Mirror Symmetry and Spinor–Vector Duality

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- Torodial lattices of heterotic–strings: G, B, W
- Mirror symmetry: $G, B \rightarrow \tilde{G}, \tilde{B}$

  Kähler $\leftrightarrow$ Complex structure moduli $\qquad \chi \leftrightarrow -\chi$
- Spinor–vector duality: $W \rightarrow \tilde{W}$

AEF, C Kounnas, J Rizos, PLB2007; NPB2007
C Angelantonj, AEF, M. Tsulaia, JHEP
AEF, I Florakis, T Mohaupt, M Tsulaia NPB2011
AEF, S Groot–Nibbelink, M Hurtado Heredia, PRD2021; NPB2021; ...

String theory journal club, CERN, 24 March 2023

Fermionic $Z_2 \times Z_2$ orbifolds

'Phenomenology of the Standard Model and Unification'

- Minimal Superstring Standard Model — NPB 335 (1990) 347 (with Nanopoulos & Yuan)
- Top quark mass $\sim$ 175–180GeV — PLB 274 (1992) 47
- Generation mass hierarchy — NPB 407 (1993) 57
- CKM mixing — NPB 416 (1994) 63 (with Halyo)
- Stringy seesaw mechanism — PLB 307 (1993) 311 (with Halyo)
- Gauge coupling unification — NPB 457 (1995) 409 (with Dienes)
- Proton stability — NPB 428 (1994) 111
- Squark degeneracy — NPB 526 (1998) 21 (with Pati)
- Moduli fixing — NPB 728 (2005) 83
- Classification — 2003 – $\cdots$ (with Kounnas, Rizos & ... Percival, Matyas)

$Z_2 \times Z_2$ orbifolds

torus: One complex parameter $Z = Z + n\, e_1 + m\, e_2$

$T^2 \times T^2 \times T^2 \longrightarrow$ Three complex coordinates z_1, z_2 and z_3

Z_2 orbifold : $Z = -Z + \sum_i m_i e_i \longrightarrow$ 4 fixed points

$$Z = \{ 0 , 1/2\, e_1 , 1/2\, e_2 , 1/2 (e_1 + e_2) \}$$

$$\frac{T^2 \times T^2 \times T^2}{Z_2 \times Z_2}$$

$\alpha : (z1 , z2 , z3) \to (-z1 , -z2 , +z3) \to 16$

$\beta : (z1 , z2 , z3) \to (+z1 , -z2 , -z3) \to 16$

$\alpha\beta : (z1 , z2 , z3) \to (-z1 , +z2 , -z3) \to 16$

48

↓

$\gamma : (z1, z2, z3) \to (z1+1/2, z2+1/2, z3+1/2) \longrightarrow 24$

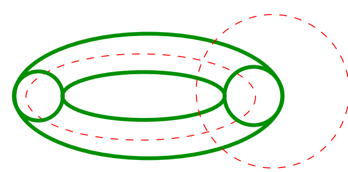
Classification: Donagi, AEF (2004); Donagi, Wendland (2008)

Fermionic Construction

Left-Movers: $\psi^{\mu=1,2}$, χ_i, y_i, ω_i $(i = 1, \cdots, 6)$

Right-Movers

$$\bar{\phi}_{A=1,\cdots,44} = \begin{cases} \bar{y}_i \ , \ \bar{\omega}_i & i = 1, \cdots, 6 \\ \\ \bar{\eta}_i & U(1)_i \quad i = 1, 2, 3 \\ \bar{\psi}_{1,\cdots,5} & SO(10) \\ \bar{\phi}_{1,\cdots,8} & SO(16) \end{cases}$$



$V \longrightarrow V$ $\qquad f \longrightarrow -e^{i\pi\alpha(f)} f$

$$Z = \sum_{\substack{all\ spin \\ structures}} c\binom{\vec{\alpha}}{\vec{\beta}}\ Z\binom{\vec{\alpha}}{\vec{\beta}}$$

Models $\longleftrightarrow$ Basis vectors $+$ one–loop phases

Model building – Construction of the physical states

$$b_j \quad j=1,\cdots,N \qquad \to \qquad \Xi \;=\; \sum_j n_j b_j$$

For $\vec{\alpha} \;=\; (\vec{\alpha}_L;\vec{\alpha}_R) \;\in\; \Xi \quad \Rightarrow \quad \mathrm{H}_{\vec{\alpha}}$

$$\alpha(f)=1 \quad \Rightarrow \quad |\pm\rangle \quad ; \quad \alpha(f)\neq 1 \quad \Rightarrow \quad f|0\rangle, f^*|0\rangle \quad , \quad \nu_{f,f^*}=\frac{1\mp\alpha(f)}{2}$$

$$M_L^2=-\frac{1}{2}+\frac{\vec{\alpha}_L\cdot\vec{\alpha}_L}{8}+N_L=-1+\frac{\vec{\alpha}_R\cdot\vec{\alpha}_R}{8}+N_R=M_R^2 \;\;(\;\equiv\;\; 0\;)$$

<u>GSO projections</u>

$$e^{i\pi(\vec{b}_i\cdot\vec{F}_\alpha)}|s\rangle_{\vec{\alpha}}=\delta_\alpha c^*\begin{pmatrix}\vec{\alpha}\\ \vec{b}_i\end{pmatrix}|s\rangle_{\vec{\alpha}}$$

$F_\alpha(f) \;\to\;$ fermion # operator $\quad = \begin{cases}-1, & |-\rangle\\ 0, & |+\rangle\end{cases} \quad = \begin{cases}+1, & f\\ -1, & f^*\end{cases}$

$Q(f)=\frac{1}{2}\alpha(f)+F(f) \quad \to \quad$ U(1) charges

Example : $\vec{\alpha} = \vec{S} = (\underbrace{1, \cdots, 1}_{\psi^{\mu}_{12}, \chi^{12}, \chi^{34}, \chi^{56}}, 0, \cdots, 0 | 0, \cdots, 0).$

$$(\vec{S}_L \cdot \vec{S}_L = 4 \quad \vec{S}_R \cdot \vec{S}_R = 0)$$

For $\alpha(f) = 1 \rightarrow$ periodic BC $\Rightarrow F : |\pm\rangle = \begin{cases} -1, & F : |-\rangle \\ 0, & F : |+\rangle \end{cases}$

otherwise $F(f|0\rangle; f^*|0\rangle) = \pm 1|0\rangle \quad \nu_{f;f^*} = \frac{1 \pm \alpha(f)}{2}$

Mass formula $M_L^2 = -\frac{1}{2} + \frac{4}{8} + N_L = -1 + \frac{0}{8} + N_R = M_R^2$

$$\nu_f = \frac{1 \pm 0}{2} = \frac{1}{2} \quad \Rightarrow \quad N_R = \frac{1}{2} + \frac{1}{2} = 1$$

$|S\rangle_S = |D\rangle_L \bar{\phi}_{\frac{1}{2}} \bar{\phi}_{\frac{1}{2}} |0\rangle_R \qquad |D\rangle_L = \left[\binom{4}{0} + \binom{4}{1} + \binom{4}{2} + \binom{4}{3} + \binom{4}{4} \right]$

apply GSO projections : $e^{i\pi \vec{S} \cdot \vec{F}_S} |S\rangle_S = \delta_S c^* \binom{S}{S} |S\rangle_S = \pm |S\rangle_S$

$$\Rightarrow \left[\binom{4}{0} + \binom{4}{2} + \binom{4}{4} \right]_+ \quad \text{or} \quad \left[\binom{4}{1} + \binom{4}{3} \right]_-$$

$$Q(\bar{f}) = \frac{1}{2} \cdot 0 \pm 1 = \pm 1$$

Classification of fermionic $Z_2 \times Z_2$ orbifolds

Basis vectors:

$$
\begin{aligned}
1 &= \{\psi^\mu,\ \chi^{1,\dots,6}, y^{1,\dots,6}, \omega^{1,\dots,6} \mid \bar{y}^{1,\dots,6}, \bar{\omega}^{1,\dots,6}, \bar{\eta}^{1,2,3}, \bar{\psi}^{1,\dots,5}, \bar{\phi}^{1,\dots,8}\} \\
S &= \{\psi^\mu, \chi^{1,\dots,6}\}, \\
z_1 &= \{\bar{\phi}^{1,\dots,4}\}, \\
z_2 &= \{\bar{\phi}^{5,\dots,8}\}, \\
e_i &= \{y^i, \omega^i | \bar{y}^i, \bar{\omega}^i\},\ i = 1, \dots, 6, \qquad N = 4 \text{ Vacua} \\
b_1 &= \{\chi^{34}, \chi^{56}, y^{34}, y^{56} | \bar{y}^{34}, \bar{y}^{56}, \bar{\eta}^1, \bar{\psi}^{1,\dots,5}\}, \qquad N = 4 \to N = 2 \\
b_2 &= \{\chi^{12}, \chi^{56}, y^{12}, y^{56} | \bar{y}^{12}, \bar{y}^{56}, \bar{\eta}^2, \bar{\psi}^{1,\dots,5}\}, \qquad N = 2 \to N = 1
\end{aligned}
$$

Vector bosons: NS, $z_{1,2}$, $z_1 + z_2$, $\quad X = 1 + s + \sum e_i + z_1 + z_2$

impose: $c\left[{z_1 \atop z_2}\right] \ = \ -1$ $\quad$ & $\quad$ Gauge group $SO(10) \times U(1)^3 \times \mathrm{hidden}$

Independent phases $c\begin{bmatrix} v_i \\ v_j \end{bmatrix} = \exp[i\pi(v_i|v_j)]$: upper block

$$
\begin{array}{c} \\ 1 \\ S \\ e_1 \\ e_2 \\ e_3 \\ e_4 \\ e_5 \\ e_6 \\ z_1 \\ z_2 \\ b_1 \\ b_2 \end{array}
\begin{array}{c}
\begin{array}{cccccccccccc} 1 & S & e_1 & e_2 & e_3 & e_4 & e_5 & e_6 & z_1 & z_2 & b_1 & b_2 \end{array} \\
\begin{pmatrix}
-1 & -1 & \pm & \pm & \pm & \pm & \pm & \pm & \pm & \pm & \pm & \pm \\
 & & -1 & -1 & -1 & -1 & -1 & -1 & -1 & -1 & 1 & 1 \\
 & & & \pm & \pm & \pm & \pm & \pm & \pm & \pm & \pm & \pm \\
 & & & & \pm & \pm & \pm & \pm & \pm & \pm & \pm & \pm \\
 & & & & & \pm & \pm & \pm & \pm & \pm & \pm & \pm \\
 & & & & & & \pm & \pm & \pm & \pm & \pm & \pm \\
 & & & & & & & \pm & \pm & \pm & \pm & \pm \\
 & & & & & & & & \pm & \pm & \pm & \pm \\
 & & & & & & & & & \pm & \pm & \pm \\
 & & & & & & & & & & \pm & \pm \\
 & & & & & & & & & & & \pm \\
 & & & & & & & & & & & -1
\end{pmatrix}
\end{array}
$$

A priori 55 independent coefficients $\rightarrow$ 2^{55} distinct vacua

PLB2021, Percival *et al* $\rightarrow$ Satisfiability Modulo Theories $\longrightarrow$ $t \times 10^{-3}$

Mirror symmetry

Enhance $SO(10) \to E_6$

from $X = 1 + S + \sum_i e_i + z_1 + z_2 = \{\bar{\psi}^{1,\cdots,5}, \bar{\eta}^{1,2,3}\}$

Euler characteristic $\chi = \#(\mathbf{27} - \overline{\mathbf{27}}) \longrightarrow -\chi$

Exchanges complex and Kähler structure moduli

Moduli of the internal compactified space

Vafa–Witten 1994: Mirror symmetry in terms of discrete torsion

$$c\binom{b_1}{b_2} = +1 \to c\binom{b_1}{b_2} = -1$$

Spinor–vector duality: AEF, Kounnas, Rizos, NPB774 (2007) 208

Duality under exchange of spinors and vectors.

First Plane			Second plane			Third Plane			
s	$\bar{s}$	v	s	$\bar{s}$	v	s	$\bar{s}$	v	# of models
2	0	0	0	0	0	0	0	0	1325963712
0	2	0	0	0	0	0	0	0	1340075584
1	1	0	0	0	0	0	0	0	3718991872
0	0	2	0	0	0	0	0	0	6385031168

# of models with $\#(16+\overline{16}) = \#$ of models with $\#(10)$

For every model with $\#(16+\overline{16})$ & $\#(10)$

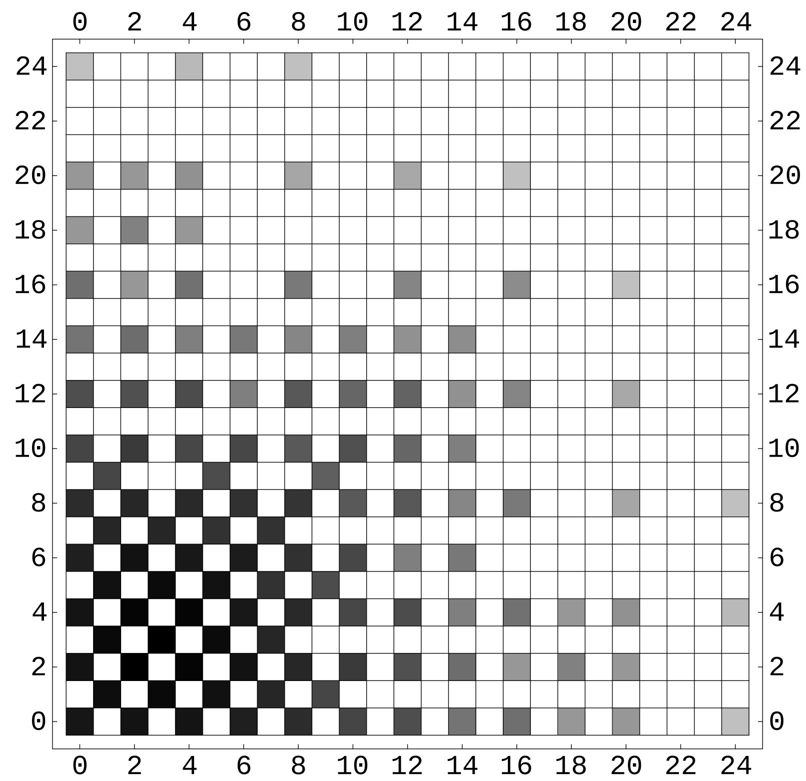
There exist another model in which they are interchanged

Reflects discrete exchange of phases

	0	1	2	3	4	5	6	7	8
0	14424168320	0	19155093504	0	17251226688	0	5722036224	0	16635982
1	0	35042893824	0	54063267840	0	24984354816	0	3050569728	0
2	19155093504	0	138128891904	0	80400635904	0	22541905920	0	25932533
3	0	54063267840	0	128713392128	0	43913576448	0	3064725504	0
4	17251226688	0	80400635904	0	78871289088	0	11554105344	0	22462053
5	0	24984354816	0	43913576448	0	21663891456	0	856424448	0
6	5722036224	0	22541905920	0	11554105344	0	8043915264	0	9377280
7	0	3050569728	0	3064725504	0	856424448	0	866942976	0
8	1663598208	0	2593253376	0	2246205312	0	937728000	0	7030003
9	0	113541120	0	0	0	67829760	0	0	0
10	135948288	0	406695936	0	107403264	0	104902656	0	1746739
11	0	0	0	0	0	0	0	0	0
12	50867584	0	42448896	0	65853312	0	387072	0	1859020
13	0	0	0	0	0	0	0	0	0
14	1210368	0	2420736	0	387072	0	774144	0	202752
15	0	0	0	0	0	0	0	0	0
16	1854336	0	36864	0	1514688	0	0	0	714816
17	0	0	0	0	0	0	0	0	0
18	36864	0	313344	0	36864	0	0	0	0
19	0	0	0	0	0	0	0	0	0
20	33984	0	36864	0	62784	0	0	0	7680
21	0	0	0	0	0	0	0	0	0
22	0	0	0	0	0	0	0	0	0
23	0	0	0	0	0	0	0	0	0
24	576	0	0	0	1152	0	0	0	576

Spinor–vector duality:

Invariance under exchange of $\#(16+\overline{16}) \;\; <-> \;\; \#(10)$



Symmetric under exchange of rows and columns

$$E_6 \;\; : \qquad\qquad 27 = 16 + 10 + 1 \qquad\qquad \overline{27} = \overline{16} + 10 + 1$$

Self-dual: $\#(16+\overline{16}) \;\; = \;\; \#(10)$ without E_6 symmetry

Using the level-one $\mathrm{SO}(2n)$ characters:

$$O_{2n} = \frac{1}{2}\left(\frac{\theta_3^n}{\eta^n} + \frac{\theta_4^n}{\eta^n}\right) , \qquad V_{2n} = \frac{1}{2}\left(\frac{\theta_3^n}{\eta^n} - \frac{\theta_4^n}{\eta^n}\right) ,$$

$$S_{2n} = \frac{1}{2}\left(\frac{\theta_2^n}{\eta^n} + i^{-n}\frac{\theta_1^n}{\eta^n}\right) , \qquad C_{2n} = \frac{1}{2}\left(\frac{\theta_2^n}{\eta^n} - i^{-n}\frac{\theta_1^n}{\eta^n}\right) .$$

where

$$\theta_3 \equiv Z_f\binom{0}{0} \quad \theta_4 \equiv Z_f\binom{0}{1}$$

$$\theta_2 \equiv Z_f\binom{1}{0} \quad \theta_1 \equiv Z_f\binom{1}{1}$$

Spinor–Vector duality in Orbifolds:

Starting from: $$Z_+ = (V_8 - S_8) \left(\sum_{m,n} \Lambda_{m,n} \right)^{\otimes 6} E_8 \times E_8 \,,$$

apply $Z_2 \times Z_2' \; : \; g \times g'$

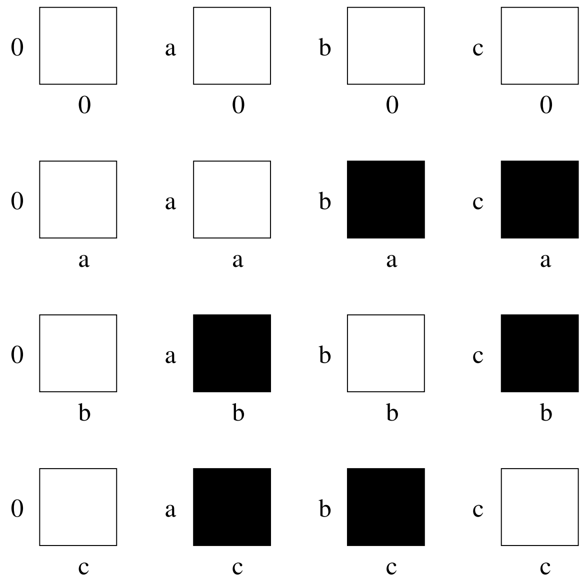
$$g \; : \; (0^7, 1|1, 0^7) \; \rightarrow \; \text{Wilson line} \; \rightarrow \; E_8 \times E_8 \rightarrow SO(16) \times SO(16)$$

$$g' \; : \; (x_4, x_5, x_6, x_7, x_8, x_9) \; \longrightarrow \; (-x_4, -x_5, -x_6, -x_7, +x_8, +x_9)$$

Note: A single space twisting Z_2' $\Rightarrow$ $N = 4 \rightarrow N = 2$

$$E_7 \rightarrow SO(12) \times SU(2)$$

$\Rightarrow$ Analyze $Z = \left(\frac{Z_+}{Z_g \times Z_{g'}} \right) = \left[\frac{(1+g)}{2} \frac{(1+g')}{2} \right] Z_+$



a = g ; b = g' ; c = gg'

P.F. = (□ + ε ■) = $\Lambda_{m,n}$ • ( ) + $\Lambda_{m,n+1/2}$ • ( )

ε = ± 1

massless massive

• sector b

$$\Lambda_{p,q}\left\{\frac{1}{2}\left(\left|\frac{2\eta}{\theta_4}\right|^4+\left|\frac{2\eta}{\theta_3}\right|^4\right)\left[P_\epsilon^+ Q_s\overline{V}_{12}\overline{C}_4\overline{O}_{16}+P_\epsilon^- Q_s\overline{S}_{12}\overline{O}_4\overline{O}_{16}\right]+\right.$$
$$\left.\frac{1}{2}\left(\left|\frac{2\eta}{\theta_4}\right|^4-\left|\frac{2\eta}{\theta_3}\right|^4\right)\left[P_\epsilon^+ Q_s\overline{O}_{12}\overline{S}_4\overline{O}_{16}\right]\right\}\quad+\quad\text{massive}$$

where

$$P_\epsilon^+=\left(\frac{1+\epsilon(-1)^m}{2}\right)\Lambda_{m,n}\quad P_\epsilon^-=\left(\frac{1-\epsilon(-1)^m}{2}\right)\Lambda_{m,n}$$

$$\epsilon=+1\;\Rightarrow\;P_\epsilon^+=\Lambda_{2m,n}\qquad P_\epsilon^-=\Lambda_{2m+1,n}$$

$$\epsilon=-1\;\Rightarrow\;P_\epsilon^+=\Lambda_{2m+1,n}\qquad P_\epsilon^-=\Lambda_{2m,n}$$

and $12\cdot 2\;+\;4\cdot 2\;=\;32$

Spinor-Vector duality on Calabi–Yau threefolds with vector bundles :

- From the "Land" to the "Swamp" w Groot–Nibellink & Hurtado-Heredia, arXiv:2103.13442, spinor–vector duality on a resolved orbifold. The role of the discrete torsion in the effective field theory limit

- Vafa–Witten 1994, the role of a discrete torsion in the $Z_2 \times Z_2$ orbifold in mirror symmetry

- In similar spirit $\rightarrow$ the imprint of the worldsheet modular properties in the effective field theory limit

Interactions

correlators between vertex operators $\langle V_1^f V_2^f V_3^b \cdots \cdot V_N^b \rangle$

Vertex operators

$$V^f_{(-\frac{1}{2})} = \mathrm{e}^{(-\frac{c}{2})}\ \mathcal{L}^\ell\ \mathrm{e}^{(i\alpha\chi_{12})}\ \mathrm{e}^{(i\beta\chi_{34})}\ \mathrm{e}^{(i\gamma\chi_{56})}$$

$$\left(\prod_j \mathrm{e}^{(iq_i\zeta_j)}\ \ \{\sigma' s\} \prod_j \mathrm{e}^{(i\bar{q}_i\bar{\zeta}_j)} \right)$$

$$\mathrm{e}^{(i\bar{\alpha}\bar{\eta}_1)}\ \mathrm{e}^{(i\bar{\beta}\bar{\eta}_2)}\ \mathrm{e}^{(i\bar{\gamma}\bar{\eta}_3)}\ \mathrm{e}^{(iW_R\cdot\bar{J})}\mathrm{e}^{(i\frac{1}{2}KX)}\ \mathrm{e}^{(\mathrm{i}\frac{1}{2}K\cdot\bar{X})}$$

Non–vanishing correlators $\longrightarrow$ invariant under all the string symmetries

Mirror symmetry $\quad\longrightarrow\quad 27\cdot 27\cdot 27 \longleftrightarrow \overline{27}\cdot\overline{27}\cdot\overline{27}$

Mathematical implications

- On Calabi–Yau threefolds, the couplings correspond to intersections of curves $\longleftrightarrow$ rational curves on CY manifolds
- mirror symmetry is instrumental in counting of rational curves on CY 3–folds $\longleftrightarrow$ instrumental in enumerative geometry
- A tool developed for that purpose are the Gromov–Witten invariants

Questions

- Perform a similar analysis of correlators on spinor–vector dual vacua;
- What are the analogue of the Gromov–Witten invariants in the case of spinor–vector duality
- spinor–vector duality $\longrightarrow$ a tool to study CY manifolds with bundles
- moduli spaces of $(2,0)$ string compactifications
- Is it complete? Is it constraining the viable effective field limit of stringy quantum gravity.
- ...

A Swamplane Conjecture(?): AEF, EPJC 79 (2019) 703

- Every EFT $(2,0)$ heterotic–string compactification has to be connected to a (2,2) heterotic–string compactification by an orbifold or by continuous interpolation. If not $\rightarrow$ it is in the swampland

- Completeness?!

Novel Basis

$$
\begin{aligned}
S &= \{\psi^\mu, \chi^{1,\dots,6}\},\\
z_1 &= \{\bar\phi^{1,\dots,4}\},\\
z_2 &= \{\bar\phi^{5,\dots,8}\},\\
z_3 &= \{\bar\psi^{1,\dots,4}\},\\
z_4 &= \{\bar\eta^{0,\dots,3}\},\\
e_i &= \{y^i, \omega^i | \bar y^i, \bar\omega^i\},\ i = 1, \dots, 6,
\end{aligned}
$$

$\bar\eta^0 \equiv \bar\psi^5$

$N = 4$ Vacua

$$
\begin{aligned}
1 &= S + \sum e_i + z_1 + z_2 + z_3 + z_4\\
b_1 &= \{\chi^{34}, \chi^{56}, y^{34}, y^{56} | \bar y^{34}, \bar y^{56}, \bar\eta^2, \bar\eta^3\},
\end{aligned}
$$

$N = 4 \to N = 2$

Vector bosons: NS, $z_{1,2,3,4}$, $z_i + z_j$

NS $\leftrightarrow$ $SO(8)^4$ $\to$ $SO(8) \times SO(4) \times SO(4) \times SO(8)^2$

$SO(12)$–GUT $\to$ from enhancement

Duality picture is facilitated

$SO(12)$ enhancement $\longrightarrow$ $B \longleftrightarrow B + z_3$

Spinor $\longleftrightarrow$ Vector map $\longrightarrow$ $B \longleftrightarrow B + z_4$

$z_4 \rightarrow$ right–moving spectral flow operator $\rightarrow$ “modular map”

The picture extends to compactifications with Interacting Internal CFT

(P. Athanasopoulos, AEF, D. Gepner, PLB 735 (2014) 357)

"Modular maps" in two dimensions

Left-Movers: $\chi_i, \quad y_i, \quad \omega_i \quad (i = 1, \cdots, 8)$

Right-Movers

$$\bar{\phi}_{A=1,\cdots,48} = \begin{cases} \bar{y}_i \;,\; \bar{\omega}_i & i = 1, \cdots, 8 \\ & \\ \bar{\eta}_i & i = 0, 1, 2, 3 \\ \bar{\psi}_{1,\cdots,4} & \\ \bar{\phi}_{1,\cdots,8} & \end{cases}$$

Novel Basis

$$
\begin{aligned}
1 &= \{\chi^{1,\dots,8}, y^{1,\dots,8}, \omega^{1,\dots,8} | \bar{y}^{1,\dots,8}, \bar{\omega}^{1,\dots,8}, \bar{\eta}^{0,\dots,3}, \bar{\psi}^{1,\dots,4} \bar{\phi}^{1,\dots,8}\},\\
H_L &= \{\chi^{1,\dots,8}, y^{1,\dots,8}, \chi^{1,\dots,8}\},\\
z_1 &= \{\bar{\phi}^{1,\dots,4}\},\\
z_2 &= \{\bar{\phi}^{5,\dots,8}\},\\
z_3 &= \{\bar{\psi}^{1,\dots,4}\},\\
z_4 &= \{\bar{\eta}^{0,\dots,3}\},\\
z_5 &= \{\bar{\rho}_i = \bar{y}^i + i\bar{\omega}^i\},\ i = 1,\dots,4,
\end{aligned}
$$

$$z_6 = \mathbf{1} + H_L + z_1 + z_2 + z_3 + z_4 + z_5 = \{\bar{\rho}^{5,6,7,8}\}.$$

massless bosons: NS, $z_{1,2,3,4,5,6}$, $z_i + z_j \;\; i,j = 1,\dots,6,\ i \neq j$

24 dimensional lattices $\;\rightarrow\;$ from enhancements

$c\binom{z_1}{H_L}$	$c\binom{z_2}{H_L}$	$c\binom{z_3}{H_L}$	$c\binom{z_4}{H_L}$	$c\binom{z_1}{z_2}$	$c\binom{z_1}{z_3}$	$c\binom{z_1}{z_4}$	$c\binom{z_2}{z_3}$	$c\binom{z_2}{z_4}$	$c\binom{z_3}{z_4}$	Gauge group G
+	+	+	+	+	+	+	+	+	+	$E_8 \times SO(32)$
+	+	+	+	+	+	+	+	+	−	$SO(16) \times SO(16) \times SO(16)$
+	+	+	+	+	+	+	+	−	−	$SO(16) \times SO(8) \times SO(8) \times SO(16)$
+	+	+	+	+	+	+	−	−	−	$E_8 \times SO(24) \times SO(8)$
+	+	+	+	−	+	+	−	+	−	$SO(16) \times SO(8) \times SO(8) \times SO(8) \times SO(8)$
+	+	+	+	−	−	−	+	+	+	$SO(24) \times SO(16) \times SO(8)$
+	+	+	+	+	−	−	−	−	+	$E_8 \times SO(16) \times SO(16)$
−	+	+	+	+	+	+	+	+	+	$SO(24) \times SO(24)$
+	+	+	+	−	−	−	−	−	−	$SO(32) \times SO(16)$
−	−	+	+	−	+	+	+	+	+	$E_8 \times E_8 \times SO(16)$
−	−	−	−	+	+	+	+	+	+	$SO(48)$
−	−	−	+	+	+	+	+	+	+	$SO(40) \times SO(8)$
−	−	−	−	−	+	+	+	+	−	$E_8 \times E_8 \times E_8$

Table 1: The configuration of the symmetry group with six basis vectors.

From a subset with six basis vector

A subset of enhanced lattices

Additionally obtain representation of 24 dimensional Niemeier lattices

in terms of free fermion data (with Panos Athanasopoulos arXiv:1610.04898)

Conclusions

- Mirror Symmetry $\longrightarrow$ pure mathematical interest

- Spinor–vector duality $\longrightarrow$ extension of mirror symmetry

- $(G, B, W) \quad \longrightarrow \tilde{G}, \tilde{B}, \tilde{W}$

- Spinor–vector duality $\longleftrightarrow$ pure mathematical interest???
- Physical application : String derived extra Z' model

  AEF, J Rizos, NPB 895 (2015) 233