Minimal Standard Heterotic String Models

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- Recent results from free fermion heterotic–string model building

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DATA $\longrightarrow$ STANDARD MODEL

$SU(3) \times SU(2) \times U(1)_Y \longrightarrow SU(5) \longrightarrow SO(10)$

$$\left[\begin{pmatrix} \nu \\ e \end{pmatrix} + D^c_L \right] + \left[U^c_L + \begin{pmatrix} u \\ d \end{pmatrix} + E^c_L \right] + N^c_L$$

$$\bar{5} \quad + \quad 10 \quad\quad 1$$

$$\overline{}$$
$$16$$

STANDARD MODEL -> UNIFICATION

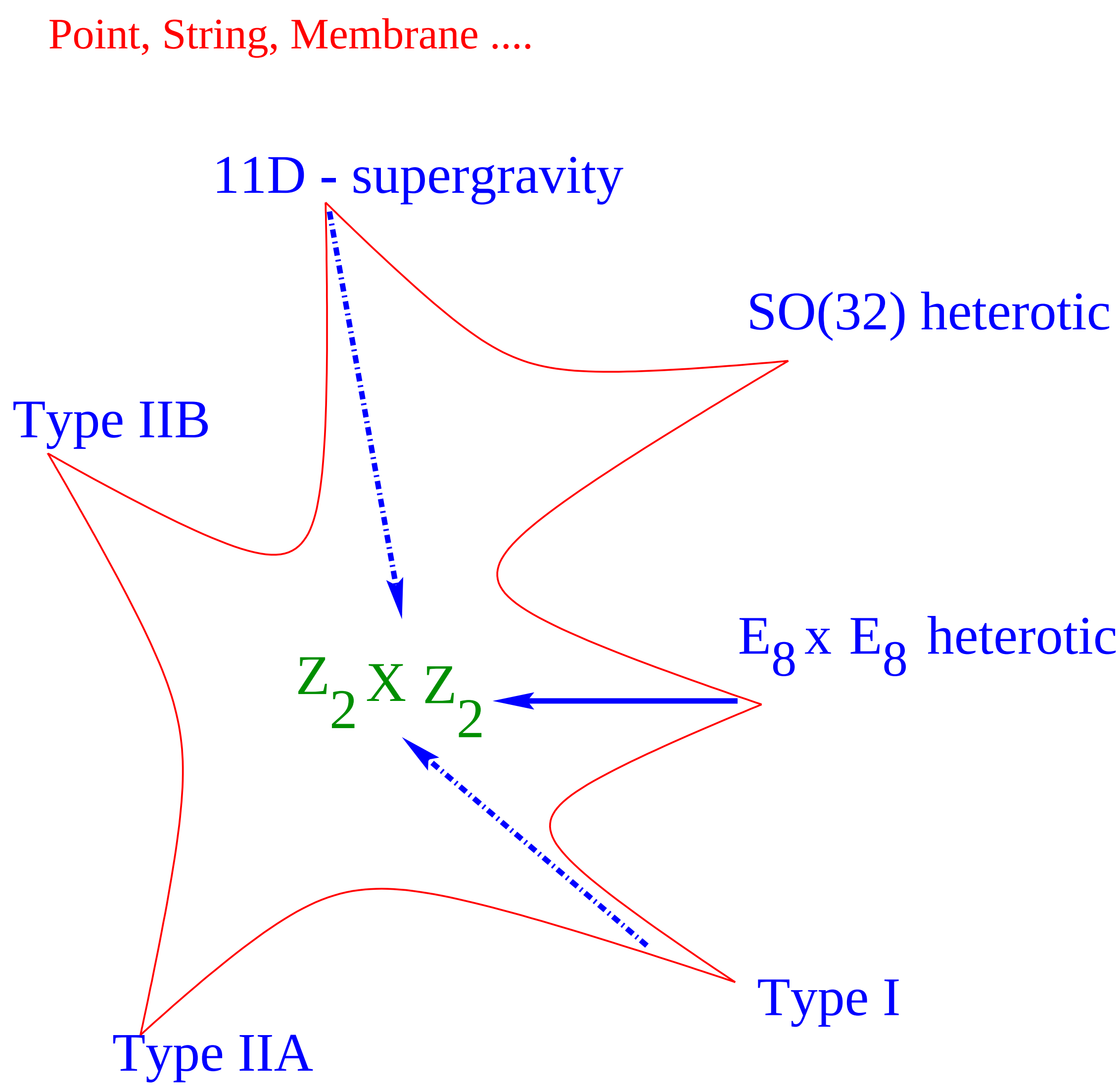
ADDITIONAL EVIDENCE:

Logarithmic running, proton longevity, neutrino masses

PRIMARY GUIDES:

3 generations

SO(10) embedding


Point, String, Membrane ....
11D - supergravity
SO(32) heterotic
Type IIB
E8 x E8 heterotic
Z2 X Z2
Type I
Type IIA

Realistic free fermionic models

'Phenomenology of the Standard Model and string unification'

- Top quark mass $\sim$ 175–180GeV — PLB 274 (1992) 47
- Generation mass hierarchy — NPB 407 (1993) 57
- CKM mixing — NPB 416 (1994) 63 (with Halyo)
- Stringy seesaw mechanism — PLB 307 (1993) 311 (with Halyo)
- Gauge coupling unification — NPB 457 (1993) 409 (with Dienes)
- Proton stability — NPB 428 (1994) 111
- Squark degeneracy — NPB 526 (1998) 21 (with Pati)
- Minimal Superstring Standard Model — PLB 455 (1999) 135 (with Cleaver & Nanopoulos)
- Moduli fixing — NPB 728 (2005) 83

Free Fermionic Construction

Left-Movers: $\psi^{\mu}_{1,2}, \quad \chi_i, \quad y_i, \ \omega_i \quad (i=1,\cdots,6)$

Right-Movers

$$\bar{\phi}_{A=1,\cdots,44} = \begin{cases} \bar{y}_i \ , \ \bar{\omega}_i & i=1,\cdots,6 \\ \\ \bar{\eta}_i & i=1,2,3 \\ \bar{\psi}_{1,\cdots,5} & \\ \bar{\phi}_{1,\cdots,8} & \end{cases}$$

$$f \longrightarrow -e^{i\pi\alpha(f)} f$$

$$V \longrightarrow V \qquad\qquad Z = \sum_{\substack{all\ spin \\ structures}} c\binom{\vec{\alpha}}{\vec{\beta}} Z\binom{\vec{\alpha}}{\vec{\beta}}$$

Models $\longleftrightarrow$ Basis vectors $+$ one–loop phases

The NAHE set : $\{ \mathbf{1}, \; S \;, \quad b_1 \;, \quad b_2 \;, \quad b_3 \; \}$

$$N = \quad 4 \quad \rightarrow \quad 2 \qquad\quad 1 \qquad\quad 1$$ vacua

$Z_2 \times Z_2$ orbifold compactification

$\Longrightarrow$ Gauge group $SO(10) \times SO(6)^{1,2,3} \times E_8$

beyond the NAHE set Add $\{\alpha, \beta, \gamma\}$

number of generations is reduced to three

$$SO(10) \quad \longrightarrow \quad SU(3) \times SU(2) \times U(1)_{T_{3_R}} \times U(1)_{B-L}$$

$$U(1)_Y \; = \; \tfrac{1}{2}(B-L) \; + \; T_{3_R} \; \in \; SO(10) \; !$$

$$SO(6)^{1,2,3} \quad \longrightarrow \quad U(1)^{1,2,3} \; \times \; U(1)^{1,2,3}$$

The massless spectrum

Three twisted generations $b_1, \quad b_2, \quad b_3$

Untwisted Higgs doublets

$$h_{1_{1,0,0}} \qquad \bar{h}_{1_{-1,0,0}}$$

$$h_{2_{0,1,0}} \qquad \bar{h}_{2_{0,-1,0}}$$

$$h_{3_{0,0,1}} \qquad \bar{h}_{3_{0,0,-1}}$$

"standard" $SO(10)$ representations

NAHE $+ \{ \alpha , \beta , \gamma \} \rightarrow$ exotic vector–like matter $\rightarrow$ superheavy

$\oplus$ Quasi-realistic phenomenology

Fermion mass hierarchy

Fermion mass terms

$$c g f_i f_j h \left(\frac{\langle\phi\rangle}{M}\right)^{N-3}$$

c - calculable coefficients g - gauge coupling

$$f_i,\, f_j \;\in\; b_j \quad j \;=\; 1, 2, 3$$

$h \rightarrow$ light Higgs multiplets

$M \;\sim\; 10^{18}\, GeV$

$\langle\phi\rangle$ generalized VEVs, several sources

Top quark mass prediction

only $\lambda_t = \langle t^c Q_t \bar{h}_1 \rangle = \sqrt{2} g \neq 0$ at $N = 3$

$$W_4 \longrightarrow b^c Q_b h_{\alpha\beta} \Phi_1 + \tau^c L_\tau h_{\alpha\beta} \Phi_1$$

$$\Longrightarrow \quad \lambda_b = \left(c_b \frac{\langle \phi \rangle}{M} \right) \qquad \lambda_\tau = \left(c_\tau \frac{\langle \phi \rangle}{M} \right)$$

$$\longrightarrow \quad \lambda_b = \lambda_\tau = 0.35 g^3 \sim \frac{1}{8} \lambda_t$$

Evolve λ_t , λ_b to low energies

$$m_t = \lambda_t v_1 = \lambda_t \frac{v_0}{\sqrt{2}} \sin\beta \qquad m_b = \lambda_b v_2 = \lambda_b \frac{v_0}{\sqrt{2}} \cos\beta$$

where $v_0 = \dfrac{2 m_W}{g_2(M_Z)} = 246 \mathrm{GeV}$ and $(v_1^2 + v_2^2) = \dfrac{v_0^2}{2}$

$$m_t = \lambda_t(m_t) \frac{v_0}{\sqrt{2}} \frac{\tan\beta}{(1 + \tan^2\beta)^{\frac{1}{2}}} \Longrightarrow \qquad m_t \sim 175 \mathrm{GeV} \quad \mathrm{PLB274(1992)47}$$

Exotics : (in FNY model, NPB 335 (1990) 347)

$$W_2 = \frac{1}{\sqrt{2}}\{ H_1H_2\phi_4 + H_3H_4\bar\phi_4 + H_5H_6\bar\phi_4 + (H_7H_8 + H_9H_{10})\phi'_4 + (H_{11} + H_{12})(H_{13} + H_{14})\bar\phi'_4 + V_{41}V_{42}\bar\phi_4 + V_{43}V_{44}\bar\phi_4 + V_{45}V_{46}\phi_4 + (V_{47}V_{48} + V_{49}V_{50})\bar\phi'_4 + V_{51}V_{52}\phi'_4\}$$

$\langle\, \bar\phi_4, \bar\phi'_4, \phi_4, \phi'_4 \,\rangle \rightarrow$ massive exotic states at N=3 (PRD46 (1993) 3204)

CFN $\rightarrow$ Classification of flat directions (PLB 455 (1999) 135)

All Standard Model charged states beyond MSSM $\rightarrow$ $\approx$ $M_{\rm string}$

MINIMAL STANDARD HETEROTIC STRING MODEL

John Rizos $\rightarrow$ Exophobic Pati–Salam Models

MINIMAL DOUBLET HIGGS CONTENT

(w Manno & Timirgaziu (SLM); Christodoulides (FSU5) in progress)

	ψ^μ	χ^{12}	χ^{34}	χ^{56}	y^3y^6	$y^4\bar{y}^4$	$y^5\bar{y}^5$	$\bar{y}^3\bar{y}^6$	$y^1\omega^5$	$y^2\bar{y}^2$	$\omega^6\bar{\omega}^6$	$\bar{y}^1\bar{\omega}^5$	$\omega^2\omega^4$	$\omega^1\bar{\omega}^1$	$\omega^3\bar{\omega}^3$	$\bar{\omega}^2\bar{\omega}^4$	$\bar{\psi}^{1,\dots,5}$	$\bar{\eta}^1$	$\bar{\eta}^2$	$\bar{\eta}^3$	$\bar{\phi}$
α	0	0	0	0	1	0	0	1	0	0	1	1	0	0	1	1	1 1 1 0 0	1	0	0	1 1 0
β	0	0	0	0	0	0	1	1	1	0	0	1	0	1	0	1	1 1 1 0 0	0	1	0	0 0 1
γ	0	0	0	0	0	1	0	0	0	1	0	0	1	0	0	0	$\frac{1}{2}$ $\frac{1}{2}$ $\frac{1}{2}$ $\frac{1}{2}$ $\frac{1}{2}$	$\frac{1}{2}$	$\frac{1}{2}$	$\frac{1}{2}$	0 0 0

SYMMETRIC $\leftrightarrow$ ASYMMETRIC

with respect to b_1 & b_2

$h_1,\ \bar{h}_1,\ D_1,\ \bar{D}_1$, $h_2,\ \bar{h}_2,\ D_2,\ \bar{D}_2$ are projected out

$h_3,\ \bar{h}_3$ remain in the spectrum

$$\lambda_t Q_3 t_3^c \bar{h}_3 \quad \text{with} \quad \lambda_t\ O(1)$$

No Phenomenologically viable flat directions

Cleaver, Faraggi, Manno, Timirgaziu, PRD 78 (2008) 046009

Classification of F and D flat directions in EMT reduced Higgs model

No D flat direction which is F-flat up to order eight in the superpotential

no stringent flat directions to all orders

Suggesting no supersymmetric flat directions in this model (class of models)

implying no supersymmetric moduli

only remaining perturbative moduli is the dilaton

quasi–realistic model: SLM; 3 gen; $SO(10)$ embed; Higgs & $\lambda_t \sim 1$; ...

vanishing one–loop partition function,

Fixed geometrical, twisted and SUSY moduli

Correspondence with $Z_2 \times Z_2$ orbifold

PLB 326 (1994) 62

NAHE $\oplus$ $(\xi_2 = \{\bar{\psi}_{1,\cdots,5}, \bar{\eta}^1, \bar{\eta}^2, \bar{\eta}^3\} = 1)$ $\rightarrow$ $\{1, S, \xi_1, \xi_2, b_1, b_2\}$

Gauge group: $SO(4)^3 \times E_6 \times U(1)^2 \times E_8$ and 24 generations.

toroidal compactification $(6_L + 6_R)$ g_{ij}, b_{ij}

$$g_{ij} = \begin{pmatrix} 2 & -1 & 0 & 0 & 0 & 0 \\ -1 & 2 & -1 & 0 & 0 & 0 \\ 0 & -1 & 2 & -1 & 0 & 0 \\ 0 & 0 & -1 & 2 & -1 & -1 \\ 0 & 0 & 0 & -1 & 2 & 0 \\ 0 & 0 & 0 & -1 & 0 & 2 \end{pmatrix} \qquad b_{ij} = \begin{cases} g_{ij} & i < j \\ 0 & i = j \\ -g_{ij} & i > j \end{cases}$$

R_i $\rightarrow$ the free fermionic point $\rightarrow$G.G. $SO(12) \times E_8 \times E_8$

mod out by a $Z_2 \times Z_2$ with standard embedding

$\Rightarrow SO(4)^3 \times E_6 \times U(1)^2 \times E_8$ with 24 generations

Exact correspondence

In the realistic free fermionic models

replace $X = \{\bar{\psi}^{1,\cdots,5}, \bar{\eta}^1, \bar{\eta}^2, \bar{\eta}^3\} = 1$

with $2\gamma = \{\bar{\psi}^{1,\cdots,5}, \bar{\eta}^1, \bar{\eta}^2, \bar{\eta}^3, \bar{\phi}^{1,\cdots,4}\} = 1$

Then $\{\vec{1}, \vec{S}, \vec{\xi}_1 = \vec{1} + \vec{b}_1 + \vec{b}_2 + \vec{b}_3, 2\gamma\} \rightarrow$ N=4 SUSY and

$SO(12) \times SO(16) \times SO(16)$

apply $b_1 \times b_2 \rightarrow Z_2 \times Z_2 \rightarrow$ N=1 SUSY and

$SO(4)^3 \times SO(10) \times U(1)^3 \times SO(16)$

b_1, b_2, b_3 $\Rightarrow (3 \times 8) \cdot 16$ of $SO(10)_O$

$b_1 + 2\gamma$, $b_2 + 2\gamma$, $b_3 + 2\gamma$ $\Rightarrow (3 \times 8) \cdot 16$ of $SO(16)_H$

Alternatively, $c\binom{\xi_1}{\xi_2} = +1 \rightarrow -1$

$Z_2 \times Z_2$ orbifolds

torus: One complex parameter $Z = Z + n e_1 + m e_2$

$T^2 \times T^2 \times T^2 \longrightarrow$ Three complex coordinates z_1, z_2 and z_3

Z_2 orbifold : $Z = -Z + \sum_i m_i e_i \longrightarrow$ 4 fixed points

$$Z = \{ 0 , 1/2\, e_1 , 1/2\, e_2 , 1/2\, (e_1 + e_2) \}$$

$$\frac{T^2 \times T^2 \times T^2}{Z_2 \times Z_2}$$

$\alpha : (z1 , z2 , z3) \to (-z1 , -z2 , +z3) \to 16$

$\beta : (z1 , z2 , z3) \to (+z1 , -z2 , -z3) \to 16$

$\alpha\beta : (z1 , z2 , z3) \to (-z1 , +z2 , -z3) \to 16$

48

↓

$\gamma : (z1, z2, z3) \to (z1+1/2, z2+1/2, z3+1/2) \longrightarrow$ 24

Classification of fermionic $Z_2 \times Z_2$ orbifolds (AEF, Kounnas, Rizos)

Basis vectors: consistent modular blocks 4,8 periodic fermions

$$1 = \{\psi^\mu,\ \chi^{1,\dots,6}, y^{1,\dots,6}, \omega^{1,\dots,6} \mid \bar{y}^{1,\dots,6}, \bar{\omega}^{1,\dots,6}, \bar{\eta}^{1,2,3}, \bar{\psi}^{1,\dots,5}, \bar{\phi}^{1,\dots,8}\}$$

$$S = \{\psi^\mu, \chi^{1,\dots,6}\},$$

$$z_1 = \{\bar{\phi}^{1,\dots,4}\},$$

$$z_2 = \{\bar{\phi}^{5,\dots,8}\},$$

$$e_i = \{y^i, \omega^i | \bar{y}^i, \bar{\omega}^i\},\ i = 1, \dots, 6,$$ $N = 4$ Vacua

$$b_1 = \{\chi^{34}, \chi^{56}, y^{34}, y^{56} | \bar{y}^{34}, \bar{y}^{56}, \bar{\eta}^1, \bar{\psi}^{1,\dots,5}\},$$ $N = 4 \to N = 2$

$$b_2 = \{\chi^{12}, \chi^{56}, y^{12}, y^{56} | \bar{y}^{12}, \bar{y}^{56}, \bar{\eta}^2, \bar{\psi}^{1,\dots,5}\},$$ $N = 2 \to N = 1$

Vector bosons: NS, $z_{1,2}$, $z_1 + z_2$, $x = 1 + s + \sum e_i + z_1 + z_2$

impose: Gauge group $SO(10) \times U(1)^3 \times \text{hidden}$

Independent phases $c{vi \brack v_j} \; = \; \exp[i\pi(v_i|v_j)]$: upper block

$$
\begin{array}{c}
 \\ 1 \\ S \\ e_1 \\ e_2 \\ e_3 \\ e_4 \\ e_5 \\ e_6 \\ z_1 \\ z_2 \\ b_1 \\ b_2
\end{array}
\begin{array}{cccccccccccc}
1 & S & e_1 & e_2 & e_3 & e_4 & e_5 & e_6 & z_1 & z_2 & b_1 & b_2 \\
-1 & -1 & \pm & \pm & \pm & \pm & \pm & \pm & \pm & \pm & \pm & \pm \\
 & & -1 & -1 & -1 & -1 & -1 & -1 & -1 & -1 & 1 & 1 \\
 & & & \pm & \pm & \pm & \pm & \pm & \pm & \pm & \pm & \pm \\
 & & & & \pm & \pm & \pm & \pm & \pm & \pm & \pm & \pm \\
 & & & & & \pm & \pm & \pm & \pm & \pm & \pm & \pm \\
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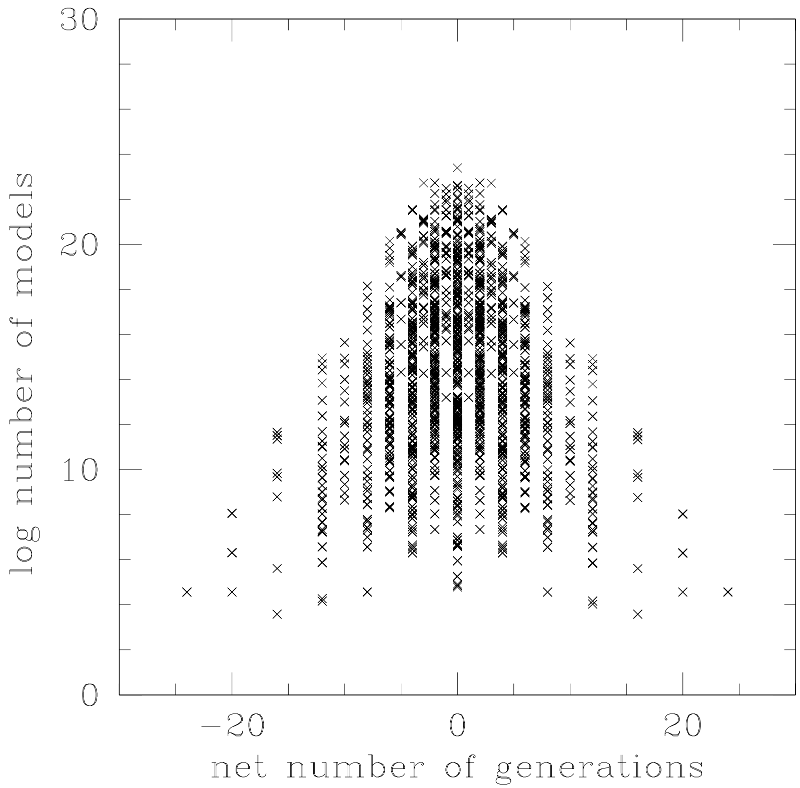
 & & & & & & & & & & \pm & \pm \\
 & & & & & & & & & & & \pm \\
 & & & & & & & & & & &
\end{array}
$$

Apriori 55 independent coefficients $\rightarrow$ 2^{55} distinct vacua

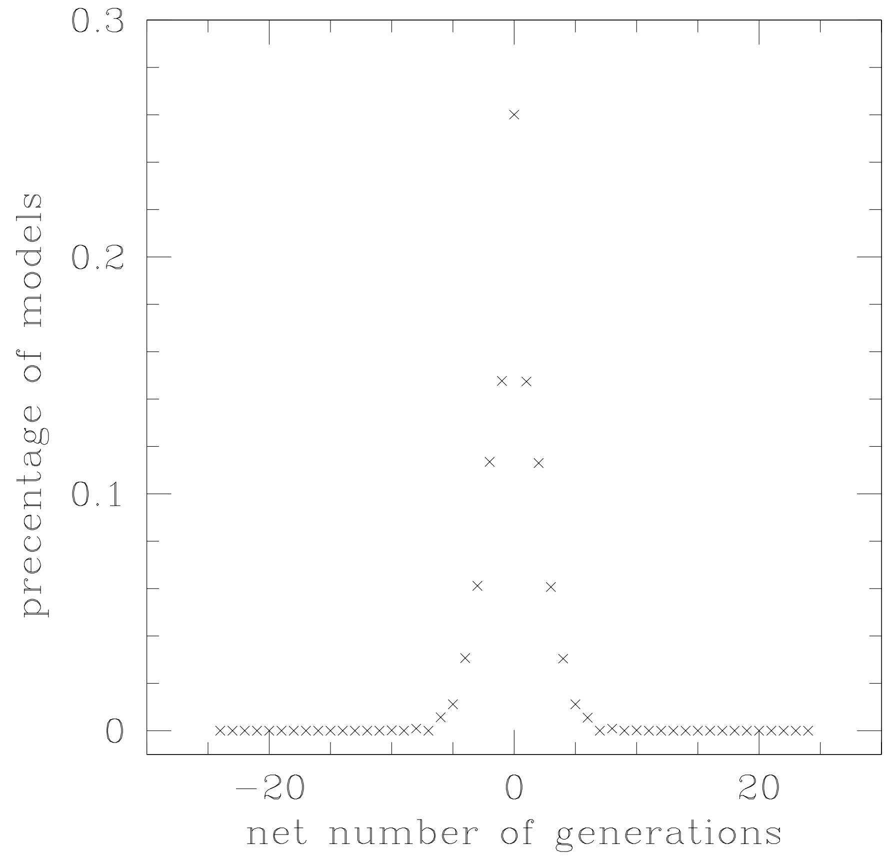
Impose: Gauge group $SO(10) \times U(1)^3 \times SO(8)^2$

$\longrightarrow$ 40 independent coefficients

RESULTS:



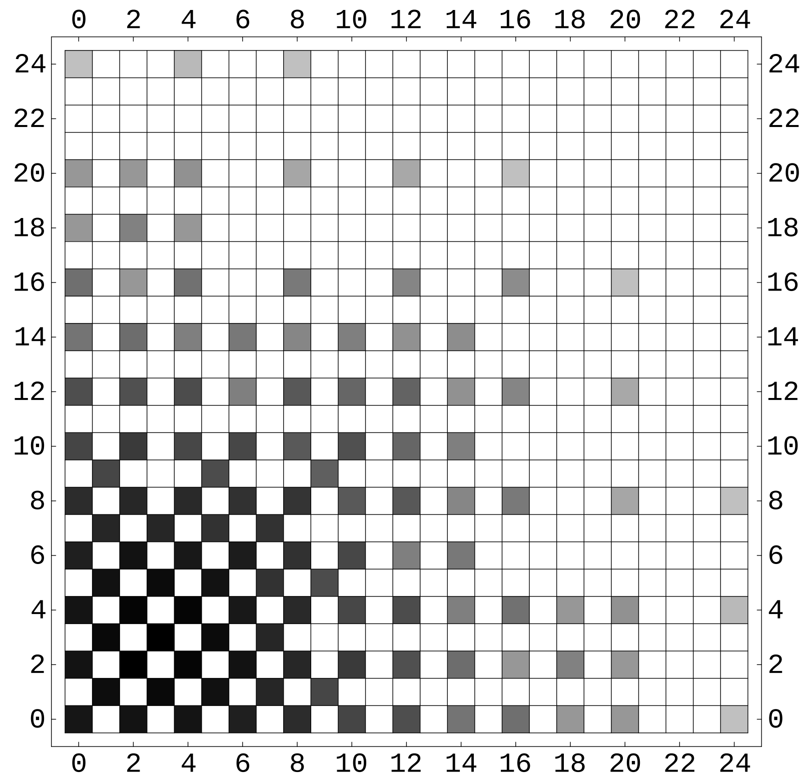
30
20
10
0
log number of models
−20
0
20
net number of generations



7×10^9 models $\sim$ 15% with 3 gen FKRI

Spinor–vector duality:

Invariance under exchange of $\#(16+\overline{16}) \;<->\; \#(10)$



Symmetric under exchange of rows and columns

$$E_6 \;:\qquad 27 = 16 + 10 + 1 \qquad \overline{27} = \overline{16} + 10 + 1$$

Self-dual: $\#(16+\overline{16}) \;=\; \#(10)$ without E_6 symmetry

NAHE-based partition functions: w Carlo Angelantonj, Mirian Tsulaia

Question:

$$\frac{T^6}{Z_2 \times Z_2} \;\rightarrow\; 48 \text{ fixed points} \qquad\qquad \frac{SO(12)}{Z_2 \times Z_2} \;\rightarrow\; 24 \text{ fixed points}$$

$$Z_2 \text{ shift } : 48 \longleftrightarrow 24$$

Is this the same model? In general, no.

shift that reproduces the $SO(12)$ lattice at the free fermionic point?

Possible shifts:

$$A_1 \;:\; X_{\rm L,R} \to X_{\rm L,R} + \tfrac{1}{2}\pi R \,,$$

$$A_2 \;:\; X_{\rm L,R} \to X_{\rm L,R} + \tfrac{1}{2}\left(\pi R \pm \frac{\pi\alpha'}{R}\right) \,,$$

$$A_3 \;:\; X_{\rm L,R} \to X_{\rm L,R} \pm \tfrac{1}{2}\frac{\pi\alpha'}{R} \,.$$

Using the level-one $\mathrm{SO}(2n)$ characters

$$O_{2n} = \tfrac{1}{2}\left(\frac{\vartheta_3^n}{\eta^n} + \frac{\vartheta_4^n}{\eta^n}\right) \,, \qquad V_{2n} = \tfrac{1}{2}\left(\frac{\vartheta_3^n}{\eta^n} - \frac{\vartheta_4^n}{\eta^n}\right) \,,$$

$$S_{2n} = \tfrac{1}{2}\left(\frac{\vartheta_2^n}{\eta^n} + i^{-n}\frac{\vartheta_1^n}{\eta^n}\right) \,, \qquad C_{2n} = \tfrac{1}{2}\left(\frac{\vartheta_2^n}{\eta^n} - i^{-n}\frac{\vartheta_1^n}{\eta^n}\right) \,.$$

The partition function of the heterotic string on $SO(12)$ lattice:

$$Z_+ = (V_8 - S_8)\left[|O_{12}|^2 + |V_{12}|^2 + |S_{12}|^2 + |C_{12}|^2\right](\bar{O}_{16} + \bar{S}_{16})(\bar{O}_{16} + \bar{S}_{16}) \; ,$$

and

$$\begin{aligned} Z_- \; = \; (V_8 - S_8) \Big[& \left(|O_{12}|^2 + |V_{12}|^2\right)(\bar{O}_{16}\bar{O}_{16} + \bar{C}_{16}\bar{C}_{16}) \\ & + \left(|S_{12}|^2 + |C_{12}|^2\right)(\bar{S}_{16}\bar{S}_{16} + \bar{V}_{16}\bar{V}_{16}) \\ & + (O_{12}\bar{V}_{12} + V_{12}\bar{O}_{12})(\bar{S}_{16}\bar{V}_{16} + \bar{V}_{16}\bar{S}_{16}) \\ & + (S_{12}\bar{C}_{12} + C_{12}\bar{S}_{12})(\bar{O}_{16}\bar{C}_{16} + \bar{C}_{16}\bar{O}_{16}) \Big] \; . \end{aligned}$$

where $\pm$ refers to

$$c\binom{\xi_1}{\xi_2} \; = \; \pm 1$$

connected by : $\qquad Z_- = Z_+ / a \otimes b \, ,$

$$a = (-1)^{F_{\rm L}^{\rm int} + F_\xi^1} \, , \qquad\qquad b = (-1)^{F_{\rm L}^{\rm int} + F_\xi^2} \, .$$

Starting from:

$$Z_+ = (V_8 - S_8)\left(\sum_{m,n} \Lambda_{m,n}\right)^{\otimes 6} (\bar{O}_{16} + \bar{S}_{16})\,(\bar{O}_{16} + \bar{S}_{16})\ ,$$

where as usual, for each circle,

$$p^i_{\mathrm{L,R}} = \frac{m_i}{R_i} \pm \frac{n_i R_i}{\alpha'}\,,$$

and

$$\Lambda_{m,n} = \frac{q^{\frac{\alpha'}{4}p^2_{\mathrm{L}}}\,\bar{q}^{\frac{\alpha'}{4}p^2_{\mathrm{R}}}}{|\eta|^2}\,.$$

Add shifts : (A_1, A_1, A_1) , (A_3, A_3, A_3)

($48 \to 24$ yes)

($SO(12)$? no)

Uniquely:

$$g \ : \ (A_2, A_2, 0)\,,$$

$$h \ : \ (0, A_2, A_2)\,,$$

where each A_2 acts on a complex coordinate

($48 \longrightarrow 24$ yes)

($SO(12)$? yes)

$$R = \sqrt{\alpha'}$$

Uniquely in 10D:

$$c\binom{\xi_1}{\xi_2} = -1 \quad \Longrightarrow \quad N=1 \rightarrow N=0 \ \text{SUSY}$$

Question : What is the relation between the $10D$ and $D<10$ cases?

Answer : $\rightarrow$

Interpolation among SUSY and non–SUSY vacua

Start with: $$Z_+ = (V_8 - S_8) \left(\sum_{m,n} \Lambda_{m,n} \right)^{\otimes 6} \left(\overline{O}_{16} + \overline{S}_{16} \right) \left(\overline{O}_{16} + \overline{S}_{16} \right)$$

apply $$Z_2 \times Z_2' : g \times g' = (-1)^{F_{\xi^1}} \delta_1 \times (-1)^{F_{\xi^2}} \delta_1$$

with $$\delta_1 X^9 = X_9 + \pi R_9 ,$$

$$\begin{aligned} Z_-^{9d} = (V_8 - S_8) \, [\quad & \Lambda_{2m,n} \left(\overline{O}_{16} \overline{O}_{16} + \overline{C}_{16} \overline{C}_{16} \right) \\ + \; & \Lambda_{2m+1,n} \left(\overline{S}_{16} \overline{S}_{16} + \overline{V}_{16} \overline{V}_{16} \right) \\ + \; & \Lambda_{2m,n+\frac{1}{2}} \left(\overline{S}_{16} \overline{V}_{16} + \overline{V}_{16} \overline{S}_{16} \right) \\ + \; & \Lambda_{2m+1,n+\frac{1}{2}} \left(\overline{O}_{16} \overline{C}_{16} + \overline{C}_{16} \overline{O}_{16} \right) \Big] \; . \end{aligned}$$

Next, compactify on S^1_2 moded by

$$Z''_2 \; : \; g'' = (-1)^{F+F_{\xi^1}+F_{\xi^2}} \delta_2$$

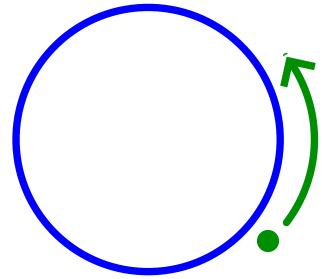
with $\delta_2 \; : \; X_8 \; \rightarrow \; X_8 + \pi R_8$

$\lim R_8 \rightarrow 0 \Rightarrow$ non–SUSY $(SO(16) \times SO(16))/(Z_2 \times Z'_2)$

$\lim R_8 \rightarrow \infty \Rightarrow$ SUSY $(SO(16) \times SO(16))/(Z_2 \times Z'_2)$

T1 − COMPACTIFICATION

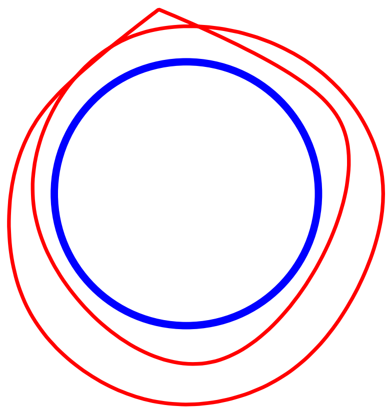
$X \longrightarrow X \sim X + 2\pi R m$



Point particle

$\Psi \sim \mathrm{Exp}(i\,P\,X) \Rightarrow P = \frac{m}{R}$

String



$$P_{L,R} = \frac{m}{R} \pm \frac{n R}{\alpha'}$$

T − DUALITY

$$\text{mass}^2 = \left(\frac{n}{R}\right)^2 + \left(\frac{m\,R}{\alpha'}\right)^2$$

Invariant under

$$\frac{1}{R} \longleftrightarrow \frac{R}{\alpha'} \quad \text{with} \quad m \longleftrightarrow n$$

An exact symmetry in string perturbation theory!

Self−dual point $R = \frac{\alpha'}{R}$ = free fermionic point

Conclusions

Phenomenological string models produce interesting lessons

Spinor–vector duality

relevance of non–standard geometries

Free Fermionic Models $\longrightarrow$ $Z_2 \times Z_2$ orbifold near the self–dual point

Duality & Self–Duality $\Leftrightarrow$ String Vacuum Selection