

# A New Perspective on Quantum Field Theory

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Sussex U.

October 2010

# Outline of this Lecture

- 1 Qualitative Aspects of the ERG
- 2 Renormalizability
- 3 Correlation Functions in the ERG

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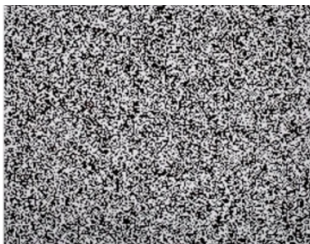
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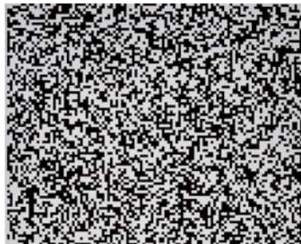
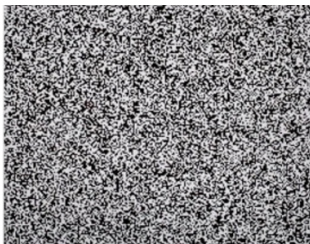
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# When should we use the Exact Renormalization Group?

The ERG is of use for systems with

- A large number of degrees of freedom per correlation length
- Local interactions

Easy Problems (no need for ERG)

- A small number of degrees of freedom per correlation length
- The subsystem is not well understood
- The subsystem captures the behaviour of the whole system
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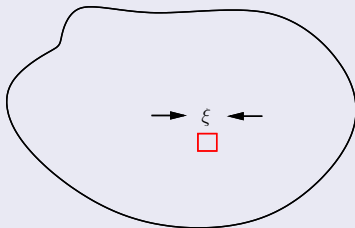
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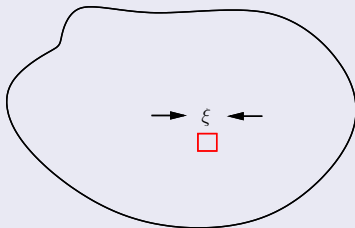
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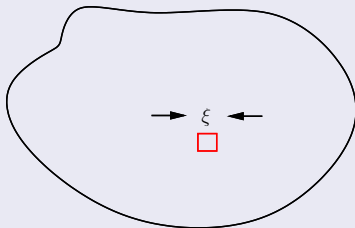
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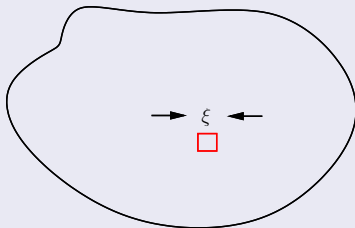
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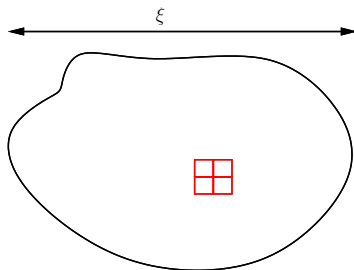
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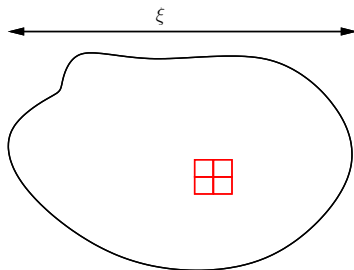
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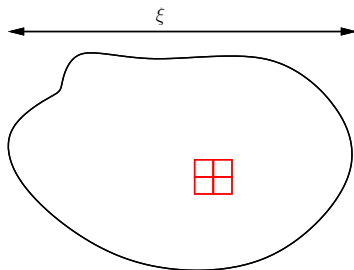


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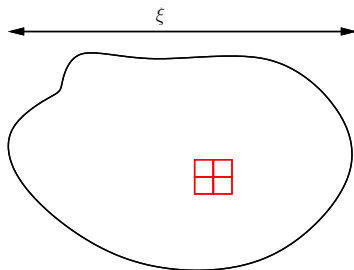
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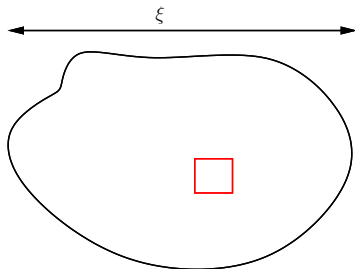
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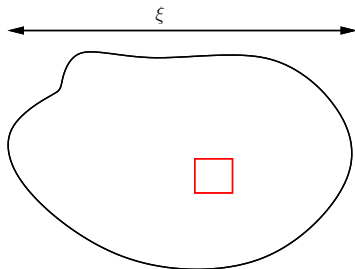
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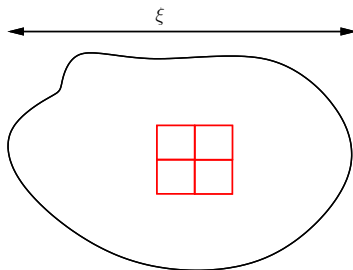
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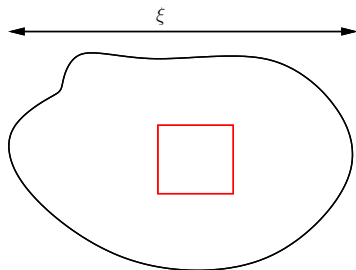
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# Practicalities

## Applications

• Statistical field theory

• Critical phenomena

• Quantum field theory of gauge fields, Higgs physics

## What has the ERG given us?

• A deep understanding of renormalization and universality

• A systematic way to compute observables

## What's the catch?

• The renormalization procedure cannot be done exactly

• The ERG is a powerful, nonperturbative approximation scheme

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- Kondo effect, ultra-cold gases, nuclear physics,...

## What has the ERG given us?

- A deep understanding of renormalization and universality
- A tool for performing real calculations

## What's the catch?

- The coarse-graining procedure cannot be done exactly
  - The renormalization group is only an approximation
  - For many purposes of interest, this is still a good approximation
- The ERG supports nonperturbative approximation schemes



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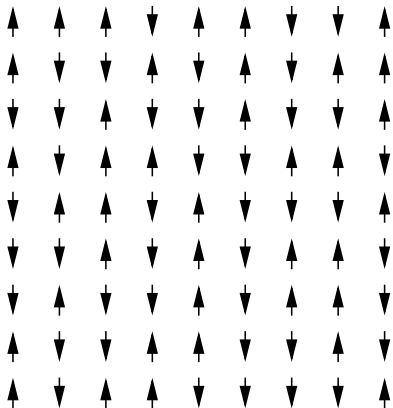
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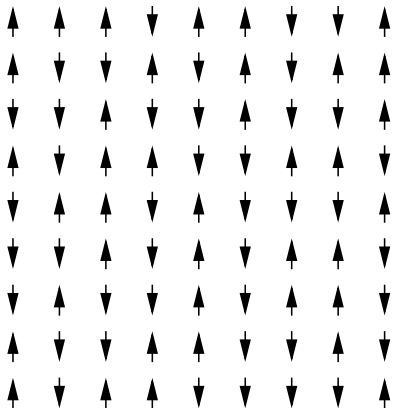
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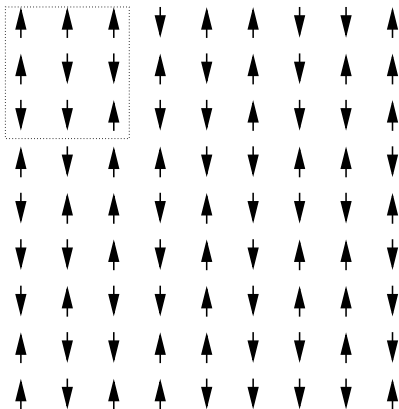
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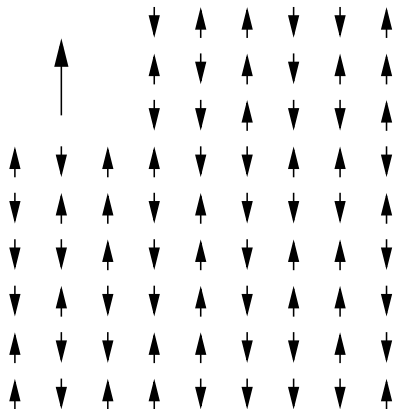
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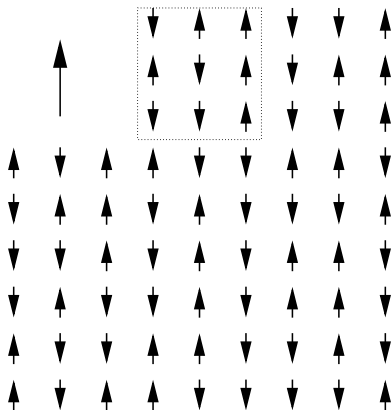
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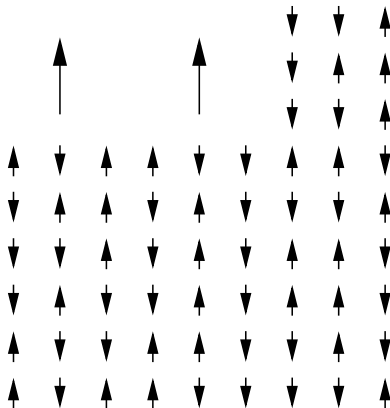
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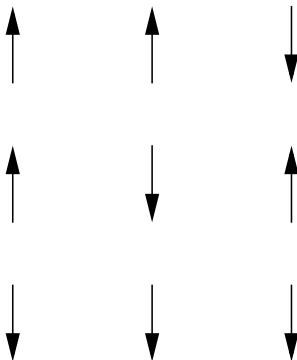
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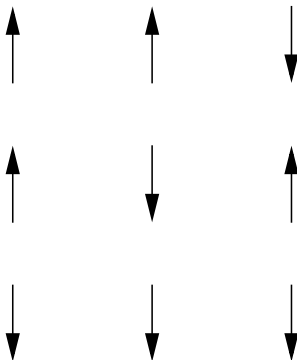
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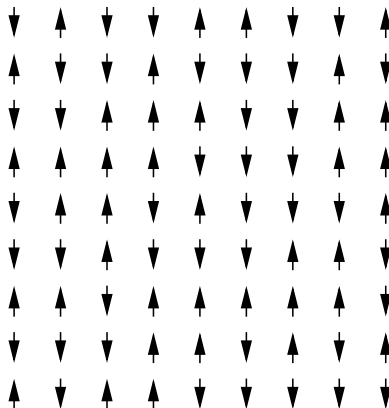
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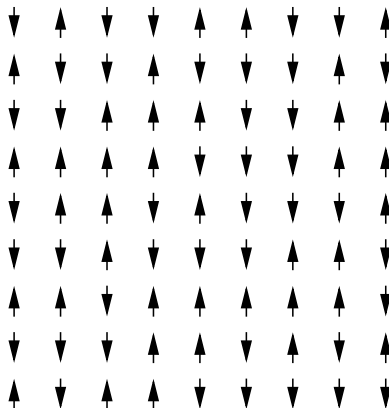
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The ERG implements the continuous version of blocking

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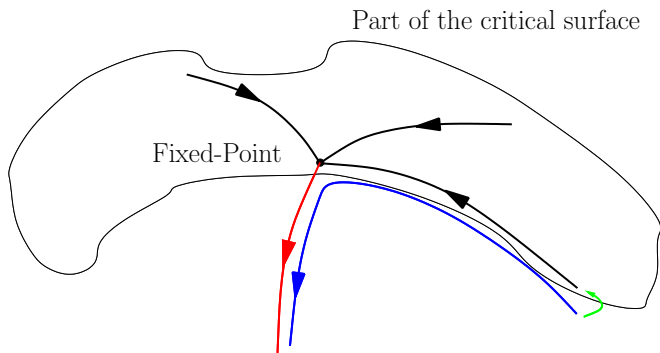
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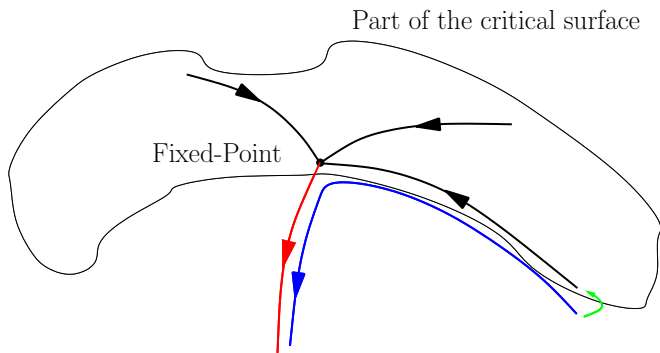
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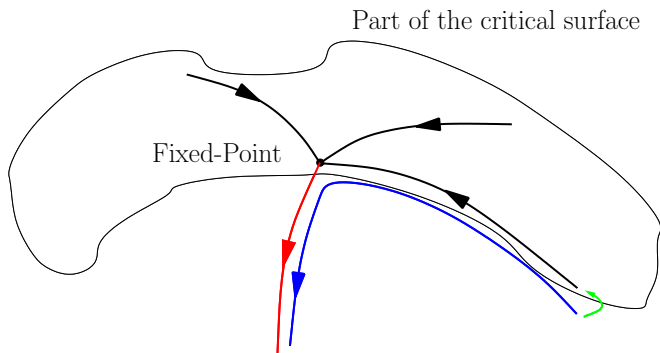


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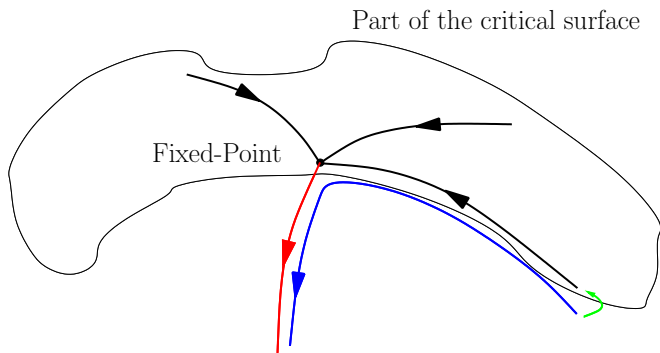
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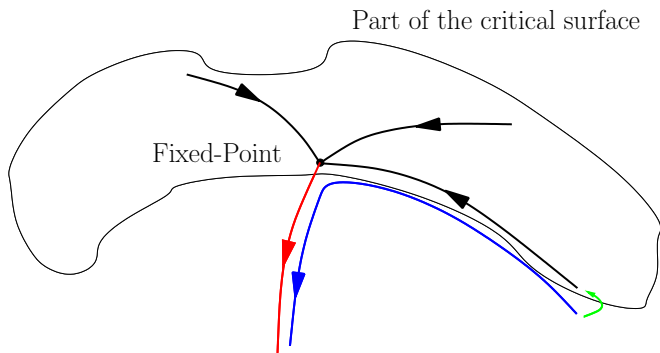
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- Trajectories in the critical surface flow into the fixed-point
- The critical manifold is spanned by the irrelevant operators
- Flows along the relevant directions leave the critical surface
- If there are  $n$  relevant directions, then we must tune  $n$  quantities to get on to the critical surface

# The Wilsonian Effective Action

Start with the partition function

$$Z = \int_{\Lambda_0} \mathcal{D}\phi e^{-S_{\Lambda_0}[\phi]} = \int_{\Lambda} \mathcal{D}\phi e^{-S_{\Lambda}[\phi]}$$

- The bare scale
  - High energy (short distance) scale
  - Modes above this scale are cut off (regularized)
- The bare (classical) action
- Integrate out modes between the bare scale and an intermediate scale,  $\Lambda$ 
  - The partition function stays the same
  - The effects of the high energy modes must be taken into account
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$$Z = \int_{\Lambda_0} \mathcal{D}\phi e^{-S_{\Lambda_0}[\phi]} = \int_{\Lambda} \mathcal{D}\phi e^{-S_{\Lambda}[\phi]}$$

- The bare scale
  - High energy (short distance) scale
  - Modes above this scale are cut off (regularized)
- The bare (classical) action
- Integrate out modes between the bare scale and an intermediate scale,  $\Lambda$ 
  - The partition function stays the same
  - The effects of the high energy modes must be taken into account
  - The action evolves  $\Rightarrow$  Wilsonian effective action

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# Very General ERGs

## Formulation

$$-\Lambda \partial_\Lambda e^{-S[\phi]} = \int_x \frac{\delta}{\delta\phi(x)} \left( \Psi_x[\phi] e^{-S[\phi]} \right)$$

• effective action

• set of fields

• Wilsonian effective action

• partition function,  $\int \mathcal{D}\phi e^{-S[\phi]}$ , invariant under the flow

• Parametrisation blocking procedure

## Flow Equation

$$-\Lambda \partial_\Lambda S = \int_x \frac{\delta S}{\delta\phi(x)} \Psi_x - \int_x \frac{\delta \Psi_x}{\delta\phi(x)}$$

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# Rescaling

## Ingredients of ERG Transformation

- Sliding (or rescaling)

- Rescaling

## Implementing Rescaling

- Rescaling of dimensional quantities in units of  $\Lambda$

- Rescaling to take account of anomalous dimensions

- Rescaling of dimensionless quantities

- Rescaling of dimensional,  $\omega$  dimensionless

- Rescaling of  $\omega$  with  $\omega = \ln \mu/\Lambda$

## What we need for this talk

- Dimensional analysis of  $S_{\text{eff}} = \dots$

- Dimensional analysis of  $S_{\text{eff}} = 0$

# Rescaling

## Ingredients of ERG Transformation

- Blocking (coarse-graining)
- Rescaling

## Implementing Rescaling

- Measure all dimensionful quantities in units of  $\Lambda$
- Remember to take account of anomalous dimensions!

$$X \rightarrow X \Lambda^{\text{full scaling dimension}}$$

- Notation:  $\phi$  dimensionful,  $\varphi$  dimensionless
- $-\Lambda \partial_\Lambda \rightarrow \partial_t$ , with  $t = \ln \mu/\Lambda$

## What we need for this talk

- ERG Equation:  $\partial_t S[\varphi] = \dots$
- Fixed-points:  $\partial_t S_*[\varphi] = 0$

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# Relevance/Irrelevance

- At a fixed-point we have  $\partial_t S_x = 0$
- Consider an infinitesimal perturbation

## First order classification

- Operators that grow with  $t$  are relevant
- Operators that shrink with  $t$  are irrelevant
- Operators that stay the same are marginal

## Marginal Operators

- An  $\mathcal{O}(g_{\text{marginal}})$  is a fixed-point up to  $\mathcal{O}(g^2)$
- It might not be true beyond leading order
- In the two-point coupling in  $D = 4$  scalar field theory is marginally irrelevant
- An exactly marginal operator generates a line of fixed points

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- $\mathcal{S}_* + a \mathcal{O}_{\text{marginal}}$  is a fixed-point up to  $O(a^2)$
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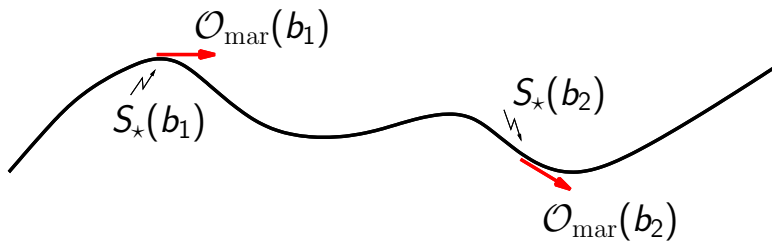
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# Relevance/Irrelevance



- 1 Qualitative Aspects of the ERG
- 2 Renormalizability**
- 3 Correlation Functions in the ERG

# Textbook renormalization

- Choose an action *e.g.*

$$S[\phi] = \int d^D x \left[ \frac{1}{2} \partial_\mu \phi \partial_\mu \phi + \frac{1}{2} m^2 \phi^2 + \frac{\lambda}{4!} \phi^4 \right]$$

- Choose a UV regulator
- Start computing the correlation functions

$$\langle \phi(x_1) \cdots \phi(x_n) \rangle = \frac{1}{Z} \int \mathcal{D}\phi \phi(x_1) \cdots \phi(x_n) e^{-S[\phi]}$$

- Adjust the action to absorb UV divergences:

$$S[\phi] \rightarrow S[\phi] + \delta S[\phi]$$

- If  $\delta S$  has the same form as  $S$ , the theory is renormalizable

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$$S[\phi] \rightarrow S[\phi] + \delta S[\phi]$$

- If  $\delta S$  has the same form as  $S$ , the theory is renormalizable

# Textbook renormalization

- Choose an action e.g.

$$S[\phi] = \int d^Dx \left[ \frac{1}{2} \partial_\mu \phi \partial_\mu \phi + \frac{1}{2} m^2 \phi^2 + \frac{\lambda}{4!} \phi^4 \right]$$

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# Wilsonian I

## The Question

What are the degrees of freedom  $\Phi$  of a Wilsonian effective action  $S_{\Lambda}$ ?

## The Simplest Answer

- Fields of all mass dimension  $\leq d$
- Only those that are renormalizable
- Fields that are in the ERG-conformal fixed-point manifold

and is independent of all scales, including  $\Lambda$

• Simplest answer:  $\Phi = \Phi_{\text{fixed}}$

# Wilsonian I

## The Question

Are there effective actions  $S_{\Lambda, \Lambda_0}[\phi]$  for which we can safely send  $\Lambda_0 \rightarrow \infty$ ?

## The Simplest Answer

- Rescale all quantities, using  $\Lambda$
- Only dimensionless variables appear
- Fixed-points of the ERG correspond to continuum limits!

- $S_*$  is independent of all scales, including  $\Lambda_0$
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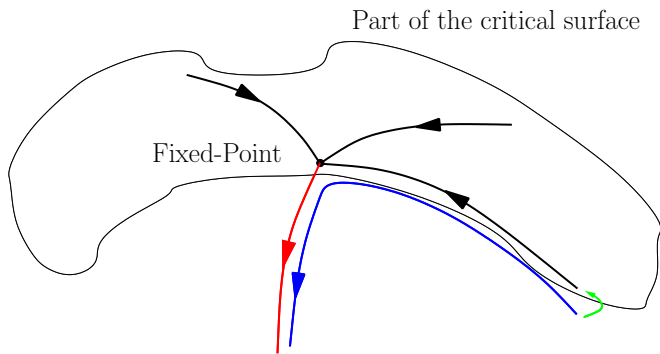
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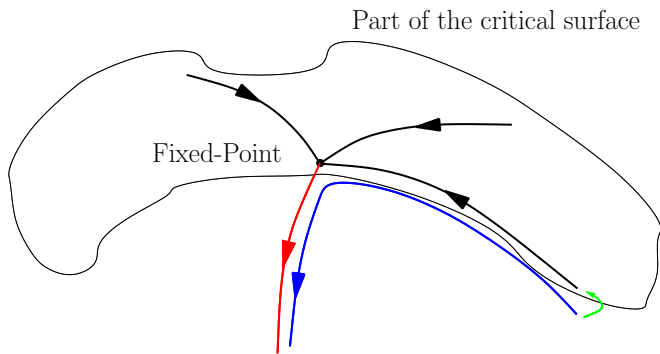
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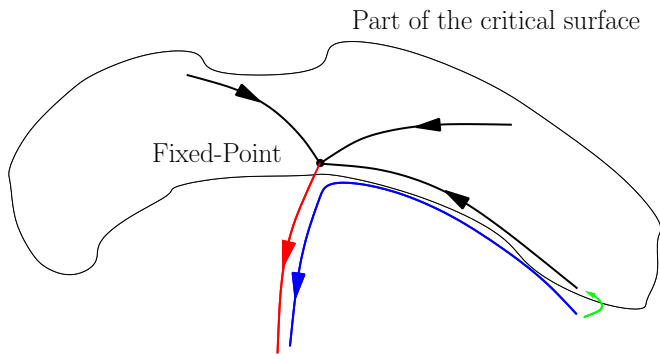


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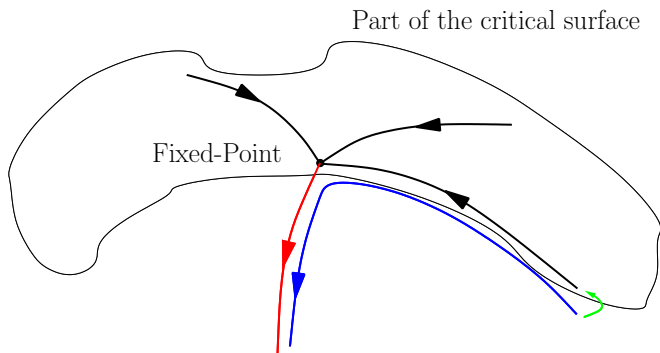
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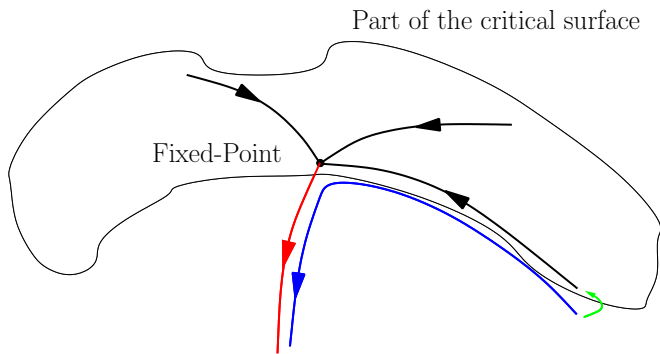
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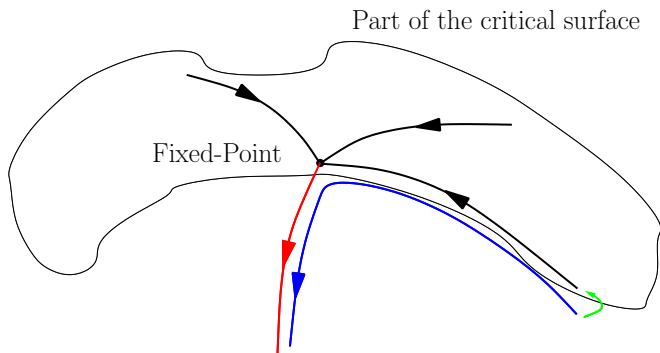


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# The Key Point

Nonperturbatively renormalizable theories follow from fixed-points

- Either directly
- Or from the renormalized trajectories emanating from them

## Theory Space

- QFTs should be understood in terms of "theory space"
- Nonrenormalizable QFTs follow from the solution to an equation

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Asymptotic Safety

NT FP



renormalizability  
determined  
in UV

(GFP)



Theory appears  
non renormalizable  
in IR



# Scalar Field Theory: Four Dimensions

## Nonperturbative

- Only one coupling constant
- The mass is relevant
- The cut-point coupling is marginaly relevant
- All other couplings are irrelevant
- The theory is asymptotically free
- The beta function is negative

## Perturbative

- Order by order is perturbative theory,  $\Phi^4$  is renormalizable
- The beta function is positive
- The mass is marginaly relevant
- The cut-point coupling is marginaly relevant
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# Scalar Field Theory: Four Dimensions

## Nonperturbative

- Only the Gaussian FP exists
- The mass is relevant
- The four point coupling is marginally irrelevant
- All other couplings are irrelevant
- The only nonperturbatively renormalizable scalar field theories in four dimensions are trivial!

## Perturbative

- Order by order in perturbation theory,  $\lambda\varphi^4$  is renormalizable
- $\lambda_{\text{bare}} = \sum a_n \lambda_{\text{renorm}}^n + f(\Lambda_{\text{renorm}}/\Lambda_0)$
- Formally,  $\lim_{\Lambda_0 \rightarrow \infty} f(\Lambda_{\text{renorm}}/\Lambda_0) = 0$
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## Gaussian Fixed-point

- The mass term is relevant
- The four-point coupling is relevant
- Non-trivial renormalizable theories exist along the  $g_4$  direction

## Wilson-Fisher Fixed-point

- In addition to the Gaussian FP, there is a non-trivial FP
- The  $W$ -FP possesses a single relevant direction
- This can also be used to construct an RG

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# Scalar Field Theory: Three Dimensions

Example of a Continuum Limit in  $D=3$

Wilson-Fisher FP

Gaussian FP



# Textbook versus Wilsonian

Question: What is the link?

- Textbook renormalization is in terms of the renormalization group
- Textbook renormalization is in terms of the ERG

My aims in the rest of this talk

- To explain why the question is relevant

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- 1 Qualitative Aspects of the ERG
- 2 Renormalizability
- 3 Correlation Functions in the ERG**

# Polchinski's Equation

- Polchinski made a particular choice

$$\Psi = \Psi_{\text{Pol}}$$

## Pros

- The flow equation is simple
- The correlation functions can be extracted from  $\Psi$
- The renormalizability of  $S$  is renormalizability of  $\Psi$  (and  $\Psi_{\text{Pol}}$ )

## Cons

- It is inconvenient for finding fixed points

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$$\Psi = \Psi_{\text{Pol}}$$

## Pros

- The flow equation is simple
- The correlation functions can be extracted from  $S_{\Lambda=0}$
- Renormalizability of  $S \Rightarrow$  renormalizability of  $\langle \phi(x_1) \cdots \phi(x_n) \rangle$

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# A modified version of Polchinski's equation

- Allow for an extra field redefinition along the flow

$$\Psi = \Psi_{\text{Pol}} + \psi$$

- Choose

$$\psi = -\frac{1}{2}\eta\phi, \quad \eta \equiv \Lambda \frac{d \ln Z}{d\Lambda}$$

- Since  $\psi$  is a field redefinition, this choice ensures canonical normalization of the kinetic term
- The redundant coupling,  $Z$ , is removed from the action

## Pros

- Easy to find fixed points with  $\eta_* \neq 0$

## Cons

- The link between  $\xi$  and  $\ln(\xi)$  is explicitly changed

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# Introducing a source: Textbook

## The Standard Correlation Functions

• The standard correlation functions are defined as

$$G_n(x_1, \dots, x_n) = \int \mathcal{D}\phi \phi(x_1) \dots \phi(x_n) e^{-S[\phi] + \int \phi(x) J(x)} \quad (1)$$

• The source  $J(x)$  is considered a combined correlation function from  $\mathcal{W}[J] = \ln Z[J]$

$$G_n(x_1, \dots, x_n) = \frac{\delta^n \mathcal{W}[J]}{\delta J(x_1) \dots \delta J(x_n)} \bigg|_{J=0} \quad (2)$$

## Composite Operators

• The standard correlation functions are

• The composite operators with momenta  $\{k_i\}$  are defined as

$$G_n(k_1, \dots, k_n) = \int \mathcal{D}\phi \phi(k_1) \dots \phi(k_n) e^{-S[\phi] + \int \phi(x) J(x)} \quad (3)$$

• The composite operators are defined as

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# New ERG Approach

- Introduce an external field,  $J$ , with undetermined scaling dimension,  $d_J$
- Allow for  $J$ -dependence of the action

$$S_\Lambda[\phi] \rightarrow T_\Lambda[\phi, J]$$

- The flow equation follows as before

$$-\Lambda \partial_\Lambda e^{-T_\Lambda[\phi, J]} = \int d^D x \frac{\delta}{\delta \phi(x)} \left\{ \Psi(x) e^{-T_\Lambda[\phi, J]} \right\}$$

- A sensible boundary condition would be

$$\lim_{\Lambda \rightarrow \Lambda_0} T_\Lambda[\phi, J] - S_\Lambda[\phi] = -J \cdot \phi$$

- But we will not implement the bc in this way

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# Source-Dependent Renormalization

## The game

• The goal is to find a renormalization scheme, source-dependent renormalization

## The strategy

• Develop a renormalization scheme, which does not follow from the ERG

• The renormalization scheme is

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## Notation

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# Source-Dependent Renormalization

## The game

- Search for renormalizable, source-dependent solutions

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Nonperturbatively renormalizable solutions follow from fixed-points

- Either directly:  $\partial_t T_*[\varphi, j] = 0$
- Or from relevant (source-dependent) perturbations

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- $\phi, J$  dimensional,  $\varphi, j$  dimensionless

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# A Source-Dependent Fixed-Point

- Suppose that we have found a critical fixed-point

$$\partial_t S_*[\varphi] = 0$$

- Then there is always a source-dependent f-p

$$T_*[\varphi, j] = S_*[\varphi] + \left[ e^{-\vec{j} \cdot \varphi \delta / \delta \varphi} - 1 \right] \left[ S_*[\varphi] + \frac{1}{2} \varphi \cdot f \cdot \varphi \right]$$

$$\partial_t T_*[\varphi, j] = 0$$

$$\partial_t \varphi = \alpha(\varphi), \quad f = f(\varphi)$$

## Two crucial points

- The solution only works if  $d_j = (D + 2 - \eta_\varphi)/2$
- in dimensional variables

$$\lim_{t \rightarrow \infty} T_*[\varphi, j] = S_j[\varphi] = -\vec{j} \cdot \varphi$$

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# And more. . .

- For each critical f-p, we can find the eigenperturbations

$$S_t[\varphi] = S_*[\varphi] + \sum_i \alpha_i e^{\lambda_i t} \mathcal{O}_i[\varphi]$$

- Every eigenperturbation,  $\mathcal{O}_i$  has a source-dependent extension

$$\tilde{\mathcal{O}}_i[\varphi, j] = e^{\vec{j} \cdot \varphi} \delta / \delta \varphi \mathcal{O}_i$$

- At the linear level

$$T_t[\varphi, j] = T_*[\varphi, j] + \sum_i \alpha_i e^{\tilde{\lambda}_i t} \tilde{\mathcal{O}}_i[\varphi, j]$$

where  $\tilde{\lambda}_i = \lambda_i + \vec{j} \cdot \vec{\gamma}_i$

# And more . . .

- For each critical f-p, we can find the eigenperturbations

$$S_t[\varphi] = S_*[\varphi] + \sum_i \alpha_i e^{\lambda_i t} \mathcal{O}_i[\varphi]$$

- Every eigenperturbation,  $\mathcal{O}_i$  has a source-dependent extension

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- Every critical f-p has a particular source-dependent extension
- Every renormalized trajectory has a source-dependent extension
- This source-dependence corresponds to the boundary condition

$$\lim_{\Lambda \rightarrow \infty} T_\Lambda[\phi, J] - S_\Lambda[\phi] = -J \cdot \phi$$

## Conclusion

How does the modified Polchinski equation with  $\phi = \omega_{\text{RG}}/\Lambda$

Renormalizability of  $S_\Lambda$  implies renormalizability of the extended correlation functions

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# A new perspective on QFT?

- Decide which correlation functions to compute
- Introduce appropriate source term *e.g.*  $J \cdot \phi$
- Analyse renormalizability of correlation functions
- Allow arbitrary source dependence
- Search for fixed-point solutions
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## Philosophy

- The Wilsonian effective action is fundamental
- QFT determines which quantities we should compute

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# Questions

## Modified Polchinski Equation $\psi = -\eta_\psi/2$

- Is this a more renormalizable source-dependent action than usual?
- How does the CFE play a role?
- Can this be generalized to include all CFEs?

## Other flow equations

- What happens for other flow equations?
- How does this apply to gauge theories?

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## Modified Polchinski Equation $\psi = -\eta\varphi/2$

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- It is possible to construct a gauge invariant cutoff, using
  - Covariant higher derivatives
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- $\Psi$  can be chosen to give a
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- No gauge fixing is required at any stage!
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- How do manifestly gauge invariant operators renormalize?
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