Anders Basbøll (University of Sussex): SUSY flat directions: To get a Vacuum Expectation Value or not?

"If SOTV is so intriguing, it does not shine by a flat direction, taken over by the actors"

from review of movie "Shadow of the Vampire" on anticool.com

University of Liverpool 10.01.20

Outline

- MSSM and Flat Directions (FDs)
- Cosmological role of FDs
- Counting and categorising FDs
- Particle Production from FDs
- Problems
- Expanding the Superpotential
- Outlook

Supersymmetry (SUSY) in 1 slide

- Every elementary boson has a fermionic superpartner, vice versa. (sparticles, gauginos)
- Multiplets: equal number of bosons and fermions. (partner with ½ lower spin – if possible)
- Space-time+ 2 anti-commuting coordinates.
- If local, Super Gravity.
- SUSY: all 3 SM couplings equal at high energy – Grand Unified Theory (GUT) scale.
- High energy divergencies in Quantum Field Theory cancelled: fermionic, bosonic loops: equal contributions, opposite sign.

Minimal SUSY SM (MSSM)

Names		spin 0	spin 1/2	$SU(3)_c$, $SU(2)_L$, $U(1)_y$
squarks, quarks	Q	$(\tilde{u}_L, \tilde{d}_L)$	(u_L, d_L)	$\mathbf{3}$, $\mathbf{2}$, 1/3
($\times$ 3 families)	$\bar{u}$	$\tilde{\bar{u}}_L (\tilde{u}_R)$	$\bar{u}_L \sim (u_R)^c$	$\bar{\mathbf{3}}$, $\mathbf{1}$, -4/3
	$\bar{d}$	$\tilde{\bar{d}}_L (\tilde{d}_R)$	$\bar{d}_L \sim (d_R)^c$	$\bar{\mathbf{3}}$, $\mathbf{1}$, 2/3
sleptons, leptons	L	$(\tilde{\nu}_{eL}, \tilde{e}_L)$	(ν_{eL}, e_L)	$\mathbf{1}$, $\mathbf{2}$, -1
($\times$ 3 families)	$\bar{e}$	$\tilde{\bar{e}}_L (\tilde{e}_R)$	$\bar{e}_L \sim (e_R)^c$	$\mathbf{1}$, $\mathbf{1}$, 2
higgs, higgsinos	H_u	(H_u^+, H_u^0)	$(\tilde{H}_u^+, \tilde{H}_u^0)$	$\mathbf{1}$, $\mathbf{2}$, 1
	H_d	(H_d^0, H_d^-)	$(\tilde{H}_d^0, \tilde{H}_d^-)$	$\mathbf{1}$, $\mathbf{2}$, -1

Table 1: Chiral supermultiplet fields in the MSSM.

Names	spin 1/2	spin 1	$SU(3)_c$, $SU(2)_L$, $U(1)_y$
gluinos, gluons	$\tilde{g}$	g	$\mathbf{8}$, $\mathbf{1}$, 0
winos, W bosons	$\widetilde{W}^\pm, \widetilde{W}^0$	$W^\pm, W^0$	$\mathbf{1}$, $\mathbf{3}$, 0
bino, B boson	$\tilde{B}$	B	$\mathbf{1}$, $\mathbf{1}$, 0

Table 2: Gauge supermultiplet fields in the MSSM.

- table taken from Aitchison hep-ph/0505105
- Add Right Handed Neutrinos N: charges 1,1,0

(Super)potential and flatness MSSM

- Superpotential - renormalisable part

$$W = y_{\mathrm{u}}^{ij}\bar{u}_i Q_j \cdot H_{\mathrm{u}} - y_{\mathrm{d}}^{ij}\bar{d}_i Q_j \cdot H_{\mathrm{d}} - y_{\mathrm{e}}^{ij}\bar{e}_i L_j \cdot H_{\mathrm{d}} + \mu H_{\mathrm{u}} \cdot H_{\mathrm{d}}$$

- Yukawas as in SM. HERE: generation defined by diagonal SUSY breaking mass terms: 1/2 $m_i^* |\varphi_i|^2$

Scalar potential: F- and D-term
T: symmetry generators

$$V = \sum_i |F_i|^2 + \frac{1}{2}\sum_a g_a^2 D^a D^a$$

$$F_i \equiv \frac{\partial W_{\mathrm{MSSM}}}{\partial \chi_i}, \quad D^a = \chi_i^* T_{ij}^a \chi_j$$

Flatness: V=0 nonzero fields:
All F- and D-terms vanish

- V=0 (nonzero field values) only for exact SUSY (unbroken), no non-renormalisable terms

Superpartners not seen

- R-parity: All particles and gauge bosons: +1. Sparticles, gauginos: -1.

- Can be defined: (-1)^(2j+3B+L)

- Thus, superpartners can only be created or destructed in pairs.

- Thus: Lightest Super Partner (LSP) odds-on? favorite candidate as Dark Matter

 - as Weakly Interacting Massive Particle (WIMP)

Flat direction evolution – from Dine, Randall, Thomas

Nucl.Phys.B458:291-326,1996

- Flat direction (QLD, example) $Q_1^\alpha = \frac{1}{\sqrt{3}}\begin{pmatrix}\phi \\ 0\end{pmatrix} \quad L_1 = \frac{1}{\sqrt{3}}\begin{pmatrix}0 \\ \phi\end{pmatrix} \quad \bar{d}_2^\alpha = \frac{1}{\sqrt{3}}\phi$
- Flatness easy: Colour, weak charge, hyperc. balance.

$$X = Q_1 L_1 \bar{d}_2 \ (m = 3)$$

m: dimension

- with ϕ canonical field $X = c\phi^m$

- Equation of motion
- - with Hubble friction

$$\ddot{\phi} + 3H\dot{\phi} + V'(\phi) = 0$$

Non renormalisable terms

- Non-renormalisable term: M braeking scale (Planck/GUT/?)

$$W = W_{\mathrm{renorm}} + \sum_{n>3} \frac{\lambda}{M^{n-3}} \Phi^n$$

- Particle physics:
- not forbidden: allowed, coupling order one.
- ALL terms respecting gauge invariance and R-parity will be considered (L,B conservation not demanded - couplings are small)
- below: terms breaking symmetry (k=1,2 R-parity)

$$W = \frac{\lambda}{nM^{n-3}} X^k = \frac{\lambda}{nM^{n-3}} \phi^n$$

$$W = \frac{\lambda}{M^{n-3}} \psi \phi^{n-1}$$

Flatness and monomials: $L_1L_2E_3$

- $L_1L_2E_3$ monomial: $(\nu_e{}^*\mu - e^*\nu_\mu)\tau_c$ $SU_L(2) \times U_Y(1)$ invariant.
- Flatness: (ν_e, μ, τ_c) or (e, ν_μ, τ_c)
  $=\varphi\,(e^{i\theta_1}, e^{i\theta_2}, e^{i\theta_3})$: D=0 for all D-terms.
- Always: D-flatness independent of scale.

- Lifting flat direction with F-term:
- $W_4 \supset L_1L_2E_3N_1$: $F_{N_1} \propto L_1L_2E_3$
  $|F_{N1}|^2 \propto |L_1L_2E_3|^2$: positive potential.

- A-term (scalar only)
- $[A^* e^{i\theta_A} (\nu_e{}^*\mu - e^*\nu_\mu)\tau_c + \text{h.c}]$
- Negative contribution for some phases.

Evolution during inflation

- Potential (n=k*m – k integer in X^k)

$$V(\phi) = -cH_I^2|\phi|^2 + \left(\frac{a\lambda H_I \phi^n}{nM^{n-3}} + \ h.c.\right) + |\lambda|^2 \frac{|\phi|^{2n-2}}{M^{2n-6}}$$

- c: order unity. c>0 necessary (50%chance?) - does NOT work with Minimal Kähler Potential.

- With H dominating, SUSY masses negligible
- Last term: non-renorm. SUSY-breaking terms.

- Minimum:
- β order 1 constant

$$|\phi_0| = \left(\frac{\beta H_I M^{n-3}}{\lambda}\right)^{\frac{1}{n-2}}$$

Minimum could easily be of order 10^{16} GeV!

End of inflation

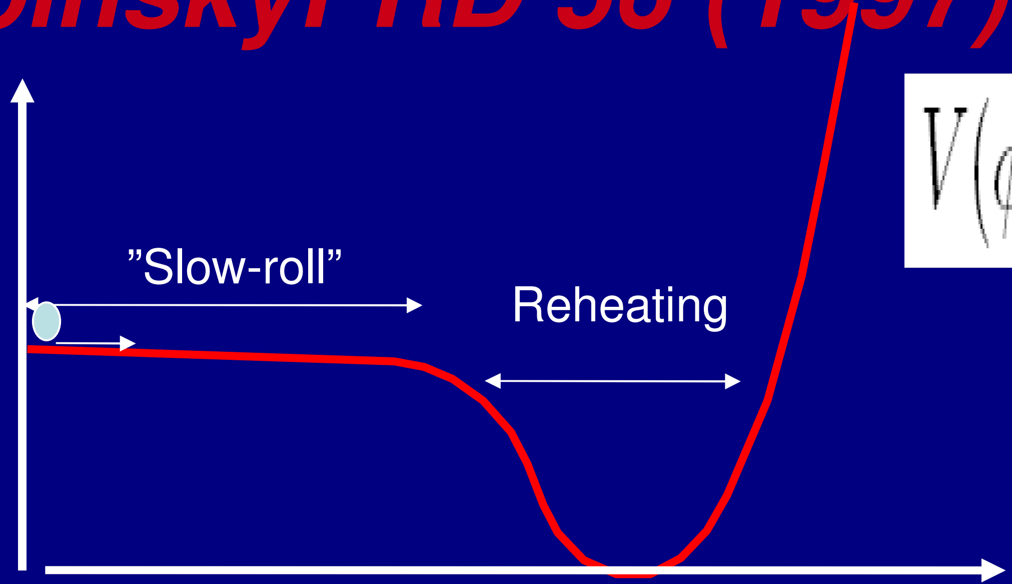
- When H is of SUSY mass order (matter dom):

$$V(\phi) = m_\phi^2|\phi|^2 - \frac{c'}{t^2}|\phi|^2 + \left(\frac{(Am_{3/2} + aH)\lambda\phi^n}{nM^{n-3}} + h.c.\right) + |\lambda|^2\frac{|\phi|^{2n-2}}{M^{2n-6}}$$

- Eventually mass term dominates, and the flat direction oscillates.

- **Potential has phase dependent part**:
- Baryon number if B-L not conserved by FD

Preheating – Kofman, Linde, StarobinskyPRD 56 (1997)



$$V(\phi) \sim \frac{1}{2}m^2(\phi - \sigma)^2$$

- Potential with minimum at ϕ=σ
- Interaction term with “massless” scalars $-\frac{1}{2}g^2\phi^2\chi^2$
- Make shift: $\phi - \sigma \rightarrow \phi$ $\quad -\frac{1}{2}g^2\phi^2\chi^2 - g^2\sigma\phi\chi^2 - \frac{1}{2}g^2\sigma^2\chi^2$
- Quantise scalars: ∃ frequencies: $n_k(t)$ α Exp(μ*t)
- Energy to scalars at once: Preheating.
- Overproduction of gravitinos.

Consequences of Flat Directions

- Allahverdi, Mazumdar(+others) hep-ph/0603244
- FD induce masses to inflaton decay products: $g\langle\phi\rangle > H_{\mathrm{I}}$
- **No preheating [no massless scalars]**
- Energy is stored in the flat direction.
- When FD decays – Reheating.
- Reheating temp. 10^3-10^7 GeV (not 10^9GeV)
- **Avoids the (lack of) gravitino problem**
- **FD itself a candidate for the Inflaton!(LLE,UDD)**

  Allahverdi, Garcia-Bellido, Enqvist, Mazumdar, PRL 97:191304(2006)

Olive, Peloso PRD 74:103514(2006):

- **Valid only if decay is nonperturbative SPOILED if FD decays rapidly - likely.**

Counting flat directions

- They can't be counted (misnomer, in my opinion)
- D-flat space: 37 complex d.o.f. (40 with N's)

(dimensionality of D-flat space)

- Gauge invariant products (made from 712(715) basis invariants=monomials - count those)

- Ex: Monomial $L_1L_2E_3$ breaks SU(2) X U(1) (4 generators). Space of L_1,L_2,E_3:
- dimensionality: 5cdof(2+2+1)-4cdof(non-flat +gauge choice for each generator)=1cdof
- i.e. with gauge choices $L_1L_2E_3$ can be described by $(\nu_{e,}\mu,\tau_c)=\varphi\ (e^{i\theta},e^{i\theta},e^{i\theta})$ - details later!
- here: 1 monomial ~ 1 flat direction

Counting of LLE

- FDs do not superimpose: (a=1,b=0 and a=0,b=1) below
- Flat subsystem $|\langle e^c\rangle| = a, |\langle \mu^c\rangle| = b$ $|\langle \nu_e\rangle| = |\langle \mu\rangle| = \sqrt{a^2+b^2}$

- Space of L_1,L_2,L_3,E_1,E_2
- Dimensionality: 8-4=4 c.d.o.f.
- Monomials: 6 (which L to omit, which E to accept)

$$\nu_e = \varphi_1 e^{i\sigma_1}, \nu_\mu = \varphi_2 e^{i\sigma_2}, e^c = \varphi_3 e^{i\sigma_3}, \mu^c = \varphi_4 e^{i\sigma_4}, \tau = 0,$$

$$e = \frac{\sqrt{\varphi_3^2+\varphi_4^2}\varphi_2}{\sqrt{\varphi_1^2+\varphi_2^2}} e^{i\sigma_1}, \mu = \frac{\sqrt{\varphi_3^2+\varphi_4^2}\varphi_1}{\sqrt{\varphi_1^2+\varphi_2^2}} e^{i(\sigma_2+\pi)}, \nu_\tau = \sqrt{\varphi_3^2+\varphi_4^2-\varphi_1^2-\varphi_2^2} e^{i\sigma_1}$$

- Monomials can be found $L_1L_2E_1$ corresponds to $\varphi_1 = \varphi_3 = 1$

$L_3L_1E_1$ corresponds to $\varphi_3 = 1, \varphi_1 = a^2, \varphi_2 = a$ for $a \to 0$

#cdof<#monomials BUT cdof harder to identify.

Categorising D-flat directions

- Gherghetta, Kolda, Martin, NPB 468, 37 (1996)

- First: Create SU(3) singlets:
- ex: $(QU)^{\alpha}=Q^{\alpha a}U^{b}\varepsilon_{ab}$, QQQ(see later), UDD, L, E..

- Second: Create SU(2) singlets of these:
- ex: $(QUL)=(QU)^{\alpha}L^{\beta}\varepsilon_{\alpha\beta}$, LL, UDD, E....

- Third: Create U(1) singlets of these:
- ex: QULE= QUL*E, LLE, UDD...

- 28 of these types necessary to form basis of everything gauge invariant. Others omitted. Ex:

QDHu(Y=1)*LHd(Y=-1) covered by QDL+HuHd

Generational structure

- Catalogue included mention of dimensionality of many of the 28 types of monomials.

- Complete Generational AB, arXiv:0910.0244

- νMSSM: include N: 3 generations.
- Most trivial: QLD: 3*3*3 dimensions.

- No selfcontraction: DDDLL 3 dim (what L not there)

- QUQUE: antisymmetric between (QU)s. 36*3 dim.
- UUUEE: symmetric between Es. 6 dimensions.

- (UUD)[(QD)(QD)]: by itself: 324 dimensions.

  \- but 90 in catalogue - rest made from UDD+QUQD.

QQQ - SU(2) doublet

- 3 Qs as doublet: 2 SU(2) indices contracted.
- QQQ_{ijk}=
- first look 27.

$$(Q_i Q_j Q_k)^\alpha \equiv Q_i^{\beta a} Q_j^{\gamma b} Q_k^{\alpha c} \epsilon_{abc} \epsilon_{\beta\gamma}$$

- correct counting: 8 combinations.

- 2+1 (2 of repeated gen.) only 1 comb. (6 in all)
- $(QQQ_{112}+QQQ_{121}+QQQ_{211})$/Sqrt[3]=0
- $(QQQ_{121}-QQQ_{211})$/Sqrt[2]=0
- $(QQQ_{211}+QQQ_{121}-2*QQQ_{112})$/Sqrt[6] free
- 1+1+1 (all different generations): 2 combinations.

- Thus: QQQL 24 dimensions.

QQQQU

$(QQQ)_2QU$:

Max 8(QQQ)*3(Q)*3(U) =24(Qs)*3(U)=72 dim.

Min 12*3=36 dim [12 ways of assigning 3 gen to 4 Q's - condition: not all same generation]

Correct 18*3=54 dim

No way, to my knowledge, to explain this - other than by investigating the basis vectors.

QQQ uncontracted SU(2)

- (QQQ)4= $(QQQ)_{\mathbf{4}}^{\alpha\beta\gamma} = Q_i^{\alpha a} Q_j^{\beta b} Q_k^{\gamma c} \epsilon_{abc} \epsilon^{ijk} / \sqrt{6}$
- $(QQQ)_4$LLLE is 30 dimensional
- Which Q contracts with which L doesn't matter
- Bottom (3) line(s): Anything gauge invariant can be written as a linear combination of the 715 catalogue elements to nonnegative powers.

Monomial catalogue

N	3	R_-
H_uH_d	1	R_+
LH_u	3	R_-
LH_dE	9	R_+
QH_dD	9	R_+
QH_uU	9	R_+
QLD	27	R_-
LLE	9	R_-
UDD	9	R_-
$UUDE$	27	R_+
$QULE$	81	R_+
$QUQD$	81	R_+
$QQQL$	24	R_+
QUH_dE	27	R_-

$QQQH_d$	8	R_-
$DDDLH_d$	3	R_+
$QQQQU$	54	R_-
$QUQUE$	108	R_-
$UUUEE$	6	R_-
$DDDLL$	3	R_-
$UUDQDH_u$	54	R_-
$(QQQ)_4LLH_u$	6	R_-
$(QQQ)_4LH_uH_d$	3	R_+
$(QQQ)_4H_uH_dH_d$	1	R_-
$(QQQ)_4LLLE$	30	R_-
$(QQQ)_4LLH_dE$	18	R_+
$(QQQ)_4LH_dH_dE$	9	R_-
$(QQQ)_4H_dH_dH_dE$	3	R_+
$UUDQDQD$	90	R_-

The framework

- AB, Maybury, Riva, West PRD 76, 065005 (2007)
- Excitations of fields $\Xi \equiv (\xi_1, \xi_2, \xi_3, \xi_4, \xi_5, \xi_6)^T$
- Lagrangian

$$\mathcal{L} \supset \frac{1}{2}|\partial_\mu \Xi|^2 - \frac{1}{2}\Xi^T \mathcal{M}^2 \Xi - \dot{\Xi}^T U \Xi + \ldots$$

- Excitations: mixed kinetic terms avoided, correctly normalised

- Orthogonal transformation $\Xi' = A\Xi$
- to make U-term disappear: $\dot{A}^T A = U$
- "New" Lagrangian

$$\mathcal{L} \supset \frac{1}{2}|\partial_\mu \Xi'|^2 - \frac{1}{2}\Xi'^T \mathcal{M}'^2 \Xi'$$

- Diagonalisation

$$\mathcal{M}'^2 = A\mathcal{M}^2 A^T = AB\mathcal{M}_d^2 B^T A^T = C\mathcal{M}_d^2 C^T \qquad C = AB$$

Non-perturbative particle production

- Conformal fields $$\chi_i = a\Xi'_i$$
- Equation of motion $$\ddot{\chi}_i + \Omega^2_{ij}(t)\chi_j = 0$$
- Mass matrix $$\Omega^2_{ij} = a^2 \mathcal{M}'^2_{ij} + k^2\delta_{ij}$$

Changing eigenvectors

$$C^T(t)\Omega^2(t)C(t) = \omega^2(t)$$

Particle production - changing vacua

- Quantise field (changing creation/annil.~ α,β)
- initial: $\alpha(\eta_0) = 1 \qquad \beta(\eta_0) = 0$
- differential equations:

$$\begin{aligned} \dot\alpha &= -i\omega\alpha + \frac{\dot\omega}{2\omega}\beta \\ \dot\beta &= \frac{\dot\omega}{2\omega}\alpha + i\omega\beta \end{aligned}$$

- Occ. number $N(\eta) =< a^\dagger a >= |\beta|^2$

Notice: adiabicity condition!

Multifield: α,β->matrices

$$\begin{aligned} \dot\alpha &= -i\omega\alpha + \frac{\dot\omega}{2\omega}\beta - I\alpha - J\beta \\ \dot\beta &= \frac{\dot\omega}{2\omega}\alpha + i\omega\beta - J\alpha - I\beta \end{aligned}$$

- Off-diagonal terms
- Rapidly changing

Eigenstates OR

Eigenvalues ->particles

$$I = \frac{1}{2}\left(\sqrt{\omega}\, C^T \dot C \frac{1}{\sqrt{\omega}} + \frac{1}{\sqrt{\omega}} C^T \dot C \sqrt{\omega}\right)$$

$$J = \frac{1}{2}\left(\sqrt{\omega}\, C^T \dot C \frac{1}{\sqrt{\omega}} - \frac{1}{\sqrt{\omega}} C^T \dot C \sqrt{\omega}\right)$$

$$n_i(t) = (\beta^* \beta^T)_{ii}$$

Nilles, Peloso, Sorbo: JHEP 0104 (2001) 004

Our framework: $C^T \dot C = B^T A^T \dot A B = -B^T U B$

LLE: flatness

- Vacuum expectation values (VEV's)

$$< \nu_e >= \varphi e^{i\sigma_1}$$
$$< \mu >= \varphi e^{i\sigma_2}$$
$$< \tau^c >= \varphi e^{i\sigma_3}$$

- (almost) generic feature: Flatness independent of phase
- Potential (no colour involved)
- Hypercharge: L: 2*(-1) E:2
  so D_H=0
- Weak charge:
- 1 up, 1 down: D_a=0

$$V = \frac{1}{2}\left(D_H^2 + \sum_a D_a^2\right)$$

$$D_H = \frac{g_1}{2}\sum_i q_i|\phi_i|^2$$
$$D_a = \frac{g_2}{2}\phi^\dagger P^a \phi$$

LLE – only VEV's

- Covariant derivative $D_i^\mu = \left[(\partial^\mu - iq_i A_0^\mu)\delta_{ij} - \sum_{k=1}^3 iP_{ij}^a A_a^\mu\right]\phi_j$

- Lagrangian

$$\mathcal{L} = \sum_{i=1}^{3} \frac{1}{2}|D_\mu \Phi_i|^2 - V - \frac{1}{4}F_{\mu\nu}^2 - \sum_i \frac{1}{4}W_{\mu\nu}^{i2}$$

- F, W: Hypercharge, Weak field strength tensors
- Mixed kinetic terms – Nambu-Goldstone Bosons (unphys.)

$$\mathcal{L} \supset -\varphi^2 A_0(\dot{\sigma}_1 + \dot{\sigma}_2 - 2\dot{\sigma}_3) - \varphi^2 A_3(\dot{\sigma}_1 - \dot{\sigma}_2)$$

- Only time derivative of phase: only 0'th Lorentz component of Gauge fields matter (index omitted)

Gauging Goldstones away - go to unitary gauge

- U(1) (hypercharge) transformation

$$\langle \Phi_i \rangle \to \langle \Phi_i' \rangle = e^{iq_i\lambda} \langle \Phi_i \rangle$$

- with

$$\lambda = \frac{2\sigma_3 - \sigma_1 - \sigma_2}{3}$$

- SU(2) (weak charge) transformation

$$\langle \Phi_i \rangle \to \langle \Phi_i' \rangle = e^{iP^3\gamma} \langle \Phi_i \rangle$$

- with

$$\gamma = \frac{\sigma_2 - \sigma_1}{2}$$

- P^3: 3rd Pauli matrix

- new VEV's (phase differences gone)

$$\begin{aligned} \langle \nu_e \rangle &= \varphi e^{i\sigma} \\ \langle \mu \rangle &= \varphi e^{i\sigma} \\ \langle \tau^c \rangle &= \varphi e^{i\sigma} \end{aligned}$$

- with

$$\sigma = (\sigma_1 + \sigma_2 + \sigma_3)/3$$

Diagonal Goldstones removed - define excitatations(incl. partners)

$$
\begin{aligned}
\nu_e &= (\varphi + \xi_2) e^{i(\sigma + \frac{\xi_1}{\sqrt{3}\varphi})} \\
e &= (\xi_5 + i\xi_6) e^{i\sigma} \\
\nu_\mu &= (\xi_7 + i\xi_8) e^{i\sigma} \\
\mu &= (\varphi + \xi_3) e^{i(\sigma + \frac{\xi_1}{\sqrt{3}\varphi})} \\
\tau^c &= (\varphi + \xi_4) e^{i(\sigma + \frac{\xi_1}{\sqrt{3}\varphi})}
\end{aligned}
$$

$$
\mathcal{L} \supset -\varphi \left(A_1(\dot{\xi}_6 + \dot{\xi}_8) + A_2(\dot{\xi}_7 - \dot{\xi}_5) \right)
$$

All Goldstones removed

$$\begin{aligned}
\nu_e &= (\varphi + \xi_2) e^{i(\sigma + \frac{\xi_1}{\sqrt{3}\varphi})} \\
e &= \frac{(\xi_5 + i\xi_6)}{\sqrt{2}} e^{i\sigma} \\
\nu_\mu &= \frac{(\xi_5 - i\xi_6)}{\sqrt{2}} e^{i\sigma} \\
\mu &= (\varphi + \xi_3) e^{i(\sigma + \frac{\xi_1}{\sqrt{3}\varphi})} \\
\tau^c &= (\varphi + \xi_4) e^{i(\sigma + \frac{\xi_1}{\sqrt{3}\varphi})}
\end{aligned}$$

results – LLE and UDD (seperately)

- Unitary gauge: phase differences removed
- Calculate M,U
- J-matrix=0, No preheating!

$$< u^{c\bar{1}} >= \varphi e^{i\sigma_1}$$
$$< s^{c\bar{2}} >= \varphi e^{i\sigma_2}$$
$$< b^{c\bar{3}} >= \varphi e^{i\sigma_3}.$$

UDD exactly the same.
3 phases, 2 gauged away.
J=0, no preheating

we found in BMRW: particle production prop. to derivative of VEV-phase differences.

$(QQQ)_4LLLE$ – VEV Fields

- "4": (QQQ) **4** under SU(2) [isospin 3/2]
- Squarks with identical SU2-charge chosen

$$
\begin{aligned}
u^{c1} &= (\varphi + \xi_4)e^{i(\sigma_1 + \frac{\xi_1}{\sqrt{7}\varphi})} \\
c^{c2} &= (\varphi + \xi_5)e^{i(\sigma_1 + \frac{\xi_1}{\sqrt{7}\varphi})} \\
t^{c3} &= (\varphi + \xi_6)e^{i(\sigma_1 + \frac{\xi_1}{\sqrt{7}\varphi})} \\
e &= (\varphi + \xi_7)e^{i(\sigma_1 + \frac{\xi_1}{\sqrt{7}\varphi} + \sigma_2 + \frac{\xi_2}{\sqrt{2}\varphi} + \sigma_3 + \frac{\xi_3}{\sqrt{6}\varphi})} \\
\mu &= (\varphi + \xi_8)e^{i(\sigma_1 + \frac{\xi_1}{\sqrt{7}\varphi} - \sigma_2 - \frac{\xi_2}{\sqrt{2}\varphi} + \sigma_3 + \frac{\xi_3}{\sqrt{6}\varphi})} \\
\tau &= (\varphi + \xi_9)e^{i(\sigma_1 + \frac{\xi_1}{\sqrt{7}\varphi} - 2\sigma_3 - \frac{2\xi_3}{\sqrt{6}\varphi})} \\
e^c &= (\varphi + \xi_{10})e^{i(\sigma_1 + \frac{\xi_1}{\sqrt{7}\varphi})}
\end{aligned}
$$

(QQQ)$_4$LLLE – other fields

$$
\begin{aligned}
u^{c2} &= \frac{\xi_{11}+i\xi_{12}}{\sqrt{2}}e^{i\sigma_1} \\
u^{c3} &= \frac{\xi_{13}+i\xi_{14}}{\sqrt{2}}e^{i\sigma_1} \\
d^{c1} &= \left(\frac{\xi_{15}+i\xi_{16}}{\sqrt{2}} - \frac{\xi_{21}+i\xi_{22}}{\sqrt{6}} - \frac{\xi_{27}+i\xi_{28}}{2\sqrt{3}} + \frac{\xi_{29}-i\xi_{30}}{2\sqrt{5}} + \frac{\xi_{31}-i\xi_{32}}{\sqrt{30}}\right) e^{i\sigma_1} \\
c^{c1} &= \frac{\xi_{11}-i\xi_{12}}{\sqrt{2}}e^{i\sigma_1} \\
c^{c3} &= \frac{\xi_{19}+i\xi_{20}}{\sqrt{2}}e^{i\sigma_1} \\
s^{c2} &= \left(\frac{\xi_{21}+i\xi_{22}}{\sqrt{\frac{3}{2}}} - \frac{\xi_{27}+i\xi_{28}}{2\sqrt{3}} + \frac{\xi_{29}-i\xi_{30}}{2\sqrt{5}} + \frac{\xi_{31}-i\xi_{32}}{\sqrt{30}}\right) e^{i\sigma_1} \\
t^{c1} &= \frac{\xi_{13}-i\xi_{14}}{\sqrt{2}}e^{i\sigma_1} \\
t^{c2} &= \frac{\xi_{19}-i\xi_{20}}{\sqrt{2}}e^{i\sigma_1} \\
b^{c3} &= \left(\frac{\xi_{27}+i\xi_{28}}{2\sqrt{\frac{1}{3}}} + \frac{\xi_{29}-i\xi_{30}}{2\sqrt{5}} + \frac{\xi_{31}-i\xi_{32}}{\sqrt{30}}\right) e^{i\sigma_1} \\
\nu_e &= \left(\frac{\xi_{29}+i\xi_{30}}{2\sqrt{5}} - \frac{\xi_{31}+i\xi_{32}}{\sqrt{30}}\right) e^{i(\sigma_1+\sigma_2+\sigma_3)} \\
\nu_\mu &= \left(\frac{\xi_{31}+i\xi_{32}}{\sqrt{\frac{6}{5}}}\right) e^{i(\sigma_1-\sigma_2+\sigma_3)} \\
\nu_\tau &= \left(\frac{\xi_{15}-i\xi_{16}}{\sqrt{2}} + \frac{\xi_{21}-i\xi_{22}}{\sqrt{6}} + \frac{\xi_{27}-i\xi_{28}}{2\sqrt{3}} - \frac{\xi_{29}+i\xi_{30}}{2\sqrt{5}} - \frac{\xi_{31}+i\xi_{32}}{\sqrt{30}}\right) e^{i(\sigma_1-2\sigma_3)}
\end{aligned}
$$

$(QQQ)_4LLLE$

- Preheating in both sectors!

- but depend on phase derivatives.

J goes as Sqrt(g_i*φ/k)*σ_i'

for φ>>k

(where σ_i is a phase difference)

$$J_{4,17} \;=\; J_{3,18} = -\frac{\sqrt{3}(-\sqrt{k}+\sqrt{k+\frac{6g_2^2\varphi^2}{k}})}{4\sqrt{10}(k^2+6g_2^2\varphi^2)^{\frac{1}{4}}}\sigma_3'$$

2 Flat directions: UDD+LLE

- Give VEV's to same fields as before!
- Now 6 phases – only 4 diagonal generators.
- However, it is 1 phase for LLE and 1 for UDD that survives.
- U-matrix block diagonal: Fields and phase of LLE in one block. Fields and phase of UDD in another block.
- J=0 – no preheating.
- Very encouraging for the cosmological role of SUSY flat directions!

QLD+LLE – overlapping directions

- QLD and LLE can co-exist. They can have VEV in the same field. A: relation between VEV's.
- ”Overlapping field” size: Root of squares .
- Preheating!

$$< d^1 >= \varphi e^{i\sigma_1}$$

$$< s^{c\bar{1}} >= \varphi e^{i\sigma_2}$$

$$< \nu_e >= \varphi\sqrt{1 + A^2}e^{i\sigma_3}$$

$$< \mu >= \varphi A e^{i\sigma_4}$$

$$< \tau^c >= \varphi A e^{i\sigma_5}$$

Problems with this picture

- Directions not independent.
- 17 (20) mass terms (Q,U,D,L,E,(N)) times 3 + Higgses - but 712(715) independent monomials

- Example: $(QQQ)_4L_1L_2L_3E$ v $(QQQ)_4L_1L_2L_2E$

$$m_\varphi^2 = 1/7*(m_{Q_1}^2+m_{Q_2}^2+m_{Q_3}^2+m_{L_1}^2+m_{L_2}^2+m_{L_3}^2+m_{E_1}^2)$$

$$m_\varphi'^2 = 1/7*(m_{Q_1}^2+m_{Q_2}^2+m_{Q_3}^2+m_{L_1}^2+2*m_{L_2}^2+m_{E_1}^2)$$

- Notice: VEV to first dir. => VEV to second dir.
- And similar equal A-terms
- QQQ_4LLLE: m=7. Broken at n>=7? - next slide.

$(QQQ)_4LLLE$ - broken when?

- Without Ns:
- Picture: R_, broken by $(QQQLLLE)^2$

- Correct: Q,L,E 18+6+3=27 complex d.o.f
- Breaks SM completely. 12 c.d.o.f removed -15 left
- W_4 includes QQQL, QULE:
- F_Q, F_L, F_U, F_E non trivial. 36 complex constrains (GKM)
- Lifted by V_6 - but A-term NOT of order 4.

- Include Ns: A-term: QQQLLLEN not n=4

- still broken by W_4 (including LLEN)

- Point: DRT formula not valid.

Investigation of potential

- Phase differences must have dynamical equations of motion to create preheating (only A-terms).

- Effective mass terms must be negative for some directions. (or A-term large compared to mass)

Kasuya, Kawasaki PRD 74 063507 (2006)

- 715 monomials, but also: LLE*UDD etc.

- Flattest: Q,U,E directions broken at W_9 (V_{16})

-- but, including N, at W_6 (V_{10}).

- Allahverdi+Mazumdar: General hierarchical VEVs
- Olive+Peloso: several large VEVs
- Goal: Write down potential to order V_{10}
- Estimate: (overcounting) 2,3 million couplings?

Statistical and numerical approach

- Statistical:
- Choose random couplings.
- Find minimum of potential. Monte Carlo.
- Try enough combinations to get a feeling of how many superfields (and which) get large VEV's.

- Analytical:
- Impose symmetries.
- Common couplings: $m_{1/2}$, m_0, A
- Will the flattest direction win?
  - n=9 is formally flattest.

Choosing Normalisation

- All couplings order 1. But what is one coupling?
- $(QQQ)_4$= $(QQQ)_{\mathbf{4}}^{\alpha\beta\gamma} = Q_i^{\alpha a} Q_j^{\beta b} Q_k^{\gamma c} \epsilon_{abc} \epsilon^{ijk} / \sqrt{6}$
- 1 dim. 36 terms.
- Choice: gauge ε tensors - not normalised
- Generation epsilon: as any linear combination:
- comb. of 6 basis vectors: 1/sqrt[6].

- 1/n! (not n) for φ^n, φ SUPERfield (not field):
- $(H_uH_d)^2$: $(H^+H^-)^2-2H^+H^-H_u^0H_d^0+(H_u^0H_d^0)^2$

Removing superfluous couplings

- Superfield products (incl. gauge ε tensors): a,b,c.

If a+b+c=0: include (a-b)/sqrt[2], (a+b-2c)/sqrt[6]

- Does this makes sense?
- Think of complex plane (or R^2):

a:(-1/2,-sqrt[3]/2) b:(-1/2,sqrt[3]/2) c:(1,0)

new basis vectors: A=(0,sqrt[3/2]), B=(-sqrt[3/2],0)

Length and orthogonality regardless of choice of c.

Had one chosen any 2 of a,b,c - orthogonality lost.

- Is boost okay? Desirable or not?

If normal distribution A,B: N(0,1) then in R^2

Average (0,0) Variance(3/2,3/2)

EXACTLY as if a,b,c: N(0,1).

BUT clear there are 2 couplings (not 3).

An example: SU(2) contractions

General formula for SU(2) contractions ():
(AB)(DC)+(AC)(BD)+(AD)(CB)=0 so define
$SU(2)_4$[A,B,C,D]={[(AB)(DC)-(AC)(BD)]/sqrt(2),
(AB)(DC)+(AC)(BD)-2(AD)(CB)/sqrt(6)}

$QL_iD+L_jH_u$'='$QL_jD+L_iH_u$'='$QDH_u+L_iL_j$ (last not hypercharge-invariant - but as good).

$(QDLLH_u)_{i,j,k,\mathbf{e}}$	$SU(2)_4[Q_i^a D_j^{\overline{a}} \delta_{a\overline{a}}, L_{k+1}, L_{k+2}, H_u]_{\mathbf{e}}$, $[1 \leq i, j, k \leq 3, 1 \leq \mathbf{e} \leq 2]$
$(QDL2H_u)_{i,j,k}$	$(QLD)_{i,k,j} * (H_u L)_k/2!$ $[1 \leq i, j, k \leq 3]$

Monomial catalogue: Efficient notation also desired.
(L2: repeated generation of L,
LL different generations. index: absent L)

W to 6th order - examples

- AB arXiv:0911.5340

- Neutrinos: Products trivial:

ex: dim(LLEN2N)=dim(LLE)*dim(N2N)=54

- Monomials: covered

- Trivial products:

ex: $(H_uH_d2QU)_{i,j}=(H_uH_d)*(Q_iH_uU_j)/2!$

- Symmetry only between monomials:
- ex: $(Hu2LL)_i=(HuL)_{i+1}*(HuL)_{i+2}/2!$

Last types of products

- More E's: as monomial UUUEE earlier.
as neutrinos: no contractions.

- 4 SU(2) fields: covered

- 4+ SU(3) superfields.
ex: $(QLUDDD)_{i,j,k,e}$ ($e \in \{7,8,9\}$) from
$(QL)_{i,j}$ contracts with one of $\{U_k, D_1, D_2, D_3\}$
1 linear combination is zero.
3 linear combinations: e parameter.

W expansion

- W: 5179 couplings order <=6 (R-parity, gauge inv)
- order 2: 7 $\lambda_{H_u H_d}(H_u H_d) * M$
- order 3: 36 $\lambda^{i,j}_{Q H_d D}(Q H_d D)_{i,j}$
- order 4: 376 $\lambda^{i,j,k}_{LLEN}(LLE)_{i,j} * N_k / M$
- order 5: 468 $\lambda^{i}_{DDDLH_d}(DDDLH_d)_i / M^2$
- order 6: 4293 $\lambda^{i,j,k,l,\mathbf{m}}_{QLDN2N}(QLD)_{i,j,k} * (N2N)_{l,\mathbf{m}} / M^3$
- A-terms: same structure, different power of M

Perspectives

- Immense degeneracy. 52-12=40 c.d.o.f.

- but lifted by higher order terms.

- Today vacuum is: zero for all fields.

- Earlier 2 ways for new physics:
- Universe in false minimum -> Tunneling OR
- True minimum were different (induced parameters dominated - can have different minimum)

- One can add something to (ν)MSSM to lift degeneracy.

Anomaly mediated SUSY-breaking - current work with Jones, Hindmarsh

- Hodgson, Jack, Jones, Ross: NPB 728 (2005)
- $W = W_{\nu MSSM}(\text{excl. } H_u H_d) + H_u H_d S + M^2 S + \bar{\Phi}\Phi S + \Phi N N$

- S inflaton, <S> given.
- New U´(1): Anomaly free, S uncharged. Fayet-Iliopoulos ξ
- Minimum (from F-term of S): M^4.
- F-terms restricts only H_u,H_d,$\bar{\Phi}$,Φ,N

- Flat space: 45-13=32 c.d.o.f.

-- (All fields Vanishing NOT in it)
- Loop corrections $\Sigma m_i^4 \log(m_i^2/\mu^2)$ breaks degeneracy (under investigation)

Conclusions

- SUSY Flat directions can have crucial influence on (p)reheating, the gravitino problem etc.

- Preheating serious threat. Must look at specific cases - no general rules to predict production.
- The potential must be investigated thoroughly!

- Any gauge invariant term can be expressed by product of 712(715) mon's to nonnegative powers.
- There are 5179 couplings of order <=6 in W.

- Massive degeneracy - lifted by higher order terms -- or by new physics.

Extra: Terms from Potential

- generalised A-terms:
- cross in Kähler deriv (Superpots)

$$W_\phi K^{\phi\bar{\phi}} K_{\bar{\phi}} \frac{W^*(I)}{M_p^2} + h.c.$$

- cross between Superpots

$$\left(\frac{1}{M_p^2} K_I K^{I\bar{I}} K_I - 3\right)\left(\frac{W(\phi)^* W(I)}{M_p^2} + h.c.\right)$$

- Kähler pot couplings

$$W_\phi K^{\phi\bar{I}} D_{\bar{I}} W^*(I) + h.c.$$

- generalised mass terms:
- Exponential prefactor $e^{K(\phi^\dagger,\phi)/M_p^2} V(I)$
- cross in Kähler deriv (FD K, Infl W)

$$K_\phi K^{\phi\bar{\phi}} K_{\bar{\phi}} \frac{|W(I)|^2}{M_p^4}$$

- Kähler pot couplings

$$K_\phi K^{\phi\bar{I}} D_{\bar{I}} W^*(I) \frac{W(I)}{M_p^2} + h.c.$$

Dine, Randall, Thomas

Extra: Preheating details

- Quantum fluctuations of “massless” scalars

$$\hat{\chi}(t,\vec{x}) = \frac{1}{(2\pi)^{3/2}} \int d^3k \left(\hat{a}_{\vec{k}} \chi_{\vec{k}}(t) \exp\left(-i\vec{k}\cdot\vec{x}\right) + \hat{a}^{\dagger}_{\vec{k}} \chi^{*}_{\vec{k}}(t) \exp\left(i\vec{k}\cdot\vec{x}\right) \right)$$

- Oscillator with varying energy $\ddot{\chi}_k + 3H\dot{\chi}_k + \left(\frac{k^2}{a(t)^2} + g^2\phi\right)\chi_k = 0$

def: amplitude Φ, $z = m_\phi t/2 + \pi/4$, $q = \frac{4g^2\sigma\Phi}{m_\phi^2}$, $A(k) = 4\frac{k^2+g^2\sigma^2}{m_\phi^2}$

- Mathieu equation: $\chi_k'' + \left(A(k) - 2q\cos(2z)\right)\chi_k = 0$

- Instabilities: existence of band of frequncies: $\Delta k^{(n)}$
- Bose enhancement: $n_k(t) \propto \exp(2\mu_k^{(n)} z)$

Exponential particle production

- This is a parametric resonance – all energy tranferred to scalars: **Preheating**
-

Extra: $(QQQ)_2QU$ counting

3-1 Min 6, correct 6, max 6 (2-1-0, add 0-0-1)
2-2 Min 3, correct 3, max 6=6*1 [2-1 add 0-1])
2-1-1 Min 3, correct 9, max 12=6*1[2-1-0 add 0-0-1]+2*3[1-1-1 add 1-0-0])

Extra: Adding normal functions

- Notice: All 3 couplings Distribution: $(\Theta,\sigma^2)=(0,1)$ gives 3 times (0,1/Sqrt[3]) in relevant direction.
- If GAUSSIAN: 3*N(0,1/Sqrt[3])=N(0,1) so okay!
- But would like to know there is 1 not 3 couplings.

Extra: W: neutrinos

TABLE I: Neutrino products.

Name	Expression	Dimension	c.d.o.f.	R-parity
$(N)_i$	$N_i, [1 \leq i \leq 3]$	1	3	$-$
$(N2)_i$	$N_i^2/2!, [1 \leq i \leq 3]$	2	3	$+$
$(NN)_i$	$N_{i+1} * N_{i+2}, [1 \leq i \leq 3]$	2	3	$+$
$(N3)_i$	$N_i^3/3!, [1 \leq i \leq 3]$	3	3	$-$
$(N2N)_{i,\mathbf{j}}$	$N_i^2 * N_{i+\mathbf{j}}/2!, [1 \leq i \leq 3, 1 \leq \mathbf{j} \leq 2]$	3	6	$-$
(NNN)	$N_1 * N_2 * N_3$	3	1	$-$
$(N4)_i$	$N_i^4/4!, [1 \leq i \leq 3]$	4	3	$+$
$(N3N)_{i,\mathbf{j}}$	$N_i^3 * N_{i+\mathbf{j}}/3!, [1 \leq i \leq 3, 1 \leq \mathbf{j} \leq 2]$	4	6	$+$
$(N2N2)_i$	$N_{i+1}^2 * N_{i+2}^2/(2!2!), [1 \leq i \leq 3]$	4	3	$+$
$(N2NN)_i$	$N_i^2 * N_{i+1} * N_{i+2}/2!, [1 \leq i \leq 3]$	4	3	$+$
$(N5)_i$	$N_i^5/5!, [1 \leq i \leq 3]$	5	3	$-$
$(N4N)_{i,\mathbf{j}}$	$N_i^4 * N_{i+\mathbf{j}}/4!, [1 \leq i \leq 3, 1 \leq \mathbf{j} \leq 2]$	5	6	$-$
$(N3N2)_{i,\mathbf{j}}$	$N_i^3 * N_{i+\mathbf{j}}^2/(3!2!), [1 \leq i \leq 3, 1 \leq \mathbf{j} \leq 2]$	5	6	$-$
$(N3NN)_i$	$N_i^3 * N_{i+1} * N_{i+2}/3!, [1 \leq i \leq 3]$	5	3	$-$
$(N2N2N)_i$	$N_i * N_{i+1}^2 * N_{i+2}^2/(2!2!), [1 \leq i \leq 3]$	5	3	$-$
$(N6)_i$	$N_i^6/6!, [1 \leq i \leq 3]$	6	3	$+$
$(N5N1)_{i,\mathbf{j}}$	$N_i^5 * N_{i+\mathbf{j}}/(5!), [1 \leq i \leq 3, 1 \leq \mathbf{j} \leq 2]$	6	6	$+$
$(N4N2)_{i,\mathbf{j}}$	$N_i^4 * N_{i+\mathbf{j}}^2/(4!2!), [1 \leq i \leq 3, 1 \leq \mathbf{j} \leq 2]$	6	6	$+$
$(N4NN)_i$	$N_i^4 * N_{i+1} * N_{i+2}/4!, [1 \leq i \leq 3]$	6	3	$-$
$(N3N3)_i$	$N_{i+1}^3 * N_{i+2}^3/(3!3!), [1 \leq i \leq 3]$	6	3	$+$
$(N3N2N)_{i,\mathbf{j}}$	$N_i^3 * N_{i+\mathbf{j}}^2 * N_{i-\mathbf{j}}/(3!2!), [1 \leq i \leq 3, 1 \leq \mathbf{j} \leq 2]$	6	6	$+$
$(N2N2N2)$	$N_1^2 * N_2^2 * N_3^2/(2!2!2!)$	6	1	$+$

Extra: W: Monomials

TABLE II: Monomials. – means wrong R-parity or lower dimension than the entry, () means c.d.o.f. of entry, but wrong R-parity. For each dimension, the c.d.o.f. is listed. In the first line, the indices must sum to the number of the dimension.

Name	Expression	Dim 2	Dim 3	Dim 4	Dim 5	Dim 6
$(N)_i$ summary of table I	$N_1^{n1} N_2^{n2} N_3^{n3}/(n1!n2!n3!)$	$\mathbf{6}$	(10)	$\mathbf{15}$	(21)	$\mathbf{28}$
$(H_u H_d)$	$H_u^\alpha H_d^\beta \epsilon_{\alpha\beta}$	$\mathbf{1}$	–	$\mathbf{6}(2N)$	–	$\mathbf{15}(4N)$
$(H_u L)_i$	$H_u^\alpha L_i^\beta \epsilon_{\alpha\beta}, [1 \leq i \leq 3]$	(3)	$\mathbf{9}(N)$	–	$\mathbf{30}(3N)$	–
$(Q H_u U)_{i,j}$	$Q_i^{\alpha,a} H_u^\beta U_j^{\overline{a}} \epsilon_{\alpha\beta} \delta_{a\overline{a}}, [1 \leq i,j \leq 3]$	–	$\mathbf{9}$	–	$\mathbf{54}(2N)$	–
$(Q H_d D)_{i,j}$	$Q_i^{\alpha,a} H_d^\beta D_j^{\overline{a}} \epsilon_{\alpha\beta} \delta_{a\overline{a}}, [1 \leq i,j \leq 3]$	–	$\mathbf{9}$	–	$\mathbf{54}(2N)$	–
$(L H_d E)_{i,j}$	$L_i^\alpha H_d^\beta E_j \epsilon_{\alpha\beta}, [1 \leq i,j \leq 3]$	–	$\mathbf{9}$	–	$\mathbf{54}(2N)$	–
$(QLD)_{i,j,k}$	$Q_i^{\alpha,a} L_j^\beta D_k^{\overline{a}} \epsilon_{\alpha\beta} \delta_{a\overline{a}}, [1 \leq i,j,k \leq 3]$	–	(27)	$\mathbf{81}(N)$	–	$\mathbf{270}(3N)$
$(LLE)_{i,j}$	$L_{i+1}^\alpha L_{i+2}^\beta E_j \epsilon_{\alpha\beta}, [1 \leq i,j \leq 3]$	–	(9)	$\mathbf{27}(N)$	–	$\mathbf{90}(3N)$
$(UDD)_{i,j}$	$U_i^{\overline{a}} D_{j+1}^{\overline{b}} D_{j+2}^{\overline{c}} \epsilon_{\overline{a}\overline{b}\overline{c}}, [1 \leq i,j \leq 3]$	–	(9)	$\mathbf{27}(N)$	–	$\mathbf{90}(3N)$
$(UUDE)_{i,j,k}$	$U_{i+1}^{\overline{a}} U_{i+2}^{\overline{b}} D_j^{\overline{c}} E_k \epsilon_{\overline{a}\overline{b}\overline{c}}, [1 \leq i,j,k \leq 3]$	–	–	$\mathbf{27}$	–	$\mathbf{162}(2N)$
$(QLUE)_{i,j,k,l}$	$Q_i^{\alpha,a} L_j^\beta U_k^{\overline{a}} E_l \epsilon_{\alpha\beta} \delta_{a\overline{a}}, [1 \leq i,j,k,l \leq 3]$	–	–	$\mathbf{81}$	–	$\mathbf{486}(2N)$
$(Q2UD)_{i,j,k}$	$Q_i^{\alpha a} U_j^{\overline{a}} Q_i^{\beta b} D_k^{\overline{b}} \delta_{a\overline{a}} \delta_{b\overline{b}} \epsilon_{\alpha\beta}/2!, [1 \leq i,j,k \leq 3]$	–	–	$\mathbf{27}$	–	$\mathbf{162}(2N)$
$(QQUD)_{i,j,k,\mathbf{e}}$	$Q_{i+\mathbf{e}}^{\alpha a} U_k^{\overline{a}} Q_{i-\mathbf{e}}^{\beta b} D_l^{\overline{b}} \delta_{a\overline{a}} \delta_{b\overline{b}} \epsilon_{\alpha\beta}, [1 \leq i,j,k \leq 3, 1 \leq \mathbf{e} \leq 2]$	–	–	$\mathbf{54}$	–	$\mathbf{324}(2N)$
$(Q H_d U E)_{i,j,k}$	$Q_i^{\alpha,a} H_d^\beta U_j^{\overline{a}} E_k \epsilon_{\alpha\beta} \delta_{a\overline{a}}, [1 \leq i,j,k \leq 3]$	–	–	(27)	$\mathbf{81}(N)$	–
$(Q2QL)_{i,\mathbf{j},k}$	$QQQ_{i\mathbf{jh}}^\alpha L_k^\beta \epsilon_{\alpha\beta}/2!, [1 \leq i,k \leq 3, 1 \leq \mathbf{j} \leq 2]$	–	–	$\mathbf{18}$	–	$\mathbf{108}(2N)$
$(QQQL)_{\mathbf{e},k}$	$QQQ_{\mathbf{e}}^\alpha L_k^\beta \epsilon_{\alpha\beta}, [1 \leq k \leq 3, 7 \leq e \leq 8]$	–	–	$\mathbf{6}$	–	$\mathbf{36}(2N)$
$(Q2QH_d)_{i,\mathbf{j}}$	$QQQ_{i\mathbf{jh}}^\alpha H_d^\beta \epsilon_{\alpha\beta}/2!, [1 \leq i \leq 3, 1 \leq \mathbf{j} \leq 2]$	–	–	(6)	$\mathbf{18}(N)$	–
$(QQQH_d)_{\mathbf{e}}$	$QQQ_{\mathbf{e}}^\alpha H_d^\beta \epsilon_{\alpha\beta}, [7 \leq e \leq 8]$	–	–	(2)	$\mathbf{6}(N)$	–
$(DDDLH_d)_i$	$D_1^{\overline{a}} D_2^{\overline{b}} D_3^{\overline{c}} L_i^\alpha H_d^\beta \epsilon_{\overline{a}\overline{b}\overline{c}} \epsilon_{\alpha\beta}, [1 \leq i \leq 3]$	–	–	–	$\mathbf{3}$	–
$(DDDLL)_i$	$D_1^{\overline{a}} D_2^{\overline{b}} D_3^{\overline{c}} L_{i+1}^\alpha L_{i+2}^\beta \epsilon_{\overline{a}\overline{b}\overline{c}} \epsilon_{\alpha\beta}, [1 \leq i \leq 3]$	–	–	–	(3)	$\mathbf{9}(N)$
$(QQUUE)_{i,j\mathbf{e},k}$	$Q_{i+1}^{\alpha a} U_{j+\mathbf{e}}^{\overline{a}} Q_{i+2}^{\beta b} U_{j-\mathbf{e}}^{\overline{b}} E_k \delta_{a\overline{a}} \delta_{b\overline{b}} \epsilon_{\alpha\beta},$ $[1 \leq i,j,k \leq 3, 1 \leq \mathbf{e} \leq 2]$	–	–	–	(54)	$\mathbf{162}(N)$
$(Q2UUE)_{i,j,k}$	$Q_i^{\alpha a} U_{j+1}^{\overline{a}} Q_i^{\beta b} U_{j+2}^{\overline{b}} E_k \delta_{a\overline{a}} \delta_{b\overline{b}} \epsilon_{\alpha\beta}/2!,$ $[1 \leq i,j,k \leq 3]$	–	–	–	(27)	$\mathbf{81}(N)$
$(QQU2E)_{i,j,k}$	$Q_{i+1}^{\alpha a} U_j^{\overline{a}} Q_{i+2}^{\beta b} U_j^{\overline{b}} E_m \delta_{a\overline{a}} \delta_{b\overline{b}} \epsilon_{\alpha\beta}/2!,$ $[1 \leq i,j,k \leq 3]$	–	–	–	(27)	$\mathbf{81}(N)$
$(UUUE2)_i$	$U_1^{\overline{a}} U_2^{\overline{b}} U_3^{\overline{c}} E_i E_j \epsilon_{\overline{a}\overline{b}\overline{c}}/2!, [1 \leq i \leq 3]$	–	–	–	(3)	$\mathbf{9}(N)$
$(UUUEE)_i$	$U_1^{\overline{a}} U_2^{\overline{b}} U_3^{\overline{c}} E_{i+1} E_{i+2} \epsilon_{\overline{a}\overline{b}\overline{c}}, [1 \leq i \leq 3]$	–	–	–	(3)	$\mathbf{9}(N)$
$(QQQ_{\mathbf{4}} L H_u H_d)_i$	$(QQQ)_{\mathbf{4}}^{\alpha\beta\gamma} L_i^{\alpha'} H_u^{\beta'} H_d^{\gamma'} \epsilon_{\alpha\alpha'} \epsilon_{\beta\beta'} \epsilon_{\gamma\gamma'}, [1 \leq i \leq 3]$	–	–	–	–	$\mathbf{3}$
$Q2Q2U_{i,j}$	$(Q2Q2)_{i\mathbf{d}}^a U_j^{\overline{a}} \delta_{a\overline{a}}/(2!2!)[1 \leq i,j \leq 3]$	–	–	–	(9)	$\mathbf{27}(N)$
$Q3QU_{i,\mathbf{j},k}$	$(Q2Q_{i\mathbf{jh}}^\alpha Q_i^{\beta a} U_k^{\overline{a}} \epsilon_{\alpha\beta} \delta_{a\overline{a}}/3!,$ $[1 \leq i,k \leq 3, 1 \leq \mathbf{j} \leq 2]$	–	–	–	(18)	$\mathbf{54}(N)$
$Q2QQU_{i,\mathbf{e},j}$	$QQQQ_{i\mathbf{e}}^a U_j^{\overline{a}} \delta_{a\overline{a}}/2!,$ $[1 \leq i,j \leq 3, 7 \leq e \leq 9]$	–	–	–	(27)	$\mathbf{81}(N)$

Extra: W: Trivial products

TABLE III: Trivial products. – means wrong R-parity or lower dimension than the entry, () means c.d.o.f. of entry, but wrong R-parity. For each dimension, the c.d.o.f. is listed.

Name	Expression	Dim 2	Dim 3	Dim 4	Dim 5	Dim 6
$H_u 2 H_d 2$	$(H_u H_d) * (H_u H_d)/2!$	–	–	**1**	–	**6**$(2N)$
$(H_u 2 H_d L)_i$	$(H_u H_d) * (H_u L)_i/2!$, $[1 \leq i \leq 3]$	–	–	(3)	**9**(N)	–
$(H_u H_d 2 LE)_{i,j}$	$(H_u H_d) * (L H_d E)_{i,j}/2!$, $[1 \leq i, j \leq 3]$	–	–	–	**9**	–
$(H_u H_d 2 QD)_{i,j}$	$(H_u H_d) * (Q H_d D)_{i,j}/2!$, $[1 \leq i, j \leq 3]$	–	–	–	**9**	–
$(H_u H_d 2 QU)_{i,j}$	$(H_u H_d) * (Q H_u U)_{i,j}/2!$, $[1 \leq i, j \leq 3]$	–	–	–	**9**	–
$(UDDH_u L)_{i,j,k}$	$(UDD)_{i,j} * (H_u L)_k$, $[1 \leq i, j, k \leq 3]$	–	–	–	**27**	–
$(UDDH_u H_d)_{i,j}$	$(UDD)_{i,j} * (H_u H_d)$, $[1 \leq i, j \leq 3]$	–	–	–	(9)	**27**(N)
$(QUH_u 2L)_{i,j,k}$	$(Q H_u U)_{i,j} * (H_u L)_k/2!$, $[1 \leq i, j, k \leq 3]$	–	–	–	(27)	**81**(N)
$(QDH_d 2LE)_{i,j,k,l}$	$(Q H_d D)_{i,j} * (L H_d E)_{k,l}/2!$, $[1 \leq i, j, k, l \leq 3]$	–	–	–	–	**81**
$(UDDLLE)_{i,j,k,l}$	$(UDD)_{i,j} * (LLE)_{k,l}$, $[1 \leq i, j, k, l \leq 3]$	–	–	–	–	**81**
$(UUDEH_u H_d)_{i,j,k}$	$(UUDE)_{i,j,k} * (H_u H_d)$, $[1 \leq i, j, k \leq 3]$	–	–	–	–	**27**
$H_u 3 H_d 3$	$(H_u H_d)^3/6!$	–	–	–	–	**1**

Extra: W: Symmetry only between mon's

TABLE IV: Symmetry only between monomials. – means wrong R-parity or lower dimension than the entry, () means c.d.o.f. of entry, but wrong R-parity. For each dimension, the c.d.o.f. is listed.

Name	Expression	Dim 2	Dim 3	Dim 4	Dim 5	Dim 6
$(H_u2L2)_i$	$(H_uL)_i^2/(2!2!), [1 \leq i \leq 3]$	–	–	**3**	–	**18**$(2N)$
$(H_u2LL)_i$	$(H_uL)_{i+1} * (H_uL)_{i+2}/(2!), [1 \leq i \leq 3]$	–	–	**3**	–	**18**$(2N)$
$(H_u3L2H_d)_i$	$(H_uL)_i^2(H_uH_d)/(3!2!), [1 \leq i \leq 3]$	–	–	–	–	**3**
$(H_u3LLH_d)_i$	$(H_uL)_{i+1} * (H_uL)_{i+2} * (H_uH_d)/(3!), [1 \leq i \leq 3]$	–	–	–	–	**3**
$(Q2H_d2D2)_{i,j}$	$(QH_dD)_{i,j}^2 * /(2!2!2!), [1 \leq i,j \leq 3]$	–	–	–	–	**9**
$(Q2H_d2DD)_{i,j}$	$(QH_dD)_{i,j+1} * (QH_dD)_{i,j+2}/(2!2!), [1 \leq i,j \leq 3]$	–	–	–	–	**9**
$(QQH_d2D2)_{i,j}$	$(QH_dD)_{i+1,j} * (QH_dD)_{i+2,j}/(2!2!), [1 \leq i,j \leq 3]$	–	–	–	–	**9**
$(QQH_d2DD)_{i,j,\mathbf{e}}$	$(QH_dD)_{i+1,j+\mathbf{e}} * (QH_dD)_{i+2,j-\mathbf{e}}/(2!), [1 \leq i,j \leq 3, 1 \leq \mathbf{e} \leq 2]$	–	–	–	–	**18**
$(Q2H_u2U2)_{i,j}$	$(QH_uU)_{i,j}^2 * /(2!2!2!), [1 \leq i,j \leq 3]$	–	–	–	–	**9**
$(Q2H_u2UU)_{i,j}$	$(QH_uU)_{i,j+1} * (QH_dD)_{i,j+2}/(2!2!), [1 \leq i,j \leq 3]$	–	–	–	–	**9**
$(QQH_u2U2)_{i,j}$	$(QH_uU)_{i+1,j} * (QH_uU)_{i+2,j}/(2!2!), [1 \leq i,j \leq 3]$	–	–	–	–	**9**
$(QQH_u2UU)_{i,j,\mathbf{e}}$	$(QH_uU)_{i+1,j+\mathbf{e}} * (QH_uU)_{i+2,j-\mathbf{e}}/(2!), [1 \leq i,j \leq 3, 1 \leq \mathbf{e} \leq 2]$	–	–	–	–	**18**
$(Q2L2D2)_{i,j,k}$	$(QLD)_{i,j,k}^2/(2!2!2!),$ $[1 \leq i,j,k \leq 3]$	–	–	–	–	**27**
$(Q2L2DD)_{i,j,k}$	$(QLD)_{i,j,k+1} * (QLD)_{i,j,k+2}/(2!2!),$ $[1 \leq i,j,k \leq 3]$	–	–	–	–	**27**
$(Q2LLD2)_{i,j,k}$	$(QLD)_{i,j+1,k} * (QLD)_{i,j+2,k}/(2!2!),$ $[1 \leq i,j,k \leq 3]$	–	–	–	–	**27**
$(QQL2D2)_{i,j,k}$	$(QLD)_{i+1,j,k} * (QLD)_{i+2,j,k}/(2!2!),$ $[1 \leq i,j,k \leq 3]$	–	–	–	–	**27**
$(Q2LLDD)_{i,j,k,\mathbf{e}}$	$(QLD)_{i,j+1,k+\mathbf{e}} * (QLD)_{i,j+2,k-\mathbf{e}}/(2!),$ $[1 \leq i,j,k \leq 3, 1 \leq \mathbf{e} \leq 2]$	–	–	–	–	**54**
$(QQL2DD)_{i,j,k,\mathbf{e}}$	$(QLD)_{i+1,j,k+\mathbf{e}} * (QLD)_{i+2,j,k-\mathbf{e}}/(2!),$ $[1 \leq i,j,k \leq 3, 1 \leq \mathbf{e} \leq 2]$	–	–	–	–	**54**
$(QQLLD2)_{i,j,k,\mathbf{e}}$	$(QLD)_{i+1,j+\mathbf{e},k} * (QLD)_{i+2,j-\mathbf{e},k}/(2!),$ $[1 \leq i,j,k \leq 3, 1 \leq \mathbf{e} \leq 2]$	–	–	–	–	**54**
$(QQLLDD)_{i,j,k,\mathbf{e},\mathbf{f}}$	$(QLD)_{i+1,j+\mathbf{e},k+\mathbf{f}} * (QLD)_{i+2,j-\mathbf{e},k-\mathbf{f}},$	–	–	–	–	**108**

Extra: W: SU(2) Products

TABLE VI: 4 $SU(2)$ Superfields . – means wrong R-parity or lower dimension than the entry, () means c.d.o.f. of entry, but wrong R-parity. For each dimension, the c.d.o.f. is listed.

Name	Expression	Dim 2	Dim 3	Dim 4	Dim 5	Dim 6
$(Q2UDH_uH_d)_{i,j,k,\mathbf{e}}$	$SU(2)_{\mathbf{4}}[Q_i^aU_j^{\overline{a}}\delta_{a\overline{a}}, Q_i^bD_k^{\overline{b}}\delta_{b\overline{b}}H_uH_d]_{\mathbf{e}}/2!$, $[1 \leq i,j,k \leq 3, 1 \leq \mathbf{e} \leq 2]$	–	–	–	–	**54**
$(QQUDH_uH_d)_{i,j,k,\mathbf{e},\mathbf{f}}$	$SU(2)_{\mathbf{4}}[Q_{i+\mathbf{e}}^aU_j^{\overline{a}}\delta_{a\overline{a}}, Q_{i-\mathbf{e}}^bD_k^{\overline{b}}\delta_{b\overline{b}}H_uH_d]_{\mathbf{f}}$, $[1 \leq i,j,k \leq 3, 1 \leq \mathbf{e},\mathbf{f} \leq 2]$	–	–	–	–	**108**
$(QULEH_uH_d)_{i,j,k,\mathbf{e},l}$	$SU(2)_{\mathbf{4}}[Q_i^aU_j^{\overline{a}}\delta_{a\overline{a}}, L_k, H_u, H_d]_{\mathbf{e}}E_l$, $[1 \leq i,j,k,l \leq 3, 1 \leq \mathbf{e} \leq 2]$	–	–	–	–	**162**
$(QDLH_uH_d)_{i,j,k,\mathbf{e}}$	$SU(2)_{\mathbf{4}}[Q_i^aD_j^{\overline{a}}\delta_{a\overline{a}}, L_k, H_u, H_d]_{\mathbf{e}}$, $[1 \leq i,j,k \leq 3, 1 \leq \mathbf{e} \leq 2]$	–	–	–	(54)	**162**(N)
$(LLLH_uE)_{\mathbf{e},i}$	$SU(2)_{\mathbf{4}}[L_1, L_2, L_3, H_u]_{\mathbf{e}}E_i$, $[1 \leq i \leq 3, 1 \leq \mathbf{e} \leq 2]$	–	–	–	**6**	–
$(L2LH_uE)_{i,\mathbf{j},k}$	$(LLE)_{i+j,k} * (H_uL)_i/2!$ $[1 \leq i,k \leq 3, 1 \leq \mathbf{j} \leq 2]$	–	–	–	**18**	–
$(QDLLH_u)_{i,j,k,\mathbf{e}}$	$SU(2)_{\mathbf{4}}[Q_i^aD_j^{\overline{a}}\delta_{a\overline{a}}, L_{k+1}, L_{k+2}, H_u]_{\mathbf{e}}$, $[1 \leq i,j,k \leq 3, 1 \leq \mathbf{e} \leq 2]$	–	–	–	**54**	–
$(QDL2H_u)_{i,j,k}$	$(QLD)_{i,k,j} * (H_uL)_k/2!$ $[1 \leq i,j,k \leq 3]$	–	–	–	**27**	–
$(QDLLLE)_{i,j,k,\mathbf{e}}$	$SU(2)_{\mathbf{4}}[Q_i^aD_j^{\overline{a}}\delta_{a\overline{a}}, L_1, L_2, L_3]_{\mathbf{e}}E_k$, $[1 \leq i,j,k \leq 3, 1 \leq \mathbf{e} \leq 2]$	–	–	–	–	**54**
$(QDL2LE)_{i,j,k,\mathbf{l},m}$	$(QLD)_{i,k,j} * (LLE)_{k+1} * E_m/2!$ $[1 \leq i,j,k,m \leq 3, 1 \leq \mathbf{l} \leq 2]$	–	–	–	–	**162**
$(H_uLLH_dE)_{i,\mathbf{e},j}$	$SU(2)_{\mathbf{4}}[H_u, L_{i+1}, L_{i+2}, H_d]_{\mathbf{e}}E_j$, $[1 \leq i,j \leq 3, 1 \leq \mathbf{e} \leq 2]$	–	–	–	(18)	**54**(N)
$(H_uL2H_dE)_{i,j}$	$(H_uL)_i * (LH_dE)_{i,j}/2!$ $[1 \leq i,j \leq 3]$	–	–	–	(9)	**27**(N)
$((Q2Q)_2LH_uH_d)_{i,\mathbf{j},k,\mathbf{e}}$	$SU(2)_{\mathbf{4}}[Q2Q_{i\mathbf{jh}}, L_k, H_u, H_d]_{\mathbf{e}}/2!$, $[1 \leq i,k \leq 3, 1 \leq \mathbf{j},\mathbf{e} \leq 2]$	–	–	–	–	**36**
$((QQQ)_2LH_uH_d)_{\mathbf{e},i,\mathbf{f}}$	$SU(2)_{\mathbf{4}}[QQQ_{\mathbf{e}}, L_i, H_u, H_d]_{\mathbf{f}}$, $[1 \leq i \leq 3, 1 \leq \mathbf{e},\mathbf{f} \leq 2]$	–	–	–	–	**12**

Extra: W: 2 Es

TABLE V: 2 Es – means wrong R-parity or lower dimension than the entry, () means c.d.o.f. of entry, but wrong R-parity. For each dimension, the c.d.o.f. is listed.

Name	Expression	Dim 2	Dim 3	Dim 4	Dim 5	Dim 6
$(L2H_d2E2)_{i,j}$	$(LH_dE)^2_{i,j}/(2!2!2!)$, $[1 \le i \le j, \le 3]$	–	–	–	–	**9**
$(L2H_d2EE)_{i,j}$	$(LH_dE)_{i,j+1} * (LH_dE)_{i,j+2}/(2!2!)$, $[1 \le i \le j, \le 3]$	–	–	–	–	**9**
$(LLH_d2E2)_{i,j}$	$(LH_dE)_{i+1,j} * (LH_dE)_{i+2,j}/(2!2!)$, $[1 \le i \le j, \le 3]$	–	–	–	–	**9**
$(LLH_d2EE)_{i,j}$	$(LH_dE)_{i+1,j+1} * (LH_dE)_{i+2,j+2}/2!$, $[1 \le i \le j, \le 3]$	–	–	–	–	**9**
$(L2L2E2)_{i,j}$	$(LLE)^2_{i,j}/(2!2!2!)$, $[1 \le i\ j \le 3]$	–	–	–	–	**9**
$(L2L2EE)_{i,j}$	$(LLE)_{i,j+1} * (LLE)_{i,j+2}/(2!2!)$, $[1 \le i\ j \le 3]$	–	–	–	–	**9**
$(L2LLE2)_{i,j}$	$(LLE)_{i+1,j} * (LLE)_{i+2,j}/(2!2!)$, $[1 \le i\ j \le 3]$	–	–	–	–	**9**
$(L2LLEE)_{i,j}$	$(LLE)_{i+1,j+1} * (LLE)_{i+2,j+2}/(2!)$, $[1 \le i, j \le 3]$	–	–	–	–	**9**

Extra: W: 4+ SU(3) Fields

TABLE VII: More than 3 $SU(3)$ Superfields . – means wrong R-parity or lower dimension than the entry, () means c.d.o.f. of entry, but wrong R-parity. For each dimension, the c.d.o.f. is listed.

Name	Expression	Dim 2	Dim 3	Dim 4	Dim 5	Dim 6
$(U2D2D2)_{i,j}$	$(UDD)^2_{i,j}/(2!2!2!)$, $[1 \le i,j \le 3]$	–	–	–	–	9
$(U2D2DD)_{i,j}$	$(UDD)_{i,j+1}(UDD)_{i,j+2}/(2!2!)$, $[1 \le i,j \le 3]$	–	–	–	–	9
$(UUD2D2)_{i,j}$	$(UDD)_{i+1,j}(UDD)_{i+2,j}/(2!2!)$, $[1 \le i,j \le 3]$	–	–	–	–	9
$(UUD2DD)_{i,j,\mathbf{e}}$,	$[1 \le i,j,k \le 3, 7 \le \mathbf{e} \le 8]$ with	–	–	–	–	18
$(UUD2DD)_{i,j,\mathbf{e}=7}$	$((UDD)_{i+1,j+1}(UDD)_{i+2,j+2}) - (UDD)_{i+1,j+2}(UDD)_{i+2,j+1})/(2!\sqrt{2})$,					
$(UUD2DD)_{i,j,\mathbf{e}=8}$	$((UDD)_{i+1,j+1} * (UDD)_{i+2,j+2} + (UDD)_{i+1,j+2} * (UDD)_{i+2,j+1}$ $-2(UUD)_{i,j} * (DDD))/(2!\sqrt{6})$,					
$(QLUD2D)_{i,j,k,l,\mathbf{m}}$	$(UDD)_{k,l+\mathbf{m}} * (QLD)_{i,j,l}/2!$, $[1 \le i,j,k,l \le 3, 1 \le \mathbf{m} \le 2]$	–	–	–	–	162
$(QLUDDD)_{i,j,k,\mathbf{e}}$	$with[1 \le i,j,k \le 3, 7 \le \mathbf{e} \le 9]$	–	–	–	–	81
$(QLUDDD)_{i,j,k,\mathbf{e}=7}$	$((UDD)_{k,1} * (QLD)_{i,j,1} - (UDD)_{k,2} * (QLD)_{i,j,2})/\sqrt{2}$,					
$(QLUDDD)_{i,j,k,\mathbf{e}=8}$	$((UDD)_{k,1} * (QLD)_{i,j,1} + (UDD)_{k,2} * (QLD)_{i,j,2}$ $-2(UDD)_{k,3} * (QLD)_{i,j,3})/\sqrt{6}$,					
$(QLUDDD)_{i,j,k,\mathbf{e}=9}$	$((UDD)_{k,1} * (QLD)_{i,j,1} + (UDD)_{k,2} * (QLD)_{i,j,2}+$, $(UDD)_{k,3} * (QLD)_{i,j,3} + 3 * (QLU)_{i,j,k} * (DDD)/\sqrt{12}$					