Spinor–vector duality and BSM phenomenology

- 2006 – · · · Spinor–vector duality · · ·
- 1990 – · · · String derived Z' · · ·
- 2019 – · · · non–SUSY string phenomenology from 10D tachyonic vacua

AEF, S Groot–Nibbelink, M Hurtado Heredia, arXiv:2103:13442
AEF, J Rizos, NPB895 (2015) 233
AEF, B Percival, V Matyas, EPJC80 (2020) 337; NPB961 (2020) 115231;
PLB 814 (2021) 136080; 2010.06637; 2011.04113.

BSM–2021, Zoom, 29 March 2021

Fermionic $Z_2 \times Z_2$ orbifolds

‘Phenomenology of the Standard Model and Unification’

• Minimal Superstring Standard Model	NPB 335 (1990) 347 (with Nanopoulos & Yuan)
• Top quark mass $\sim$ 175–180GeV	PLB 274 (1992) 47
• Generation mass hierarchy	NPB 407 (1993) 57
• CKM mixing	NPB 416 (1994) 63 (with Halyo)
• Stringy seesaw mechanism	PLB 307 (1993) 311 (with Halyo)
• Gauge coupling unification	NPB 457 (1995) 409 (with Dienes)
• Proton stability	NPB 428 (1994) 111
• Squark degeneracy	NPB 526 (1998) 21 (with Pati)
• Moduli fixing	NPB 728 (2005) 83
• Classification	2003 – · · · (with Kounnas, Rizos & ... Harries, Percival)

Other approaches

Geometrical

Greene, Kirklin, Miron, Ross (1987)
Donagi, Ovrut, Pantev, Waldram (1999)
Blumenhagen, Moster, Reinbacher, Weigand (2006)
Heckman, Vafa (2008)
....

Orbifolds

Ibanez, Nilles, Quevedo (1987)
Bailin, Love, Thomas (1987)
Kobayashi, Raby, Zhang (2004)
Lebedev, Nilles, Raby, Ramos-Sanchez, Ratz, Vaudrevange, Wingerter (2007)
Blaszczyk, Groot–Nibbelink, Ruehle, Trapletti, Vaudrevange (2010)
....

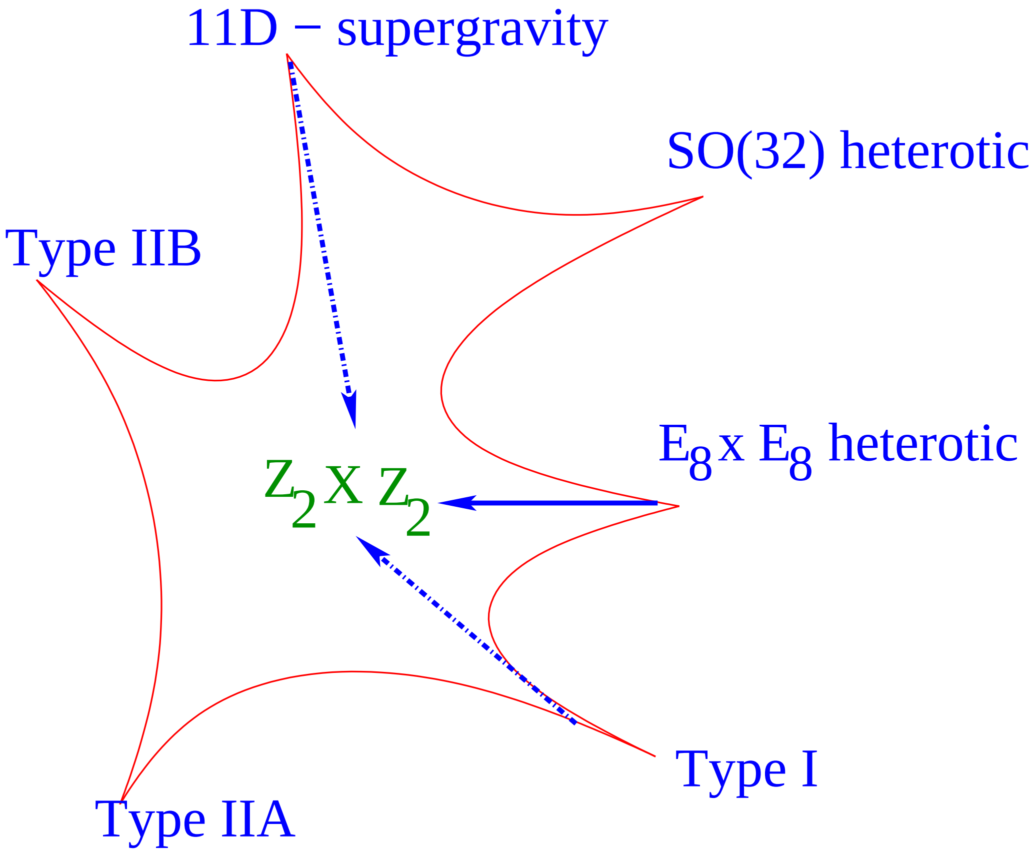
Other CFTs

Gepner (1987)
Schellekens, Yankielowicz (1989)
Gato–Rivera, Schellekens (2009)
....

Orientifolds

Cvetic, Shiu, Uranga (2001)
Ibanez, Marchesano, Rabadan (2001)
Kiristis, Schellekens, Tsulaia (2008)
....

Point, String, Membrane ....



11D − supergravity

SO(32) heterotic

Type IIB

E_8 x E_8 heterotic

Z_2 X Z_2

Type I

Type IIA

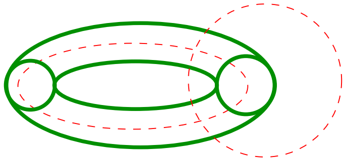
\+ ... SO(16)xSO(16), E8, SO(16)xE8 + ...

Free Fermionic Construction

Left-Movers: $\psi^{\mu}_{1,2},\quad \chi_i,\quad y_i,\ \omega_i\quad (i=1,\cdots,6)$

Right-Movers

$$\bar{\phi}_{A=1,\cdots,44} = \begin{cases} \bar{y}_i\ ,\ \bar{\omega}_i & i=1,\cdots,6 \\ & \\ \bar{\eta}_i & U(1)_i \quad i=1,2,3 \\ \bar{\psi}_{1,\cdots,5} & SO(10) \\ \bar{\phi}_{1,\cdots,8} & SO(16) \end{cases}$$



$V \longrightarrow V$ $\qquad$ $f \longrightarrow -e^{i\pi\alpha(f)}f$

$$Z = \sum_{\substack{all\ spin\\ structures}} c\binom{\vec{\alpha}}{\vec{\beta}}\ Z\binom{\vec{\alpha}}{\vec{\beta}}$$

Models $\longleftrightarrow$ Basis vectors $+$ one–loop phases

Classification of fermionic $Z_2 \times Z_2$ orbifolds (Modern School)

Basis vectors:

$$\begin{aligned}
1 &= \{\psi^\mu,\ \chi^{1,\dots,6}, y^{1,\dots,6}, \omega^{1,\dots,6} \mid \bar{y}^{1,\dots,6}, \bar{\omega}^{1,\dots,6}, \bar{\eta}^{1,2,3}, \bar{\psi}^{1,\dots,5}, \bar{\phi}^{1,\dots,8}\} \\
S &= \{\psi^\mu, \chi^{1,\dots,6}\}, \\
z_1 &= \{\bar{\phi}^{1,\dots,4}\}, \\
z_2 &= \{\bar{\phi}^{5,\dots,8}\}, \\
e_i &= \{y^i, \omega^i | \bar{y}^i, \bar{\omega}^i\},\ i = 1, \dots, 6,
\end{aligned}$$

$N = 4$ Vacua

$$\begin{aligned}
b_1 &= \{\chi^{34}, \chi^{56}, y^{34}, y^{56} | \bar{y}^{34}, \bar{y}^{56}, \bar{\eta}^1, \bar{\psi}^{1,\dots,5}\}, \qquad N = 4 \rightarrow N = 2 \\
b_2 &= \{\chi^{12}, \chi^{56}, y^{12}, y^{56} | \bar{y}^{12}, \bar{y}^{56}, \bar{\eta}^2, \bar{\psi}^{1,\dots,5}\}, \qquad N = 2 \rightarrow N = 1
\end{aligned}$$

$$\alpha = \{\bar{\psi}^{4,5}, \bar{\phi}^{1,2}\} \qquad \& \quad SO(10) \rightarrow SO(6) \times SO(4) \times \cdots$$

$$\beta = \{\bar{\psi}^{1,\cdots,5} \equiv \tfrac{1}{2}, \cdots\} \qquad \& \quad SO(10) \rightarrow SU(5) \times U(1) \times \cdots$$

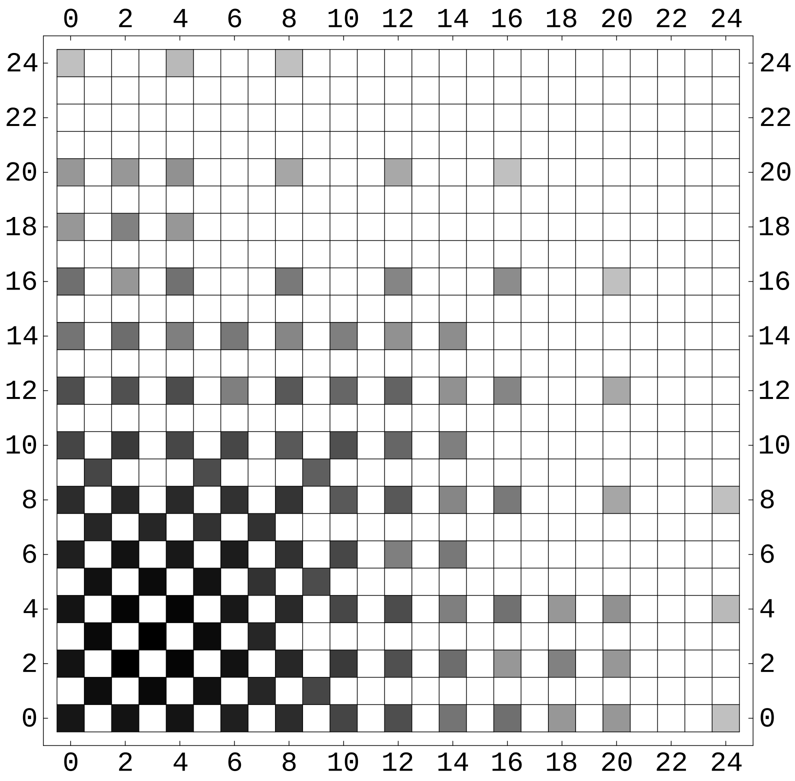
Independent phases $c\begin{bmatrix} v_i \\ v_j \end{bmatrix} = \exp[i\pi(v_i|v_j)]$: upper block

$$\begin{array}{cccccccccccccc}
 & 1 & S & e_1 & e_2 & e_3 & e_4 & e_5 & e_6 & z_1 & z_2 & b_1 & b_2 & \alpha \\
1 & -1 & -1 & \pm & \pm & \pm & \pm & \pm & \pm & \pm & \pm & \pm & \pm & \pm \\
S & & & -1 & -1 & -1 & -1 & -1 & -1 & -1 & -1 & 1 & 1 & -1 \\
e_1 & & & & \pm & \pm & \pm & \pm & \pm & \pm & \pm & \pm & \pm & \pm \\
e_2 & & & & & \pm & \pm & \pm & \pm & \pm & \pm & \pm & \pm & \pm \\
e_3 & & & & & & \pm & \pm & \pm & \pm & \pm & \pm & \pm & \pm \\
e_4 & & & & & & & \pm & \pm & \pm & \pm & \pm & \pm & \pm \\
e_5 & & & & & & & & \pm & \pm & \pm & \pm & \pm & \pm \\
e_6 & & & & & & & & & \pm & \pm & \pm & \pm & \pm \\
z_1 & & & & & & & & & & \pm & \pm & \pm & \pm \\
z_2 & & & & & & & & & & & \pm & \pm & \pm \\
b_1 & & & & & & & & & & & & \pm & \pm \\
b_2 & & & & & & & & & & & & -1 & \pm \\
\alpha & & & & & & & & & & & & &
\end{array}$$

A priori 66 independent coefficients → 2^{66} distinct vacua

Spinor–vector duality:

Invariance under exchange of $\#(16+\overline{16}) \;\; <-> \;\; \#(10)$



Symmetric under exchange of rows and columns

$$E_6 \;\; : \qquad 27 = 16 + 10 + 1 \qquad \overline{27} = \overline{16} + 10 + 1$$

Self-dual: $\#(16+\overline{16}) \;\; = \;\; \#(10)$ without E_6 symmetry

Spinor–Vector duality in Orbifolds:

Starting from: $$Z_+ = (V_8 - S_8) \left(\sum_{m,n} \Lambda_{m,n} \right)^{\otimes 6} E_8 \times E_8 \, ,$$

apply $Z_2 \times Z_2' \; : \; g \times g'$

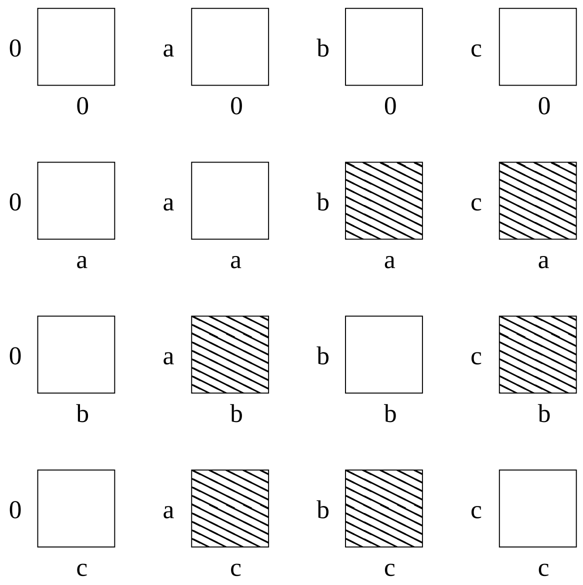
$$g \; : \; (0^7, 1|1, 0^7) \; \rightarrow \; \text{Wilson line} \; \rightarrow \; E_8 \times E_8 \rightarrow SO(16) \times SO(16)$$

$$g' \; : \; (x_4, x_5, x_6, x_7, x_8, x_9) \; \longrightarrow \; (-x_4, -x_5, -x_6, -x_7, +x_8, +x_9)$$

Note: A single space twisting Z_2' $\Rightarrow$ $N = 4 \rightarrow N = 2$

$$E_7 \rightarrow SO(12) \times SU(2)$$

⇒ Analyze $$Z = \left(\frac{Z_+}{Z_g \times Z_{g'}}\right) = \left[\frac{(1+g)}{2}\frac{(1+g')}{2}\right] Z_+$$



a = g ; b = g' ; c = gg'

P.F. = (□ + ε ▨) = $\Lambda_{m,n}$ • ( ) + $\Lambda_{m,n+1/2}$ • ( )

massless massive

$\varepsilon = \pm 1$

- sector b

$$\Lambda_{p,q}\left\{\frac{1}{2}\left(\left|\frac{2\eta}{\theta_4}\right|^4+\left|\frac{2\eta}{\theta_3}\right|^4\right)\left[P_\epsilon^+ Q_s\overline{V}_{12}\overline{C}_4\overline{O}_{16}+P_\epsilon^- Q_s\overline{S}_{12}\overline{O}_4\overline{O}_{16}\right]+\right.$$

$$\left.\frac{1}{2}\left(\left|\frac{2\eta}{\theta_4}\right|^4-\left|\frac{2\eta}{\theta_3}\right|^4\right)\left[P_\epsilon^+ Q_s\overline{O}_{12}\overline{S}_4\overline{O}_{16}\right]\right\}\quad+\quad\text{massive}$$

where

$$P_\epsilon^+=\left(\frac{1+\epsilon(-1)^m}{2}\right)\Lambda_{m,n}\qquad P_\epsilon^-=\left(\frac{1-\epsilon(-1)^m}{2}\right)\Lambda_{m,n}$$

$$\epsilon\;=\;+1\quad\Rightarrow\quad P_\epsilon^+\;=\;\Lambda_{2m,n}\qquad P_\epsilon^-\;=\;\Lambda_{2m+1,n}$$

$$\epsilon\;=\;-1\quad\Rightarrow\quad P_\epsilon^+\;=\;\Lambda_{2m+1,n}\qquad P_\epsilon^-\;=\;\Lambda_{2m,n}$$

and $12\cdot 2\;+\;4\cdot 2\;=\;32$

Further :

- From the “Land” to the “Swamp” w Groot–Nibellink & Hurtado-Heredia, arXiv:2103.13442, spinor–vector duality on a resolved orbifold. The role of the discrete torsion in the effective field theory limit
- Vafa–Witten 1994, the role of a discrete torsion in the $Z_2 \times Z_2$ orbifold in mirror symmetry
- In similar spirit $\longrightarrow$ the imprint of the worldsheet modular properties in the effective field theory limit

Low scale Z' in free fermionic models:

- $\frac{3}{2}U(1)_{B-L} - 2U(1)_R \in SO(10)$ @ $1TeV$ MPL A6 (1991) 61

  (with Nanopoulos)

- But $m_t = m_{\nu_\tau}$ & $1TeV$ $Z' \Rightarrow m_{\nu_\tau} \approx 10MeV$ PLB 245 (1990) 435

  $\underline{E_6 \rightarrow SO(10) \times U(1)_A} \Longrightarrow U(1)_A$ is anomalous!

  $\Longrightarrow U(1)_A \notin$ low scale $U(1)_{Z'}$

- 1996-2013, Pati, AEF, Guzzi, Mehta, Athanasopoulos, $U(1) \notin E_6$

- On the other hand ....(AEF, Viraf Mehta, PRD88 (2013) 025006)

$$\sin^2\theta_W(M_Z)\ ,\ \alpha_s(M_Z) \Longrightarrow U(1)_{Z'} \in E_6$$

- Z' string derived model, (with Rizos) NPB 895 (2015) 233

light Z' heterotic–string model $c\left[\begin{matrix} vi \\ v_j \end{matrix}\right] = \exp[i\pi(v_i|v_j)]$:

$$(v_i|v_j) \;=\; \begin{array}{c|ccccccccccccc} & 1 & S & e_1 & e_2 & e_3 & e_4 & e_5 & e_6 & b_1 & b_2 & z_1 & z_2 & \alpha \\ \hline 1 & 1 & 1 & 1 & 1 & 1 & 1 & 1 & 1 & 1 & 1 & 1 & 1 & 1 \\ S & 1 & 1 & 1 & 1 & 1 & 1 & 1 & 1 & 1 & 1 & 1 & 1 & 1 \\ e_1 & 1 & 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 \\ e_2 & 1 & 1 & 0 & 0 & 0 & 0 & 0 & 1 & 0 & 0 & 0 & 1 & 0 \\ e_3 & 1 & 1 & 0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 & 1 & 1 \\ e_4 & 1 & 1 & 0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 & 1 & 0 & 0 \\ e_5 & 1 & 1 & 0 & 0 & 0 & 0 & 0 & 1 & 0 & 0 & 0 & 1 & 1 \\ e_6 & 1 & 1 & 0 & 1 & 0 & 0 & 1 & 0 & 0 & 0 & 1 & 0 & 0 \\ b_1 & 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 & 1 & 0 & 0 & 0 \\ b_2 & 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 & 1 & 0 & 0 & 1 \\ z_1 & 1 & 1 & 0 & 0 & 0 & 1 & 0 & 1 & 0 & 0 & 1 & 1 & 0 \\ z_2 & 1 & 1 & 0 & 1 & 1 & 0 & 1 & 0 & 0 & 0 & 1 & 1 & 0 \\ \alpha & 1 & 1 & 1 & 0 & 1 & 0 & 1 & 0 & 1 & 0 & 1 & 0 & 1 \end{array}$$

Observable gauge group: $SO(6) \times SO(4) \times U(1)_{1,2,3}$

$U(1)_\zeta = U(1)_1 + U(1)_2 + U(1)_3$ is anomaly free

sector	field	$SU(4)\times SU(2)_L\times SU(2)_R$	$U(1)_1$	$U(1)_2$	$U(1)_3$	$U(1)_\zeta$
$S+b_1$	$\bar{F}_{1R}$	$(\bar{\mathbf{4}},\mathbf{1},\mathbf{2})$	$1/2$	0	0	$1/2$
$S+b_1+e_3+e_5$	F_{1R}	$(\mathbf{4},\mathbf{1},\mathbf{2})$	$1/2$	0	0	$1/2$
$S+b_2$	F_{1L}	$(\mathbf{4},\mathbf{2},\mathbf{1})$	0	$1/2$	0	$1/2$
$S+b_2+e_1+e_2+e_5$	F_{2L}	$(\mathbf{4},\mathbf{2},\mathbf{1})$	0	$1/2$	0	$1/2$
$S+b_2+e_1$	$\bar{F}_{2R}$	$(\bar{\mathbf{4}},\mathbf{1},\mathbf{2})$	0	$1/2$	0	$1/2$
$S+b_2+e_2+e_5$	$\bar{F}_{3R}$	$(\bar{\mathbf{4}},\mathbf{1},\mathbf{2})$	0	$1/2$	0	$1/2$
$S+b_3+e_1+e_2$	F_{3L}	$(\mathbf{4},\mathbf{2},\mathbf{1})$	0	0	$1/2$	$1/2$
$S+b_3+e_2$	$\bar{F}_{4R}$	$(\bar{\mathbf{4}},\mathbf{1},\mathbf{2})$	0	0	$1/2$	$1/2$
$S+b_3+x$	h_1	$(\mathbf{1},\mathbf{2},\mathbf{2})$	$-1/2$	$-1/2$	0	-1
$S+b_2+x+e_5$	h_2	$(\mathbf{1},\mathbf{2},\mathbf{2})$	$-1/2$	0	$-1/2$	-1
$S+b_2+x+e_1+e_2$	h_3	$(\mathbf{1},\mathbf{2},\mathbf{2})$	$-1/2$	0	$-1/2$	-1
$S+b_3+x+e_1$	D_4	$(\mathbf{6},\mathbf{1},\mathbf{1})$	$-1/2$	$-1/2$	0	-1
	χ_1^+	$(\mathbf{1},\mathbf{1},\mathbf{1})$	$1/2$	$1/2$	1	$+2$
	χ_1^-	$(\mathbf{1},\mathbf{1},\mathbf{1})$	$1/2$	$1/2$	-1	0
	$\zeta_a, a=2,3$	$(\mathbf{1},\mathbf{1},\mathbf{1})$	$1/2$	$-1/2$	0	0
	$\bar{\zeta}_a, a=2,3$	$(\mathbf{1},\mathbf{1},\mathbf{1})$	$-1/2$	$1/2$	0	0
$S+b_2+x+e_1+e_5$	D_5	$(\mathbf{6},\mathbf{1},\mathbf{1})$	$-1/2$	0	$-1/2$	-1
	χ_2^+	$(\mathbf{1},\mathbf{1},\mathbf{1})$	$1/2$	1	$1/2$	$+2$
	χ_2^-	$(\mathbf{1},\mathbf{1},\mathbf{1})$	$1/2$	-1	$1/2$	0
	$\zeta_a, a=4,5$	$(\mathbf{1},\mathbf{1},\mathbf{1})$	$1/2$	0	$-1/2$	0
	$\bar{\zeta}_a, a=4,5$	$(\mathbf{1},\mathbf{1},\mathbf{1})$	$-1/2$	0	$1/2$	0
$S+b_2+x+e_2$	D_6	$(\mathbf{6},\mathbf{1},\mathbf{1})$	$-1/2$	0	$-1/2$	-1
	χ_3^+	$(\mathbf{1},\mathbf{1},\mathbf{1})$	$1/2$	1	$1/2$	$+2$
	χ_3^-	$(\mathbf{1},\mathbf{1},\mathbf{1})$	$1/2$	-1	$1/2$	0
	$\zeta_a, a=6,7$	$(\mathbf{1},\mathbf{1},\mathbf{1})$	$1/2$	0	$-1/2$	0
	$\bar{\zeta}_a, a=6,7$	$(\mathbf{1},\mathbf{1},\mathbf{1})$	$-1/2$	0	$1/2$	0
$S+b_1+x+e_3$	$\bar{D}_6$	$(\mathbf{6},\mathbf{1},\mathbf{1})$	0	$1/2$	$1/2$	$+1$
	$\bar{\chi}_4^+$	$(\mathbf{1},\mathbf{1},\mathbf{1})$	-1	$-1/2$	$-1/2$	-2
	$\bar{\chi}_4^-$	$(\mathbf{1},\mathbf{1},\mathbf{1})$	1	$-1/2$	$-1/2$	0
	$\zeta_a, a=8,9$	$(\mathbf{1},\mathbf{1},\mathbf{1})$	0	$1/2$	$-1/2$	0
	$\bar{\zeta}_a, a=8,9$	$(\mathbf{1},\mathbf{1},\mathbf{1})$	0	$-1/2$	$1/2$	0
$S+b_1+x+e_5$	D_7	$(\mathbf{6},\mathbf{1},\mathbf{1})$	0	$-1/2$	$-1/2$	-1
	χ_5^+	$(\mathbf{1},\mathbf{1},\mathbf{1})$	1	$1/2$	$1/2$	$+2$
	χ_5^-	$(\mathbf{1},\mathbf{1},\mathbf{1})$	-1	$1/2$	$1/2$	0
	$\zeta_a, a=10,11$	$(\mathbf{1},\mathbf{1},\mathbf{1})$	0	$1/2$	$-1/2$	0
	$\bar{\zeta}_a, a=10,11$	$(\mathbf{1},\mathbf{1},\mathbf{1})$	0	$-1/2$	$1/2$	0
$S+b_3+x+e_2+e_3$	ζ_1	$(\mathbf{1},\mathbf{1},\mathbf{1})$	$1/2$	$-1/2$	0	0
	$\bar{\zeta}_1$	$(\mathbf{1},\mathbf{1},\mathbf{1})$	$-1/2$	$1/2$	0	0
$S+b_1+x+e_3+e_4+e_6$	ϕ_1	$(\mathbf{1},\mathbf{1},\mathbf{1})$	0	$1/2$	$1/2$	$+1$
	$\bar{\phi}_1$	$(\mathbf{1},\mathbf{1},\mathbf{1})$	0	$-1/2$	$-1/2$	-1
$S+b_1+x+e_4+e_5+e_6$	ϕ_2	$(\mathbf{1},\mathbf{1},\mathbf{1})$	0	$1/2$	$1/2$	$+1$
	$\bar{\phi}_2$	$(\mathbf{1},\mathbf{1},\mathbf{1})$	0	$-1/2$	$-1/2$	-1

Table 1: *Observable twisted matter spectrum and* $SU(4)\times SU(2)_L\times SU(2)_R\times U(1)^3$ *quantum numbers.*

Z' model at low scales Heavy Higgs $\langle \mathcal{N} \rangle \sim M_{\text{String}}$ $\longrightarrow$ high seesaw $\longrightarrow$

Field	$SU(3)_C$	$\times SU(2)_L$	$U(1)_Y$	$U(1)_{Z'}$
Q^i_L	3	2	$+\frac{1}{6}$	$-\frac{2}{5}$
u^i_L	$\bar{3}$	1	$-\frac{2}{3}$	$-\frac{2}{5}$
d^i_L	$\bar{3}$	1	$+\frac{1}{3}$	$-\frac{4}{5}$
e^i_L	1	1	$+1$	$-\frac{2}{5}$
L^i_L	1	2	$-\frac{1}{2}$	$-\frac{4}{5}$
D^i	3	1	$-\frac{1}{3}$	$+\frac{4}{5}$
$\bar{D}^i$	$\bar{3}$	1	$+\frac{1}{3}$	$+\frac{6}{5}$
H^i	1	2	$-\frac{1}{2}$	$+\frac{6}{5}$
$\bar{H}^i$	1	2	$+\frac{1}{2}$	$+\frac{4}{5}$
S^i	1	1	0	-2
h	1	2	$-\frac{1}{2}$	$-\frac{4}{5}$
$\bar{h}$	1	2	$+\frac{1}{2}$	$+\frac{4}{5}$
ϕ	1	1	0	-1
$\bar{\phi}$	1	1	0	$+1$
ζ^i	1	1	0	0

Additional matter states at $U(1)_{Z'}$ breaking scale

NON–SUSY String Phenomenology:

Starting with: $Z^+_{10d} = (V_8 - S_8)\left(\overline{O}_{16} + \overline{S}_{16}\right)\left(\overline{O}_{16} + \overline{S}_{16}\right)\,,$

using the level–one $SO(2n)$ characters

$$O_{2n} = \tfrac{1}{2}\left(\frac{\theta_3^n}{\eta^n} + \frac{\theta_4^n}{\eta^n}\right)\,, \qquad V_{2n} = \tfrac{1}{2}\left(\frac{\theta_3^n}{\eta^n} - \frac{\theta_4^n}{\eta^n}\right)\,,$$

$$S_{2n} = \tfrac{1}{2}\left(\frac{\theta_2^n}{\eta^n} + i^{-n}\frac{\theta_1^n}{\eta^n}\right)\,, \qquad C_{2n} = \tfrac{1}{2}\left(\frac{\theta_2^n}{\eta^n} - i^{-n}\frac{\theta_1^n}{\eta^n}\right)\,.$$

where

$$\theta_3 \equiv Z_f\binom{0}{0} \qquad \theta_4 \equiv Z_f\binom{0}{1} \qquad \theta_2 \equiv Z_f\binom{1}{0} \qquad \theta_1 \equiv Z_f\binom{1}{1}$$

Apply $g = (-1)^{F+F_{z_1}+F_{z_2}}$

$$\begin{aligned} Z^-_{10d} = \; & \Big[\; V_8\left(\overline{O}_{16}\overline{O}_{16} + \overline{S}_{16}\overline{S}_{16}\right) - S_8\left(\overline{O}_{16}\overline{S}_{16} + \overline{S}_{16}\overline{O}_{16}\right) \\ + \; & \underline{O_8\left(\overline{C}_{16}\overline{V}_{16} + \overline{V}_{16}\overline{C}_{16}\right)} - C_8\left(\overline{C}_{16}\overline{C}_{16} + \overline{V}_{16}\overline{V}_{16}\right)\Big]\,. \end{aligned}$$

In fermionic language: $\{ \mathbf{1} , z_1 , z_2 \}$

$$\text{where} \quad z_1 = \{\bar\psi^{1,\cdots,5}, \bar\eta^{1,2,3}\} \quad ; \quad z_2 = \{\bar\phi^{1,\cdots,8}\} \quad \Rightarrow \quad S = \mathbf{1} + z_1 + z_2$$

$$c\binom{z_1}{z_2} = +1 \quad \Longrightarrow \quad E_8 \times E_8 \qquad ; \qquad c\binom{z_1}{z_2} = -1 \quad \Longrightarrow \quad SO(16) \times SO(16)$$

non-SUSY string phenomenology

Alternatively: Apply $g = (-1)^{F+F_{z_1}}$

$$Z^-_{10d} \;=\; \left(V_8\overline{O}_{16} - S_8\overline{S}_{16} + \underline{O_8\overline{V}_{16}} - C_8\overline{C}_{16}\right)\left(\overline{O}_{16} + \overline{S}_{16}\right) ,$$

$O_8\overline{V}_{16}\overline{O}_{16} \quad \Longrightarrow \quad$ tachyon

In fermionic language: $\{ \mathbf{1} , z_2 \} \quad \Longrightarrow \quad$ No S

Tachyon free models: $S \longleftrightarrow \tilde{S}$–map

Modified NAHE $\longleftrightarrow$ $\overline{\mathrm{NAHE}}$

	ψ^μ	χ^{12}	χ^{34}	χ^{56}	$y^{3,\ldots,6}$	$\bar{y}^{3,\ldots,6}$	$y^{1,2},\omega^{5,6}$	$\bar{y}^{1,2},\bar{\omega}^{5,6}$	$\omega^{1,\ldots,4}$	$\bar{\omega}^{1,\ldots,4}$	$\bar{\psi}^{1,\ldots,5}$	$\bar{\eta}^1$	$\bar{\eta}^2$	$\bar{\eta}^3$	$\bar{\phi}^{1,\ldots,8}$
$\mathbf{1}$	1	1	1	1	1,…,1	1,…,1	1,…,1	1,…,1	1,…,1	1,…,1	1,…,1	1	1	1	1,1,1,1,1,1,1,1
$\tilde{S}$	1	1	1	1	0,…,0	0,…,0	0,…,0	0,…,0	0,…,0	0,…,0	0,…,0	0	0	0	1,1,1,1,0,0,0,0
b_1	1	1	0	0	1,…,1	1,…,1	0,…,0	0,…,0	0,…,0	0,…,0	1,…,1	1	0	0	0,0,0,0,0,0,0,0
b_2	1	0	1	0	0,…,0	0,…,0	1,…,1	1,…,1	0,…,0	0,…,0	1,…,1	0	1	0	0,0,0,0,0,0,0,0
b_3	1	0	0	1	0,…,0	0,…,0	0,…,0	0,…,0	1,…,1	1,…,1	1,…,1	0	0	1	0,0,0,0,0,0,0,0

Beyond the $\overline{\mathrm{NAHE}}$–set

	ψ^μ	χ^{12}	χ^{34}	χ^{56}	y^3y^6	$y^4\bar{y}^4$	$y^5\bar{y}^5$	$\bar{y}^3\bar{y}^6$	$y^1\omega^5$	$y^2\bar{y}^2$	$\omega^6\bar{\omega}^6$	$\bar{y}^1\bar{\omega}^5$	$\omega^2\omega^4$	$\omega^1\bar{\omega}^1$	$\omega^3\bar{\omega}^3$	$\bar{\omega}^2\bar{\omega}^4$	$\bar{\psi}^{1,\ldots,5}$	$\bar{\eta}^1$	$\bar{\eta}^2$	$\bar{\eta}^3$	$\bar{\phi}$
α	0	0	0	0	1	0	0	1	0	0	1	1	0	0	1	1	1 1 1 0 0	1	0	0	0 0 0
β	0	0	0	0	0	0	1	1	1	0	0	1	0	1	0	1	1 1 1 0 0	0	1	0	1 1 0
γ	0	0	0	0	0	1	0	0	0	1	0	0	1	0	0	0	$\frac{1}{2}$ $\frac{1}{2}$ $\frac{1}{2}$ $\frac{1}{2}$ $\frac{1}{2}$	$\frac{1}{2}$	$\frac{1}{2}$	$\frac{1}{2}$	0 0 $\frac{1}{2}$

Up to the $S \longleftrightarrow \tilde{S}$–map

Same model as published with

with Cleaver, Manno and Timirgazi in PRD78 (2008) 046009

Stable non–SUSY heterotic–string vacuum?

Toward Classification of tachyon free models

Basis vectors:

$$\begin{aligned}
1 &= \{\psi^\mu,\ \chi^{1,\dots,6}, y^{1,\dots,6}, \omega^{1,\dots,6} \mid \bar{y}^{1,\dots,6}, \bar{\omega}^{1,\dots,6}, \bar{\eta}^{1,2,3}, \bar{\psi}^{1,\dots,5}, \bar{\phi}^{1,\dots,8}\} \\
\tilde{S} &= \{\psi^\mu, \chi^{1,\dots,6} \mid \bar{\phi}^{3,\cdots,6}\}, \\
z_1 &= \{\bar{\phi}^{1,\dots,4}\}, \\
z_2 &= \{\bar{\phi}^{5,\dots,8}\}, \\
e_i &= \{y^i, \omega^i | \bar{y}^i, \bar{\omega}^i\},\ i = 1, \dots, 6,
\end{aligned}$$

$N = 4$ Vacua

$$b_1 = \{\chi^{34}, \chi^{56}, y^{34}, y^{56} | \bar{y}^{34}, \bar{y}^{56}, \bar{\eta}^1, \bar{\psi}^{1,\dots,5}\}, \qquad N = 4 \rightarrow N = 2$$

$$b_2 = \{\chi^{12}, \chi^{56}, y^{12}, y^{56} | \bar{y}^{12}, \bar{y}^{56}, \bar{\eta}^2, \bar{\psi}^{1,\dots,5}\}, \qquad N = 2 \rightarrow N = 1$$

with Viktor Matyas and Ben Percival, NPB 961 (2020) 115231; 2011.04113

Partition functions and the cosmological constant

Full Partition Function for Free Fermionic models:

$$Z_{ToT} \quad = \quad \int_{\mathfrak{F}} \frac{d^2\tau}{\tau_2^2} Z_B\, Z_F \quad \equiv \quad \Lambda$$

Integral over the inequivalent tori

- Fermionic contribution:

$$Z_F = \sum_{Sp.Str.} c\binom{\alpha}{\beta} \prod_f Z\begin{bmatrix}\alpha(f)\\ \beta(f)\end{bmatrix}$$

$$Z\begin{bmatrix}1\\1\end{bmatrix} = \sqrt{\frac{\theta_1}{\eta}}, \qquad Z\begin{bmatrix}1\\0\end{bmatrix} = \sqrt{\frac{\theta_2}{\eta}}, \qquad Z\begin{bmatrix}0\\0\end{bmatrix} = \sqrt{\frac{\theta_3}{\eta}}, \qquad Z\begin{bmatrix}0\\1\end{bmatrix} = \sqrt{\frac{\theta_4}{\eta}},$$

- Bososnic : $Z_B = \frac{1}{\tau_2}\frac{1}{\eta^2\bar{\eta}^2}$ from spacetime Bosons.

Evaluated using $q \equiv e^{2\pi i\tau}$ expansion

$$Z = \sum_{n.m} a_{mn} \int_{\mathfrak{F}} \frac{d^2\tau}{\tau_2^3} q^m \bar{q}^n \qquad \begin{cases} d\tau_1 & \longrightarrow \textit{analytic} \\ d\tau_2 & \longrightarrow \textit{numeric} \end{cases}$$

$q -$ expansion of Z

$$I_{mn} = \begin{cases} \infty & \text{if} \quad m+n<0 \ \wedge \ m-n \notin \mathbb{Z}\backslash\{0\} \\ \text{Finite} & \text{Otherwise.} \end{cases}$$

- On-Shell Tachyons cause divergence

- Off-Shell Tachyons allowed (necessary)

Modular invariance $\longrightarrow m - n \in \mathbb{Z}$.

Allowed states

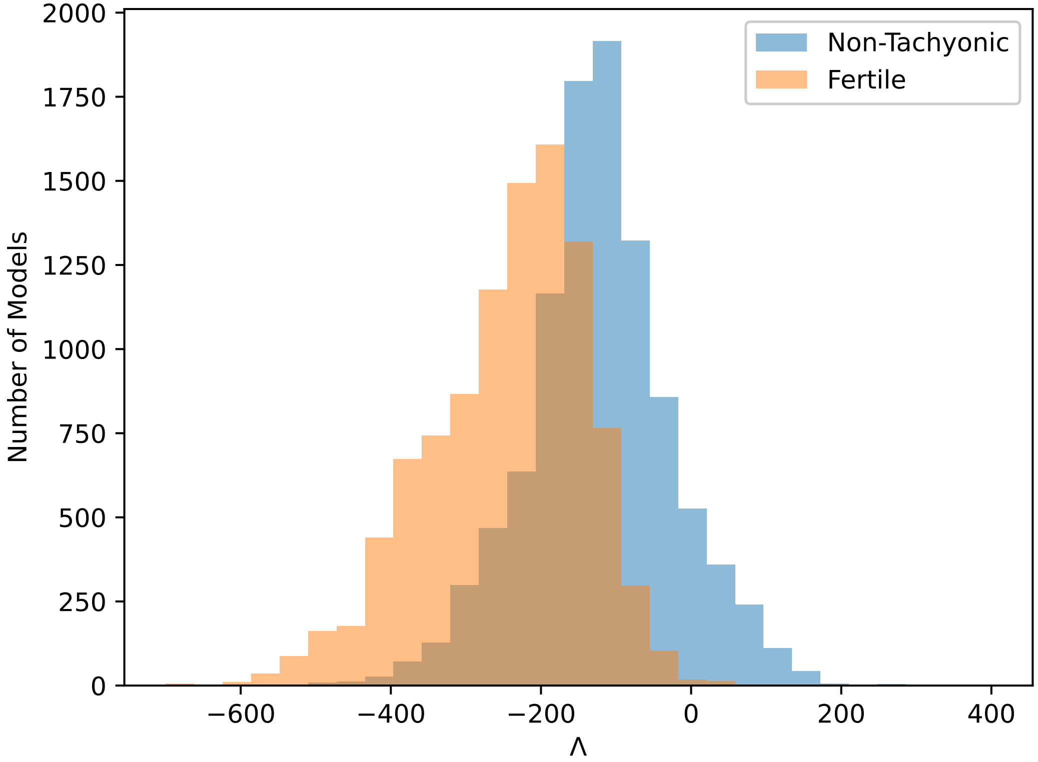
$$a_{mn} = \begin{pmatrix} 0 & 0 & a_{-\frac{1}{2}-\frac{1}{2}} & 0 & 0 & 0 & a_{-\frac{1}{2}\frac{1}{2}} & 0 & 0 & 0 \\ 0 & 0 & 0 & a_{-\frac{1}{4}-\frac{1}{4}} & 0 & 0 & 0 & a_{-\frac{1}{4}\frac{3}{4}} & 0 & 0 \\ a_{0-1} & 0 & 0 & 0 & a_{00} & 0 & 0 & 0 & a_{01} & 0 \\ 0 & a_{\frac{1}{4}-\frac{3}{4}} & 0 & 0 & 0 & a_{\frac{1}{4}\frac{1}{4}} & 0 & 0 & 0 & \ddots \\ 0 & 0 & a_{\frac{1}{2}-\frac{1}{2}} & 0 & 0 & 0 & a_{\frac{1}{2}\frac{1}{2}} & 0 & 0 & 0 \\ 0 & 0 & 0 & a_{\frac{3}{4}-\frac{1}{4}} & 0 & 0 & 0 & a_{\frac{3}{4}\frac{3}{4}} & 0 & 0 \\ a_{1-1} & 0 & 0 & 0 & a_{10} & 0 & 0 & 0 & a_{11} & 0 \\ 0 & \ddots & 0 & 0 & 0 & \ddots & 0 & 0 & 0 & \ddots \end{pmatrix}$$

Coefficients $a_{mn} = N_b - N_f$ at specific mass level.

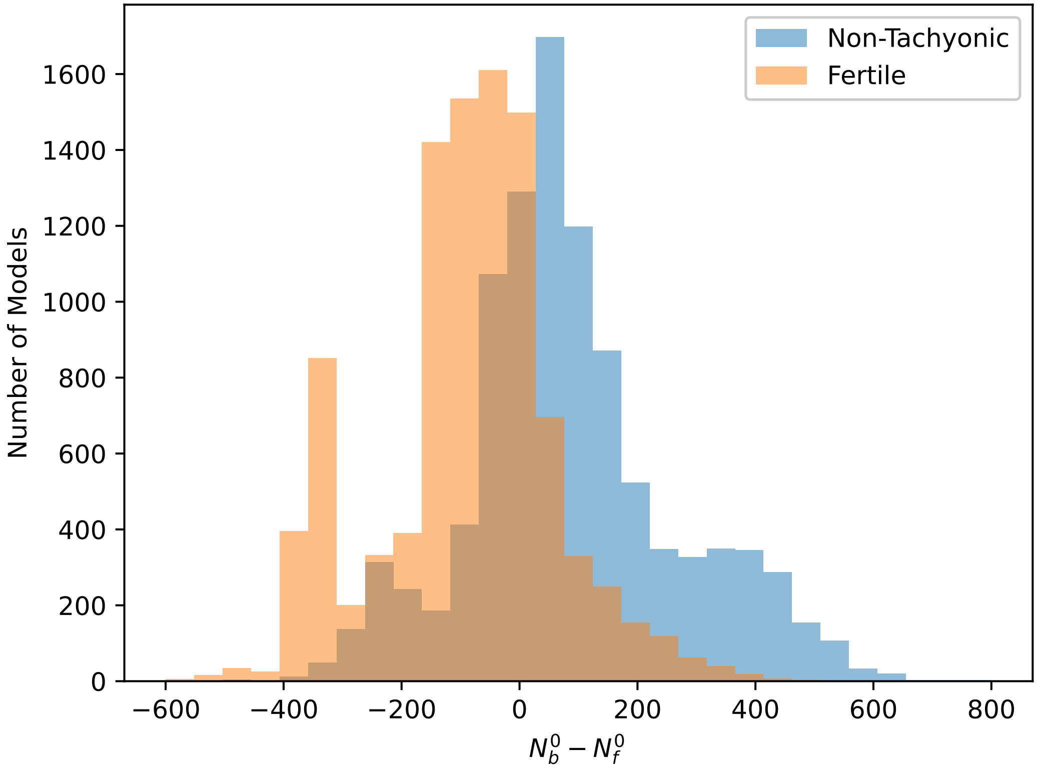
For SUSY Theories $a_{mn} = 0 \; \forall m, n$

Some interesting results

Distribution of Λ



Distribution of a_{00}



Conclusions

- DATA ⟶ UNIFICATION
- STRINGS THEORY ⟶ GAUGE & GRAVITY UNIFICATION
- STRINGS PHENOMENOLOGY ⟶ AT ITS INFANCY
- Moduli spaces of (2,0) string compactifications

  ⟶ from the "land" to the "swamp"
- String derived Z' at LHCb
- Non–SUSY string phenomenology $\cdots\cdots\cdots$
- String Phenomenology ⟶ Physics of the third millennium

  e.g. Aristarchus to Copernicus