Free fermion models and asymmetric orbifolds

Alon E. Faraggi

- Old and new results from free fermion heterotic–string model building

Collaborators: ..., Panos Athanasopoulos, Viraf Mehta, Hasan Sonmez ...

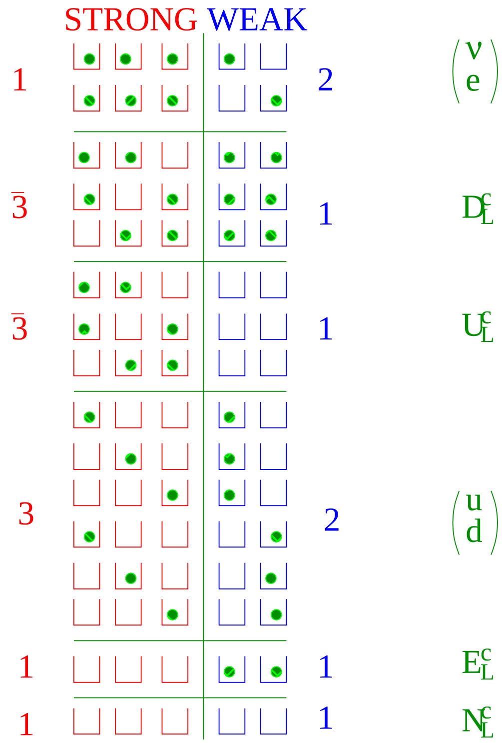
Non-geometry, asymmetric orbifolds and model building, Bonn, 11 June 2014

Disclaimer:

World–sheet CFT constructions and geometrical compactifications are intimately related in string theory. This is exhibited in the correspondence of Gepner models with Calabi–Yau manifolds at special points. The simplest CFT constructions include free world-sheet fermions and bosons. The correspondence between two dimensional world–sheet fermions and bosons translates to compactifications based on these worldsheet CFTs to be mathematically identical. Nevertheless, to write a detailed dictionary in specific cases remains a challenge and is, in general, unknown. Recipes exist.

STANDARD MODEL → UNIFICATION

STRONG WEAK ELECTROMAGNETIC → SO(10)



$$\bar{5} = \binom{5}{4} = \frac{5!}{4!1!}$$

+

$$10 = \binom{5}{2}$$

+

$$1 = \binom{5}{0}$$

16

Additional evidence: Log running , τ_p , m_ν

Guides: 3 Generations & SO(10) embedding

Realistic free fermionic models

'Phenomenology of the Standard Model and string unification'

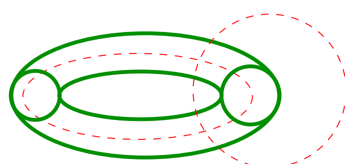
• Top quark mass $\sim$ 175–180GeV	PLB 274 (1992) 47
• Generation mass hierarchy	NPB 407 (1993) 57
• CKM mixing	NPB 416 (1994) 63 (with Halyo)
• Stringy seesaw mechanism	PLB 307 (1993) 311 (with Halyo)
• Gauge coupling unification	NPB 457 (1993) 409 (with Dienes)
• Proton stability	NPB 428 (1994) 111
• Squark degeneracy	NPB 526 (1998) 21 (with Pati)
• Minimal Superstring Standard Model	PLB 455 (1999) 135 (with Cleaver & Nanopoulos)
• Moduli fixing	NPB 728 (2005) 83

Free Fermionic Construction

Left-Movers: $\psi^{\mu}_{1,2}, \quad \chi_i, \quad y_i, \ \ \omega_i \quad (i=1,\cdots,6)$

Right-Movers

$$\bar\phi_{A=1,\cdots,44} = \begin{cases} \bar y_i \ , \ \bar\omega_i & i=1,\cdots,6 \\ \\ \bar\eta_i & i=1,2,3 \\ \bar\psi_{1,\cdots,5} & \\ \bar\phi_{1,\cdots,8} & \end{cases}$$



$$V \longrightarrow V \qquad\qquad f \longrightarrow -e^{i\pi\alpha(f)} f$$

$$Z = \sum_{\substack{all\ spin\\ structures}} c\binom{\vec\alpha}{\vec\beta}\ Z\binom{\vec\alpha}{\vec\beta}$$

Models $\longleftrightarrow$ Basis vectors $+$ one–loop phases

Model building – Construction of the physical states

$$b_j \quad j = 1, \cdots, N \quad \rightarrow \quad \Xi = \sum_j n_j b_j$$

For $\vec{\alpha} = (\vec{\alpha}_L; \vec{\alpha}_R) \in \Xi \Rightarrow \mathrm{H}_{\vec{\alpha}}$

$$\alpha(f) = 1 \Rightarrow |\pm\rangle \quad ; \quad \alpha(f) \neq 1 \Rightarrow f, f^* \quad , \quad \nu_{f,f^*} = \frac{1 \mp \alpha(f)}{2}$$

$$M_L^2 = -\frac{1}{2} + \frac{\vec{\alpha}_L \cdot \vec{\alpha}_L}{8} + N_L = -1 + \frac{\vec{\alpha}_R \cdot \vec{\alpha}_R}{8} + N_R = M_R^2 \quad (\equiv 0)$$

GSO projections $\quad e^{i\pi(\vec{b}_i \cdot \vec{F}_\alpha)} |s\rangle_{\vec{\alpha}} = \delta_\alpha c^* \binom{\vec{\alpha}}{\vec{b}_i} |s\rangle_{\vec{\alpha}}$

$F_\alpha(f) \rightarrow$ fermion # operator $= \begin{cases} -1, & |-\rangle \\ 0, & |+\rangle \end{cases} = \begin{cases} +1, \\ -1, \end{cases}$

$Q(f) = \frac{1}{2}\alpha(f) + F(f) \rightarrow$ U(1) charges

Calculation of Mass Terms

nonvanishing correlators

$$\langle V_1^f V_2^f V_3^b \cdot \cdots V_N^b \rangle$$

gauge & string invariant

"anomalous" $U(1)_A$

$$\text{Tr}Q_A \neq 0 \Rightarrow \quad D_A = 0 = A + \sum Q_k^A |\langle \phi_k \rangle|^2$$

$$D_j \;= 0 = \qquad \sum Q_k^j |\langle \phi_k \rangle|^2$$

$$\langle W \rangle = \langle \frac{\partial W_N}{\partial \eta_i} \rangle = 0 \quad N = 3 \cdots \cdot$$

Supersymmetric vacuum $\langle F \rangle = \langle D \rangle = 0.$

nonrenormalizable terms $\longrightarrow$ effective renormalizable operators

$$V_1^f V_2^f V_3^b \cdot \cdots V_N^b \;\rightarrow\; V_1^f V_2^f V_3^b \frac{\langle V_4^b \cdots V_N^b \rangle}{M^{N-3}}$$

The NAHE set : $\{ \mathbf{1}, \ S \ , \quad b_1 \ , \quad b_2 \ , \quad b_3 \ \}$

$N = \quad 4 \quad \rightarrow \quad 2 \qquad 1 \qquad 1$ vacua

$Z_2 \times Z_2$ orbifold compactification

$\Longrightarrow$ Gauge group $SO(10) \times SO(6)^{1,2,3} \times E_8$

beyond the NAHE set Add $\{\alpha, \beta, \gamma\}$

number of generations is reduced to three

$$SO(10) \quad \longrightarrow \quad SU(3) \times SU(2) \times U(1)_{T_{3_R}} \times U(1)_{B-L}$$

$$U(1)_Y \ = \ \frac{1}{2}(B-L) \ + \ T_{3_R} \ \in \ SO(10) \ !$$

$$SO(6)^{1,2,3} \quad \longrightarrow \quad U(1)^{1,2,3} \ \times \ U(1)^{1,2,3}$$

Patterns of $SO(10)$ symmetry breaking

The $SO(10) \to$ subgroup $b(\bar\psi^{1\cdots5}_{\frac{1}{2}})$:

1. $b\{\bar\psi^{1\cdots5}_{\frac{1}{2}}\ \bar\eta^1\ \bar\eta^2\ \bar\eta^3\} = \{\frac{1}{2}\frac{1}{2}\frac{1}{2}\frac{1}{2}\frac{1}{2}\frac{1}{2}\frac{1}{2}\frac{1}{2}\} \Rightarrow\ SU(5)\times U(1)\ \ U(1)\ U(1)\ U(1)$
2. $b\{\bar\psi^{1\cdots5}_{\frac{1}{2}}\ \bar\eta^1\ \bar\eta^2\ \bar\eta^3\} = \{\ 11100\ 000\ \} \Rightarrow SO(6)\times SO(4)\ U(1)\ U(1)\ U(1)$

$$(1. + 2.)\ \Rightarrow\ SO(10)\ \ \to SU(3)_C\times SU(2)_L\times U(1)_C\times U(1)_L$$

$$SO(10)\to SU(3)_C\times SU(2)_L\times SU(2)_R\times U(1)_{B-L}$$

2. $b\{\bar\psi^{1\cdots5}_{\frac{1}{2}}\bar\eta^1\ \bar\eta^2\ \bar\eta^3\} = \{\ 11100000\ \} \Rightarrow SO(6)\times SO(4)\ U(1)\ U(1)\ U(1)$
3. $b\{\bar\psi^{1\cdots5}_{\frac{1}{2}}\ \bar\eta^1\ \bar\eta^2\ \bar\eta^3\} = \{\frac{1}{2}\frac{1}{2}\frac{1}{2}00\frac{1}{2}\frac{1}{2}\frac{1}{2}\} \Rightarrow$

$$SU(\ 3\ )_C\times U(1)_C\times SU(2)_L\times SU(2)_R\ U(1)\ U(1)\ U(1)$$

The massless spectrum

Three twisted generations $b_1, \quad b_2, \quad b_3$

Untwisted Higgs doublets

$$h_{1_{1,0,0}} \qquad \bar{h}_{1_{-1,0,0}}$$
$$h_{2_{0,1,0}} \qquad \bar{h}_{2_{0,-1,0}}$$
$$h_{3_{0,0,1}} \qquad \bar{h}_{3_{0,0,-1}}$$

Sector $b_1 + b_2 + \alpha + \beta$ $\qquad h_{\alpha\beta_{-\frac{1}{2},-\frac{1}{2},0,0,0,0}} \qquad \bar{h}_{\alpha\beta_{\frac{1}{2},\frac{1}{2},0,0,0,0}}$

$\oplus \quad SO(10)$ singlets

Sectors $b_j + 2\gamma \quad j = 1, 2, 3 \quad \longrightarrow$ hidden matter multiplets

"standard" $SO(10)$ representations

NAHE + $\{ \alpha \ , \ \beta \ , \ \gamma \} \ \rightarrow$ exotic vector–like matter $\rightarrow$ superheavy

$\oplus$ Quasi-realistic phenomenology

Correspondence with $Z_2 \times Z_2$ orbifold

PLB 326 (1994) 62

NAHE $\oplus$ $(\xi_2 = \{\bar{\psi}_{1,\cdots,5}, \bar{\eta}^1, \bar{\eta}^2, \bar{\eta}^3\} = 1) \quad \rightarrow \quad \{1, S, \xi_1, \xi_2, b_1, b_2\}$

Gauge group: $SO(4)^3 \times E_6 \times U(1)^2 \times E_8$ and 24 generations.

toroidal compactification

$$g_{ij} = \begin{pmatrix} 2 & -1 & 0 & 0 & 0 & 0 \\ -1 & 2 & -1 & 0 & 0 & 0 \\ 0 & -1 & 2 & -1 & 0 & 0 \\ 0 & 0 & -1 & 2 & -1 & -1 \\ 0 & 0 & 0 & -1 & 2 & 0 \\ 0 & 0 & 0 & -1 & 0 & 2 \end{pmatrix} \qquad b_{ij} = \begin{cases} g_{ij} & i < j \\ 0 & i = j \\ -g_{ij} & i > j \end{cases}$$

$R_i \quad \rightarrow$ the free fermionic point $\rightarrow$ G.G. $SO(12) \times E_8 \times E_8$

mod out by a $Z_2 \times Z_2$ with standard embedding

$\Rightarrow$ Exact correspondence

In the realistic free fermionic models

replace $\xi_2 \equiv x = \{\bar\psi^{1,\cdots,5}, \bar\eta^1, \bar\eta^2, \bar\eta^3\} = 1$

with $2\gamma = \{\bar\psi^{1,\cdots,5}, \bar\eta^1, \bar\eta^2, \bar\eta^3, \bar\phi^{1,\cdots,4}\} = 1$

Then $\{\vec{1}, \vec{S}, \vec{\xi}_1 = \vec{1} + \vec{b}_1 + \vec{b}_2 + \vec{b}_3, 2\gamma\} \rightarrow$ N=4 SUSY and

$$SO(12) \times SO(16) \times SO(16)$$

apply $b_1 \times b_2 \rightarrow Z_2 \times Z_2 \rightarrow$ N=1 SUSY and

$$SO(4)^3 \times SO(10) \times U(1)^3 \times SO(16)$$

b_1, b_2, b_3 $\Rightarrow (3 \times 8) \cdot 16$ of $SO(10)_O$

$b_1 + 2\gamma$, $b_2 + 2\gamma$, $b_3 + 2\gamma$ $\Rightarrow (3 \times 8) \cdot 16$ of $SO(16)_H$

Alternatively, $c\binom{\xi_1}{\xi_2} = +1 \rightarrow -1$

$Z_2 \times Z_2$ orbifolds

torus: One complex parameter $Z = Z + n e_1 + m e_2$

$T^2 \times T^2 \times T^2 \longrightarrow$ Three complex coordinates z_1 , z_2 and z_3

Z_2 orbifold : $Z = - Z + \sum_i m_i e_i \longrightarrow$ 4 fixed points

$$Z = \{ 0 , 1/2\, e_1 , 1/2\, e_2 , 1/2\, (e_1 + e_2) \}$$

$$\frac{T^2 \times T^2 \times T^2}{Z_2 \times Z_2}$$

$\alpha : (z1 , z2 , z3) \to (-z1 , -z2 , +z3) \to 16$

$\beta : (z1 , z2 , z3) \to (+z1 , -z2 , -z3) \to 16$

$\alpha\beta : (z1 , z2 , z3) \to (-z1 , +z2 , -z3) \to 16$

48

$\downarrow$

$\gamma : (z1, z2, z3) \to (z1+1/2, z2+1/2, z3+1/2) \longrightarrow$ 24

NAHE-based partition functions:

Question:

$$\frac{T^6}{Z_2 \times Z_2} \rightarrow 48 \text{ fixed points} \qquad\qquad \frac{SO(12)}{Z_2 \times Z_2} \rightarrow 24 \text{ fixed points}$$

$$Z_2 \text{ shift } : 48 \longleftrightarrow 24$$

Is this the same model? In general, no.

shift that reproduces the $SO(12)$ lattice at the free fermionic point?

Possible shifts:

$$A_1 \;:\; X_{\rm L,R} \to X_{\rm L,R} + \tfrac{1}{2}\pi R\,,$$

$$A_2 \;:\; X_{\rm L,R} \to X_{\rm L,R} + \tfrac{1}{2}\left(\pi R \pm \frac{\pi\alpha'}{R}\right)\,,$$

$$A_3 \;:\; X_{\rm L,R} \to X_{\rm L,R} \pm \tfrac{1}{2}\frac{\pi\alpha'}{R}\,.$$

Using the level-one $\mathrm{SO}(2n)$ characters

$$O_{2n} = \tfrac{1}{2}\left(\frac{\vartheta_3^n}{\eta^n} + \frac{\vartheta_4^n}{\eta^n}\right)\,, \qquad V_{2n} = \tfrac{1}{2}\left(\frac{\vartheta_3^n}{\eta^n} - \frac{\vartheta_4^n}{\eta^n}\right)\,,$$

$$S_{2n} = \tfrac{1}{2}\left(\frac{\vartheta_2^n}{\eta^n} + i^{-n}\frac{\vartheta_1^n}{\eta^n}\right)\,, \qquad C_{2n} = \tfrac{1}{2}\left(\frac{\vartheta_2^n}{\eta^n} - i^{-n}\frac{\vartheta_1^n}{\eta^n}\right)\,.$$

Starting from:

$$Z_+ = (V_8 - S_8)\left(\sum_{m,n} \Lambda_{m,n}\right)^{\otimes 6} (\bar{O}_{16} + \bar{S}_{16})\,(\bar{O}_{16} + \bar{S}_{16}) \ ,$$

where as usual, for each circle,

$$p^i_{\mathrm{L,R}} = \frac{m_i}{R_i} \pm \frac{n_i R_i}{\alpha'} \,,$$

and

$$\Lambda_{m,n} = \frac{q^{\frac{\alpha'}{4} p_{\mathrm{L}}^2}\, \bar{q}^{\frac{\alpha'}{4} p_{\mathrm{R}}^2}}{|\eta|^2} \,.$$

Add shifts : (A_1, A_1, A_1) , (A_3, A_3, A_3)

($48 \rightarrow 24$ yes)

($SO(12)$? no)

Uniquely:

$$g \ : \ (A_2, A_2, 0)\,,$$

$$h \ : \ (0, A_2, A_2)\,,$$

where each A_2 acts on a complex coordinate

($48 \longrightarrow 24$ yes)

($SO(12)$? yes)

$$R = \sqrt{\alpha'}$$

Moduli?

Untwisted moduli $->$ shape & size of the internal dimensions

Twisted moduli $->$ arise from the twisted sectors

models

$$\frac{T^6}{Z_2 \times Z_2}$$

$$T^6 \;:\quad G_{IJ} \quad ; \quad B_{IJ} \qquad I, J \;=\; 1, \cdots, 6 \;.$$

untwisted moduli: coefficients of exactly marginal operators

moduli fields: massless chiral superfields with flat scalar potential

Scalar couplings of $N = 4$ SUGRA

$$\frac{SO(6,6)}{SO(6) \times SO(6)} \quad \times \quad \frac{SU(1,1)}{U(1)}$$

internal manifold dilaton

Up to orbifold projections

$$\frac{T^6}{Z_2 \times Z_2} \Longrightarrow \frac{SO(6,6)}{SO(6)\times SO(6)} \longrightarrow \left(\frac{SO(2,2)}{SO(2)\times SO(2)}\right)^3$$

$\Rightarrow$ 3 complex structures $+$ 3 Kähler moduli

In all symmetric $Z_2 \times Z_2$ orbifolds

We would like to identify these moduli in the fermionic formalism

In symmetric orbifolds:

EMO $\rightarrow \; \partial X^I \bar{\partial} X^J$

$$X^I \quad I = 1, \cdots, 6 \quad \rightarrow \quad T^6$$

$$S = \frac{1}{8\pi}\int d^2\sigma \;\; (G_{IJ}\, \partial X^I \bar{\partial} X^J \; + \; B_{IJ}\, \partial X^I \bar{\partial} X^J)$$

In FFF $\partial X_L^I \;\rightarrow\; y^I\omega^I$

$i\partial X_L^I \;\rightarrow\; U(1)$ current in the Cartan subalgebra

In the fermionic language

$$i\partial X_L^I \;\rightarrow\; J_L^I = y^I\omega^I$$

$$\Rightarrow \;\; \partial X^I \bar{\partial} X^J \rightarrow J_L^I(z)\bar{J}_R^J(\bar{z})$$

$\rightarrow$ WS Thirring interactions $\quad (R - \frac{1}{R}) J_L(z)\bar{J}(\bar{z})$

Thirring interactions vanish at free fermionic point with $R = \frac{1}{R}$

To identify the untwisted moduli in the free fermionic models

$\rightarrow$ find the operators of the form

$$J_L^I(z)\bar{J}_R^J(\bar{z})$$

that are allowed by the orbifold (fermionic) symmetry group

Bosonization

$$\xi_i = \sqrt{\frac{1}{2}}(y_i + i\omega_i) = e^{iX_i}, \eta_i = \sqrt{\frac{i}{2}}(y_i - i\omega_i) = ie^{-iX_i}$$

simlarly for the right-movers

Boundary conditions translate to twists and shifts

$$X^I(z,\bar{z}) = X_L^I(z) + X_R^I(\bar{z})$$

Complex internal coordinates

$$Z_k^{\pm} = (X^{2k-1} \pm iX^{2k}), \quad \psi_k^{\pm} = (\chi^{2k-1} \pm i\chi^{2k}) \quad (k = 1, 2, 3)$$

Untwisted moduli

$$S = \int d^2 z h_{ij}(X) J_L^i(z) \bar{J}_R^j(\bar{z})$$

$J_L^i \ \sim \ y^i \omega^i$ $\qquad$ $i = 1, \cdots, 6$ are chiral currents of $U(1)_L^6$

$J_R^j \ \sim \ \bar{\phi}^j \bar{\phi}^{*j}$ $\qquad$ $j = 1, \cdots, 22$ are chiral currents of $U(1)_R^{22}$

$h_{ij} \ \longrightarrow$ scalar components of untwisted moduli

some of these operators are projected out in concrete models

$\Rightarrow$ some of the EMO may not be invariant

Models $\{\mathbf{1}, S\}$ $N=4$ $SO(44)$

$$\frac{SO(6,22)}{SO(6)\times SO(22)}$$ Moduli space

$$\chi^i \otimes \bar\phi^a \bar\phi^{*^a} |0\rangle$$ moduli fields

$$6 \times 22$$ scalar fields

$Z_2\times Z_2$ $\{\ 1\ ,\ S\ ,\ \xi_1\ ,\ \xi_2\ \} \ + \ \{\ b_1\ ,\ b_2\ \}$

$$SO(12)\times E_8\times E_8 \qquad Z_2\times Z_2$$

$$\rightarrow \quad SO(4)^3\times E_6\times U(1)^2\times E_8$$

The Thirring interactions that remain invariant are

$$J_L^{1,2}\bar J_R^{1,2} \quad ; \quad J_L^{3,4}\bar J_R^{3,4} \quad ; \quad J_L^{5,6}\bar J_R^{5,6}$$

$$y^{1,2}\omega^{1,2}\bar y^{1,2}\bar\omega^{1,2} \quad ; \quad y^{3,4}\omega^{3,4}\bar y^{3,4}\bar\omega^{3,4} \quad ; \quad y^{5,6}\omega^{5,6}\bar y^{5,6}\bar\omega^{5,6}$$

These moduli are always present in symmetric $Z_2\times Z_2$ orbifolds in realistic models

$$\{\ 1\ ,\ S\ ,\ \xi_1\ ,\ \xi_2\ \}\ \oplus\ \{\ b_1\ ,\ b_2\ \}\ \oplus\ \{\ \alpha\ ,\ \beta\ ,\ \gamma\}$$

$$N=4 \qquad\qquad N=1$$

$$E_8\times E_8 \qquad\qquad Z_2\times Z_2$$

new feature Asymmetric orbifold

the key focus: boundary conditions of the internal fermions

$$\{\ y\ ,\ \omega\ \mid\ \bar{y}\ ,\ \omega\ \}$$

WS fermions that have same B.C. in all basis vectors are paired

pairing of LR fermions $\longrightarrow$ Ising model $\longrightarrow$ symmetric real fermions

pairing of LL & RR fermions $\longrightarrow$ complex fermions $\longrightarrow$ asymmetric

STRING DERIVED STANDARD-LIKE MODEL (PLB278)

	ψ^μ	χ^{12}	χ^{34}	χ^{56}	$y^{3,...,6}$	$\bar{y}^{3,...,6}$	$y^{1,2},\omega^{5,6}$	$\bar{y}^{1,2},\bar{\omega}^{5,6}$	$\omega^{1,...,4}$	$\bar{\omega}^{1,...,4}$	$\bar{\psi}^{1,...,5}$	$\bar{\eta}^1$	$\bar{\eta}^2$	$\bar{\eta}^3$	$\bar{\phi}^{1,...,8}$
1	1	1	1	1	1,...,1	1,...,1	1,...,1	1,...,1	1,...,1	1,...,1	1,...,1	1	1	1	1,...,1
S	1	1	1	1	0,...,0	0,...,0	0,...,0	0,...,0	0,...,0	0,...,0	0,...,0	0	0	0	0,...,0
b_1	1	1	0	0	1,...,1	1,...,1	0,...,0	0,...,0	0,...,0	0,...,0	1,...,1	1	0	0	0,...,0
b_2	1	0	1	0	0,...,0	0,...,0	1,...,1	1,...,1	0,...,0	0,...,0	1,...,1	0	1	0	0,...,0
b_3	1	0	0	1	0,...,0	0,...,0	0,...,0	0,...,0	1,...,1	1,...,1	1,...,1	0	0	1	0,...,0

	ψ^μ	χ^{12}	χ^{34}	χ^{56}	y^3y^6	$y^4\bar{y}^4$	$y^5\bar{y}^5$	$\bar{y}^3\bar{y}^6$	$y^1\omega^5$	$y^2\bar{y}^2$	$\omega^6\bar{\omega}^6$	$\bar{y}^1\bar{\omega}^5$	$\omega^2\omega^4$	$\omega^1\bar{\omega}^1$	$\omega^3\bar{\omega}^3$	$\bar{\omega}^2\bar{\omega}^4$	$\bar{\psi}^{1,...,5}$	$\bar{\eta}^1$	$\bar{\eta}^2$	$\bar{\eta}^3$	
α	0	0	0	0	1	0	0	0	0	0	1	1	0	0	1	1	1 1 1 0 0	0	0	0	1 1
β	0	0	0	0	0	0	1	1	1	0	0	0	0	1	0	1	1 1 1 0 0	0	0	0	1 1
γ	0	0	0	0	0	1	0	1	0	1	0	1	1	0	0	0	$\frac{1}{2}$ $\frac{1}{2}$ $\frac{1}{2}$ $\frac{1}{2}$ $\frac{1}{2}$	$\frac{1}{2}$	$\frac{1}{2}$	$\frac{1}{2}$	$\frac{1}{2}$ 0 1

Asymmetric $BC \Rightarrow$ all untwisted moduli are projected out!

all $y_i\omega_i\bar{y}_i\bar{\omega}_i$ are disallowed

can be translated to asymmetric bosonic identifications

$$X_L + X_R \rightarrow X_L - X_R$$

moduli fixed at enhanced symmetry point

Twisted moduli

(2,2) $b_j \oplus b_j + \xi_2 \quad \longrightarrow \quad (16 \oplus 10 + 1) + 1 = 27 + 1 \Rightarrow$ twisted moduli

(2,0) $b_j + b_j + 2\gamma \quad \longrightarrow 16_{SO(10)} + 16_{SO(16)}$

all twisted moduli are projected out

A STRINGY DOUBLET–TRIPLET SPLITTING MECHANISM

NAHE $\rightarrow\quad \chi_j\bar\psi^{1,\cdots,5}\bar\eta_j + c.c. \rightarrow (5+\bar5)_j = 10_j$ of $SO(10)$

$$\alpha \quad \rightarrow \quad SO(10) \quad \rightarrow \quad SO(6)\times SO(4)$$

$y_3\bar y_3$	$y_4\bar y_4$	$y_5\bar y_5$	$y_6\bar y_6$		y_3y_6	$y_4\bar y_4$	$y_5\bar y_5$	$\bar y_3\bar y_6$
1	0	0	1		1	0	0	0

$$\Delta_j = |\alpha_L(T_2^j) - \alpha_R(T_2^j)|$$

$$\Delta_j = 0 \rightarrow D_j, \bar D_j \qquad\qquad \Delta_j = 1 \rightarrow h_j, \bar h_j$$

$$\Delta_{1,2,3} = 1 \Rightarrow h_j \quad \bar h_j \qquad j = 1, 2, 3$$

<u>A superstring solution to the GUT hierarchy problem</u>

MINIMAL DOUBLET HIGGS CONTENT

	ψ^μ	χ^{12}	χ^{34}	χ^{56}	y^3y^6	$y^4\bar{y}^4$	$y^5\bar{y}^5$	$\bar{y}^3\bar{y}^6$	$y^1\omega^5$	$y^2\bar{y}^2$	$\omega^6\bar{\omega}^6$	$\bar{y}^1\bar{\omega}^5$	$\omega^2\omega^4$	$\omega^1\bar{\omega}^1$	$\omega^3\bar{\omega}^3$	$\bar{\omega}^2\bar{\omega}^4$	$\bar{\psi}^{1,...,5}$	$\bar{\eta}^1$	$\bar{\eta}^2$	$\bar{\eta}^3$	$\bar{\phi}$
α	0	0	0	0	1	0	0	1	0	0	1	1	0	0	1	1	1 1 1 0 0	1	0	0	1 1 0
β	0	0	0	0	0	0	1	1	1	0	0	1	0	1	0	1	1 1 1 0 0	0	1	0	0 0 1
γ	0	0	0	0	0	1	0	0	0	1	0	0	1	0	0	0	$\frac{1}{2}$ $\frac{1}{2}$ $\frac{1}{2}$ $\frac{1}{2}$ $\frac{1}{2}$	$\frac{1}{2}$	$\frac{1}{2}$	$\frac{1}{2}$	0 0 0

SYMMETRIC $\leftrightarrow$ ASYMMETRIC

with respect to b_1 & b_2

$h_1,\ \bar{h}_1,\ D_1,\ \bar{D}_1$, $h_2,\ \bar{h}_2,\ D_2,\ \bar{D}_2$ are projected out

$h_3,\ \bar{h}_3$ remain in the spectrum

$$\lambda_t Q_3 t_3^c \bar{h}_3 \quad \text{with} \quad \lambda_t\ O(1)$$

No Phenomenologically viable flat directions

Cleaver, Faraggi, Manno, Timirgaziu, PRD 78 (2008) 046009

Classification of F and D flat directions in FMT reduced Higgs model

No D flat direction which is F-flat up to order eight in the superpotential

no stringent flat directions to all orders

Suggesting no supersymmetric flat directions in this model (class of models)

implying no supersymmetric moduli

only remaining perturbative moduli is the dilaton

quasi–realistic model: SLM; 3 gen; $SO(10)$ embed; Higgs & $\lambda_t \sim 1$; ...

vanishing one–loop partition function, perturbatively broken SUSY

Fixed geometrical, twisted and SUSY moduli

Cleaver $etal$, $SO(10)$ and FSU5 analysis $->$ stringent flat directions

Classification of fermionic $Z_2 \times Z_2$ orbifolds

Basis vectors:

$$
\begin{aligned}
1 &= \{\psi^\mu,\ \chi^{1,\dots,6}, y^{1,\dots,6}, \omega^{1,\dots,6} \mid \bar{y}^{1,\dots,6}, \bar{\omega}^{1,\dots,6}, \bar{\eta}^{1,2,3}, \bar{\psi}^{1,\dots,5}, \bar{\phi}^{1,\dots,8}\}\\
S &= \{\psi^\mu, \chi^{1,\dots,6}\},\\
z_1 &= \{\bar{\phi}^{1,\dots,4}\},\\
z_2 &= \{\bar{\phi}^{5,\dots,8}\},\\
e_i &= \{y^i, \omega^i | \bar{y}^i, \bar{\omega}^i\},\ i = 1, \dots, 6, \qquad N = 4 \text{ Vacua}\\
b_1 &= \{\chi^{34}, \chi^{56}, y^{34}, y^{56} | \bar{y}^{34}, \bar{y}^{56}, \bar{\eta}^1, \bar{\psi}^{1,\dots,5}\}, \qquad N = 4 \to N = 2\\
b_2 &= \{\chi^{12}, \chi^{56}, y^{12}, y^{56} | \bar{y}^{12}, \bar{y}^{56}, \bar{\eta}^2, \bar{\psi}^{1,\dots,5}\}, \qquad N = 2 \to N = 1\\
\alpha &= \{\bar{\psi}^{4,5}, \bar{\phi}^{1,2}\} \qquad \& \quad SO(10) \to SO(6) \times SO(4) \times \cdots\\
\beta &= \{\bar{\psi}^{1,\cdots,5} \equiv \tfrac{1}{2}, \cdots\} \qquad \& \quad SO(10) \to SU(5) \times U(1) \times \cdots
\end{aligned}
$$

Independent phases $c\begin{bmatrix} v_i \\ v_j \end{bmatrix} = \exp[i\pi(v_i|v_j)]$: upper block

	1	S	e_1	e_2	e_3	e_4	e_5	e_6	z_1	z_2	b_1	b_2	α
1	-1	-1	$\pm$	$\pm$	$\pm$	$\pm$	$\pm$	$\pm$	$\pm$	$\pm$	$\pm$	$\pm$	$\pm$
S			-1	-1	-1	-1	-1	-1	-1	-1	1	1	-1
e_1				$\pm$	$\pm$	$\pm$	$\pm$	$\pm$	$\pm$	$\pm$	$\pm$	$\pm$	$\pm$
e_2					$\pm$	$\pm$	$\pm$	$\pm$	$\pm$	$\pm$	$\pm$	$\pm$	$\pm$
e_3						$\pm$	$\pm$	$\pm$	$\pm$	$\pm$	$\pm$	$\pm$	$\pm$
e_4							$\pm$	$\pm$	$\pm$	$\pm$	$\pm$	$\pm$	$\pm$
e_5								$\pm$	$\pm$	$\pm$	$\pm$	$\pm$	$\pm$
e_6									$\pm$	$\pm$	$\pm$	$\pm$	$\pm$
z_1										$\pm$	$\pm$	$\pm$	$\pm$
z_2											$\pm$	$\pm$	$\pm$
b_1												$\pm$	$\pm$
b_2												-1	$\pm$
α													

A priori 66 independent coefficients $\rightarrow$ 2^{66} distinct vacua

The twisted matter spectrum:

$$B^1_{\ell^1_3\ell^1_4\ell^1_5\ell^1_6} = S + b_1 + \ell^1_3 e_3 + \ell^1_4 e_4 + \ell^1_5 e_5 + \ell^1_6 e_6$$

$$B^2_{\ell^2_1\ell^2_2\ell^2_5\ell^2_6} = S + b_2 + \ell^2_1 e_1 + \ell^2_2 e_2 + \ell^2_5 e_5 + \ell^2_6 e_6$$

$$B^3_{\ell^3_1\ell^3_2\ell^3_3\ell^3_4} = S + b_3 + \ell^3_1 e_1 + \ell^3_2 e_2 + \ell^3_3 e_3 + \ell^3_4 e_4 \qquad l^j_i = 0,1$$

sectors B^i_{pqrs} $\quad \rightarrow$ 16 or $\overline{16}$ of $SO(10)$ with multiplicity $(1,0,-1)$

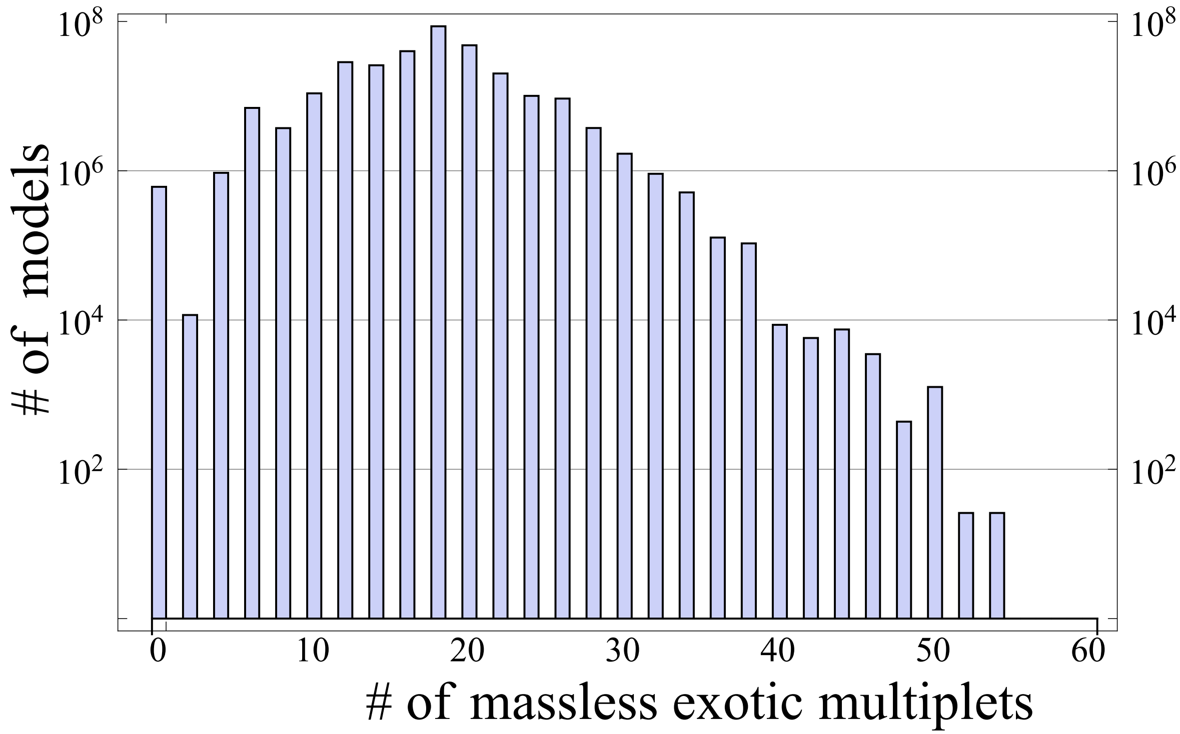
$B^i_{pqrs} + x \quad \rightarrow \quad 10$ of $SO(10)$ with multiplicity $(1,0)$

$x = \{\bar\psi^{1,\cdots,5}, \bar\eta^1, \bar\eta^2, \bar\eta^3\}$ $\qquad x -$ map $\leftrightarrow$ spinor–vector map

Algebraic formulas for $\quad S = \sum_{i=1}^3 S_+^{(i)} - S_-^{(i)}$ $\quad$ and $\quad V = \sum_{i=1}^3 V^{(i)}$

Pati–Salam class: with Assel, Christodoulides, Kounnas, Rizos

RESULTS: of random search of over 10^{11} vacua



Number of 3-generation models versus total number of exotic multiplets

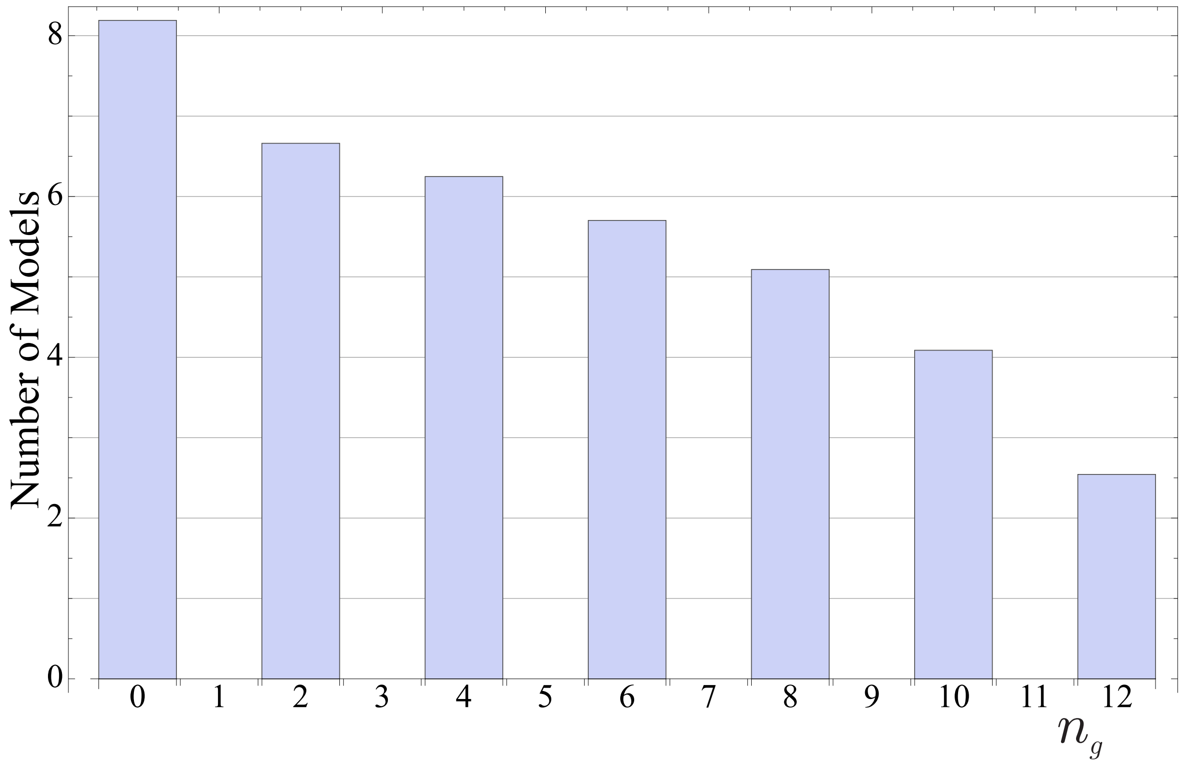
Pati–Salam models statistics with respect to phenomenological constraints

constraint	# of models	probability	# of models
None	100000000000	1	2.25×10^{15}
+ No gauge group enhancements.	78977078333	7.90×10^{-1}	1.78×10^{15}
+ Complete families	22497003372	2.25×10^{-1}	5.07×10^{14}
+ 3 generations	298140621	2.98×10^{-3}	6.71×10^{12}
+ PS breaking Higgs	23694017	2.37×10^{-4}	5.34×10^{11}
+ SM breaking Higgs	19191088	1.92×10^{-4}	4.32×10^{11}
+ No massless exotics	121669	1.22×10^{-6}	2.74×10^{9}

Constraints in second column act additionally.

flipped $SU(5)$ class: with Sonmez, Rizos

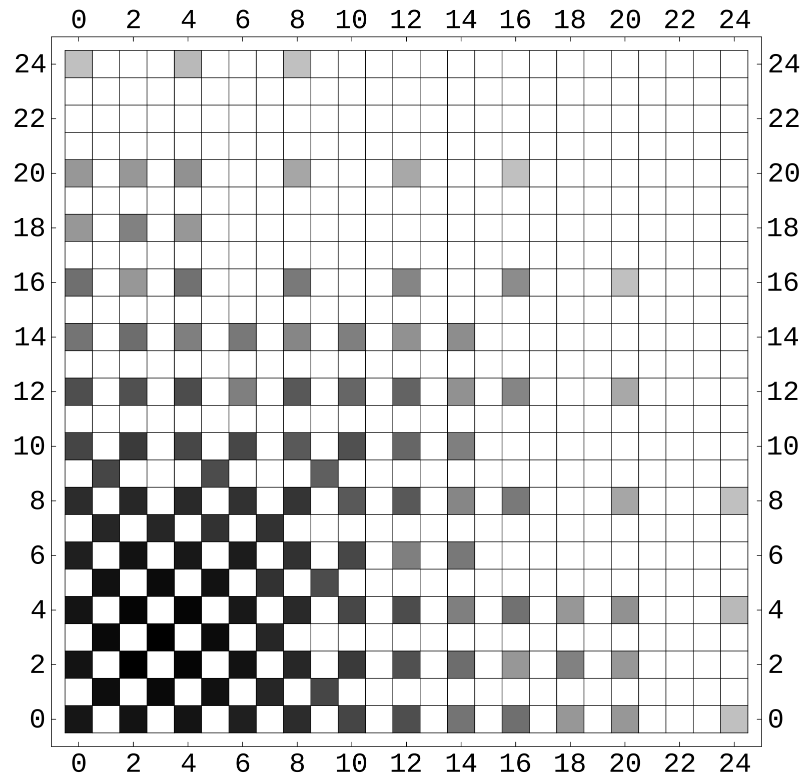
RESULTS: of random search of over 10^{12} vacua



Number of exophobic models versus the number of generations

Spinor–vector duality:

Invariance under exchange of $\#(16+\overline{16}) \;<-> \;\#(10)$



Symmetric under exchange of rows and columns

$$E_6 \;: \qquad 27 = 16 + 10 + 1 \qquad \overline{27} = \overline{16} + 10 + 1$$

Self-dual: $\#(16+\overline{16}) \;=\; \#(10)$ without E_6 symmetry

Spinor–Vector duality in Orbifolds:

Using the level-one $\mathrm{SO}(2n)$ characters

$$O_{2n} = \frac{1}{2}\left(\frac{\theta_3^n}{\eta^n} + \frac{\theta_4^n}{\eta^n}\right), \qquad V_{2n} = \frac{1}{2}\left(\frac{\theta_3^n}{\eta^n} - \frac{\theta_4^n}{\eta^n}\right),$$

$$S_{2n} = \frac{1}{2}\left(\frac{\theta_2^n}{\eta^n} + i^{-n}\frac{\theta_1^n}{\eta^n}\right), \qquad C_{2n} = \frac{1}{2}\left(\frac{\theta_2^n}{\eta^n} - i^{-n}\frac{\theta_1^n}{\eta^n}\right).$$

where

$$\theta_3 \equiv Z_f\binom{0}{0} \quad \theta_4 \equiv Z_f\binom{0}{1}$$

$$\theta_2 \equiv Z_f\binom{1}{0} \quad \theta_1 \equiv Z_f\binom{1}{1}$$

Starting from:

$$\mathrm{Z}_{+} = (V_8 - S_8) \left(\sum_{m,n} \Lambda_{m,n} \right)^{\otimes 6} (\bar{O}_{16} + \bar{S}_{16}) \, (\bar{O}_{16} + \bar{S}_{16}) \ ,$$

where as usual, for each circle,

$$p^i_{\mathrm{L,R}} = \frac{m_i}{R_i} \pm \frac{n_i R_i}{\alpha'} \,,$$

and

$$\Lambda_{m,n} = \frac{q^{\frac{\alpha'}{4} p^2_{\mathrm{L}}} \, \bar{q}^{\frac{\alpha'}{4} p^2_{\mathrm{R}}}}{|\eta|^2} \,.$$

apply $\quad Z_2 \times Z_2' : g \times g'$

$$g \; : \; (-1)^{(F_1+F_2)}\delta \qquad : \quad E_8 \times E_8 \longrightarrow SO(16) \times SO(16$$

$$F_{1,2} \; : \; (\, \overline{O}_{16}^{1,2},\, \overline{V}_{16}^{1,2},\, \overline{S}_{16}^{1,2},\, \overline{C}_{16}^{1,2}\,) \longrightarrow (\, \overline{O}_{16}^{1,2},\, \overline{V}_{16}^{1,2},\, -\overline{S}_{16}^{1,2},\, -\overline{C}_{16}^{1,2}\,)$$

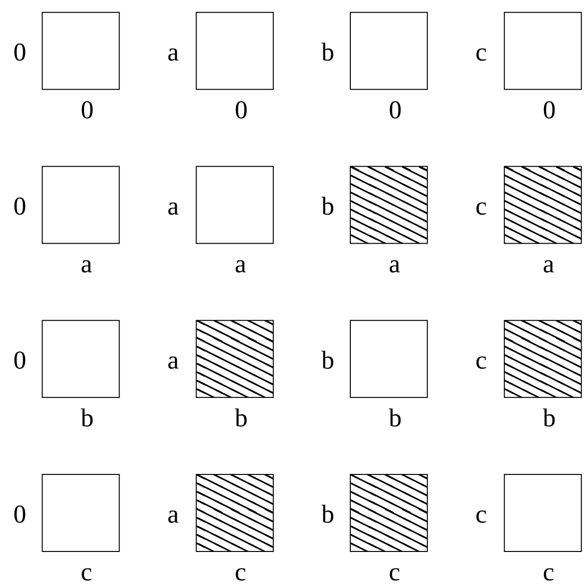
$$\delta \; : \; X_9 = X_9 + \pi R_9 \longrightarrow \delta \; : \; \Lambda^9_{m,n} \longrightarrow (-1)^m \Lambda^9_{m,n}$$

$$g' \; : \; (x_4, x_5, x_6, x_7, x_8, x_9) \longrightarrow (-x_4, -x_5, -x_6, -x_7, +x_8, +x_9)$$

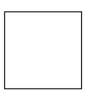
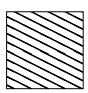
Note: A single space twisting Z_2' $\Longrightarrow$ $N = 4 \rightarrow N = 2$

$$E_7 \rightarrow SO(12) \times SU(2)$$

$\Rightarrow$ Analyze $\quad Z = \left(\dfrac{Z_+}{Z_g \times Z_{g'}} \right) = \left[\dfrac{(1+g)}{2} \dfrac{(1+g')}{2} \right] Z_+$



a = g ; b = g' ; c = gg'

P.F. = ( + ε ) = $\Lambda_{m,n} \bullet$ ( ) + $\Lambda_{m,n+1/2} \bullet$ ( )

ε = $\pm$ 1

massless massive

- sector b

$$\Lambda_{p,q}\left\{\frac{1}{2}\left(\left|\frac{2\eta}{\theta_4}\right|^4+\left|\frac{2\eta}{\theta_3}\right|^4\right)\left[P_\epsilon^+ Q_s\overline{V}_{12}\overline{C}_4\overline{O}_{16}+P_\epsilon^- Q_s\overline{S}_{12}\overline{O}_4\overline{O}_{16}\right]+\right.$$
$$\left.\frac{1}{2}\left(\left|\frac{2\eta}{\theta_4}\right|^4-\left|\frac{2\eta}{\theta_3}\right|^4\right)\left[P_\epsilon^+ Q_s\overline{O}_{12}\overline{S}_4\overline{O}_{16}\right]\right\} \quad + \quad \text{massive}$$

where

$$P_\epsilon^+ = \left(\frac{1+\epsilon(-1)^m}{2}\right)\Lambda_{m,n} \qquad P_\epsilon^- = \left(\frac{1-\epsilon(-1)^m}{2}\right)\Lambda_m$$

$$\epsilon = +1 \Rightarrow P_\epsilon^+ = \Lambda_{2m,n} \qquad P_\epsilon^- = \Lambda_{2m+1,n}$$

$$\epsilon = -1 \Rightarrow P_\epsilon^+ = \Lambda_{2m+1,n} \qquad P_\epsilon^- = \Lambda_{2m,n}$$

and $12\cdot 2 + 4\cdot 2 = 32$

Further :

- The spinor–vector duality in this model is realised in terms of a continuous interpolation between two discrete Wilson lines.

- The spinor-vector duality is realised in terms of a spectral flow operator that operates in the bosonic side of the heterotic string. In the case of enhanced E_6 symmetry, the spectral flow operator acts as an internal E_6 generator. When E_6 is broken the spectral flow operator induces the spinor–vector duality map.

Away from the free fermionic point:

$$Z = \int \frac{d^2\tau}{\tau_2^2} \frac{\tau_2^{-1}}{\eta^{12}\bar\eta^{24}} \frac{1}{2^3} \left(\sum (-)^{a+b+ab} \vartheta \begin{bmatrix} a \\ b \end{bmatrix} \vartheta \begin{bmatrix} a+h_1 \\ b+g_1 \end{bmatrix} \vartheta \begin{bmatrix} a+h_2 \\ b+g_2 \end{bmatrix} \vartheta \begin{bmatrix} a+h_3 \\ b+g_3 \end{bmatrix}\right)_{\psi^\mu,}$$

$$\times \left(\frac{1}{2}\sum_{\epsilon,\xi} \bar\vartheta \begin{bmatrix} \epsilon \\ \xi \end{bmatrix}^5 \bar\vartheta \begin{bmatrix} \epsilon+h_1 \\ \xi+g_1 \end{bmatrix} \bar\vartheta \begin{bmatrix} \epsilon+h_2 \\ \xi+g_2 \end{bmatrix} \bar\vartheta \begin{bmatrix} \epsilon+h_3 \\ \xi+g_3 \end{bmatrix}\right)_{\bar\psi^{1\ldots5},\bar\eta^{1,2,3}}$$

$$\times \left(\frac{1}{2}\sum_{H_1,G_1} \frac{1}{2}\sum_{H_2,G_2} (-)^{H_1G_1+H_2G_2} \bar\vartheta \begin{bmatrix} \epsilon+H_1 \\ \xi+G_1 \end{bmatrix}^4 \bar\vartheta \begin{bmatrix} \epsilon+H_2 \\ \xi+G_2 \end{bmatrix}^4\right)_{\bar\phi^{1\ldots8}}$$

$$\times \left(\sum_{s_i,t_i} \Gamma_{6,6} \begin{bmatrix} h_i|s_i \\ g_i|t_i \end{bmatrix}\right)_{(y\omega\bar y\bar\omega)^{1\ldots6}} \times e^{i\pi\Phi(\gamma,\delta,s_i,t_i,\epsilon,\xi,h_i,g_i,H_1,G_1,H_2,G_2)}$$

$$\Gamma_{1,1}\left[{}^h_g\right] = \frac{R}{\sqrt{\tau_2}} \sum_{\tilde m,n} \exp\left[-\frac{\pi R^2}{\tau_2} \left|(2\tilde m + g) + (2n+h)\,\tau\right|^2\right]$$

Fermion mass hierarchy

Fermion mass terms

$$c g f_i f_j h \left(\frac{\langle\phi\rangle}{M}\right)^{N-3}$$

c - calculable coefficients g - gauge coupling

$$f_i,\, f_j \;\in\; b_j \quad j \;=\; 1, 2, 3$$

$h \rightarrow$ light Higgs multiplets

$$M \;\sim\; 10^{18}\, GeV$$

$\langle\phi\rangle$ generalized VEVs, several sources

Up/Down–type Yukawa Selection Mechanism

	ψ^μ	χ^{12}	χ^{34}	χ^{56}	y^3y^6	$y^4\bar{y}^4$	$y^5\bar{y}^5$	$\bar{y}^3\bar{y}^6$	$y^1\omega^5$	$y^2\bar{y}^2$	$\omega^6\bar{\omega}^6$	$\bar{y}^1\bar{\omega}^5$	$\omega^2\omega^4$	$\omega^1\bar{\omega}^1$	$\omega^3\bar{\omega}^3$	$\bar{\omega}^2\bar{\omega}^4$	$\bar{\psi}^{1,...,5}$	$\bar{\eta}^1$	$\bar{\eta}^2$	$\bar{\eta}^3$	$\bar{\phi}$
α	0	0	0	0	1	0	0	0	0	0	1	1	0	0	1	1	1 1 1 0 0	0	0	0	1 1 1
β	0	0	0	0	0	0	1	1	1	0	0	0	0	1	0	1	1 1 1 0 0	0	0	0	1 1 1
γ	0	0	0	0	0	1	0	1	0	1	0	1	1	0	0	0	$\frac{1}{2}$ $\frac{1}{2}$ $\frac{1}{2}$ $\frac{1}{2}$ $\frac{1}{2}$	$\frac{1}{2}$	$\frac{1}{2}$	$\frac{1}{2}$	0 0 0

At the cubic level $\longrightarrow$ γ selects trilevel Yukawa coupling:

$$\Delta_j = |\gamma(U(1)_{L_{j+3}}) - \gamma(U(1)_{R_{j+3}})| = 0, 1 \quad j = 1, 2, 3$$

$$\Delta_j = 1 \Rightarrow u_j Q_j \bar{h}_j \qquad \Delta_j = 0 \Rightarrow d_j Q_j h_j \; ; \; e_j L_j h_j$$

b_1 : y_3y_6 $y_4\bar{y}_4$ $y_5\bar{y}_5$ $\bar{y}_3\bar{y}_6$ — y_3y_6 $y_4\bar{y}_4$ $y_5\bar{y}_5$ $\bar{y}_3\bar{y}_6$

γ : 1 0 0 0 — 1 0 0 1

$\Delta_1 = 1$ — $\Delta_1 = 0$

$\lambda_j t^c_j Q_j \bar{h}_j$

Top quark mass prediction

only $\lambda_t = \langle t^c Q_t \bar{h}_1 \rangle = \sqrt{2} g \neq 0$ at $N = 3$

$$W_4 \Longrightarrow \qquad \lambda_b = \left(c_b \frac{\langle \phi \rangle}{M} \right) \qquad \lambda_\tau = \left(c_\tau \frac{\langle \phi \rangle}{M} \right)$$

$$\longrightarrow \qquad \lambda_b = \lambda_\tau = 0.35 g^3 \sim \frac{1}{8} \lambda_t$$

$$m_t = \lambda_t(m_t) v_1 \qquad m_b = \lambda_b(m_b) v_2$$

where $(v_1^2 + v_2^2) = \frac{v_0^2}{2}$ and $v_0 = \frac{2 m_W}{g_2(M_Z)} = 246\text{GeV}$

$$m_t = \lambda_t(m_t) \frac{v_0}{\sqrt{2}} \frac{\tan\beta}{(1 + \tan^2\beta)^{\frac{1}{2}}} \Longrightarrow \qquad m_t \sim 175\text{GeV} \quad \text{PLB274(1992)47}$$

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PHYSICS LETTERS B

Hierarchical top–bottom mass relation in a superstring derived standard-like model

Alon E. Faraggi

Center For Theoretical Physics, Texas A&M University, College Station, TX 77843-4242, USA
and Astroparticle Physics Group, Houston Advanced Research Center (HARC), The Woodlands, TX 77381, USA

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I propose a mechanism in a class of superstring standard-like models which explains the mass hierarchy between the top and bottom quarks. At the trilinear level of the superpotential only the top quark gets a nonvanishing mass term while the bottom quarks and tau lepton mass terms are obtained from nonrenormalizable terms. I construct a model which realized this mechanism. In this model the bottom quark and tau lepton Yukawa couplings are obtained from quartic order terms. I show that $\lambda_b = \lambda_\tau \sim \frac{1}{8}\lambda_t$ at the unification scale. A naive estimate yields $m_t \sim 175$–180 GeV.

One of the unresolved puzzles of the standard model is the mass splitting between the top quark and the lighter ıuarks and leptons. Especially difficult to understand within the context of the standard model is the big splitıng in the heaviest generation. Experimental limits [1] indicate the top mass to be above 80 GeV, while the ›ottom and tau lepton masses are found at 5 GeV and 1.78 GeV respectively. Possible extensions to the standard nodel are grand unified theories. Although the main prediction of GUTs, proton decay, has not yet been oberved, calculations of $\sin^2\theta_W$ and of the mass ratio m_b/m_τ support their validity. Recent calculations seem to upport supersymmetric GUTs versus nonsupersymmetric ones [2]. In spite of the success of SUSY GUTs in onfronting LEP data [2], an understanding of the mass splitting between the top quark and the lighter quarks ınd leptons is still lacking. The next level in which such an understanding may be developed is in the context of uperstring theory [3].

Cabibbo mixing — PLB 307 (1993) 305

Find anomaly free solution

$$M_d \sim \begin{pmatrix} \epsilon & \frac{V_2 \bar{V}_3 \Phi_{\alpha\beta}}{M^3} & 0 \\ \frac{V_2 \bar{V}_3 \Phi_{\alpha\beta} \xi_1}{M^4} & \frac{\bar{\Phi}_2^- \xi_1}{M^2} & 0 \\ 0 & 0 & \frac{\Phi_1^+ \xi_2}{M^2} \end{pmatrix} v_2,$$

$$\epsilon < 10^{-8} \qquad \frac{V_2 \bar{V}_3 \Phi_{\alpha\beta}}{M^3} = \frac{\sqrt{5}\, g^6}{64\, \pi^3} \approx 2 - 3 \times 10^{-4}.$$

$$\Rightarrow \quad |V| \sim \begin{pmatrix} 0.98 & 0.2 & 0 \\ 0.2 & 0.98 & 0 \\ 0 & 0 & 1 \end{pmatrix}$$

Three generation mixing $\longrightarrow$ NPB 416 (1994) 63 $\quad |J| \sim 10^{-6}$

Conclusions

Phenomenological string models produce interesting lessons

relevance of non–standard geometries

Spinor–vector duality

Exophobia

Free Fermionic Models $\longrightarrow$ $Z_2 \times Z_2$ orbifold near the self–dual point

Duality $\Leftrightarrow$ Phase–Space Duality $\longrightarrow$ String Vacuum Selection