Mirror Symmetry and Spinor–Vector Duality

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- Torodial lattices of heterotic–strings: G, B, W
- Mirror symmetry: $G, B \to \tilde{G}, \tilde{B}$

  Kähler $\leftrightarrow$ Complex structure moduli $\chi \leftrightarrow -\chi$
- Spinor–vector duality: $W \to \tilde{W}$

  AEF, C Kounnas, J Rizos, PLB2007; NPB2007
  C Angelantonj, AEF, M. Tsulaia, JHEP
  AEF, I Florakis, T Mohaupt, M Tsulaia NPB2011
  AEF, S Groot–Nibbelink, M Hurtado Heredia, PRD2021; NPB2021; ...

12^{th} Bangkok workshop on HET, Chulalongkorn University, 24 January 2025

Fermionic $Z_2 \times Z_2$ orbifolds

'Phenomenology of the Standard Model and Unification'

- Minimal Superstring Standard Model — NPB 335 (1990) 347 (with Nanopoulos & Yuan)
- Top quark mass $\sim$ 175–180GeV — PLB 274 (1992) 47
- Generation mass hierarchy — NPB 407 (1993) 57
- CKM mixing — NPB 416 (1994) 63 (with Halyo)
- Stringy seesaw mechanism — PLB 307 (1993) 311 (with Halyo)
- Gauge coupling unification — NPB 457 (1995) 409 (with Dienes)
- Proton stability — NPB 428 (1994) 111
- Squark degeneracy — NPB 526 (1998) 21 (with Pati)
- Moduli fixing — NPB 728 (2005) 83
- Classification & Vacuum structure — 2003 – $\cdots$

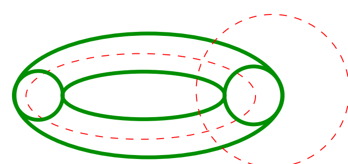
  (with Kounnas, Rizos & ... Percival, Matyas, Avalos–Diaz ...)

Fermionic Construction

Left-Movers: $\psi^{\mu=1,2}\ ,\quad \chi_i,\quad y_i,\ \ \omega_i\quad (i=1,\cdots,6)$

Right-Movers

$$\bar\phi_{A=1,\cdots,44} = \begin{cases} \bar y_i\ ,\ \bar\omega_i & i=1,\cdots,6 \\ \\ \bar\eta_i & U(1)_i \quad i=1,2,3 \\ \bar\psi_{1,\cdots,5} & SO(10) \\ \bar\phi_{1,\cdots,8} & SO(16) \end{cases}$$



$$V \longrightarrow V \qquad\qquad f \longrightarrow -e^{i\pi\alpha(f)} f$$

$$Z = \sum_{\substack{all\ spin\\ structures}} c\binom{\vec\alpha}{\vec\beta}\ Z\binom{\vec\alpha}{\vec\beta}$$

Models $\longleftrightarrow$ Basis vectors $+$ one–loop phases

Classification of fermionic $Z_2 \times Z_2$ orbifolds

Basis vectors:

$$\begin{aligned}
1 &= \{\psi^\mu,\ \chi^{1,\dots,6}, y^{1,\dots,6}, \omega^{1,\dots,6} \mid \bar{y}^{1,\dots,6}, \bar{\omega}^{1,\dots,6}, \bar{\eta}^{1,2,3}, \bar{\psi}^{1,\dots,5}, \bar{\phi}^{1,\dots,8}\} \\
S &= \{\psi^\mu, \chi^{1,\dots,6}\}, \\
z_1 &= \{\bar{\phi}^{1,\dots,4}\}, \\
z_2 &= \{\bar{\phi}^{5,\dots,8}\}, \\
e_i &= \{y^i, \omega^i | \bar{y}^i, \bar{\omega}^i\},\ i = 1, \dots, 6, && N = 4 \text{ Vacua} \\
b_1 &= \{\chi^{34}, \chi^{56}, y^{34}, y^{56} | \bar{y}^{34}, \bar{y}^{56}, \bar{\eta}^1, \bar{\psi}^{1,\dots,5}\}, && N = 4 \to N = 2 \\
b_2 &= \{\chi^{12}, \chi^{56}, y^{12}, y^{56} | \bar{y}^{12}, \bar{y}^{56}, \bar{\eta}^2, \bar{\psi}^{1,\dots,5}\}, && N = 2 \to N = 1
\end{aligned}$$

Vector bosons: NS, $z_{1,2}$, $z_1 + z_2$, $\quad X = 1 + s + \sum e_i + z_1 + z_2$

impose: $c\begin{bmatrix} z_1 \\ z_2 \end{bmatrix} = -1$ $\quad$ & $\quad$ Gauge group $SO(10) \times U(1)^3 \times \text{hidden}$

Independent phases $c\binom{v_i}{v_j} = \exp[i\pi(v_i|v_j)]$: upper block

$$
\begin{array}{c}
 \\ 1 \\ S \\ e_1 \\ e_2 \\ e_3 \\ e_4 \\ e_5 \\ e_6 \\ z_1 \\ z_2 \\ b_1 \\ b_2
\end{array}
\begin{array}{c}
\begin{array}{cccccccccccc}
1 & S & e_1 & e_2 & e_3 & e_4 & e_5 & e_6 & z_1 & z_2 & b_1 & b_2
\end{array} \\
\begin{pmatrix}
-1 & -1 & \pm & \pm & \pm & \pm & \pm & \pm & \pm & \pm & \pm & \pm \\
 & & -1 & -1 & -1 & -1 & -1 & -1 & -1 & -1 & 1 & 1 \\
 & & & \pm & \pm & \pm & \pm & \pm & \pm & \pm & \pm & \pm \\
 & & & & \pm & \pm & \pm & \pm & \pm & \pm & \pm & \pm \\
 & & & & & \pm & \pm & \pm & \pm & \pm & \pm & \pm \\
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 & & & & & & & & & & \pm & \pm \\
 & & & & & & & & & & & \pm \\
 & & & & & & & & & & & -1 \\
 & & & & & & & & & & &
\end{pmatrix}
\end{array}
$$

A priori 55 independent coefficients $\rightarrow$ 2^{55} distinct vacua

PLB2021, Percival *et al* $\rightarrow$ Satisfiability Modulo Theories $\longrightarrow$ $t \times 10^{-3}$

Mirror symmetry

Enhance $SO(10) \to E_6$

from $X = 1 + S + \sum_i e_i + z_1 + z_2 = \{\bar\psi^{1,\cdots,5}, \bar\eta^{1,2,3}\}$

Euler characteristic $\chi \; = \; \#(\mathbf{27} - \overline{\mathbf{27}}) \quad \longrightarrow \quad -\chi$

Exchanges Complex and Kähler structure moduli

Moduli of the internal compactified space

Vafa–Witten 1994: Mirror symmetry in terms of discrete torsion

$$c\binom{b_1}{b_2} = +1 \to c\binom{b_1}{b_2} = -1$$

Spinor–vector duality: AEF, Kounnas, Rizos, NPB774 (2007) 208

Duality under exchange of spinors and vectors.

First Plane			Second plane			Third Plane			
s	$\bar{s}$	v	s	$\bar{s}$	v	s	$\bar{s}$	v	# of models
2	0	0	0	0	0	0	0	0	1325963712
0	2	0	0	0	0	0	0	0	1340075584
1	1	0	0	0	0	0	0	0	3718991872
0	0	2	0	0	0	0	0	0	6385031168

$\#$ of models with $\#(16+\overline{16}) = \#$ of models with $\#(10)$

For every model with $\#(16+\overline{16})$ & $\#(10)$

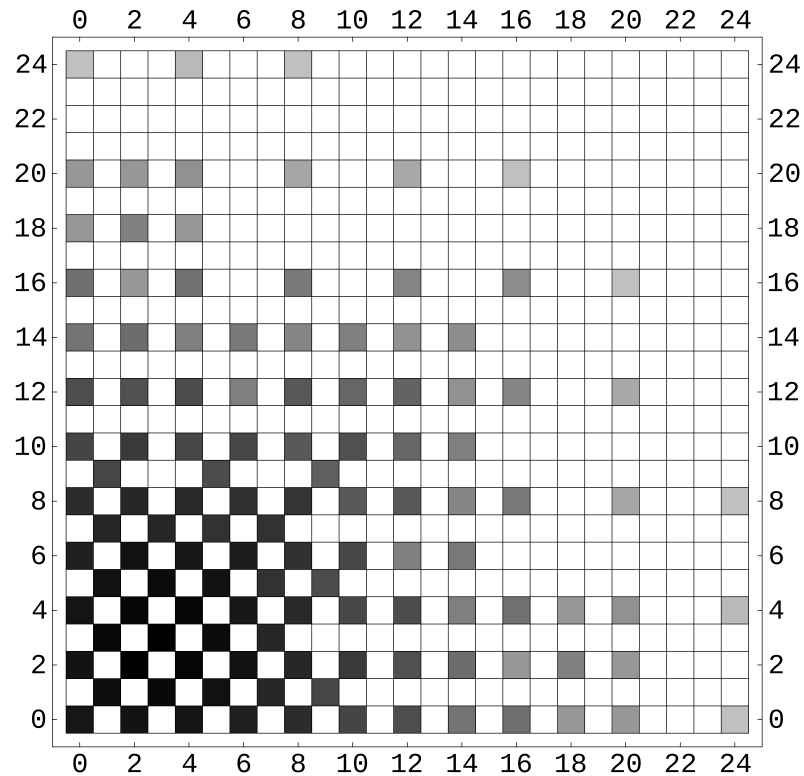
There exist another model in which they are interchanged

Reflects discrete exchange of phases

	0	1	2	3	4	5	6	7	8
0	14424168320	0	19155093504	0	17251226688	0	5722036224	0	16635982
1	0	35042893824	0	54063267840	0	24984354816	0	3050569728	0
2	19155093504	0	138128891904	0	80400635904	0	22541905920	0	25932533
3	0	54063267840	0	128713392128	0	43913576448	0	3064725504	0
4	17251226688	0	80400635904	0	78871289088	0	11554105344	0	22462053
5	0	24984354816	0	43913576448	0	21663891456	0	856424448	0
6	5722036224	0	22541905920	0	11554105344	0	8043915264	0	9377280
7	0	3050569728	0	3064725504	0	856424448	0	866942976	0
8	1663598208	0	2593253376	0	2246205312	0	937728000	0	7030003
9	0	113541120	0	0	0	67829760	0	0	0
10	135948288	0	406695936	0	107403264	0	104902656	0	1746739
11	0	0	0	0	0	0	0	0	0
12	50867584	0	42448896	0	65853312	0	387072	0	1859020
13	0	0	0	0	0	0	0	0	0
14	1210368	0	2420736	0	387072	0	774144	0	202752
15	0	0	0	0	0	0	0	0	0
16	1854336	0	36864	0	1514688	0	0	0	714816
17	0	0	0	0	0	0	0	0	0
18	36864	0	313344	0	36864	0	0	0	0
19	0	0	0	0	0	0	0	0	0
20	33984	0	36864	0	62784	0	0	0	7680
21	0	0	0	0	0	0	0	0	0
22	0	0	0	0	0	0	0	0	0
23	0	0	0	0	0	0	0	0	0
24	576	0	0	0	1152	0	0	0	576

Spinor–vector duality:

Invariance under exchange of $\#(16+\overline{16}) \;\; <-> \;\; \#(10)$



Symmetric under exchange of rows and columns

$$E_6 \;\; : \qquad 27 = 16 + 10 + 1 \qquad \overline{27} = \overline{16} + 10 + 1$$

Self-dual: $\#(16+\overline{16}) \;\; = \;\; \#(10)$ without E_6 symmetry

Using the level-one $\mathrm{SO}(2n)$ characters:

$$O_{2n} = \frac{1}{2}\left(\frac{\theta_3^n}{\eta^n} + \frac{\theta_4^n}{\eta^n}\right), \qquad V_{2n} = \frac{1}{2}\left(\frac{\theta_3^n}{\eta^n} - \frac{\theta_4^n}{\eta^n}\right),$$

$$S_{2n} = \frac{1}{2}\left(\frac{\theta_2^n}{\eta^n} + i^{-n}\frac{\theta_1^n}{\eta^n}\right), \qquad C_{2n} = \frac{1}{2}\left(\frac{\theta_2^n}{\eta^n} - i^{-n}\frac{\theta_1^n}{\eta^n}\right).$$

where

$$\theta_3 \equiv Z_f\begin{pmatrix}0\\0\end{pmatrix} \quad \theta_4 \equiv Z_f\begin{pmatrix}0\\1\end{pmatrix}$$

$$\theta_2 \equiv Z_f\begin{pmatrix}1\\0\end{pmatrix} \quad \theta_1 \equiv Z_f\begin{pmatrix}1\\1\end{pmatrix}$$

Spinor–Vector duality in Orbifolds:

Starting from: $$Z_+ = (V_8 - S_8)\left(\sum_{m,n}\Lambda_{m,n}\right)^{\otimes 6} E_8 \times E_8\,,$$

apply $Z_2 \times Z_2' \,:\, g \times g'$

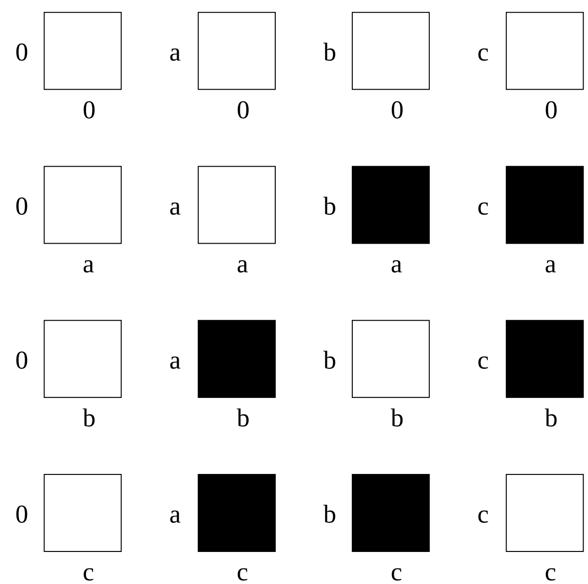
$$g \;:\; (0^7,1|1,0^7) \;\rightarrow\; \text{Wilson line} \;\rightarrow\; E_8\times E_8 \rightarrow SO(16)\times SO(16)$$

$$g' \;:\; (x_4,x_5,x_6,x_7,x_8,x_9) \;\longrightarrow\; (-x_4,-x_5,-x_6,-x_7,+x_8,+x_9)$$

Note: A single space twisting Z_2' $\Rightarrow$ $N=4 \rightarrow N=2$

$$E_7 \rightarrow SO(12)\times SU(2)$$

$\Rightarrow$ Analyze $\quad Z = \left(\dfrac{Z_+}{Z_g \times Z_{g'}} \right) = \left[\dfrac{(1+g)}{2} \dfrac{(1+g')}{2} \right] Z_+$



a = g ; b = g’ ; c = gg’

$$\mathrm{P.F.} = (\square + \varepsilon \blacksquare) = \Lambda_{m,n} \bullet (\quad) + \Lambda_{m,n+1/2} \bullet (\quad)$$

$\varepsilon = \pm 1$

massless massive

- sector b

$$\Lambda_{p,q}\left\{\frac{1}{2}\left(\left|\frac{2\eta}{\theta_4}\right|^4+\left|\frac{2\eta}{\theta_3}\right|^4\right)\left[P_\epsilon^+ Q_s\overline{V}_{12}\overline{C}_4\overline{O}_{16}+P_\epsilon^- Q_s\overline{S}_{12}\overline{O}_4\overline{O}_{16}\right]+\right.$$
$$\left.\frac{1}{2}\left(\left|\frac{2\eta}{\theta_4}\right|^4-\left|\frac{2\eta}{\theta_3}\right|^4\right)\left[P_\epsilon^+ Q_s\overline{O}_{12}\overline{S}_4\overline{O}_{16}\right]\right\}\quad + \quad \text{massive}$$

where

$$P_\epsilon^+=\left(\frac{1+\epsilon(-1)^m}{2}\right)\Lambda_{m,n}\qquad P_\epsilon^-=\left(\frac{1-\epsilon(-1)^m}{2}\right)\Lambda_{m,n}$$

$$\epsilon=+1\quad\Rightarrow\quad P_\epsilon^+=\Lambda_{2m,n}\qquad P_\epsilon^-=\Lambda_{2m+1,n}$$
$$\epsilon=-1\quad\Rightarrow\quad P_\epsilon^+=\Lambda_{2m+1,n}\qquad P_\epsilon^-=\Lambda_{2m,n}$$

and $\quad 12\cdot 2 \;+\; 4\cdot 2 \;=\; 32$

w Florakis, Mohaput, Tsulaia (2011) $\epsilon \longleftrightarrow$ Wilson line & spectral flow

Spinor-Vector duality on Calabi–Yau threefolds with vector bundles :

- From the “Land” to the “Swamp” w Groot–Nibellink & Hurtado-Heredia, arXiv:2103.13442, spinor–vector duality on a resolved orbifold. The role of the discrete torsion in the effective field theory limit

- Vafa–Witten 1994, the role of a discrete torsion in the $Z_2 \times Z_2$ orbifold in mirror symmetry

- In similar spirit $\rightarrow$ the imprint of the worldsheet modular properties in the effective field theory limit

Interactions

correlators between vertex operators $\langle V_1^f V_2^f V_3^b \cdots V_N^b \rangle$

Vertex operators

$$V^f_{(-\frac{1}{2})} = \mathrm{e}^{(-\frac{c}{2})} \, \mathcal{L}^\ell \, \mathrm{e}^{(i\alpha\chi_{12})} \, \mathrm{e}^{(i\beta\chi_{34})} \, \mathrm{e}^{(i\gamma\chi_{56})}$$
$$\left(\prod_j \mathrm{e}^{(iq_i\zeta_j)} \;\; \{\sigma' s\} \prod_j \mathrm{e}^{(i\bar{q}_i\bar{\zeta}_j)} \right)$$
$$\mathrm{e}^{(i\bar{\alpha}\bar{\eta}_1)} \, \mathrm{e}^{(i\bar{\beta}\bar{\eta}_2)} \, \mathrm{e}^{(i\bar{\gamma}\bar{\eta}_3)} \, \mathrm{e}^{(iW_R\cdot\bar{J})} \mathrm{e}^{(i\frac{1}{2}KX)} \, \mathrm{e}^{(\mathrm{i}\frac{1}{2}K\cdot\bar{X})}$$

Non–vanishing correlators $\longrightarrow$ invariant under all the string symmetries

Mirror symmetry $\quad \longrightarrow \quad 27 \cdot 27 \cdot 27 \longleftrightarrow \overline{27} \cdot \overline{27} \cdot \overline{27}$

Mathematical implications

- On Calabi–Yau threefolds, the couplings correspond to intersections of curves $\longleftrightarrow$ rational curves on CY manifolds
- mirror symmetry is instrumental in counting of rational curves on CY 3–folds $\longleftrightarrow$ instrumental in enumerative geometry
- A tool developed for that purpose are the Gromov–Witten invariants

Questions

- Perform a similar analysis of correlators on spinor–vector dual vacua;
- What are the analogue of the Gromov–Witten invariants in the case of spinor–vector duality
- spinor–vector duality $\longrightarrow$ a tool to study CY manifolds with bundles
- moduli spaces of $(2,0)$ string compactifications
- Is it complete? Is it constraining the viable effective field limit of stringy quantum gravity.
- ...

A Swampland Conjecture(?): AEF, EPJC 79 (2019) 703

- Every EFT $(2,0)$ heterotic–string compactification has to be connected to a (2,2) heterotic–string compactification by an orbifold or by continuous interpolation. If not $\rightarrow$ it is in the swampland

- Completeness?!

Conclusions

- Mirror Symmetry $\longrightarrow$ pure mathematical interest
- Spinor–vector duality $\longrightarrow$ extension of mirror symmetry
- $(G, B, W) \quad \longrightarrow \tilde{G}, \tilde{B}, \tilde{W}$
- Spinor–vector duality $\longleftrightarrow$ pure mathematical interest???
- Physical application : String derived extra Z' model
  AEF, J Rizos, NPB 895 (2015) 233

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