

# String Phenomenology: past, present and future

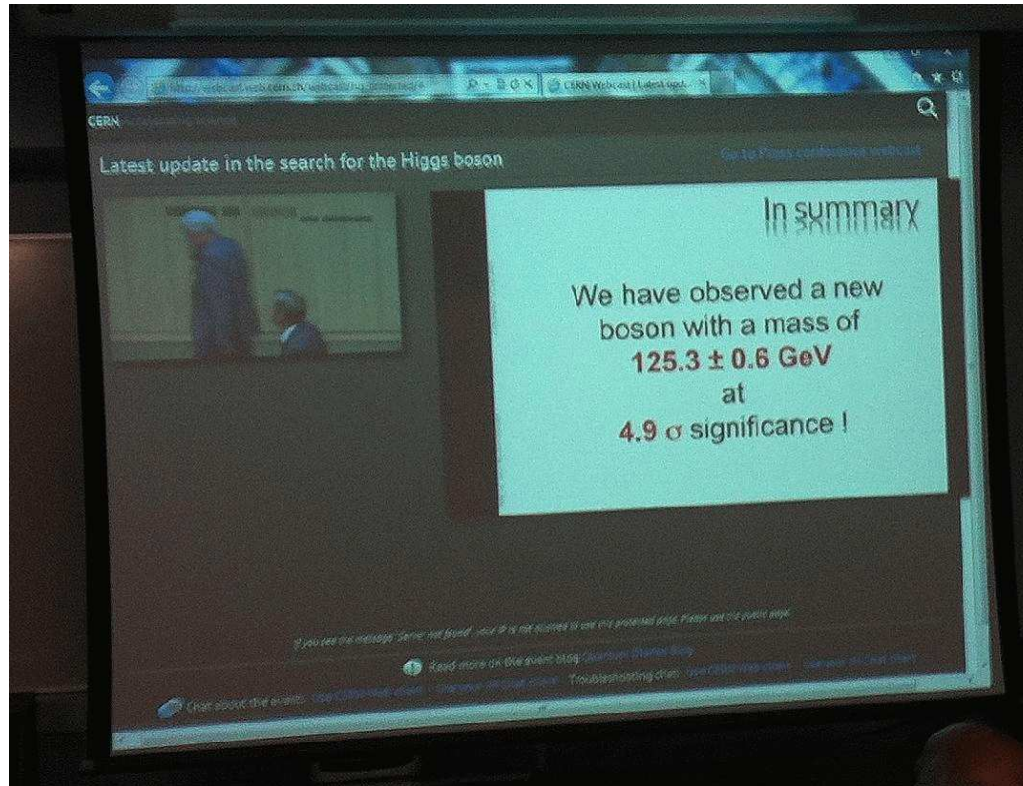


## Progress Report: The early years 1989–2012

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String Phenomenology Institute, CERN, 10 July 2011

... Before



4 July 2012

After ...

# DATA $\rightarrow$ STANDARD MODEL

$$SU(3) \times SU(2) \times U(1)_Y \longrightarrow SU(5) \longrightarrow SO(10)$$

$$\left[ \begin{pmatrix} \nu \\ e \end{pmatrix} + D_L^c \right] + \left[ U_L^c + \begin{pmatrix} u \\ d \end{pmatrix} + E_L^c \right] + N_L^c$$
$$\bar{5} \quad + \quad 10 \quad + \quad 1 \quad \quad \quad \frac{\quad}{16}$$

## STANDARD MODEL $\rightarrow$ UNIFICATION

### ADDITIONAL EVIDENCE:

Logarithmic running, proton longevity, neutrino masses

### PRIMARY GUIDES:

3 generations

SO(10) embedding

## Realistic free fermionic models

### 'Phenomenology of the Standard Model and string unification'

- Top quark mass  $\sim 175\text{--}180\text{GeV}$  PLB 274 (1992) 47
- Generation mass hierarchy NPB 407 (1993) 57
- CKM mixing NPB 416 (1994) 63 (with Halyo)
- Stringy seesaw mechanism PLB 307 (1993) 311 (with Halyo)
- Gauge coupling unification NPB 457 (1995) 409 (with Dienes)
- Proton stability NPB 428 (1994) 111
- Squark degeneracy NPB 526 (1998) 21 (with Pati)
- Minimal Superstring Standard Model PLB 455 (1999) 135  
(with Cleaver & Nanopoulos)
- Moduli fixing NPB 728 (2005) 83
- Exophobia PLB 683 (2010) 306

(with Assel, Christodoulides, Kounnas & Rizos)

## Other approaches

### Geometrical

Greene, Kirklin, Miron, Ross (1987)

Donagi, Ovrut, Pantev, Waldram (1999)

Blumenhagen, Moster, Reinbacher, Weigand (2006)

Heckman, Vafa (2008)

.....

### Orbifolds

Ibanez, Nilles, Quevedo (1987)

Bailin, Love, Thomas (1987)

Kobayashi, Raby, Zhang (2004)

Lebedev, Nilles, Raby, Ramos-Sanchez, Ratz, Vaudrevange, Wingerter (2007)

Blaszczyk, Groot-Nibbelink, Ruehle, Trapletti, Vaudrevange (2010)

.....

### Other CFTs

Gepner (1987)

Schellekens, Yankielowicz (1989)

Gato-Rivera, Schellekens (2009)

.....

### Orientifolds

Cvetic, Shiu, Uranga (2001)

Ibanez, Marchesano, Rabadan (2001)

Kiristis, Schellekens, Tsulaia (2008)

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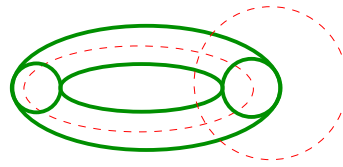
# Free Fermionic Construction

Left-Movers:  $\psi_{1,2}^\mu$ ,  $\chi_i$ ,  $y_i$ ,  $\omega_i$  ( $i = 1, \dots, 6$ )

Right-Movers

$$\bar{\phi}_{A=1, \dots, 44} = \left\{ \begin{array}{ll} \bar{y}_i, \bar{\omega}_i & i = 1, \dots, 6 \\ \bar{\eta}_i & i = 1, 2, 3 \\ \bar{\psi}_{1, \dots, 5} \\ \bar{\phi}_{1, \dots, 8} \end{array} \right.$$

$$V \longrightarrow V$$



$$f \longrightarrow -e^{i\pi\alpha(f)} f$$

$$Z = \sum_{\text{all spin structures}} c\left(\begin{smallmatrix} \vec{\alpha} \\ \vec{\beta} \end{smallmatrix}\right) Z\left(\begin{smallmatrix} \vec{\alpha} \\ \vec{\beta} \end{smallmatrix}\right)$$

Models  $\longleftrightarrow$  Basis vectors + one-loop phases

## The NAHE set:

$$1 = \{\psi^\mu, \chi^{1,\dots,6}, y^{1,\dots,6}, \omega^{1,\dots,6} \mid \bar{y}^{1,\dots,6}, \bar{\omega}^{1,\dots,6}, \bar{\eta}^{1,2,3}, \bar{\psi}^{1,\dots,5}, \bar{\phi}^{1,\dots,8}\}$$

$$S = \{\psi^\mu, \chi^{1,\dots,6}\},$$

$N = 4$  Vacua

$$b_1 = \{\chi^{34}, \chi^{56}, y^{34}, y^{56} \mid \bar{y}^{34}, \bar{y}^{56}, \bar{\eta}^1, \bar{\psi}^{1,\dots,5}\},$$

$N = 4 \rightarrow N = 2$

$$b_2 = \{\chi^{12}, \chi^{56}, y^{12}, \omega^{56} \mid \bar{y}^{12}, \bar{\omega}^{56}, \bar{\eta}^2, \bar{\psi}^{1,\dots,5}\},$$

$N = 2 \rightarrow N = 1$

$$b_3 = \{\chi^{12}, \chi^{34}, \omega^{12}, \omega^{34} \mid \bar{\omega}^{12}, \bar{\omega}^{34}, \bar{\eta}^3, \bar{\psi}^{1,\dots,5}\},$$

$N = 2 \rightarrow N = 1$

$Z_2 \times Z_2$  orbifold compactification

$\implies$  Gauge group  $SO(10) \times SO(6)^{1,2,3} \times E_8$

beyond the NAHE set

Add  $\{\alpha, \beta, \gamma\}$

number of generations is reduced to three

$$SO(10) \longrightarrow SU(3) \times SU(2) \times U(1)_{T_{3R}} \times U(1)_{B-L}$$

$$U(1)_Y = \frac{1}{2}(B - L) + T_{3R} \in SO(10) !$$

$$SO(6)^{1,2,3} \longrightarrow U(1)^{1,2,3} \times U(1)^{1,2,3}$$



# STRING DERIVED STANDARD-LIKE MODEL (PLB278)

	$\psi^\mu$	$\chi^{12}$	$\chi^{34}$	$\chi^{56}$	$y^{3,\dots,6}$	$\bar{y}^{3,\dots,6}$	$y^{1,2,\omega^{5,6}}$	$\bar{y}^{1,2,\bar{\omega}^{5,6}}$	$\omega^{1,\dots,4}$	$\bar{\omega}^{1,\dots,4}$	$\bar{\psi}^{1,\dots,5}$	$\bar{\eta}^1$	$\bar{\eta}^2$	$\bar{\eta}^3$	$\bar{\phi}^{1,\dots,8}$
<b>1</b>	1	1	1	1	1, ..., 1	1, ..., 1	1, ..., 1	1, ..., 1	1, ..., 1	1, ..., 1	1, ..., 1	1	1	1	1, ..., 1
<i>S</i>	1	1	1	1	0, ..., 0	0, ..., 0	0, ..., 0	0, ..., 0	0, ..., 0	0, ..., 0	0, ..., 0	0	0	0	0, ..., 0
<i>b</i> <sub>1</sub>	1	1	0	0	1, ..., 1	1, ..., 1	0, ..., 0	0, ..., 0	0, ..., 0	0, ..., 0	1, ..., 1	1	0	0	0, ..., 0
<i>b</i> <sub>2</sub>	1	0	1	0	0, ..., 0	0, ..., 0	1, ..., 1	1, ..., 1	0, ..., 0	0, ..., 0	1, ..., 1	0	1	0	0, ..., 0
<i>b</i> <sub>3</sub>	1	0	0	1	0, ..., 0	0, ..., 0	0, ..., 0	0, ..., 0	1, ..., 1	1, ..., 1	1, ..., 1	0	0	1	0, ..., 0

	$\psi^\mu$	$\chi^{12}$	$\chi^{34}$	$\chi^{56}$	$y^3 y^6$	$y^4 \bar{y}^4$	$y^5 \bar{y}^5$	$\bar{y}^3 \bar{y}^6$	$y^1 \omega^5$	$y^2 \bar{y}^2$	$\omega^6 \bar{\omega}^6$	$\bar{y}^1 \bar{\omega}^5$	$\omega^2 \omega^4$	$\omega^1 \bar{\omega}^1$	$\omega^3 \bar{\omega}^3$	$\bar{\omega}^2 \bar{\omega}^4$	$\bar{\psi}^{1,\dots,5}$	$\bar{\eta}^1$	$\bar{\eta}^2$	$\bar{\eta}^3$	
$\alpha$	0	0	0	0	1	0	0	0	0	0	1	1	0	0	1	1	1 1 1 0 0	0	0	0	1 1
$\beta$	0	0	0	0	0	0	1	1	1	0	0	0	0	1	0	1	1 1 1 0 0	0	0	0	1 1
$\gamma$	0	0	0	0	0	1	0	1	0	1	0	1	1	0	0	0	$\frac{1}{2} \frac{1}{2} \frac{1}{2} \frac{1}{2} \frac{1}{2}$	$\frac{1}{2}$	$\frac{1}{2}$	$\frac{1}{2}$	$\frac{1}{2} \ 0 \ 1$

Asymmetric  $BC \Rightarrow$  all untwisted moduli are projected out!

all  $y_i \omega_i \bar{y}_i \bar{\omega}_i$  are disallowed

can be translated to asymmetric bosonic identifications

$$X_L + X_R \rightarrow X_L - X_R$$

moduli fixed at enhanced symmetry point

## Calculation of Mass Terms

nonvanishing correlators

$$\langle V_1^f V_2^f V_3^b \cdots V_N^b \rangle$$

gauge & string invariant

“anomalous”  $U(1)_A$

$$\text{Tr} Q_A \neq 0 \Rightarrow D_A = 0 = A + \sum Q_k^A |\langle \phi_k \rangle|^2$$

$$D_j = 0 = \sum Q_k^j |\langle \phi_k \rangle|^2$$

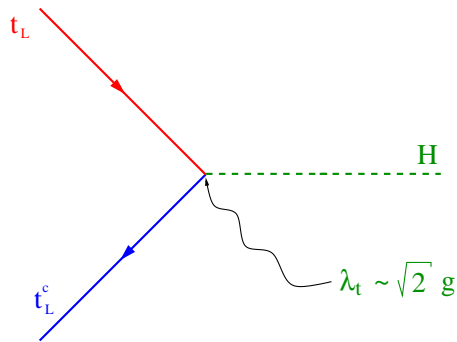
$$\langle W \rangle = \langle \frac{\partial W_N}{\partial \eta_i} \rangle = 0 \quad N = 3 \cdots$$

Supersymmetric vacuum  $\langle F \rangle = \langle D \rangle = 0$ .

nonrenormalizable terms  $\rightarrow$  effective renormalizable operators

$$V_1^f V_2^f V_3^b \cdots V_N^b \rightarrow V_1^f V_2^f V_3^b \frac{\langle V_4^b \cdots V_N^b \rangle}{M^{N-3}}$$

# Top Quark Mass Prediction



Only  $\lambda_t = \langle Q t_L^c H \rangle = \sqrt{2}g$  at  $N = 3$

mass of lighter quarks and leptons  $\rightarrow$  nonrenormalizable terms

$$\longrightarrow \lambda_b = \lambda_\tau = 0.35g^3 \sim \frac{1}{8}\lambda_t$$

Evolve  $\lambda_t$  ,  $\lambda_b$  to low energies

$$m_t = \lambda_t v_1 = \lambda_t \frac{v_0}{\sqrt{2}} \sin \beta \quad m_b = \lambda_b v_2 = \lambda_b \frac{v_0}{\sqrt{2}} \cos \beta$$

where  $v_0 = \frac{2m_W}{g_2(M_Z)} = 246\text{GeV}$  and  $v_1^2 + v_2^2 = \frac{v_0^2}{2}$

$$m_t = \lambda_t(m_t) \frac{v_0}{\sqrt{2}} \frac{\tan \beta}{(1 + \tan^2 \beta)^{\frac{1}{2}}} \implies$$

## Hierarchical top-bottom mass relation in a superstring derived standard-like model

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I propose a mechanism in a class of superstring standard-like models which explains the mass hierarchy between the top and bottom quarks. At the trilinear level of the superpotential only the top quark gets a nonvanishing mass term while the bottom quarks and tau lepton mass terms are obtained from nonrenormalizable terms. I construct a model which realized this mechanism. In this model the bottom quark and tau lepton Yukawa couplings are obtained from quartic order terms. I show that  $\lambda_b = \lambda_\tau \sim \frac{1}{3}\lambda_t$  at the unification scale. A naive estimate yields  $m_t \sim 175-180$  GeV.

One of the unresolved puzzles of the standard model is the mass splitting between the top quark and the lighter quarks and leptons. Especially difficult to understand within the context of the standard model is the big splitting in the heaviest generation. Experimental limits [1] indicate the top mass to be above 80 GeV, while the bottom and tau lepton masses are found at 5 GeV and 1.78 GeV respectively. Possible extensions to the standard model are grand unified theories. Although the main prediction of GUTs, proton decay, has not yet been observed, calculations of  $\sin^2\theta_w$  and of the mass ratio  $m_b/m_\tau$  support their validity. Recent calculations seem to support supersymmetric GUTs versus nonsupersymmetric ones [2]. In spite of the success of SUSY GUTs in confronting LEP data [2], an understanding of the mass splitting between the top quark and the lighter quarks and leptons is still lacking. The next level in which such an understanding may be developed is in the context of superstring theory [3].

Find anomaly free solution

$$M_d \sim \begin{pmatrix} \epsilon & \frac{V_2 \bar{V}_3 \Phi_{\alpha\beta}}{M^3} & 0 \\ \frac{V_2 \bar{V}_3 \Phi_{\alpha\beta} \xi_1}{M^4} & \frac{\bar{\Phi}_2^- \xi_1}{M^2} & 0 \\ 0 & 0 & \frac{\Phi_1^+ \xi_2}{M^2} \end{pmatrix} v_2,$$

$$\epsilon < 10^{-8} \quad \frac{V_2 \bar{V}_3 \Phi_{\alpha\beta}}{M^3} = \frac{\sqrt{5} g^6}{64 \pi^3} \approx 2 - 3 \times 10^{-4}.$$

$$\Rightarrow |V| \sim \begin{pmatrix} 0.98 & 0.2 & 0 \\ 0.2 & 0.98 & 0 \\ 0 & 0 & 1 \end{pmatrix}$$

Three generation mixing  $\longrightarrow$  NPB 416 (1994) 63  $|J| \sim 10^{-6}$

Cleaver, Faraggi, Manno, Timirgaziu, PRD 78 (2008) 046009

Classification of F and D flat directions in FMT reduced Higgs model

No D flat direction which is F-flat up to order eight in the superpotential  
no stringent flat directions to all orders

Suggesting no supersymmetric flat directions in this model (class of models)

implying no supersymmetric moduli

only remaining perturbative moduli is the dilaton

quasi-realistic model: SLM; 3 gen;  $SO(10)$  embed; Higgs &  $\lambda_t \sim 1$ ; ...

vanishing one-loop partition function, perturbatively broken SUSY

Fixed geometrical, twisted and SUSY moduli

Cleaver *etal*,  $SO(10)$  and FSU5 analysis —  $>$  stringent flat directions

# Classification of fermionic $Z_2 \times Z_2$ orbifolds

## Basis vectors:

$$1 = \{\psi^\mu, \chi^{1,\dots,6}, y^{1,\dots,6}, \omega^{1,\dots,6} \mid \bar{y}^{1,\dots,6}, \bar{\omega}^{1,\dots,6}, \bar{\eta}^{1,2,3}, \bar{\psi}^{1,\dots,5}, \bar{\phi}^{1,\dots,8}\}$$

$$S = \{\psi^\mu, \chi^{1,\dots,6}\},$$

$$z_1 = \{\bar{\phi}^{1,\dots,4}\},$$

$$z_2 = \{\bar{\phi}^{5,\dots,8}\},$$

$$e_i = \{y^i, \omega^i \mid \bar{y}^i, \bar{\omega}^i\}, \quad i = 1, \dots, 6, \quad N = 4 \text{ Vacua}$$

$$b_1 = \{\chi^{34}, \chi^{56}, y^{34}, y^{56} \mid \bar{y}^{34}, \bar{y}^{56}, \bar{\eta}^1, \bar{\psi}^{1,\dots,5}\}, \quad N = 4 \rightarrow N = 2$$

$$b_2 = \{\chi^{12}, \chi^{56}, y^{12}, y^{56} \mid \bar{y}^{12}, \bar{y}^{56}, \bar{\eta}^2, \bar{\psi}^{1,\dots,5}\}, \quad N = 2 \rightarrow N = 1$$

$$\alpha = \{\bar{\psi}^{4,5}, \bar{\phi}^{1,2}\}$$

& Gauge group  $SO(6) \times SO(4) \times U(1)^3 \times \text{hidden}$

Independent phases  $c \begin{bmatrix} v_i \\ v_j \end{bmatrix} = \exp[i\pi(v_i|v_j)]:$  **upper block**

---

$$\begin{array}{c}
 1 \\
 S \\
 e_1 \\
 e_2 \\
 e_3 \\
 e_4 \\
 e_5 \\
 e_6 \\
 z_1 \\
 z_2 \\
 b_1 \\
 b_2 \\
 \alpha
 \end{array}
 \begin{pmatrix}
 1 & S & e_1 & e_2 & e_3 & e_4 & e_5 & e_6 & z_1 & z_2 & b_1 & b_2 & \alpha \\
 -1 & -1 & \pm & \pm & \pm & \pm & \pm & \pm & \pm & \pm & \pm & \pm & \pm \\
 & & -1 & -1 & -1 & -1 & -1 & -1 & -1 & -1 & 1 & 1 & -1 \\
 & & & \pm & \pm & \pm & \pm & \pm & \pm & \pm & \pm & \pm & \pm \\
 & & & & \pm & \pm & \pm & \pm & \pm & \pm & \pm & \pm & \pm \\
 & & & & & \pm & \pm & \pm & \pm & \pm & \pm & \pm & \pm \\
 & & & & & & \pm & \pm & \pm & \pm & \pm & \pm & \pm \\
 & & & & & & & \pm & \pm & \pm & \pm & \pm & \pm \\
 & & & & & & & & \pm & \pm & \pm & \pm & \pm \\
 & & & & & & & & & \pm & \pm & \pm & \pm \\
 & & & & & & & & & & \pm & \pm & \pm \\
 & & & & & & & & & & & \pm & \pm \\
 & & & & & & & & & & & -1 & \pm \\
 & & & & & & & & & & & & \pm
 \end{pmatrix}$$

A priori 66 independent coefficients  $\rightarrow 2^{66}$  distinct vacua



## The twisted matter spectrum:

$$B_{l_3^1 l_4^1 l_5^1 l_6^1}^1 = S + b_1 + l_3^1 e_3 + l_4^1 e_4 + l_5^1 e_5 + l_6^1 e_6$$

$$B_{l_1^2 l_2^2 l_5^2 l_6^2}^2 = S + b_2 + l_1^2 e_1 + l_2^2 e_2 + l_5^2 e_5 + l_6^2 e_6$$

$$B_{l_1^3 l_2^3 l_3^3 l_4^3}^3 = S + b_3 + l_1^3 e_1 + l_2^3 e_2 + l_3^3 e_3 + l_4^3 e_4 \quad l_i^j = 0, 1$$

sectors  $B_{pqrs}^i \rightarrow 16$  or  $\overline{16}$  of  $SO(10)$  with multiplicity  $(1, 0, -1)$

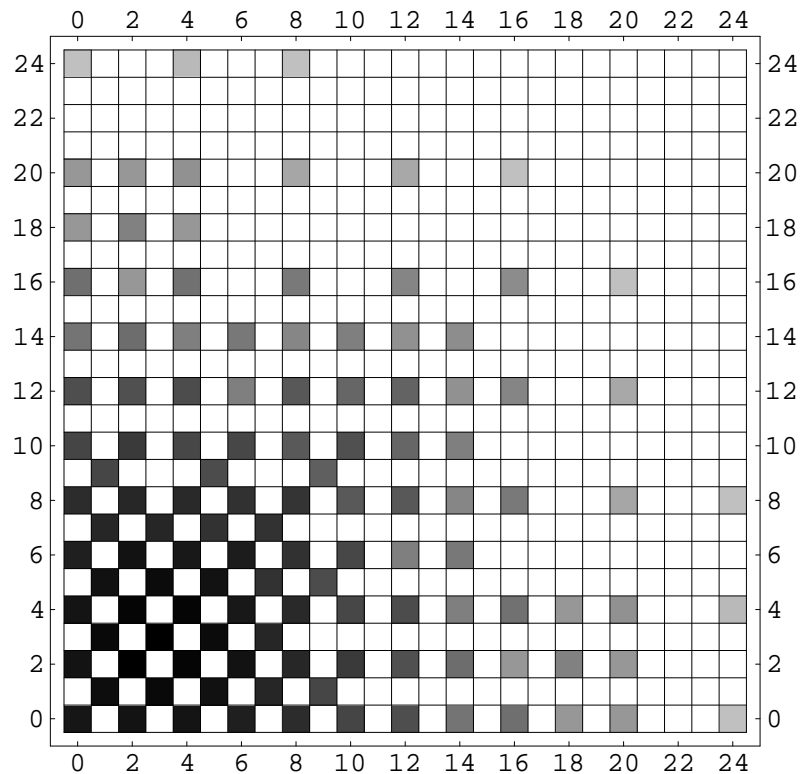
$B_{pqrs}^i + x \rightarrow 10$  of  $SO(10)$  with multiplicity  $(1, 0)$

$x = \{\bar{\psi}^{1, \dots, 5}, \bar{\eta}^1, \bar{\eta}^2, \bar{\eta}^3\}$   $x$  - map  $\leftrightarrow$  spinor-vector map

Algebraic formulas for  $S = \sum_{i=1}^3 S_+^{(i)} - S_-^{(i)}$  and  $V = \sum_{i=1}^3 V^{(i)}$

## Spinor–vector duality:

Invariance under exchange of  $\#(16 + \overline{16}) < - > \#(10)$



Symmetric under exchange of rows and columns

$$E_6 : \quad 27 = 16 + 10 + 1 \quad \overline{27} = \overline{16} + 10 + 1$$

Self-dual:  $\#(16 + \overline{16}) = \#(10)$  without  $E_6$  symmetry

## Spinor–Vector duality in Orbifolds:

Using the level-one  $SO(2n)$  characters

$$\begin{aligned} O_{2n} &= \frac{1}{2} \left( \frac{\theta_3^n}{\eta^n} + \frac{\theta_4^n}{\eta^n} \right), & V_{2n} &= \frac{1}{2} \left( \frac{\theta_3^n}{\eta^n} - \frac{\theta_4^n}{\eta^n} \right), \\ S_{2n} &= \frac{1}{2} \left( \frac{\theta_2^n}{\eta^n} + i^{-n} \frac{\theta_1^n}{\eta^n} \right), & C_{2n} &= \frac{1}{2} \left( \frac{\theta_2^n}{\eta^n} - i^{-n} \frac{\theta_1^n}{\eta^n} \right). \end{aligned}$$

where

$$\begin{aligned} \theta_3 &\equiv Z_f \begin{pmatrix} 0 \\ 0 \end{pmatrix} & \theta_4 &\equiv Z_f \begin{pmatrix} 0 \\ 1 \end{pmatrix} \\ \theta_2 &\equiv Z_f \begin{pmatrix} 1 \\ 0 \end{pmatrix} & \theta_1 &\equiv Z_f \begin{pmatrix} 1 \\ 1 \end{pmatrix} \end{aligned}$$

Starting from:

$$Z_+ = (V_8 - S_8) \left( \sum_{m,n} \Lambda_{m,n} \right)^{\otimes 6} (\bar{O}_{16} + \bar{S}_{16}) (\bar{O}_{16} + \bar{S}_{16}) ,$$

where as usual, for each circle,

$$p_{L,R}^i = \frac{m_i}{R_i} \pm \frac{n_i R_i}{\alpha'} ,$$

and

$$\Lambda_{m,n} = \frac{q^{\frac{\alpha'}{4} p_L^2} \bar{q}^{\frac{\alpha'}{4} p_R^2}}{|\eta|^2} .$$

apply  $Z_2 \times Z'_2 : g \times g'$

$$g : (-1)^{(F_1+F_2)} \delta$$

$$F_{1,2} : ( \overline{O}_{16}^{1,2}, \overline{V}_{16}^{1,2}, \overline{S}_{16}^{1,2}, \overline{C}_{16}^{1,2} ) \longrightarrow ( \overline{O}_{16}^{1,2}, \overline{V}_{16}^{1,2}, -\overline{S}_{16}^{1,2}, -\overline{C}_{16}^{1,2} )$$

$$\text{with } \delta X_9 = X_9 + \pi R_9 ,$$

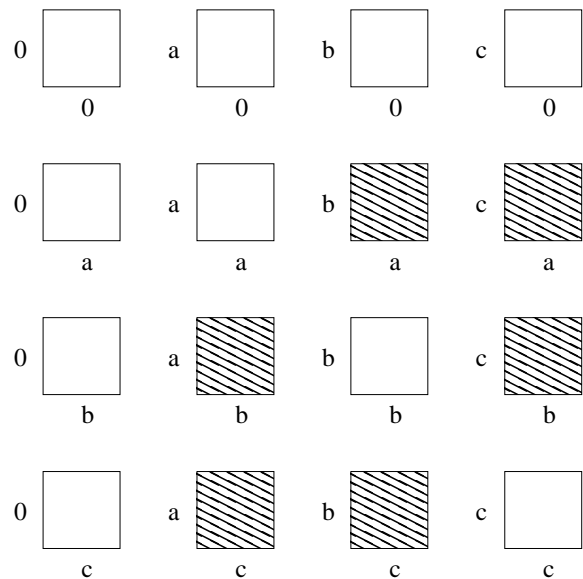
$$\delta : \Lambda_{m,n}^9 \longrightarrow (-1)^m \Lambda_{m,n}^9$$

$$g' : (x_4, x_5, x_6, x_7, x_8, x_9) \longrightarrow (-x_4, -x_5, -x_6, -x_7, +x_8, +x_9)$$

Note: A single space twisting  $Z'_2 \Rightarrow N = 4 \rightarrow N = 2$

$$E_7 \rightarrow SO(12) \times SU(2)$$

⇒ Analyze  $Z = \left( \frac{Z_+}{Z_g \times Z_{g'}} \right) = \left[ \frac{(1+g)(1+g')}{2 \cdot 2} \right] Z_+$



$a = g$  ;  $b = g'$  ;  $c = gg'$

$P.F. = ( \square + \varepsilon \text{diagonal lines} ) = \Lambda_{m,n} \bullet ( ) + \Lambda_{m,n+1/2} \bullet ( )$

$\varepsilon = \pm 1$

massless

massive

• sector  $b$

$$\Lambda_{p,q} \left\{ \frac{1}{2} \left( \left| \frac{2\eta}{\theta_4} \right|^4 + \left| \frac{2\eta}{\theta_3} \right|^4 \right) [P_\epsilon^+ Q_s \bar{V}_{12} \bar{C}_4 \bar{O}_{16} + P_\epsilon^- Q_s \bar{S}_{12} \bar{O}_4 \bar{O}_{16}] + \frac{1}{2} \left( \left| \frac{2\eta}{\theta_4} \right|^4 - \left| \frac{2\eta}{\theta_3} \right|^4 \right) [P_\epsilon^+ Q_s \bar{O}_{12} \bar{S}_4 \bar{O}_{16}] \right\} + \text{massive}$$

where

$$P_\epsilon^+ = \left( \frac{1 + \epsilon(-1)^m}{2} \right) \Lambda_{m,n} \quad P_\epsilon^- = \left( \frac{1 - \epsilon(-1)^m}{2} \right) \Lambda_{m,n}$$

$$\epsilon = +1 \Rightarrow P_\epsilon^+ = \Lambda_{2m,n} \quad P_\epsilon^- = \Lambda_{2m+1,n}$$

$$\epsilon = -1 \Rightarrow P_\epsilon^+ = \Lambda_{2m+1,n} \quad P_\epsilon^- = \Lambda_{2m,n}$$

and  $12 \cdot 2 + 4 \cdot 2 = 32$

## Further :

- The spinor–vector duality in this model is realised in terms of a continuous interpolation between two discrete Wilson lines.
- The spinor-vector duality is realised in terms of a spectral flow operator that operates in the bosonic side of the heterotic string. In the case of enhanced  $E_6$  symmetry, the spectral flow operator acts as an internal  $E_6$  generator. When  $E_6$  is broken the spectral flow operator induces the spinor–vector duality map.



## Pati–Salam models statistics with respect to phenomenological constraints

constraint	# of models	probability	# of models
None	1000000000000	1	$2.25 \times 10^{15}$
+ No gauge group enhancements.	78977078333	$7.90 \times 10^{-1}$	$1.78 \times 10^{15}$
+ Complete families	22497003372	$2.25 \times 10^{-1}$	$5.07 \times 10^{14}$
+ 3 generations	298140621	$2.98 \times 10^{-3}$	$6.71 \times 10^{12}$
+ PS breaking Higgs	23694017	$2.37 \times 10^{-4}$	$5.34 \times 10^{11}$
+ SM breaking Higgs	19191088	$1.92 \times 10^{-4}$	$4.32 \times 10^{11}$
+ No massless exotics	121669	$1.22 \times 10^{-6}$	$2.74 \times 10^9$

Constraints in second column act additionally.

- A specific choice of one-loop GSO phases
- Analysis of cubic level superpotential and flat directions
- Only one Yukawa coupling at cubic level  $\rightarrow$  heavy family
- All extra colour triplets  $\rightarrow$  massive
- One light Higgs bi-doublet
- Solely MSSM below PS breaking scale

Away from the free fermionic point:

$$\begin{aligned}
 Z = & \int \frac{d^2\tau}{\tau_2^2} \frac{\tau_2^{-1}}{\eta^{12}\bar{\eta}^{24}} \frac{1}{2^3} \left( \sum (-)^{a+b+ab} \vartheta \left[ \begin{matrix} a \\ b \end{matrix} \right] \vartheta \left[ \begin{matrix} a+h_1 \\ b+g_1 \end{matrix} \right] \vartheta \left[ \begin{matrix} a+h_2 \\ b+g_2 \end{matrix} \right] \vartheta \left[ \begin{matrix} a+h_3 \\ b+g_3 \end{matrix} \right] \right)_{\psi\mu}, \\
 & \times \left( \frac{1}{2} \sum_{\epsilon,\xi} \bar{\vartheta} \left[ \begin{matrix} \epsilon \\ \xi \end{matrix} \right]^5 \bar{\vartheta} \left[ \begin{matrix} \epsilon+h_1 \\ \xi+g_1 \end{matrix} \right] \bar{\vartheta} \left[ \begin{matrix} \epsilon+h_2 \\ \xi+g_2 \end{matrix} \right] \bar{\vartheta} \left[ \begin{matrix} \epsilon+h_3 \\ \xi+g_3 \end{matrix} \right] \right)_{\bar{\psi}^{1\dots 5}, \bar{\eta}^{1,2,3}} \\
 & \times \left( \frac{1}{2} \sum_{H_1, G_1} \frac{1}{2} \sum_{H_2, G_2} (-)^{H_1 G_1 + H_2 G_2} \bar{\vartheta} \left[ \begin{matrix} \epsilon+H_1 \\ \xi+G_1 \end{matrix} \right]^4 \bar{\vartheta} \left[ \begin{matrix} \epsilon+H_2 \\ \xi+G_2 \end{matrix} \right]^4 \right)_{\bar{\phi}^{1\dots 8}} \\
 & \times \left( \sum_{s_i, t_i} \Gamma_{6,6} \left[ \begin{matrix} h_i | s_i \\ g_i | t_i \end{matrix} \right] \right)_{(y\omega\bar{y}\bar{\omega})^{1\dots 6}} \times e^{i\pi\Phi(\gamma, \delta, s_i, t_i, \epsilon, \xi, h_i, g_i, H_1, G_1, H_2, G_2)}
 \end{aligned}$$

$$\Gamma_{1,1} \left[ \begin{matrix} h \\ g \end{matrix} \right] = \frac{R}{\sqrt{\tau_2}} \sum_{\tilde{m}, n} \exp \left[ -\frac{\pi R^2}{\tau_2} |(2\tilde{m} + g) + (2n + h) \tau|^2 \right]$$

# Towards String Predictions

## 1. Low energy supersymmetry

Specific SUSY breaking patterns  $\longrightarrow$  Collider implications

## 2. Additional (non-GUT) gauge bosons

Proton Stability and low-scale  $Z'$   $\longrightarrow$  Collider signatures

## 3. Exotic matter

### In realistic string models

Unifying gauge group  $\Rightarrow$  broken by “Wilson lines”.

$\Rightarrow$  non-GUT physical states.

$\Rightarrow$  Meta-stable heavy string relics

$\rightarrow$  Dark Matter ; UHECR candidates

## Conclusions

- DATA  $\longrightarrow$  UNIFICATION
- STRINGS  $\longrightarrow$  GAUGE & GRAVITY UNIFICATION
- EXPERIMENTAL PREDICTIONS ?
- FUNDAMENTAL PRINCIPLES ?  
*e.g.* spinor–vector duality  $\longrightarrow$  Physics & Geometry  
  
phase–space duality & the equivalence postulate of QM