Toward realistic de Sitter heterotic-string models with stable moduli

- 1989 – ··· Minimal Standard Heterotic String Models ···
- 2003 – ··· Classification of fermionic $Z_2 \times Z_2$ orbifolds ···
- 2019 – ··· 10D tachyonic vacua $\rightarrow$ phenomenology?
- 2022 – ··· toward de Sitter vacua with stable moduli – ···

AEF, DV Nanopoulos, K Yuan, NPB335 (1989) 347;
AEF, EPJC 79 (2019) 703;
AEF, B Percival, V Matyas,
EPJC 80 (2020) 337; NPB 961 (2020) 115231; PRD 104 (2021) 046002;
PLB 814 (2021) 136080; PRD 106 (2022) 026011.

KEK Annual Theory Workshop 2022, Zoom, 7 December 2022

WHY?

DATA $\longrightarrow$ STANDARD MODEL $\longleftrightarrow$ HIGGS!

EWX –> PERTUBATIVE

STANDARD MODEL –> UNIFICATION

EVIDENCE: 16 of SO(10), Log running, proton stability, neutrino masses

+ GRAVITY < –– > STRINGS

UNIFICATION of Flavour, Gravity and Hierarcchy

PRIMARY GUIDES: 3 generations
SO(10) embedding

Higgs : Fundamental? Composite? SM? Multi? SSC?!

Elements of string unification:

Classically $\quad g^{\alpha\beta} \longrightarrow \eta^{\alpha\beta}$ $\quad$ 2D WS–metric

Quantum $\quad D = 26$ (Bosonic) $\quad D = 10$ (Fermionic)

Heterotic–string $\quad D_L = 10$ $\quad D_R = 26$

REAL WORLD $\quad D = 4$

$\Rightarrow$ Bosonic $\quad \rightarrow \quad 4_{L+R} \; + \; 22_L \; + \; 22_R$

$\Rightarrow$ Fermionic $\quad \rightarrow \quad 4_{L+R} \; + \; 6_L \; + \; 6_R$

$\Rightarrow$ Heterotic–string $\quad \rightarrow \quad 4_{L+R} \; + \; (6_L \; + \; 6_R) \; + \; 16_R$

6D IM $\quad$ 16D w $R_J = \sqrt{2}$

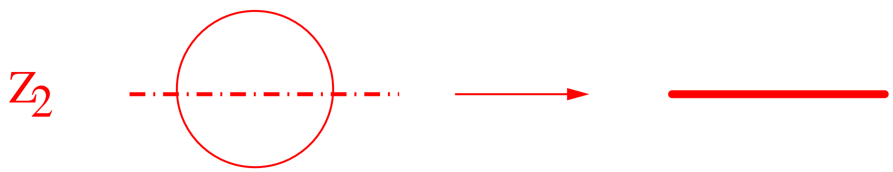
Moduli $\rightarrow$ size & shape of internal 6D manifold

REALISTIC STRING MODELS :

heterotic 10D –> heterotic 4D

6D compactifications $(T^2 \times T^2 \times T^2)$

Orbifold – twists of flat 6D torus



FREE FERMIONIC MODELS –

$Z_2 \times Z_2$ Orbifold –> $U(1)_Y \in SO(10)$

$$\frac{6}{2} = 1+1+1$$

$Z_2 \times Z_2$ orbifolds

torus: One complex parameter $Z = Z + n\, e_1 + m\, e_2$

$T^2 \times T^2 \times T^2 \longrightarrow$ Three complex coordinates z_1 , z_2 and z_3

Z_2 orbifold : $Z = -Z + \sum_i m_i e_i \longrightarrow$ 4 fixed points

$$Z = \{ \; 0 \;, \; 1/2\, e_1 \;, \; 1/2\, e_2 \;, \; 1/2\,(e_1 + e_2) \; \}$$

$\dfrac{T^2 \times T^2 \times T^2}{Z_2 \times Z_2}$

$\alpha : (z1, z2, z3) \to (-z1, -z2, +z3) \to 16$

$\beta : (z1, z2, z3) \to (+z1, -z2, -z3) \to 16$

$\alpha\beta : (z1, z2, z3) \to (-z1, +z2, -z3) \to 16$

48

$\downarrow$

$\gamma : (z1, z2, z3) \to (z1+1/2, z2+1/2, z3+1/2) \longrightarrow$ 24

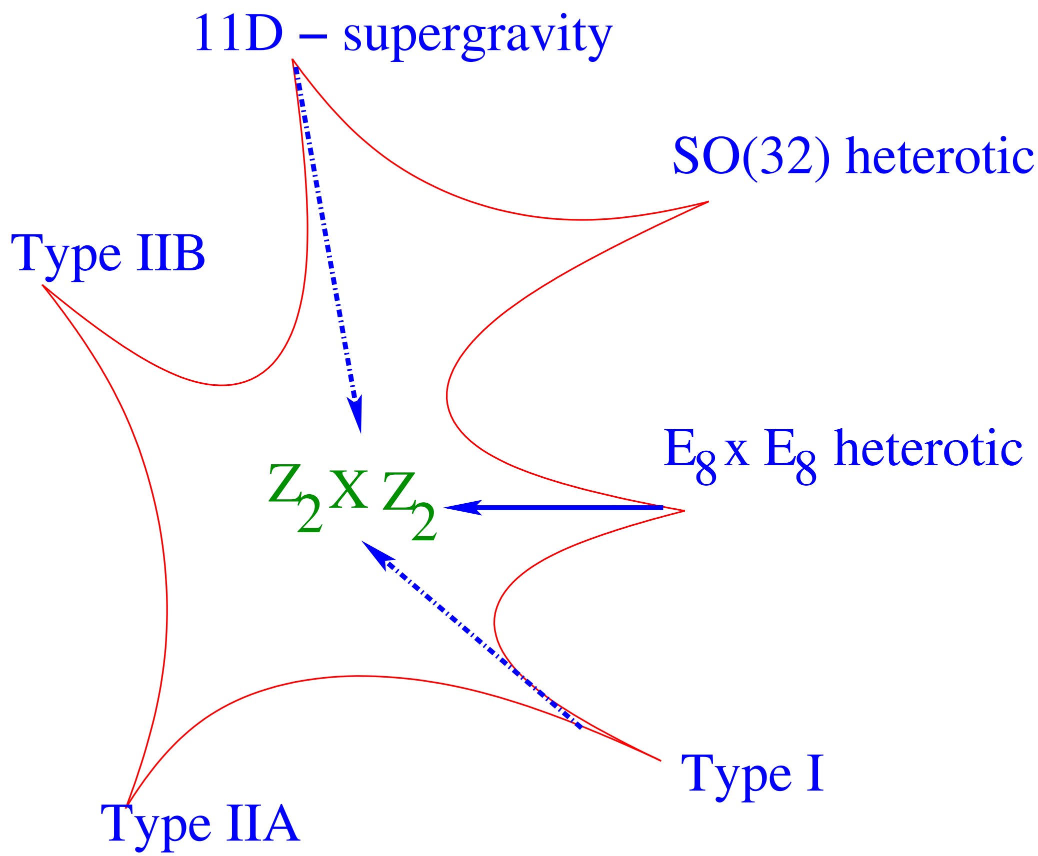
Fermionic $Z_2 \times Z_2$ orbifolds

'Phenomenology of the Standard Model and Unification'

- Minimal Superstring Standard Model — NPB 335 (1990) 347 (with Nanopoulos & Yuan)
- Top quark mass $\sim$ 175–180GeV — PLB 274 (1992) 47
- Generation mass hierarchy — NPB 407 (1993) 57
- CKM mixing — NPB 416 (1994) 63 (with Halyo)
- Stringy seesaw mechanism — PLB 307 (1993) 311 (with Halyo)
- Gauge coupling unification — NPB 457 (1995) 409 (with Dienes)
- Proton stability — NPB 428 (1994) 111
- Squark degeneracy — NPB 526 (1998) 21 (with Pati)
- Moduli fixing — NPB 728 (2005) 83
- Classification — 2003 – $\cdots$

(with Kounnas, Rizos & ... Percival, Matyas)

Point, String, Membrane ....



+ ... SO(16)xSO(16), E8, SO(16)xE8 + ...

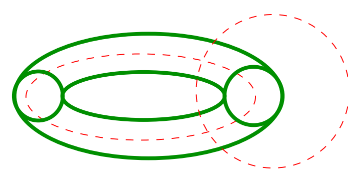
... Abel, Basile, Dienes, Kaidi, Itoyama ...

Fermionic Construction

Left-Movers: $\psi^{\mu=1,2}$, χ_i, y_i, ω_i $(i=1,\cdots,6)$

Right-Movers

$$\bar{\phi}_{A=1,\cdots,44} = \begin{cases} \bar{y}_i \ , \ \bar{\omega}_i & i=1,\cdots,6 \\ & \\ \bar{\eta}_i & U(1)_i \quad i=1,2,3 \\ \bar{\psi}_{1,\cdots,5} & SO(10) \\ \bar{\phi}_{1,\cdots,8} & SO(16) \end{cases}$$



$V \longrightarrow V$ $\qquad$ $f \longrightarrow -e^{i\pi\alpha(f)}f$

$$Z = \sum_{\substack{all\ spin \\ structures}} c\binom{\vec{\alpha}}{\vec{\beta}} Z\binom{\vec{\alpha}}{\vec{\beta}}$$

Models $\longleftrightarrow$ Basis vectors $+$ one–loop phases

Model building – Construction of the physical states

$$b_j \quad j=1,\cdots,N \qquad \rightarrow \qquad \Xi \;=\; \sum_j n_j b_j$$

For $\vec{\alpha} \;=\; (\vec{\alpha}_L;\vec{\alpha}_R) \;\in\; \Xi \quad \Rightarrow \quad \mathsf{H}_{\vec{\alpha}}$

$$\alpha(f)=1 \quad\Rightarrow\quad |\pm\rangle \quad ; \quad \alpha(f)\neq 1 \quad\Rightarrow\quad f|0\rangle, f^*|0\rangle \quad , \quad \nu_{f,f^*}=\frac{1\mp\alpha(f)}{2}$$

$$M_L^2=-\frac{1}{2}+\frac{\vec{\alpha}_L\cdot\vec{\alpha}_L}{8}+N_L=-1+\frac{\vec{\alpha}_R\cdot\vec{\alpha}_R}{8}+N_R=M_R^2 \quad (\;\equiv\;\; 0\;)$$

GSO projections

$$e^{i\pi(\vec{b}_i\cdot\vec{F}_\alpha)}|s\rangle_{\vec{\alpha}}=\delta_\alpha c^*\binom{\vec{\alpha}}{\vec{b}_i}|s\rangle_{\vec{\alpha}}$$

$F_\alpha(f) \;\rightarrow\;$ fermion # operator $\;=\begin{cases}-1, & |-\rangle\\ 0, & |+\rangle\end{cases} \;=\begin{cases}+1, & f\\ -1, & f^*\end{cases}$

$Q(f)=\frac{1}{2}\alpha(f)+F(f) \quad\rightarrow\quad$ U(1) charges

Example : $\vec{\alpha} = \vec{S} = (\underbrace{1,\cdots,1}_{\psi^{\mu}_{12},\chi^{12},\chi^{34},\chi^{56}},0,\cdots,0|0,\cdots,0)$.

$$(\vec{S}_L\cdot\vec{S}_L = 4 \quad \vec{S}_R\cdot\vec{S}_R = 0)$$

For $\alpha(f) = 1 \rightarrow$ periodic BC $\Rightarrow$ $F:|\pm\rangle = \begin{cases} -1, & F:|-\rangle \\ 0, & F:|+\rangle \end{cases}$

otherwise $F(f|0\rangle; f^*|0\rangle) = \pm 1|0\rangle \quad \nu_{f;f^*} = \frac{1\pm\alpha(f)}{2}$

Mass formula $\quad M_L^2 = -\frac{1}{2} + \frac{4}{8} + N_L = -1 + \frac{0}{8} + N_R = M_R^2$

$$\nu_f = \frac{1\pm 0}{2} = \frac{1}{2} \quad \Rightarrow \quad N_R = \frac{1}{2} + \frac{1}{2} = 1$$

$|S\rangle_S = |D\rangle_L \bar{\phi}_{\frac{1}{2}}\bar{\phi}_{\frac{1}{2}}|0\rangle_R \qquad |D\rangle_L = \left[\binom{4}{0} + \binom{4}{1} + \binom{4}{2} + \binom{4}{3} + \binom{4}{4}\right]$

apply GSO projections : $e^{i\pi\vec{S}\cdot\vec{F}_S}|S\rangle_S = \delta_S c^*\binom{S}{S}|S\rangle_S = \pm|S\rangle_S$

$$\Rightarrow \left[\binom{4}{0} + \binom{4}{2} + \binom{4}{4}\right]_+ \quad \text{or} \quad \left[\binom{4}{1} + \binom{4}{3}\right]_-$$

$$Q(\bar{f}) = \frac{1}{2}\cdot 0 \pm 1 = \pm 1$$

Classification of fermionic $Z_2 \times Z_2$ orbifolds (Modern School)

Basis vectors:

$$1 = \{\psi^\mu,\ \chi^{1,\dots,6}, y^{1,\dots,6}, \omega^{1,\dots,6} \mid \bar{y}^{1,\dots,6}, \bar{\omega}^{1,\dots,6}, \bar{\eta}^{1,2,3}, \bar{\psi}^{1,\dots,5}, \bar{\phi}^{1,\dots,8}\}$$

$$S = \{\psi^\mu, \chi^{1,\dots,6}\}, \qquad S + \Xi \longrightarrow \text{SUSY generator}$$

$$z_1 = \{\bar{\phi}^{1,\dots,4}\},$$

$$z_2 = \{\bar{\phi}^{5,\dots,8}\},$$

$$e_i = \{y^i, \omega^i | \bar{y}^i, \bar{\omega}^i\},\ i = 1, \dots, 6, \qquad N = 4 \text{ Vacua}$$

$$b_1 = \{\chi^{34}, \chi^{56}, y^{34}, y^{56} | \bar{y}^{34}, \bar{y}^{56}, \bar{\eta}^1, \bar{\psi}^{1,\dots,5}\}, \qquad N = 4 \to N = 2$$

$$b_2 = \{\chi^{12}, \chi^{56}, y^{12}, y^{56} | \bar{y}^{12}, \bar{y}^{56}, \bar{\eta}^2, \bar{\psi}^{1,\dots,5}\}, \qquad N = 2 \to N = 1$$

$$\alpha = \{\bar{\psi}^{4,5}, \bar{\phi}^{1,2}\} \qquad \& \quad SO(10) \to SO(6) \times SO(4) \times \cdots$$

$$\beta = \{\bar{\psi}^{1,\cdots,5} \equiv \tfrac{1}{2}, \cdots\} \qquad \& \quad SO(10) \to SU(5) \times U(1) \times \cdots$$

Independent phases $c\binom{v_i}{v_j} = \exp[i\pi(v_i|v_j)]$: upper block

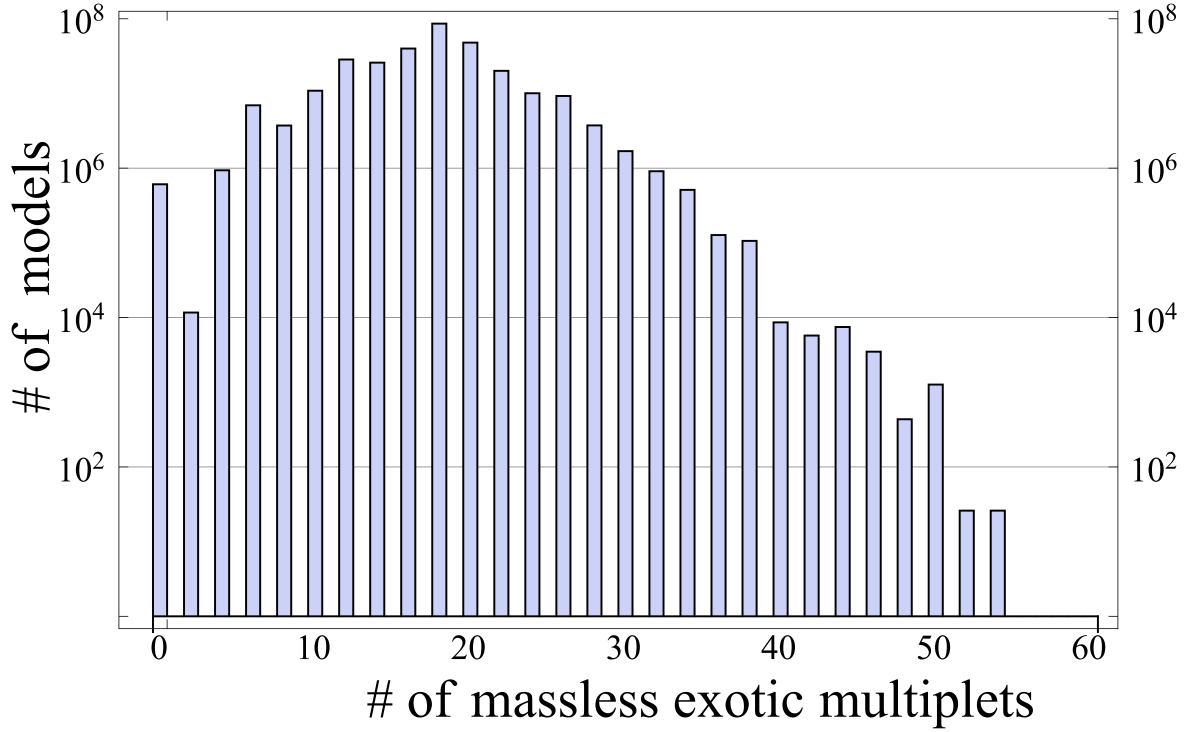
	1	S	e_1	e_2	e_3	e_4	e_5	e_6	z_1	z_2	b_1	b_2	α
1	-1	-1	$\pm$	$\pm$	$\pm$	$\pm$	$\pm$	$\pm$	$\pm$	$\pm$	$\pm$	$\pm$	$\pm$
S			-1	-1	-1	-1	-1	-1	-1	-1	1	1	-1
e_1				$\pm$	$\pm$	$\pm$	$\pm$	$\pm$	$\pm$	$\pm$	$\pm$	$\pm$	$\pm$
e_2					$\pm$	$\pm$	$\pm$	$\pm$	$\pm$	$\pm$	$\pm$	$\pm$	$\pm$
e_3						$\pm$	$\pm$	$\pm$	$\pm$	$\pm$	$\pm$	$\pm$	$\pm$
e_4							$\pm$	$\pm$	$\pm$	$\pm$	$\pm$	$\pm$	$\pm$
e_5								$\pm$	$\pm$	$\pm$	$\pm$	$\pm$	$\pm$
e_6									$\pm$	$\pm$	$\pm$	$\pm$	$\pm$
z_1										$\pm$	$\pm$	$\pm$	$\pm$
z_2											$\pm$	$\pm$	$\pm$
b_1												$\pm$	$\pm$
b_2												-1	$\pm$
α													

A priori 66 independent coefficients $\rightarrow\ 2^{66}$ distinct vacua

PLB2021, Percival *et al* $\rightarrow$ Satisfiability Modulo Theories $\longrightarrow\ t \times 10^{-3}$

Pati–Salam class: with Assel, Christodoulides, Kounnas, Rizos

RESULTS: of random search of over 10^{11} vacua



Number of 3-generation models versus total number of exotic multiplets

NON–SUSY String Phenomenology:

Starting with: $$Z^{+}_{10d} = (V_8 - S_8)\left(\overline{O}_{16} + \overline{S}_{16}\right)\left(\overline{O}_{16} + \overline{S}_{16}\right),$$

using the level–one $SO(2n)$ characters

$$O_{2n} = \tfrac{1}{2}\left(\frac{\theta_3^n}{\eta^n} + \frac{\theta_4^n}{\eta^n}\right), \qquad V_{2n} = \tfrac{1}{2}\left(\frac{\theta_3^n}{\eta^n} - \frac{\theta_4^n}{\eta^n}\right),$$

$$S_{2n} = \tfrac{1}{2}\left(\frac{\theta_2^n}{\eta^n} + i^{-n}\frac{\theta_1^n}{\eta^n}\right), \qquad C_{2n} = \tfrac{1}{2}\left(\frac{\theta_2^n}{\eta^n} - i^{-n}\frac{\theta_1^n}{\eta^n}\right).$$

where

$$\theta_3 \equiv Z_f\binom{0}{0} \qquad \theta_4 \equiv Z_f\binom{0}{1} \qquad \theta_2 \equiv Z_f\binom{1}{0} \qquad \theta_1 \equiv Z_f\binom{1}{1}$$

Apply $g = (-1)^{F+F_{z_1}+F_{z_2}}$

$$\begin{aligned} Z^{-}_{10d} = & \left[\, V_8\left(\overline{O}_{16}\overline{O}_{16} + \overline{S}_{16}\overline{S}_{16}\right) - S_8\left(\overline{O}_{16}\overline{S}_{16} + \overline{S}_{16}\overline{O}_{16}\right)\right. \\ + & \left.\underline{O_8\left(\overline{C}_{16}\overline{V}_{16} + \overline{V}_{16}\overline{C}_{16}\right)} - C_8\left(\overline{C}_{16}\overline{C}_{16} + \overline{V}_{16}\overline{V}_{16}\right)\right]. \end{aligned}$$

In fermionic language: $\{ \mathbf{1} , z_1 , z_2 \}$

where $z_1 = \{\bar\psi^{1,\cdots,5}, \bar\eta^{1,2,3}\}$; $z_2 = \{\bar\phi^{1,\cdots,8}\}$ $\Rightarrow$ $S = \mathbf{1} + z_1 + z_2$

$c\binom{z_1}{z_2} = +1 \implies E_8 \times E_8$; $c\binom{z_1}{z_2} = -1 \implies SO(16) \times SO(16)$

Tachyon free non-SUSY string phenomenology

Alternatively: Apply $g = (-1)^{F+F_{z_1}}$

$$Z^{-}_{10d} = \left(V_8\overline{O}_{16} - S_8\overline{S}_{16} + \underline{O_8\overline{V}_{16}} - C_8\overline{C}_{16}\right)\left(\overline{O}_{16} + \overline{S}_{16}\right),$$

$O_8\overline{V}_{16}\overline{O}_{16} \implies$ tachyonic 10D vacuum

In fermionic language: $\{ \mathbf{1} , z_2 \}$ $\implies$ No S

In both cases $\longrightarrow$ tachyon free 4D GSO configurations

Tachyon free models: $S \longleftrightarrow \tilde{S}$–map $\longleftarrow$ "modular map"

Modified NAHE $\longleftrightarrow$ $\overline{\mathrm{NAHE}}$

	ψ^μ	χ^{12}	χ^{34}	χ^{56}	$y^{3,\dots,6}$	$\bar{y}^{3,\dots,6}$	$y^{1,2},\omega^{5,6}$	$\bar{y}^{1,2},\bar{\omega}^{5,6}$	$\omega^{1,\dots,4}$	$\bar{\omega}^{1,\dots,4}$	$\bar{\psi}^{1,\dots,5}$	$\bar{\eta}^1$	$\bar{\eta}^2$	$\bar{\eta}^3$	$\bar{\phi}^{1,\dots,8}$
1	1	1	1	1	1,...,1	1,...,1	1,...,1	1,...,1	1,...,1	1,...,1	1,...,1	1	1	1	1,1,1,1,1,1,1,1
$\tilde{S}$	1	1	1	1	0,...,0	0,...,0	0,...,0	0,...,0	0,...,0	0,...,0	0,...,0	0	0	0	1,1,1,1,0,0,0,0
b_1	1	1	0	0	1,...,1	1,...,1	0,...,0	0,...,0	0,...,0	0,...,0	1,...,1	1	0	0	0,0,0,0,0,0,0,0
b_2	1	0	1	0	0,...,0	0,...,0	1,...,1	1,...,1	0,...,0	0,...,0	1,...,1	0	1	0	0,0,0,0,0,0,0,0
b_3	1	0	0	1	0,...,0	0,...,0	0,...,0	0,...,0	1,...,1	1,...,1	1,...,1	0	0	1	0,0,0,0,0,0,0,0

Beyond the $\overline{\mathrm{NAHE}}$–set

	ψ^μ	χ^{12}	χ^{34}	χ^{56}	y^3y^6	$y^4\bar{y}^4$	$y^5\bar{y}^5$	$\bar{y}^3\bar{y}^6$	$y^1\omega^5$	$y^2\bar{y}^2$	$\omega^6\bar{\omega}^6$	$\bar{y}^1\bar{\omega}^5$	$\omega^2\omega^4$	$\omega^1\bar{\omega}^1$	$\omega^3\bar{\omega}^3$	$\bar{\omega}^2\bar{\omega}^4$	$\bar{\psi}^{1,\dots,5}$	$\bar{\eta}^1$	$\bar{\eta}^2$	$\bar{\eta}^3$	$\bar{\phi}^{1,\dots}$
α	0	0	0	0	1	0	0	1	0	0	1	1	0	0	1	1	1 1 1 0 0	1	0	0	0 0 0 0
β	0	0	0	0	0	0	1	1	1	0	0	1	0	1	0	1	1 1 1 0 0	0	1	0	1 1 0 0
γ	0	0	0	0	0	1	0	0	0	1	0	0	1	0	0	0	$\frac{1}{2}$ $\frac{1}{2}$ $\frac{1}{2}$ $\frac{1}{2}$ $\frac{1}{2}$	$\frac{1}{2}$	$\frac{1}{2}$	$\frac{1}{2}$	0 0 $\frac{1}{2}$ $\frac{1}{2}$

Up to the $S \longleftrightarrow \tilde{S}$–map

Same model as published with

with Cleaver, Manno and Timirgaziu in PRD78 (2008) 046009

Stable non–SUSY heterotic–string vacuum?

Moduli $\to$ WS Thirring interactions $(R-\frac{1}{R})J_L^i(z)\bar{J}_R^j(\bar{z}) = (R-\frac{1}{R})y^i\omega^i\bar{y}^j\bar{\omega}^j$

To identify the untwisted moduli in the free fermionic models

$\to$ find the operators of the form

$$J_L^I(z)\bar{J}_R^J(\bar{z}) \quad \equiv \quad y^I\omega^I\bar{y}^J\bar{\omega}^J$$

that are allowed by the orbifold (fermionic) symmetry group

$Z_2 \times Z_2 \qquad \{\,1\,,\ S\,,\ z_1\,,\ z_2\,\} \ + \ \{\,b_1\,,\ b_2\,\}$

$$\to \quad SO(4)^3 \times E_6 \times U(1)^2 \times E_8$$

The Thirring interactions that remain invariant are

$$J_L^{1,2}\bar{J}_R^{1,2} \quad ; \quad J_L^{3,4}\bar{J}_R^{3,4} \quad ; \quad J_L^{5,6}\bar{J}_R^{5,6}$$

$$y^{1,2}\omega^{1,2}\bar{y}^{1,2}\bar{\omega}^{1,2} \quad ; \quad y^{3,4}\omega^{3,4}\bar{y}^{3,4}\bar{\omega}^{3,4} \quad ; \quad y^{5,6}\omega^{5,6}\bar{y}^{5,6}\bar{\omega}^{5,6}$$

These moduli are always present in symmetric $Z_2 \times Z_2$ orbifolds in realistic models

$$\{\,1\,,\ S\,,\ z_1\,,\ z_2\,\}\ \oplus\ \{\,b_1\,,\ b_2\,\}\ \oplus\ \{\,\alpha\,,\ \beta\,,\ \gamma\}$$

$$N=4 \qquad\qquad N=1$$

$$E_8\times E_8 \qquad\qquad Z_2\times Z_2$$

new feature Asymmetric orbifold $y^i\omega^i\bar{y}^i\bar{\omega}^i \ \rightarrow\ -\,y^i\omega^i\bar{y}^i\bar{\omega}^i$

the key focus: boundary conditions of the internal fermions

$$\{\ y\ ,\ \omega\ \mid\ \bar{y}\ ,\ \bar{\omega}\ \}$$

WS fermions that have same B.C. in all basis vectors are paired

pairing of LR fermions $\rightarrow$ Ising model $\rightarrow$ symmetric real fermions

pairing of LL & RR fermions $\rightarrow$ complex fermions $\rightarrow$ asymmetric

STRING DERIVED STANDARD-LIKE MODEL (PLB278)

	ψ^μ	χ^{12}	χ^{34}	χ^{56}	$y^{3,...,6}$	$\bar{y}^{3,...,6}$	$y^{1,2},\omega^{5,6}$	$\bar{y}^{1,2},\bar{\omega}^{5,6}$	$\omega^{1,...,4}$	$\bar{\omega}^{1,...,4}$	$\bar{\psi}^{1,...,5}$	$\bar{\eta}^1$	$\bar{\eta}^2$	$\bar{\eta}^3$	$\bar{\phi}^{1,...,8}$
1	1	1	1	1	1,...,1	1,...,1	1,...,1	1,...,1	1,...,1	1,...,1	1,...,1	1	1	1	1,...,1
S	1	1	1	1	0,...,0	0,...,0	0,...,0	0,...,0	0,...,0	0,...,0	0,...,0	0	0	0	0,...,0
b_1	1	1	0	0	1,...,1	1,...,1	0,...,0	0,...,0	0,...,0	0,...,0	1,...,1	1	0	0	0,...,0
b_2	1	0	1	0	0,...,0	0,...,0	1,...,1	1,...,1	0,...,0	0,...,0	1,...,1	0	1	0	0,...,0
b_3	1	0	0	1	0,...,0	0,...,0	0,...,0	0,...,0	1,...,1	1,...,1	1,...,1	0	0	1	0,...,0

	ψ^μ	χ^{12}	χ^{34}	χ^{56}	y^3y^6	$y^4\bar{y}^4$	$y^5\bar{y}^5$	$\bar{y}^3\bar{y}^6$	$y^1\omega^5$	$y^2\bar{y}^2$	$\omega^6\bar{\omega}^6$	$\bar{y}^1\bar{\omega}^5$	$\omega^2\omega^4$	$\omega^1\bar{\omega}^1$	$\omega^3\bar{\omega}^3$	$\bar{\omega}^2\bar{\omega}^4$	$\bar{\psi}^{1,...,5}$	$\bar{\eta}^1$	$\bar{\eta}^2$	$\bar{\eta}^3$	$\bar{\phi}$
α	0	0	0	0	1	0	0	0	0	0	1	1	0	0	1	1	1 1 1 0 0	0	0	0	1 1 1
β	0	0	0	0	0	0	1	1	1	0	0	0	0	1	0	1	1 1 1 0 0	0	0	0	1 1 1
γ	0	0	0	0	0	1	0	1	0	1	0	1	1	0	0	0	$\frac{1}{2}$ $\frac{1}{2}$ $\frac{1}{2}$ $\frac{1}{2}$ $\frac{1}{2}$	$\frac{1}{2}$	$\frac{1}{2}$	$\frac{1}{2}$	$\frac{1}{2}$ 0 1

Asymmetric $BC \Rightarrow$ all untwisted moduli are projected out!

all $y_i\omega_i\bar{y}_i\bar{\omega}_i$ are disallowed

can be translated to asymmetric bosonic identifications

$$X_L + X_R \rightarrow X_L - X_R$$

moduli fixed at enhanced symmetry point

Classification of tachyon free models

Basis vectors:

$$1 = \{\psi^\mu,\ \chi^{1,\dots,6}, y^{1,\dots,6}, \omega^{1,\dots,6} \mid \bar{y}^{1,\dots,6}, \bar{\omega}^{1,\dots,6}, \bar{\eta}^{1,2,3}, \bar{\psi}^{1,\dots,5}, \bar{\phi}^{1,\dots,8}\}$$

$$S = \{\psi^\mu, \chi^{1,\dots,6}\} \quad \text{and} \quad \tilde{S} = \{\psi^\mu, \chi^{1,\dots,6} \mid \bar{\phi}^{3,\cdots,6}\},$$

$$z_1 = \{\bar{\phi}^{1,\dots,4}\},$$

$$z_2 = \{\bar{\phi}^{5,\dots,8}\},$$

$$e_i = \{y^i, \omega^i | \bar{y}^i, \bar{\omega}^i\},\ i = 1, \dots, 6,$$

$$b_1 = \{\psi^{12}, \chi^{12}, y^{34}, y^{56} | \bar{y}^{34}, \bar{y}^{56}, \bar{\eta}^1, \bar{\psi}^{1,\dots,5}\}, \qquad N = 4 \to N = 2$$

$$b_2 = \{\psi^{12}, \chi^{34}, y^{12}, y^{56} | \bar{y}^{12}, \bar{y}^{56}, \bar{\eta}^2, \bar{\psi}^{1,\dots,5}\}, \qquad N = 2 \to N = 1$$

$$\alpha = \{\bar{\psi}^{4,5}, \bar{\phi}^{1,2}\}$$

with Viktor Matyas and Ben Percival, NPB 961 (2020) 115231;

PRD 104 (2021) 04600; PRD 106 (2022) 026011

Partition functions and the cosmological constant

Full Partition Function for Free Fermionic models:

$$Z_{ToT} \quad = \quad \int_{\mathfrak{F}} \frac{d^2\tau}{\tau_2^2}\, Z_B\, Z_F \quad \equiv \quad \Lambda$$

Integral over the inequivalent tori

• Fermionic contribution:

$$Z_F = \sum_{Sp.Str.} c\binom{\alpha}{\beta} \prod_f Z\begin{bmatrix}\alpha(f)\\ \beta(f)\end{bmatrix}$$

$$Z\begin{bmatrix}1\\1\end{bmatrix} = \sqrt{\frac{\theta_1}{\eta}}, \qquad Z\begin{bmatrix}1\\0\end{bmatrix} = \sqrt{\frac{\theta_2}{\eta}}, \qquad Z\begin{bmatrix}0\\0\end{bmatrix} = \sqrt{\frac{\theta_3}{\eta}}, \qquad Z\begin{bmatrix}0\\1\end{bmatrix} = \sqrt{\frac{\theta_4}{\eta}},$$

• Bosonic : $Z_B = \frac{1}{\tau_2}\frac{1}{\eta^2\bar{\eta}^2}$ from spacetime Bosons.

Evaluated using $q \equiv e^{2\pi i\tau}$ expansion

$$Z = \sum_{n.m} a_{mn} \int_{\mathcal{F}} \frac{d^2\tau}{\tau_2^3} q^m \bar{q}^n \qquad \begin{cases} d\tau_1 & \longrightarrow \textit{analytic} \\ d\tau_2 & \longrightarrow \textit{numeric} \end{cases}$$

$q-$ expansion of Z

$$I_{mn} = \begin{cases} \infty & \text{if} \quad m+n<0 \ \wedge \ m-n \notin \mathbb{Z}\backslash\{0\} \\ \text{Finite} & \text{Otherwise.} \end{cases}$$

- On-Shell Tachyons cause divergence

- Off-Shell Tachyons allowed (necessary)

Modular invariance $\longrightarrow m-n \in \mathbb{Z}$.

Allowed states

$$a_{mn} = \begin{pmatrix} 0 & 0 & a_{-\frac{1}{2}-\frac{1}{2}} & 0 & 0 & 0 & a_{-\frac{1}{2}\frac{1}{2}} & 0 & 0 & 0 \\ 0 & 0 & 0 & a_{-\frac{1}{4}-\frac{1}{4}} & 0 & 0 & 0 & a_{-\frac{1}{4}\frac{3}{4}} & 0 & 0 \\ a_{0-1} & 0 & 0 & 0 & a_{00} & 0 & 0 & 0 & a_{01} & 0 \\ 0 & a_{\frac{1}{4}-\frac{3}{4}} & 0 & 0 & 0 & a_{\frac{1}{4}\frac{1}{4}} & 0 & 0 & 0 & \ddots \\ 0 & 0 & a_{\frac{1}{2}-\frac{1}{2}} & 0 & 0 & 0 & a_{\frac{1}{2}\frac{1}{2}} & 0 & 0 & 0 \\ 0 & 0 & 0 & a_{\frac{3}{4}-\frac{1}{4}} & 0 & 0 & 0 & a_{\frac{3}{4}\frac{3}{4}} & 0 & 0 \\ a_{1-1} & 0 & 0 & 0 & a_{10} & 0 & 0 & 0 & a_{11} & 0 \\ 0 & \ddots & 0 & 0 & 0 & \ddots & 0 & 0 & 0 & \ddots \end{pmatrix}$$

Coefficients $a_{mn} = N_b - N_f$ at specific mass level.

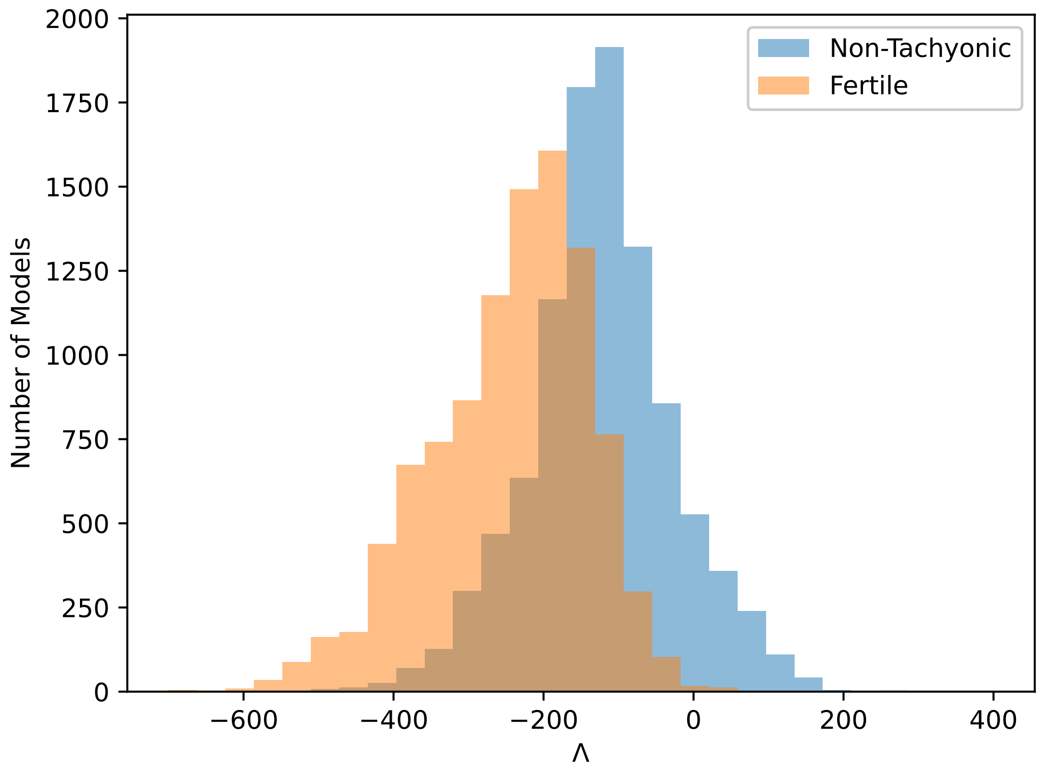
For SUSY Theories $a_{mn} = 0 \ \forall m, n$

Some interesting results

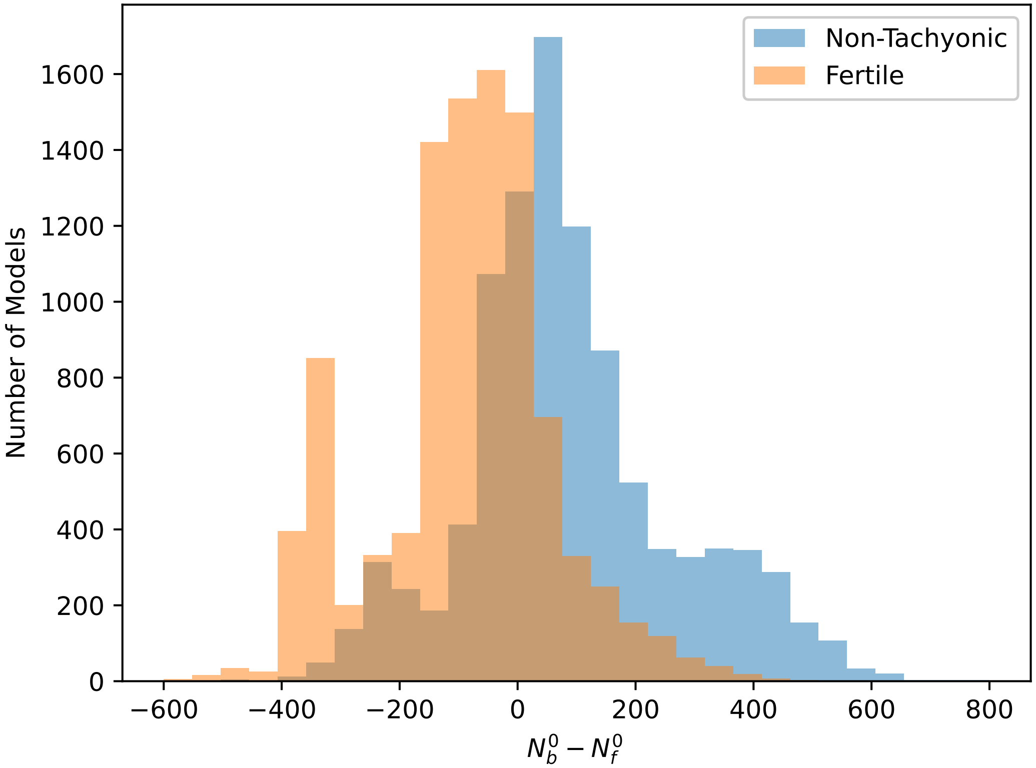
viable $\tilde{S}$–models: only SM$\times U(1)_{Z'}$.

No heavy Higgs to break FSU5 or PS symmetry; SM$\times U(1)_{Z'} \to Z'$ exotics

Distribution of Λ



Distribution of a_{00}



Toward de Sitter vacua with stable moduli

(Work in progress with Alonzo Diaz, Viktor Matyas and Benjamin Percival)

Classification of asymmetric $N = 0$ vacua (w Matyas & Percival PRD106)

Fix two tori at $R = 1$ using asymmetric boundary condition

Vary the moduli of the remaining unfixed torus (following Florakis & Rizos)

Find minima with $\Lambda_{min} > 0$

Scan for models with realistic features and stable $\Lambda_{min} > 0$

Conclusions

- DATA $\longrightarrow$ UNIFICATION $\longleftrightarrow$ HiggStructure?
- STRINGS THEORY $\longrightarrow$ GAUGE & GRAVITY UNIFICATION
- STRINGS PHENOMENOLOGY $\longrightarrow$ AT ITS INFANCY

  STILL LEARNING HOW TO WALK
- SUSY/Non–SUSY string phenomenology $\cdots\cdots\cdots$
- Vacua with/out S–SUSY generator
- Role of non–geometric backgrounds $\longleftrightarrow$ Moduli Fixing
- String Phenomenology $\longrightarrow$ Physics of the third millennium

  $e.g.$ Aristarchus to Galileo