On The Equivalence of String Vacua

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• Develop : Phenomenology $\longrightarrow$ Vacua $\longrightarrow$ Selection

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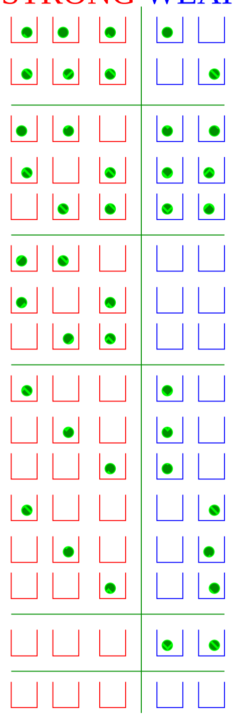
STANDARD MODEL $\longrightarrow$ UNIFICATION

STRONG WEAK ELECTROMAGNETIC $\longrightarrow$ SO(10)

STRONG WEAK



1	2	$\begin{pmatrix} \nu \\ e \end{pmatrix}$
$\bar{3}$	1	D^c_L
$\bar{3}$	1	U^c_L
3	2	$\begin{pmatrix} u \\ d \end{pmatrix}$
1	1	E^c_L
1	1	N^c_L

$$\bar{5} = \binom{5}{4} = \frac{5!}{4!1!}$$

$+$

$$10 = \binom{5}{2}$$

$+$

$$1 = \binom{5}{0}$$

16

Additional evidence: Log running , τ_p , m_ν

Guides: 3 Generations & SO(10) embedding

Realistic free fermionic models

'Phenomenology of the Standard Model and string unification'

- Top quark mass $\sim$ 175–180GeV — PLB 274 (1992) 47
- Generation mass hierarchy — NPB 407 (1993) 57
- CKM mixing — NPB 416 (1994) 63 (with Halyo)
- Stringy seesaw mechanism — PLB 307 (1993) 311 (with Halyo)
- Gauge coupling unification — NPB 457 (1993) 409 (with Dienes)
- Proton stability — NPB 428 (1994) 111
- Squark degeneracy — NPB 526 (1998) 21 (with Pati)
- Minimal Superstring Standard Model — PLB 455 (1999) 135 (with Cleaver & Nanopoulos)
- Moduli fixing — NPB 728 (2005) 83
- Exophobia — PLB 683 (2010) 306 (with Assel, Christodoulides, Kounnas & Rizos)

Other approaches

Geometrical

Greene, Kirklin, Miron, Ross (1987)
Donagi, Ovrut, Pantev, Waldram (1999)
Blumenhagen, Moster, Reinbacher, Weigand (2006)
Heckman, Vafa (2008)
....

Orbifolds

Ibanez, Nilles, Quevedo (1987)
Bailin, Love, Thomas (1987)
Kobayashi, Raby, Zhang (2004)
Lebedev, Nilles, Raby, Ramos-Sanchez, Ratz, Vaudrevange, Wingerter (2007)
Blaszczyk, Groot–Nibbelink, Ruehle, Trapletti, Vaudrevange (2010)
....

Other CFTs

Gepner (1987)
Schellekens, Yankielowicz (1989)
Gato–Rivera, Schellekens (2009)
....

Orientifolds

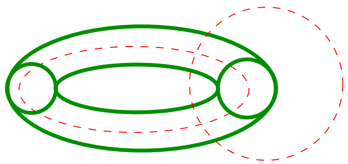
Cvetic, Shiu, Uranga (2001)
Ibanez, Marchesano, Rabadan (2001)
Kiristis, Schellekens, Tsulaia (2008)
....

Free Fermionic Construction

Left-Movers: $\psi^{\mu}_{1,2}, \quad \chi_i, \quad y_i, \;\; \omega_i \quad (i = 1, \cdots, 6)$

Right-Movers

$$\bar{\phi}_{A=1,\cdots,44} = \begin{cases} \bar{y}_i \;,\; \bar{\omega}_i & i = 1, \cdots, 6 \\ \\ \bar{\eta}_i & i = 1, 2, 3 \\ \bar{\psi}_{1,\cdots,5} & \\ \bar{\phi}_{1,\cdots,8} & \end{cases}$$



$$V \longrightarrow V \qquad\qquad f \longrightarrow -e^{i\pi\alpha(f)} f$$

$$Z = \sum_{\substack{all\ spin\\structures}} c\binom{\vec{\alpha}}{\vec{\beta}} \, Z\binom{\vec{\alpha}}{\vec{\beta}}$$

Models $\longleftrightarrow$ Basis vectors $+$ one–loop phases

The NAHE set : $\{ \mathbf{1}, \; S \;, \quad b_1 \;, \quad b_2 \;, \quad b_3 \}$

$$N = \; 4 \;\; \rightarrow \;\; 2 \qquad 1 \qquad 1 \;\; \text{vacua}$$

$Z_2 \times Z_2$ orbifold compactification

$$\Longrightarrow \quad \text{Gauge group} \quad SO(10) \times SO(6)^{1,2,3} \times E_8$$

beyond the NAHE set   Add $\{\alpha, \beta, \gamma\}$

number of generations is reduced to three

$$SO(10) \quad \longrightarrow \quad SU(3) \times SU(2) \times U(1)_{T_{3_R}} \times U(1)_{B-L}$$

$$U(1)_Y \; = \; \tfrac{1}{2}(B-L) \; + \; T_{3_R} \; \in \; SO(10) \; !$$

$$SO(6)^{1,2,3} \quad \longrightarrow \quad U(1)^{1,2,3} \; \times \; U(1)^{1,2,3}$$

Correspondence with $Z_2 \times Z_2$ orbifold   PLB 326 (1994) 62

NAHE $\oplus$ $(\xi_2 = \{\bar\psi^{1,\cdots,5}, \bar\eta^1, \bar\eta^2, \bar\eta^3\} = 1)$ $\rightarrow$ $\{1, S, \xi_1, \xi_2, b_1, b_2\}$

Gauge group: $SO(4)^3 \times E_6 \times U(1)^2 \times E_8$ and 24 generations.

toroidal compactification

$$g_{ij} = \begin{pmatrix} 2 & -1 & 0 & 0 & 0 & 0 \\ -1 & 2 & -1 & 0 & 0 & 0 \\ 0 & -1 & 2 & -1 & 0 & 0 \\ 0 & 0 & -1 & 2 & -1 & -1 \\ 0 & 0 & 0 & -1 & 2 & 0 \\ 0 & 0 & 0 & -1 & 0 & 2 \end{pmatrix} \qquad b_{ij} = \begin{cases} g_{ij} & i < j \\ 0 & i = j \\ -g_{ij} & i > j \end{cases}$$

R_i $\rightarrow$ the free fermionic point $\rightarrow$ G.G. $SO(12) \times E_8 \times E_8$

mod out by a $Z_2 \times Z_2$ with standard embedding

$\Rightarrow$ Exact correspondence

In the realistic free fermionic models

replace $\xi_2 \equiv x = \{\bar\psi^{1,\cdots,5}, \bar\eta^1, \bar\eta^2, \bar\eta^3\} = 1$

with $\quad 2\gamma = \{\bar\psi^{1,\cdots,5}, \bar\eta^1, \bar\eta^2, \bar\eta^3, \bar\phi^{1,\cdots,4}\} = 1$

Then $\{\vec{1}, \vec{S}, \vec{\xi_1} = \vec{1} + \vec{b}_1 + \vec{b}_2 + \vec{b}_3, 2\gamma\} \rightarrow$ N=4 SUSY and

$$SO(12) \times SO(16) \times SO(16)$$

apply $b_1 \times b_2 \rightarrow Z_2 \times Z_2 \rightarrow$ N=1 SUSY and

$$SO(4)^3 \times SO(10) \times U(1)^3 \times SO(16)$$

b_1, b_2, b_3 $\Rightarrow (3 \times 8) \cdot 16$ of $SO(10)_O$

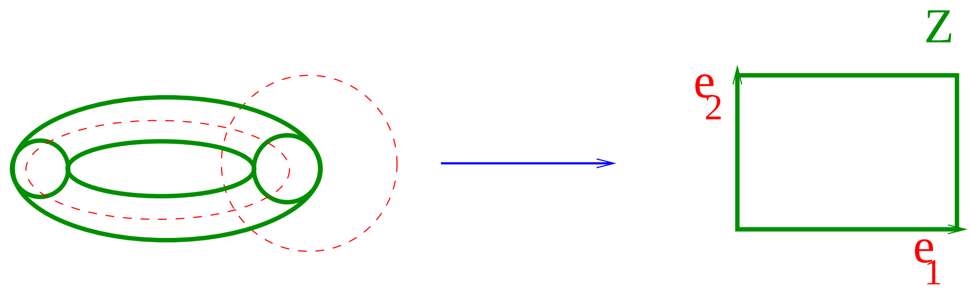
$b_1 + 2\gamma$, $b_2 + 2\gamma$, $b_3 + 2\gamma$ $\Rightarrow (3 \times 8) \cdot 16$ of $SO(16)_H$

Alternatively, $c\begin{pmatrix}\xi_1 \\ \xi_2\end{pmatrix} = +1 \rightarrow -1$

$b_1 + \xi_1$, $b_2 + \xi_1$, $b_3 + \xi_1$ $\Rightarrow (3 \times 8) \cdot 16$ of $SO(16)_H$

$Z_2 \times Z_2$ orbifolds

A torus



One complex parameter $\quad Z = Z + n\, e_1 + m\, e_2$

Z_2 orbifold : $\quad Z = -Z + \sum_i m_i e_i \longrightarrow$ 4 fixed points

$$Z = \{ \; 0 \; , \; 1/2\, e_1 \; , \; 1/2\, e_2 \; , \; 1/2\, (e_1 + e_2) \; \}$$

$$\frac{T^2 \times T^2 \times T^2}{Z_2 \times Z_2}$$

$$\alpha : (z1 , z2 , z3) \; -> \; (-z1 \; , \; -z2 \; , \; +z3) \; -> \; 16$$

$$\beta : (z1 , z2 , z3) \; -> \; (+z1 \; , \; -z2 \; , \; -z3) \; -> \; 16$$

$$\alpha\beta : (z1 , z2 , z3) \; -> \; (-z1 \; , \; +z2 \; , \; -z3) \; -> \; \underline{16}$$

$$48$$

$$\downarrow$$

$$\gamma : (z1 , z2 , z3) \; -> \; (z1+1/2 \; , z2+1/2 \; , z3+1/2) \longrightarrow \; 24$$

Classification of fermionic $Z_2 \times Z_2$ orbifolds (AEF, Kounnas, Rizos)

Basis vectors: consistent modular blocks 4,8 periodic fermions

$$
\begin{aligned}
1 &= \{\psi^\mu,\ \chi^{1,\dots,6}, y^{1,\dots,6}, \omega^{1,\dots,6} \mid \bar{y}^{1,\dots,6}, \bar{\omega}^{1,\dots,6}, \bar{\eta}^{1,2,3}, \bar{\psi}^{1,\dots,5}, \bar{\phi}^{1,\dots,8}\} \\
S &= \{\psi^\mu, \chi^{1,\dots,6}\}, \\
z_1 &= \{\bar{\phi}^{1,\dots,4}\}, \\
z_2 &= \{\bar{\phi}^{5,\dots,8}\}, \\
e_i &= \{y^i, \omega^i | \bar{y}^i, \bar{\omega}^i\},\ i = 1,\dots,6,
\end{aligned}
$$

$N = 4$ Vacua

$$
\begin{aligned}
b_1 &= \{\chi^{34}, \chi^{56}, y^{34}, y^{56} | \bar{y}^{34}, \bar{y}^{56}, \bar{\eta}^1, \bar{\psi}^{1,\dots,5}\}, \qquad N=4 \rightarrow N=2 \\
b_2 &= \{\chi^{12}, \chi^{56}, y^{12}, y^{56} | \bar{y}^{12}, \bar{y}^{56}, \bar{\eta}^2, \bar{\psi}^{1,\dots,5}\}, \qquad N=2 \rightarrow N=1
\end{aligned}
$$

Vector bosons: NS, $z_{1,2}$, z_1+z_2, $x = 1 + s + \sum e_i + z_1 + z_2$

impose: $c\begin{bmatrix} z_1 \\ z_2 \end{bmatrix} = -1$ & Gauge group $SO(10) \times U(1)^3 \times \mathrm{hidden}$

Independent phases $c\binom{v_i}{v_j} = \exp[i\pi(v_i|v_j)]$: upper block

$$
\begin{array}{c}
\\ 1 \\ S \\ e_1 \\ e_2 \\ e_3 \\ e_4 \\ e_5 \\ e_6 \\ z_1 \\ z_2 \\ b_1 \\ b_2
\end{array}
\begin{array}{c}
\begin{array}{cccccccccccc}
1 & S & e_1 & e_2 & e_3 & e_4 & e_5 & e_6 & z_1 & z_2 & b_1 & b_2
\end{array}
\\
\left(\begin{array}{cccccccccccc}
-1 & -1 & \pm & \pm & \pm & \pm & \pm & \pm & \pm & \pm & \pm & \pm \\
 & & -1 & -1 & -1 & -1 & -1 & -1 & -1 & -1 & 1 & 1 \\
 & & & \pm & \pm & \pm & \pm & \pm & \pm & \pm & \pm & \pm \\
 & & & & \pm & \pm & \pm & \pm & \pm & \pm & \pm & \pm \\
 & & & & & \pm & \pm & \pm & \pm & \pm & \pm & \pm \\
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 & & & & & & & & & & \pm & \pm \\
 & & & & & & & & & & & \pm \\
 & & & & & & & & & & &
\end{array}\right)
\end{array}
$$

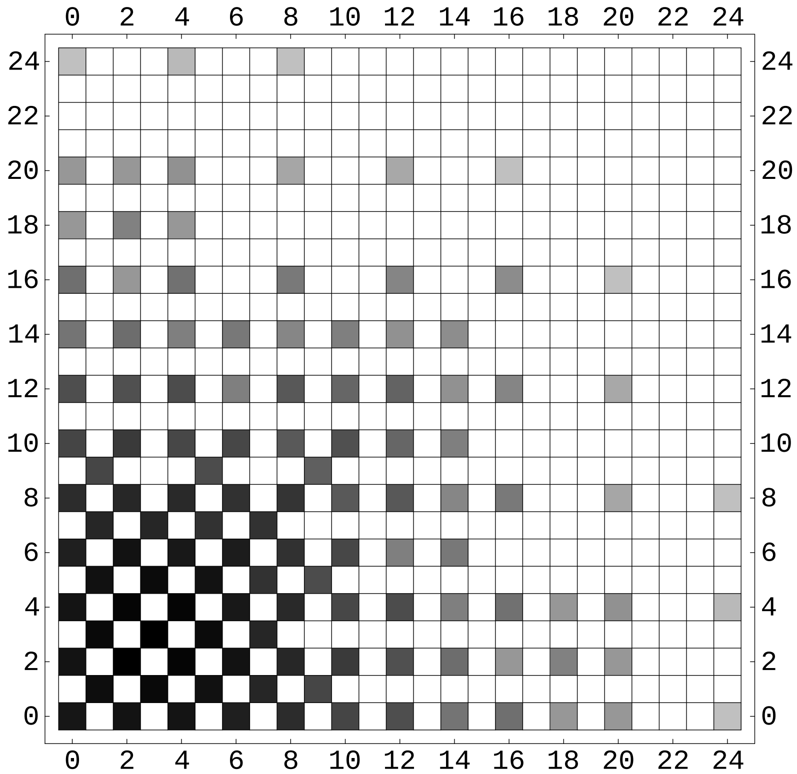
Apriori 55 independent coefficients $\rightarrow\ 2^{55}$ distinct vacua

Impose: Gauge group $SO(10) \times U(1)^3 \times SO(8)^2$

$\rightarrow$ 40 independent coefficients

Spinor–vector duality:

Invariance under exchange of $\#(16+\overline{16}) \;\; <-> \;\; \#(10)$



Symmetric under exchange of rows and columns

$$E_6 \;\; : \qquad\qquad 27 = 16 + 10 + 1 \qquad\qquad \overline{27} = \overline{16} + 10 + 1$$

Self-dual: $\#(16+\overline{16}) \;\; = \;\; \#(10)$ without E_6 symmetry

NAHE-based partition functions: PLB544 (2002) 207

Question:

$$\frac{T^6}{Z_2 \times Z_2} \rightarrow 48 \;\; \text{fixed points} \qquad\qquad \frac{SO(12)}{Z_2 \times Z_2} \rightarrow 24 \;\; \text{fixed points}$$

$$Z_2 \;\; \text{shift} \;\; : 48 \longleftrightarrow 24$$

Is this the same model? In general, no.

shift that reproduces the $SO(12)$ lattice at the free fermionic point?

Possible shifts:

$$A_1 \;:\; X_{\mathrm{L,R}} \to X_{\mathrm{L,R}} + \tfrac{1}{2}\pi R\,,$$

$$A_2 \;:\; X_{\mathrm{L,R}} \to X_{\mathrm{L,R}} + \tfrac{1}{2}\left(\pi R \pm \frac{\pi\alpha'}{R}\right)\,,$$

$$A_3 \;:\; X_{\mathrm{L,R}} \to X_{\mathrm{L,R}} \pm \tfrac{1}{2}\frac{\pi\alpha'}{R}\,.$$

Using the level-one $\mathrm{SO}(2n)$ characters:

$$O_{2n} = \tfrac{1}{2}\left(\frac{\theta_3^n}{\eta^n} + \frac{\theta_4^n}{\eta^n}\right), \qquad V_{2n} = \tfrac{1}{2}\left(\frac{\theta_3^n}{\eta^n} - \frac{\theta_4^n}{\eta^n}\right),$$

$$S_{2n} = \tfrac{1}{2}\left(\frac{\theta_2^n}{\eta^n} + i^{-n}\frac{\theta_1^n}{\eta^n}\right), \qquad C_{2n} = \tfrac{1}{2}\left(\frac{\theta_2^n}{\eta^n} - i^{-n}\frac{\theta_1^n}{\eta^n}\right).$$

where

$$\theta_3 \equiv Z_f\begin{pmatrix}0\\0\end{pmatrix} \quad \theta_4 \equiv Z_f\begin{pmatrix}0\\1\end{pmatrix}$$

$$\theta_2 \equiv Z_f\begin{pmatrix}1\\0\end{pmatrix} \quad \theta_1 \equiv Z_f\begin{pmatrix}1\\1\end{pmatrix}$$

The partition function of the heterotic string on $SO(12)$ lattice:

$$\mathrm{Z}_+ = (V_8 - S_8)\left[|O_{12}|^2 + |V_{12}|^2 + |S_{12}|^2 + |C_{12}|^2\right](\bar{O}_{16} + \bar{S}_{16})\,(\bar{O}_{16} + \bar{S}_{16})\ ,$$

and

$$\begin{aligned}\mathrm{Z}_- \ = \ (V_8 - S_8)\Big[&\left(|O_{12}|^2 + |V_{12}|^2\right)(\bar{O}_{16}\bar{O}_{16} + \bar{C}_{16}\bar{C}_{16}) \\ &+ \left(|S_{12}|^2 + |C_{12}|^2\right)(\bar{S}_{16}\bar{S}_{16} + \bar{V}_{16}\bar{V}_{16}) \\ &+ \left(O_{12}\bar{V}_{12} + V_{12}\bar{O}_{12}\right)(\bar{S}_{16}\bar{V}_{16} + \bar{V}_{16}\bar{S}_{16}) \\ &+ \left(S_{12}\bar{C}_{12} + C_{12}\bar{S}_{12}\right)(\bar{O}_{16}\bar{C}_{16} + \bar{C}_{16}\bar{O}_{16})\Big]\ .\end{aligned}$$

where $\pm$ refers to

$$c\binom{\xi_1}{\xi_2} \ = \ \pm 1$$

connected by : $\quad \mathrm{Z}_- = \mathrm{Z}_+ / a \otimes b\,,$

$$a = (-1)^{F_{\mathrm{L}}^{\mathrm{int}} + F_\xi^1}\,, \qquad\qquad b = (-1)^{F_{\mathrm{L}}^{\mathrm{int}} + F_\xi^2}\,.$$

Starting from:

$$Z_+ = (V_8 - S_8) \left(\sum_{m,n} \Lambda_{m,n} \right)^{\otimes 6} (\bar{O}_{16} + \bar{S}_{16}) (\bar{O}_{16} + \bar{S}_{16}) \ ,$$

where as usual, for each circle,

$$p^i_{\rm L,R} = \frac{m_i}{R_i} \pm \frac{n_i R_i}{\alpha'} \,,$$

and

$$\Lambda_{m,n} = \frac{q^{\frac{\alpha'}{4} p^2_{\rm L}} \, \bar{q}^{\frac{\alpha'}{4} p^2_{\rm R}}}{|\eta|^2} \,.$$

Add shifts : (A_1, A_1, A_1) , (A_3, A_3, A_3)

($48 \to 24$ yes)

($SO(12)$? no)

Uniquely:

$$g \ : (A_2, A_2, 0) \quad , \quad h \ : (0, A_2, A_2)\,,$$

where each A_2 acts on a complex coordinate

($48 \to 24$ yes)

($SO(12)$? yes)

$$R = \sqrt{\alpha'}$$

Higgs–Matter splitting: w Carlo Angelantonj, Mirian Tsulaia

$$\text{apply} \quad Z_2 \; \times \; Z_2' \; : \; g \; \times \; g' = (-1)^{F_{\xi^1}} \delta_1 \; \times \; (-1)^{F_{\xi^2}} \delta_1$$

$$\text{with} \quad \delta_1 X^9 = X_9 + \pi R_9 \; ,$$

$$\begin{aligned} Z_-^{9d} = (V_8 - S_8) \; [\quad & \Lambda_{2m,n} \qquad (\overline{O}_{16}\overline{O}_{16} + \overline{C}_{16}\overline{C}_{16}) \\ + \; & \Lambda_{2m+1,n} \quad (\overline{S}_{16}\overline{S}_{16} + \overline{V}_{16}\overline{V}_{16}) \\ + \; & \Lambda_{2m,n+\frac{1}{2}} \quad (\overline{S}_{16}\overline{V}_{16} + \overline{V}_{16}\overline{S}_{16}) \\ + \; & \Lambda_{2m+1,n+\frac{1}{2}} \, (\overline{O}_{16}\overline{C}_{16} + \overline{C}_{16}\overline{O}_{16}) \Big] \; . \end{aligned}$$

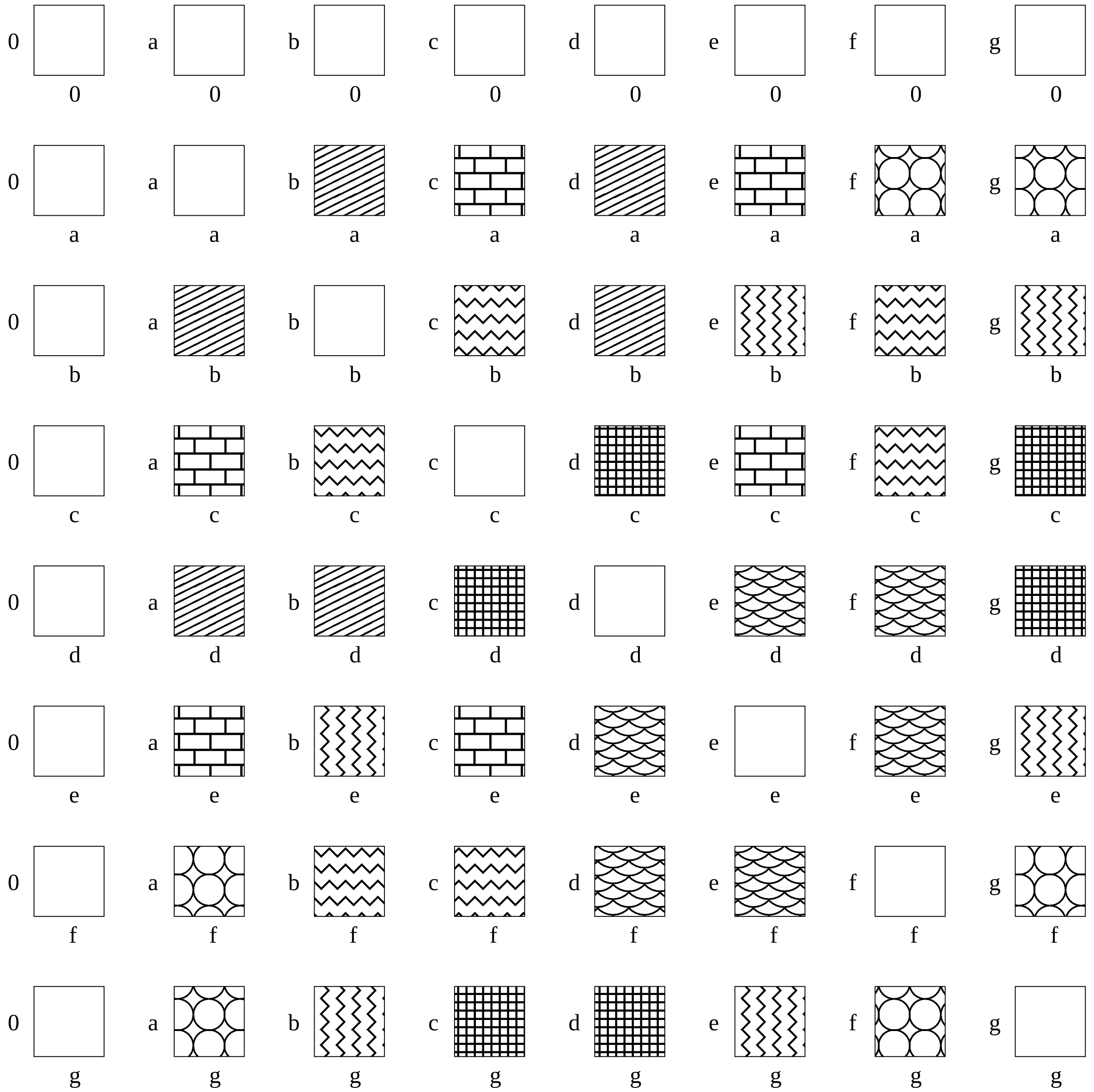
Add Z_2'' : $(x_4, x_5, x_6, x_7, x_8, x_9) \longrightarrow (+x_4, +x_5, -x_6, -x_7, -x_8, -x_9)$

Naively

$$\left(\frac{Z_+}{(Z_2 \times Z_2')}\right) / Z_2''$$

Produces massless vectorials. No massless spinorials. (Elisa Manno, 0908.3164)

$\Longrightarrow$ Analyze $$\left(\frac{Z_+}{(Z_g \times Z_{g'} \times Z_{g''})}\right)$$



eight independent orbits $\rightarrow$ ϵ_i $\quad i = 1, \cdots 7$ discrete torsions

- sector c

$$Q_c \left(\frac{1}{2} \left(\left| \frac{2\eta}{\theta_4} \right|^4 + \left| \frac{2\eta}{\theta_3} \right|^4 \right) \Lambda_{p,q} \left(\frac{\epsilon_{06}^- + \epsilon_{24}^+ (-1)^m}{2} \right) \Lambda_{m,n} \right) \overline{V}_{12} \overline{S}_4 \overline{O}_{16}$$

$$Q_c \left(\frac{1}{2} \left(\left| \frac{2\eta}{\theta_4} \right|^4 + \left| \frac{2\eta}{\theta_3} \right|^4 \right) \Lambda_{p,q} \left(\frac{\epsilon_{06}^+ - \epsilon_{24}^- (-1)^m}{2} \right) \Lambda_{m,n} \right) \overline{S}_{12} \overline{O}_4 \overline{O}_{16}$$

$$\epsilon_1 = +1,\ \epsilon_2 = -1,\ \epsilon_3 = +1,\ \epsilon_4 = +1,\ \epsilon_5 = -1,\ \epsilon_6 = +1,\ \epsilon_7 = -1,$$

No gauge enhancement. Spinors massless. Vectors Massive.

$$\epsilon_1 = +1,\ \epsilon_2 = +1,\ \epsilon_3 = +1,\ \epsilon_4 = +1,\ \epsilon_5 = +1,\ \epsilon_6 = -1,\ \epsilon_7 = +1,$$

No gauge enhancement. Spinors massive. Vectors Massless.

Spinor–Vector duality map $\{\epsilon_2, \epsilon_5, \epsilon_6, \epsilon_7\} \to - \{\epsilon_2, \epsilon_5, \epsilon_6, \epsilon_7\}$

Addionally:

$$32 \neq 2 \times 12$$

but

$$Q_c \left(\frac{1}{2} \left(\left| \frac{2\eta}{\theta_4} \right|^4 - \left| \frac{2\eta}{\theta_3} \right|^4 \right) \Lambda_{p,q} \left(\frac{\epsilon_{06}^{-} + \epsilon_{24}^{+} (-1)^m}{2} \right) \Lambda_{m,n} \overline{O}_{12} \overline{S}_4 \overline{O}_{16} \right)$$

$$\Longrightarrow \quad 32 = 2 \times (12 + 4)$$

Conclusions

Phenomenological string models produce interesting lessons

Higgs–matter splitting – A Cheshire Cat's Grin

Spinor–vector duality

All string vacua are equivalent/connected

String point of view: Organisation of low energy spectrum $\rightarrow$ secondary

Consistency $i.e.$ $number\ of\ d.o.f.$ $\rightarrow$ primary

Strings $\longrightarrow$ Duality $\longrightarrow$ Geometry ?