10^9 models $\sim$ 15% with 3 gen FKRI

Spinor–vector duality:

Invariance under exchange of $\#(16+\overline{16}) \;<-> \; \#(10)$



Symmetric under exchange of rows and columns

Self-dual: $\#(16+\overline{16}) \;=\; \#(10)$ without E_6 symmetry

Future:

Understand in geometrical terms:

Progress:

Tristan Catelin-Jullien, AEF, C. Kounnas, J. Rizos, NPB2009 $\longrightarrow$

Operational understanding in terms of partition function free phases

NAHE-based partition functions:

Question:

$$\frac{T^6}{Z_2 \times Z_2} \;\rightarrow\; 48 \;\text{ fixed points} \qquad\qquad \frac{SO(12)}{Z_2 \times Z_2} \;\rightarrow\; 24 \;\text{ fixed points}$$

$$Z_2 \;\text{ shift } \;: 48 \;\longleftrightarrow\; 24$$

Is this the same vacuum? In general, no.

shift that reproduces the $SO(12)$ lattice at the free fermionic point?

Possible shifts:

$$A_1 \ : \ \ X_{\rm L,R} \to X_{\rm L,R} + \tfrac{1}{2}\pi R \,,$$

$$A_2 \ : \ \ X_{\rm L,R} \to X_{\rm L,R} + \tfrac{1}{2}\left(\pi R \pm \frac{\pi\alpha'}{R}\right) \,,$$

$$A_3 \ : \ \ X_{\rm L,R} \to X_{\rm L,R} \pm \tfrac{1}{2}\frac{\pi\alpha'}{R} \,.$$

Using the level-one $\mathrm{SO}(2n)$ characters

$$O_{2n} = \tfrac{1}{2}\left(\frac{\vartheta_3^n}{\eta^n} + \frac{\vartheta_4^n}{\eta^n}\right) \,, \qquad V_{2n} = \tfrac{1}{2}\left(\frac{\vartheta_3^n}{\eta^n} - \frac{\vartheta_4^n}{\eta^n}\right) \,,$$

$$S_{2n} = \tfrac{1}{2}\left(\frac{\vartheta_2^n}{\eta^n} + i^{-n}\frac{\vartheta_1^n}{\eta^n}\right) \,, \qquad C_{2n} = \tfrac{1}{2}\left(\frac{\vartheta_2^n}{\eta^n} - i^{-n}\frac{\vartheta_1^n}{\eta^n}\right) \,.$$

The partition function of the heterotic string on $SO(12)$ lattice:

$$\mathrm{Z}_+ = (V_8 - S_8)\left[|O_{12}|^2 + |V_{12}|^2 + |S_{12}|^2 + |C_{12}|^2\right](\bar{O}_{16} + \bar{S}_{16})\,(\bar{O}_{16} + \bar{S}_{16}) \ ,$$

and

$$\begin{aligned}\mathrm{Z}_- \ = \ (V_8 - S_8)\Big[&\left(|O_{12}|^2 + |V_{12}|^2\right)(\bar{O}_{16}\bar{O}_{16} + \bar{C}_{16}\bar{C}_{16}) \\ &+ \left(|S_{12}|^2 + |C_{12}|^2\right)(\bar{S}_{16}\bar{S}_{16} + \bar{V}_{16}\bar{V}_{16}) \\ &+ (O_{12}\bar{V}_{12} + V_{12}\bar{O}_{12})(\bar{S}_{16}\bar{V}_{16} + \bar{V}_{16}\bar{S}_{16}) \\ &+ (S_{12}\bar{C}_{12} + C_{12}\bar{S}_{12})(\bar{O}_{16}\bar{C}_{16} + \bar{C}_{16}\bar{O}_{16})\Big] \ .\end{aligned}$$

where $\pm$ refers to

$$c\binom{\xi_1}{\xi_2} \quad = \quad \pm 1$$

connected by : $\qquad \mathrm{Z}_- = \mathrm{Z}_+ / a \otimes b \,,$

$$a = (-1)^{F_{\mathrm{L}}^{\mathrm{int}} + F_\xi^1} \,, \qquad\qquad b = (-1)^{F_{\mathrm{L}}^{\mathrm{int}} + F_\xi^2} \,.$$

Starting from:

$$Z_+ = (V_8 - S_8) \left(\sum_{m,n} \Lambda_{m,n} \right)^{\otimes 6} (\bar{O}_{16} + \bar{S}_{16}) \, (\bar{O}_{16} + \bar{S}_{16}) \ ,$$

where as usual, for each circle,

$$p^i_{\rm L,R} = \frac{m_i}{R_i} \pm \frac{n_i R_i}{\alpha'} \, ,$$

and

$$\Lambda_{m,n} = \frac{q^{\frac{\alpha'}{4} p^2_{\rm L}} \, \bar{q}^{\frac{\alpha'}{4} p^2_{\rm R}}}{|\eta|^2} \, .$$

Add shifts : (A_1, A_1, A_1) , (A_3, A_3, A_3)

($48 \to 24$ yes)

($SO(12)$? no)

Uniquely:

$$g \ : \ (A_2, A_2, 0)\,,$$
$$h \ : \ (0, A_2, A_2)\,,$$

where each A_2 acts on a complex coordinate

($48 \rightarrow 24$ yes)

($SO(12)$? yes)

A STRINGY DOUBLET-TRIPLET SPLITTING MECHANISM

	ψ^μ	χ^{12}	χ^{34}	χ^{56}	y^3y^6	$y^4\bar y^4$	$y^5\bar y^5$	$\bar y^3\bar y^6$	$y^1\omega^5$	$y^2\bar y^2$	$\omega^6\bar\omega^6$	$\bar y^1\bar\omega^5$	$\omega^2\omega^4$	$\omega^1\bar\omega^1$	$\omega^3\bar\omega^3$	$\bar\omega^2\bar\omega^4$	$\bar\psi^{1,\dots,5}$	$\bar\eta^1$	$\bar\eta^2$	$\bar\eta^3$	$\bar\phi^{1,\dots,8}$
α	0	0	0	0	1	0	0	1	0	0	1	1	0	0	1	1	1 1 1 0 0	1	0	0	1 1 0 0 0 0 0 0
β	0	0	0	0	0	0	1	1	1	0	0	1	0	1	0	1	1 1 1 0 0	0	1	0	0 0 1 1 0 0 0 0
γ	0	0	0	0	0	1	0	0	0	1	0	0	1	0	0	0	$\frac12$ $\frac12$ $\frac12$ $\frac12$ $\frac12$	$\frac12$	$\frac12$	$\frac12$	0 0 0 $\frac12$ $\frac12$ $\frac12$ $\frac12$

NAHE $\rightarrow$ $\chi_j\bar\psi^{1,\cdots,5}\bar\eta_j + c.c. \rightarrow (5+\bar 5)_j = 10_j$ of $SO(10)$

α , β $\rightarrow$ $SO(10)$ $\rightarrow$ $SO(6)\times SO(4)$

$$\Delta_j = |\alpha_L(T_2^j) - \alpha_R(T_2^j)|$$

$\Delta_j = 0 \rightarrow D_j, \bar D_j$ $\Delta_j = 1 \rightarrow h_j, \bar h_j$

$h_1,\ \bar h_1,\ D_1,\ \bar D_1$, $h_2,\ \bar h_2,\ D_2,\ \bar D_2$ are projected out

$h_3,\ \bar h_3$ remain in the spectrum

Conclusions

Phenomenological string models produce interesting lessons

Spinor–vector duality

relevance of non–standard geometries

Free Fermionic Models $\longrightarrow$ $Z_2 \times Z_2$ orbifold near the self–dual point

Duality & Self–Duality $\Leftrightarrow$ String Vacuum Selection

The massless spectrum

Three twisted generations $b_1, \quad b_2, \quad b_3$

Untwisted Higgs doublets

$h_{1_{1,0,0}} \qquad \bar{h}_{1_{-1,0,0}}$

$h_{2_{0,1,0}} \qquad \bar{h}_{2_{0,-1,0}}$

$h_{3_{0,0,1}} \qquad \bar{h}_{3_{0,0,-1}}$

Sector $b_1 + b_2 + \alpha + \beta$ $\qquad h_{\alpha\beta_{-\frac{1}{2},-\frac{1}{2},0,0,0,0}} \qquad \bar{h}_{\alpha\beta_{\frac{1}{2},\frac{1}{2},0,0,0,0}}$

$\oplus$ $SO(10)$ singlets

Sectors $b_j + 2\gamma \quad j = 1, 2, 3 \quad \longrightarrow$ hidden matter multiplets

"standard" $SO(10)$ representations

NAHE + $\{ \alpha\ ,\ \beta\ ,\ \gamma \} \ \rightarrow$ exotic vector–like matter $\rightarrow$ superheavy

$\oplus$ Quasi-realistic phenomenology

Moduli?

Untwisted moduli $->$ shape & size of the internal dimensions

Twisted moduli $->$ arise from the twisted sectors

models

$$\frac{T^6}{Z_2 \times Z_2}$$

$$T^6 \; : \qquad G_{IJ} \qquad ; \qquad B_{IJ} \qquad\qquad I, J \; = \; 1, \cdots, 6 \; .$$

untwisted moduli: coefficients of exactly marginal operators

moduli fields: massless chiral superfields with flat scalar potential

Scalar couplings of $N = 4$ SUGRA

$$\frac{SO(6,6)}{SO(6) \times SO(6)} \qquad \times \qquad \frac{SU(1,1)}{U(1)}$$

internal manifold   dilaton

Up to orbifold projections

$$\frac{T^6}{Z_2 \times Z_2} \Longrightarrow \frac{SO(6,6)}{SO(6)\times SO(6)} \longrightarrow \left(\frac{SO(2,2)}{SO(2)\times SO(2)}\right)^3$$

$\Rightarrow$ 3 complex structures $+$ 3 Kähler moduli

In all symmetric $Z_2 \times Z_2$ orbifolds

We would like to identify these moduli in the fermionic formalism

In symmetric orbifolds:

$$\text{EMO} \to \ \partial X^I \bar{\partial} X^J$$

$$X^I \quad I = 1, \cdots, 6 \ \to \ T^6$$

$$S = \frac{1}{8\pi}\int d^2\sigma \ \ (G_{IJ}\,\partial X^I \bar{\partial} X^J \ + \ B_{IJ}\,\partial X^I \bar{\partial} X^J)$$

In FFF $\partial X_L^I \;\to\; y^i\omega^I$

$i\partial X_L^I \;\to\; U(1)$ current in the Cartan subalgebra

In the fermionic language

$$i\partial X_L^I \;\to\; J_L^I = y^I\omega^I$$

$$\Rightarrow \;\; \partial X^I\bar{\partial}X^J \to J_L^I(z)\bar{J}_R(\bar{z})$$

$\to$ WS Thirring interactions $\quad (R-\frac{1}{R})J_L(z)\bar{J}(\bar{z})$

Thirring interactions vanish at free fermionic point with $R=\frac{1}{R}$

To identify the untwisted moduli in the free fermionic models

$\to$ find the operators of the form

$$J_L^I(z)\bar{J}_R^J(\bar{z})$$

that are allowed by the orbifold (fermionic) symmetry group

Bosonization

$$\xi_i = \sqrt{\frac{1}{2}}(y_i + i\omega_i) = e^{iX_i}, \eta_i = \sqrt{\frac{i}{2}}(y_i - i\omega_i) = ie^{-iX_i}$$

simlarly for the right-movers

Boundary conditions translate to twists and shifts

$$X^I(z,\bar{z}) = X_L^I(z) + X_R^I(\bar{z})$$

Complex internal coordinates

$$Z_k^{\pm} = (X^{2k-1} \pm iX^{2k}), \quad \psi_k^{\pm} = (\chi^{2k-1} \pm i\chi^{2k}) \quad (k = 1, 2, 3)$$

Untwisted moduli

$$S = \int d^2 z h_{ij}(X) J_L^i(z) \bar{J}_R^j(\bar{z})$$

$J_L^i \sim y^i \omega^i$ $\quad$ $i = 1, \cdots, 6$ are chiral currents of $U(1)_L^6$

$J_R^j \sim \bar{\phi}^j \bar{\phi}^{*j}$ $\quad$ $j = 1, \cdots, 22$ are chiral currents of $U(1)_R^{22}$

$h_{ij} \rightarrow$ scalar components of untwisted moduli

some of these operators are projected out in concrete models

$\Rightarrow$ some of the EMO may not be invariant

Models $\{\mathbf{1}, S\}$ $N=4$ $SO(44)$

$$\frac{SO(6,22)}{SO(6)\times SO(22)}$$ Moduli space

$$\chi^i \otimes \bar\phi^a \bar\phi^{*^a}|0\rangle$$ moduli fields

$$6 \times 22$$ scalar fields

$Z_2 \times Z_2$ $\{\, 1\ ,\ S\ ,\ \xi_1\ ,\ \xi_2\, \}\ +\ \{\, b_1\ ,\ b_2\, \}$

$$SO(12)\times E_8\times E_8 \qquad Z_2\times Z_2$$

$$\rightarrow \quad SO(4)^3\times E_6\times U(1)^2\times E_8$$

The Thirring interactions that remain invariant are

$$J_L^{1,2}\bar J_R^{1,2} \quad ; \quad J_L^{3,4}\bar J_R^{3,4} \quad ; \quad J_L^{5,6}\bar J_R^{5,6}$$

$$y^{1,2}\omega^{1,2}\bar y^{1,2}\bar\omega^{1,2} \quad ; \quad y^{3,4}\omega^{3,4}\bar y^{3,4}\bar\omega^{3,4} \quad ; \quad y^{5,6}\omega^{5,6}\bar y^{5,6}\bar\omega^{5,6}$$

These moduli are always present in symmetric $Z_2 \times Z_2$ orbifolds in realistic models

$$\{ 1 , S , \xi_1 , \xi_2 \} \oplus \{ b_1 , b_2 \} \oplus \{ \alpha , \beta , \gamma \}$$

$$N = 4 \qquad\qquad N = 1$$

$$E_8 \times E_8 \qquad\qquad Z_2 \times Z_2$$

new feature Asymmetric orbifold

the key focus: boundary conditions of the internal fermions

$$\{ y , \omega \mid \bar{y} , \omega \}$$

WS fermions that have same B.C. in all basis vectors are paired

pairing of LR fermions $\rightarrow$ Ising model $\rightarrow$ symmetric real fermions

pairing of LL & RR fermions $\rightarrow$ complex fermions $\rightarrow$ asymmetric

A STRINGY DOUBLET–TRIPLET SPLITTING MECHANISM

NAHE $\rightarrow$ $\chi_j\bar{\psi}^{1,\cdots,5}\bar{\eta}_j + c.c. \rightarrow (5+\bar{5})_j = 10_j$ of $SO(10)$

$\alpha \quad \rightarrow \quad SO(10) \quad \rightarrow \quad SO(6)\times SO(4)$

$y_3\bar{y}_3$	$y_4\bar{y}_4$	$y_5\bar{y}_5$	$y_6\bar{y}_6$		$y_3 y_6$	$y_4\bar{y}_4$	$y_5\bar{y}_5$	$\bar{y}_3\bar{y}_6$
1	0	0	1		1	0	0	0

$$\Delta_j = |\alpha_L(T_2^j) - \alpha_R(T_2^j)|$$

$\Delta_j = 0 \rightarrow D_j, \bar{D}_j$ $\qquad$ $\Delta_j = 1 \rightarrow h_j, \bar{h}_j$

$$\Delta_{1,2,3} = 1 \Rightarrow h_j \quad \bar{h}_j \qquad j = 1, 2, 3$$

A superstring solution to the GUT hierarchy problem

STRING DERIVED STANDARD-LIKE MODEL (PLB278)

	ψ^μ	χ^{12}	χ^{34}	χ^{56}	$y^{3,\ldots,6}$	$\bar y^{3,\ldots,6}$	$y^{1,2},\omega^{5,6}$	$\bar y^{1,2},\bar\omega^{5,6}$	$\omega^{1,\ldots,4}$	$\bar\omega^{1,\ldots,4}$	$\bar\psi^{1,\ldots,5}$	$\bar\eta^1$	$\bar\eta^2$	$\bar\eta^3$	$\bar\phi^{1,\ldots,8}$
1	1	1	1	1	1,...,1	1,...,1	1,...,1	1,...,1	1,...,1	1,...,1	1,...,1	1	1	1	1,...,1
S	1	1	1	1	0,...,0	0,...,0	0,...,0	0,...,0	0,...,0	0,...,0	0,...,0	0	0	0	0,...,0
b_1	1	1	0	0	1,...,1	1,...,1	0,...,0	0,...,0	0,...,0	0,...,0	1,...,1	1	0	0	0,...,0
b_2	1	0	1	0	0,...,0	0,...,0	1,...,1	1,...,1	0,...,0	0,...,0	1,...,1	0	1	0	0,...,0
b_3	1	0	0	1	0,...,0	0,...,0	0,...,0	0,...,0	1,...,1	1,...,1	1,...,1	0	0	1	0,...,0

	ψ^μ	χ^{12}	χ^{34}	χ^{56}	y^3y^6	$y^4\bar y^4$	$y^5\bar y^5$	$\bar y^3\bar y^6$	$y^1\omega^5$	$y^2\bar y^2$	$\omega^6\bar\omega^6$	$\bar y^1\bar\omega^5$	$\omega^2\omega^4$	$\omega^1\bar\omega^1$	$\omega^3\bar\omega^3$	$\bar\omega^2\bar\omega^4$	$\bar\psi^{1,\ldots,5}$	$\bar\eta^1$	$\bar\eta^2$	$\bar\eta^3$	$\bar\phi^{1,\ldots,8}$
α	0	0	0	0	1	0	0	0	0	0	1	1	0	0	1	1	1 1 1 0 0	0	0	0	1 1 1 1 0 0 0
β	0	0	0	0	0	0	1	1	1	0	0	0	0	1	0	1	1 1 1 0 0	0	0	0	1 1 1 1 0 0 0
γ	0	0	0	0	0	1	0	1	0	1	0	1	1	0	0	0	$\frac{1}{2}$ $\frac{1}{2}$ $\frac{1}{2}$ $\frac{1}{2}$ $\frac{1}{2}$	$\frac{1}{2}$	$\frac{1}{2}$	$\frac{1}{2}$	$\frac{1}{2}$ 0 1 1 $\frac{1}{2}$

Asymmetric BC $\Rightarrow$ all untwisted moduli are projected out!

all $y_i\omega_i\bar y_i\bar\omega_i$ are disallowed

can be translated to asymmetric bosonic identifications

$$X_L + X_R \rightarrow X_L - X_R$$

moduli fixed at enhanced symmetry point

Twisted moduli

(2,2) $b_j \oplus b_j + \xi_2 \quad \rightarrow \quad (16 \oplus 10 + 1) + 1 = 27 + 1 \Rightarrow$ twisted moduli

(2,0) $b_j + b_j + 2\gamma \quad \rightarrow 16_{SO(10)} + 16_{SO(16)}$

all twisted moduli are projected out

MINIMAL DOUBLET HIGGS CONTENT (EJPC50)

	ψ^μ	χ^{12}	χ^{34}	χ^{56}	y^3y^6	$y^4\bar{y}^4$	$y^5\bar{y}^5$	$\bar{y}^3\bar{y}^6$	$y^1\omega^5$	$y^2\bar{y}^2$	$\omega^6\bar{\omega}^6$	$\bar{y}^1\bar{\omega}^5$	$\omega^2\omega^4$	$\omega^1\bar{\omega}^1$	$\omega^3\bar{\omega}^3$	$\bar{\omega}^2\bar{\omega}^4$	$\bar{\psi}^{1,\ldots,5}$	$\bar{\eta}^1$	$\bar{\eta}^2$	$\bar{\eta}^3$	$\bar{\phi}^{1,\ldots,8}$
α	0	0	0	0	1	0	0	1	0	0	1	1	0	0	1	1	1 1 1 0 0	1	0	0	1 1 0 0 0 0 0 0
β	0	0	0	0	0	0	1	1	1	0	0	1	0	1	0	1	1 1 1 0 0	0	1	0	0 0 1 1 0 0 0 0
γ	0	0	0	0	0	1	0	0	0	1	0	0	1	0	0	0	$\frac{1}{2}$ $\frac{1}{2}$ $\frac{1}{2}$ $\frac{1}{2}$ $\frac{1}{2}$	$\frac{1}{2}$	$\frac{1}{2}$	$\frac{1}{2}$	0 0 0 $\frac{1}{2}$ $\frac{1}{2}$ $\frac{1}{2}$ $\frac{1}{2}$

(With Elisa Manno and Cristina Timirgaziu)

SYMMETRIC $\leftrightarrow$ ASYMMETRIC

with respect to b_1 & b_2

$h_1,\ \bar{h}_1,\ D_1,\ \bar{D}_1$, $h_2,\ \bar{h}_2,\ D_2,\ \bar{D}_2$ are projected out

$h_3,\ \bar{h}_3$ remain in the spectrum

$$\lambda_t Q_3 t_3^c \bar{h}_3 \ \ \text{with} \ \ \lambda_t \ O(1)$$

No Phenomenologically viable flat directions

Calculation of Mass Terms

nonvanishing correlators

$$\langle V_1^f V_2^f V_3^b \cdots \cdot V_N^b \rangle$$

gauge & string invariant

"anomalous" $U(1)_A$

$$\text{Tr}Q_A \neq 0 \Rightarrow \quad D_A = 0 = A + \sum Q_k^A |\langle \phi_k \rangle|^2$$

$$D_j \ = 0 = \qquad \sum Q_k^j |\langle \phi_k \rangle|^2$$

$$\langle W \rangle = \langle \frac{\partial W_N}{\partial \eta_i} \rangle = 0 \quad N = 3 \cdots \cdot$$

Supersymmetric vacuum $\langle F \rangle = \langle D \rangle = 0.$

nonrenormalizable terms $\rightarrow$ effective renormalizable operators

$$V_1^f V_2^f V_3^b \cdots \cdot V_N^b \ \rightarrow \ V_1^f V_2^f V_3^b \frac{\langle V_4^b \cdots V_N^b \rangle}{M^{N-3}}$$

Exotics superpotential in FNY model (NPB 335 (1990) 347)

$$W_2 = \frac{1}{\sqrt{2}}\{\ H_1H_2\phi_4 + H_3H_4\bar{\phi}_4 + H_5H_6\bar{\phi}_4 + (H_7H_8 + H_9H_{10})\phi'_4 +$$

$$(H_{11} + H_{12})(H_{13} + H_{14})\bar{\phi}'_4 + V_{41}V_{42}\bar{\phi}_4 + V_{43}V_{44}\bar{\phi}_4 +$$

$$V_{45}V_{46}\phi_4 + (V_{47}V_{48} + V_{49}V_{50})\bar{\phi}'_4 + V_{51}V_{52}\phi'_4\}$$

$\langle\ \bar{\phi}_4, \bar{\phi}'_4, \phi_4, \phi'_4\ \rangle \rightarrow$ massive exotic states at N=3 (PRD46 (1993) 3204)

CFN $\rightarrow$ Classification of flat directions (PLB 455 (1999) 135)

Example: $\{\phi_{12}, \phi_{23}, \bar{\phi}_{56}, \phi_4, \phi'_4, \bar{\phi}_4, \bar{\phi}'_4, H_{15}, H_{30}, H_{31}, H_{38}\}$

All Standard Model charged states beyond MSSM $\rightarrow\ \approx\ M_{\rm string}$

MINIMAL STANDARD HETEROTIC STRING MODEL

Cleaver, Faraggi, Manno, Timirgaziu, PRD 78 (2008) 046009

Classification of F and D flat directions in EMT reduced Higgs model

No D flat direction which is F-flat up to order eight in the superpotential

no stringent flat directions to all orders

Suggesting no supersymmetric flat directions in this model (class of models)

implying no supersymmetric moduli

only remaining perturbative moduli is the dilaton

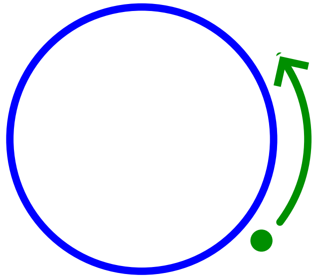
quasi–realistic model: SLM; 3 gen; $SO(10)$ embed; Higgs & $\lambda_t \sim 1$; ...

vanishing one–loop partition function, perturbatively broken SUSY

Fixed geometrical, twisted and SUSY moduli

T1 − COMPACTIFICATION

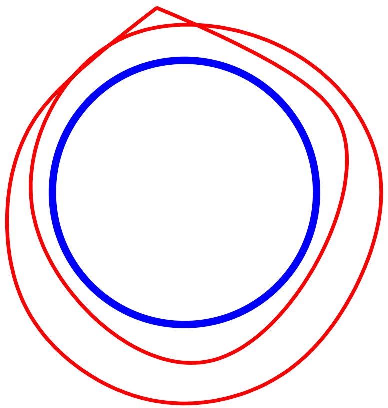
$$X \longrightarrow X \sim X + 2\pi R m$$



Point particle

$$\Psi \sim \mathrm{Exp}(i P X) \Rightarrow P = \frac{m}{R}$$

String



$$P_{L,R} = \frac{m}{R} \pm \frac{n R}{\alpha'}$$

T − DUALITY

$$\text{mass}^2 = \left(\frac{n}{R}\right)^2 + \left(\frac{m\,R}{\alpha'}\right)^2$$

Invariant under

$$\frac{1}{R} \longleftrightarrow \frac{R}{\alpha'} \qquad \text{with} \qquad m \longleftrightarrow n$$

An exact symmetry in string perturbation theory!

Self−dual point $R = \frac{\alpha'}{R}$ = free fermionic point