A New Perspective on Quantum Field Theory

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Sussex U.

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Outline of this Lecture

1. Qualitative Aspects of the ERG

2. Renormalizability

3. Correlation Functions in the ERG

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2. Renormalizability
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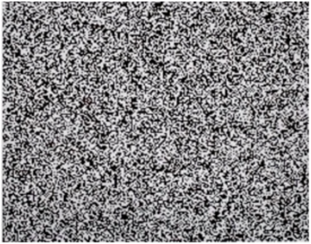

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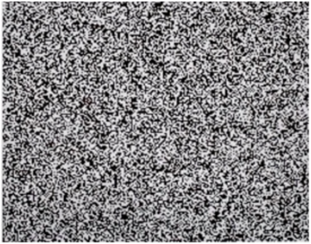

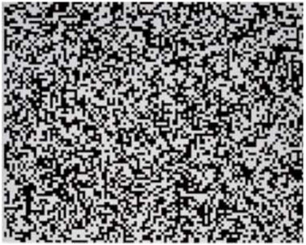

When should we use the Exact Renormalization Group?

The ERG is of use for systems with

- A large number of degrees of freedom per correlation length
- Local interactions

Easy Problems (no need for ERG)

- A small number of degrees of freedom per correlation length
- The subsystem is easy to understand
- The subsystem captures the behaviour of the whole system
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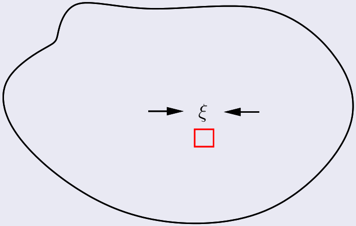

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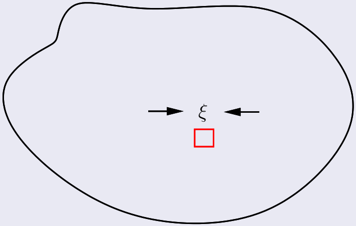

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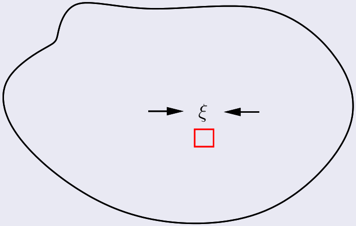

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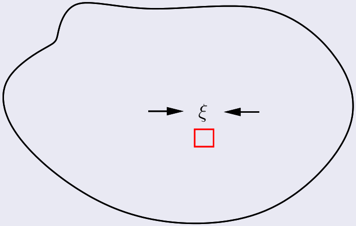

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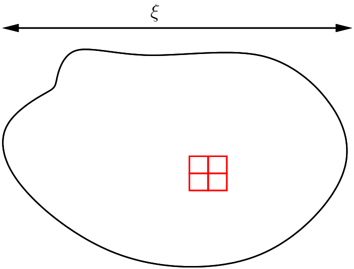

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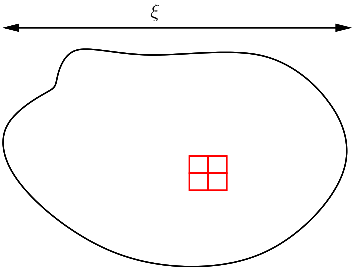

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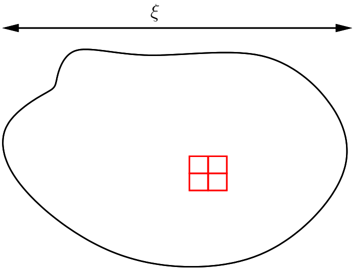

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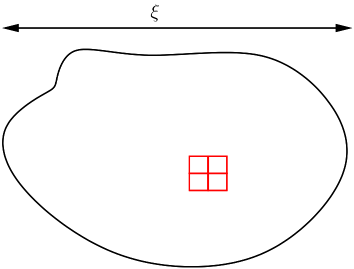

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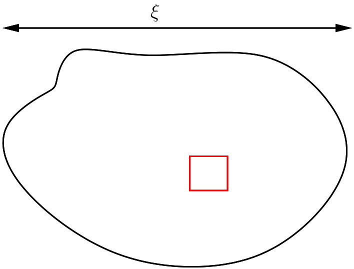

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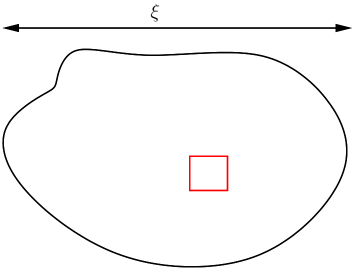

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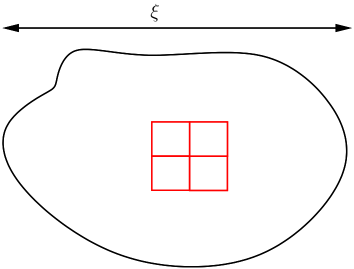

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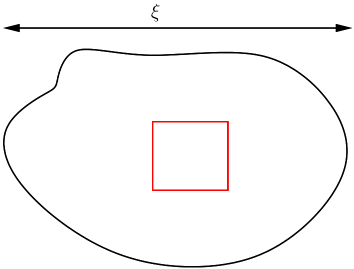

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Practicalities

Applications

- Quantum field theory
- Critical phenomena
- Kondo effect, ultra cold gases, nuclear physics

What has the ERG given us?

- A deep understanding of renormalization and universality
- A tool for performing real calculations

What's the catch?

- The coarse-graining procedure cannot be done exactly
- The ERG supports nonperturbative approximation schemes

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- To go from micro to macro, average over groups of spins
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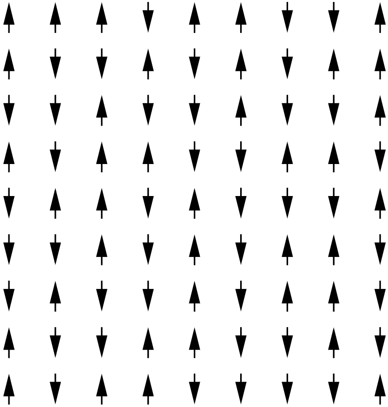

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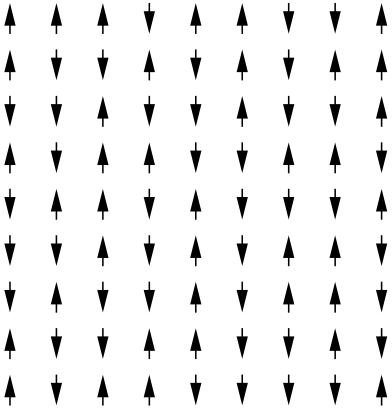

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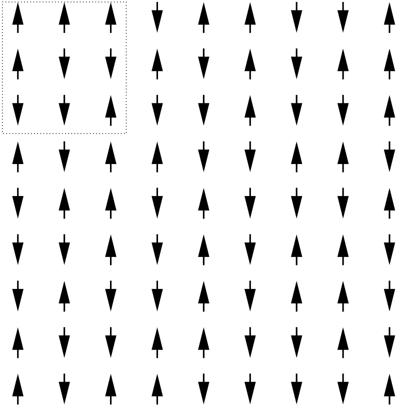

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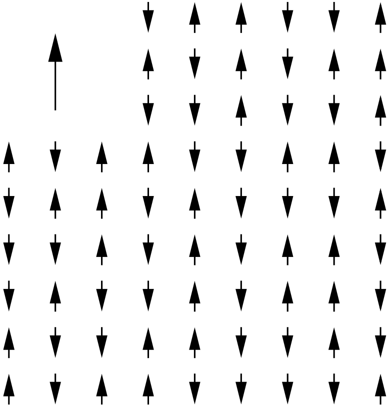

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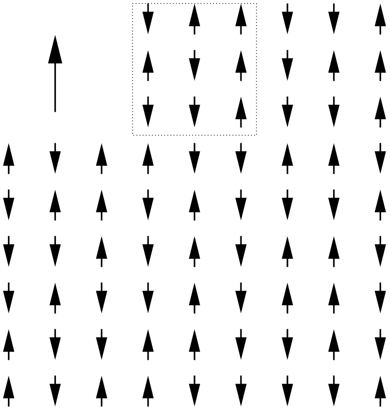

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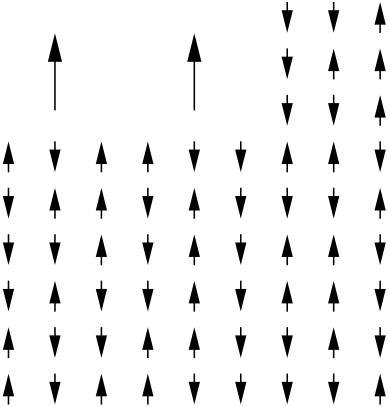

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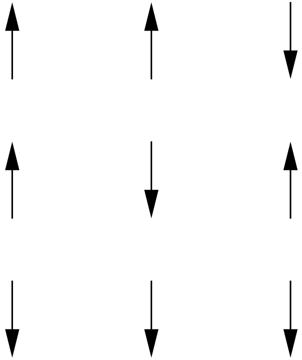

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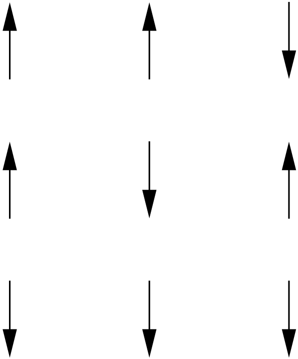

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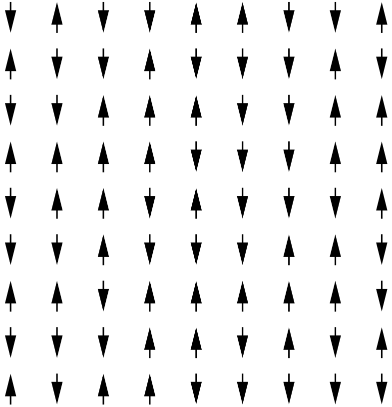

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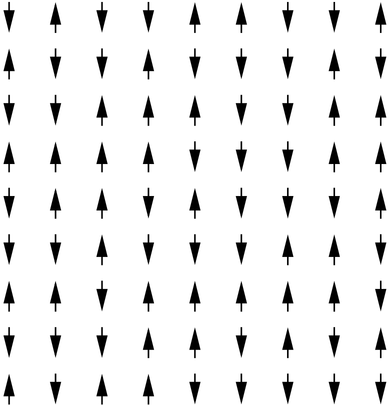

The ERG implements the continuous version of blocking

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- The transformation can have fixed-points

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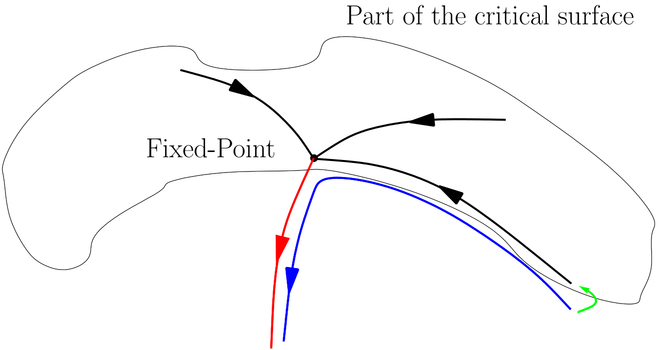

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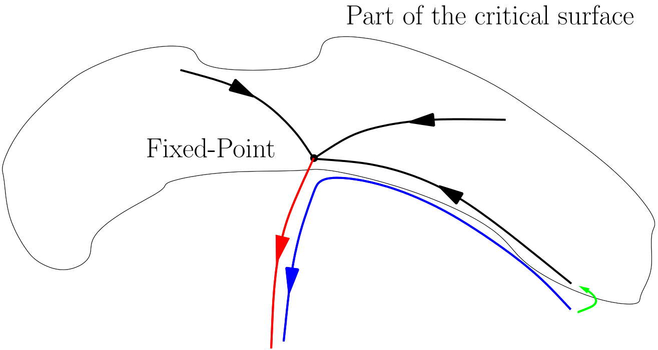

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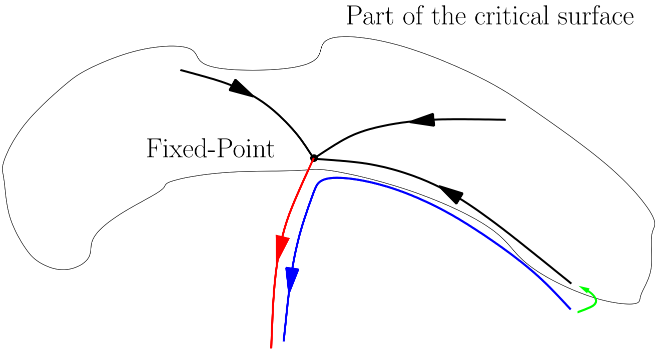

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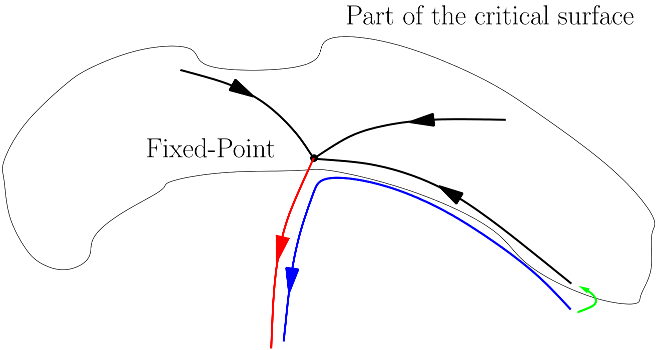

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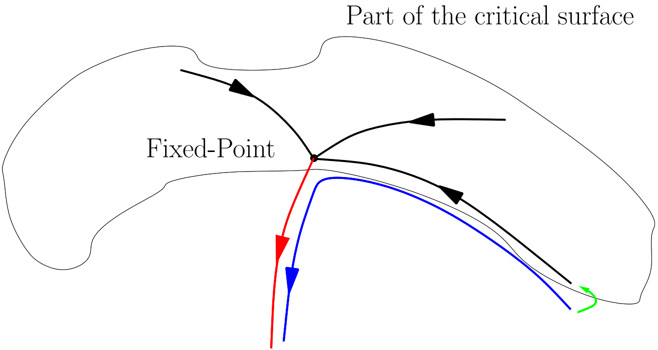

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- Flows along the relevant directions leave the critical surface
- If there are *n* relevant directions, then we must tune *n* quantities to get on to the critical surface

The Wilsonian Effective Action

Start with the partition function

$$Z = \int_{\Lambda_0} \mathcal{D}\phi e^{-S_{\Lambda_0}[\phi]} = \int_{\Lambda} \mathcal{D}\phi e^{-S_{\Lambda}[\phi]}$$

- The bare scale
  - High energy (short distance) scale
  - Modes above this scale are cut off (regularized)
- The bare (classical) action
- Integrate out modes between the bare scale and an intermediate scale, Λ
  - The partition function stays the same
  - The effects of the high energy modes must be taken into account
  - The action evolves ⇒ Wilsonian effective action

The Wilsonian Effective Action

Start with the partition function

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Very General ERGs

Formulation

$$-\Lambda\partial_\Lambda e^{-S[\phi]} = \int_x \frac{\delta}{\delta\phi(x)}\left(\Psi_x[\phi]e^{-S[\phi]}\right)$$

- effective scale
- set of fields
- Wilsonian effective action
- partition function, $\int \mathcal{D}\phi\, e^{-S[\phi]}$, invariant under the flow
- Parametrizes blocking procedure

Flow Equation

$$-\Lambda\partial_\Lambda S = \int_x \frac{\delta S}{\delta\phi(x)}\Psi_x - \int_x \frac{\delta \Psi_x}{\delta\phi(x)}$$

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Rescaling

Ingredients of ERG Transformation

- Blocking (coarse graining)
- Rescaling

Implementing Rescaling

- Measure all dimensionful quantities in units of Λ
- Remember to take account of anomalous dimensions!

$$X \to X\Lambda^{\text{full scaling dimension}}$$

- Notation: ϕ dimensionful, φ dimensionless
- $-\Lambda\partial_\Lambda \to \partial_t$, with $t = \ln \mu/\Lambda$

What we need for this talk

- ERG Equation: $\partial_t S[\varphi] = \ldots$
- Fixed-points: $\partial_t S_*[\varphi] = 0$

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Relevance/Irrelevance

- At a fixed-point we have $\partial_t S_\star = 0$
- Consider an infinitesimal perturbation

First order classification

- Operators that grow with t are relevant
- Operators that shrink with t are irrelevant
- Operators that stay the same are marginal

Marginal Operators

- $S_\star + a\mathcal{O}_{\text{marginal}}$ is a fixed-point up to $\mathcal{O}(a^2)$
- This might not be true beyond leading order
- Eg the four point coupling in $D = 4$ scalar field theory is marginally irrelevant
- An exactly marginal operator generates a line of fixed-points

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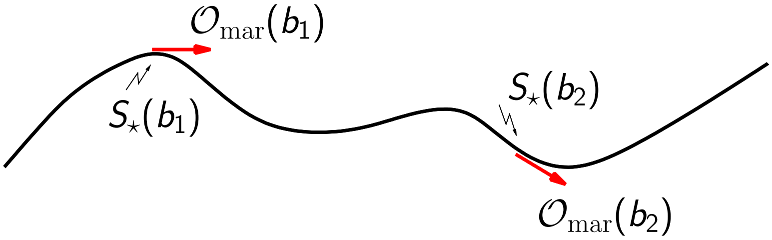

1 Qualitative Aspects of the ERG

2 Renormalizability

3 Correlation Functions in the ERG

Textbook renormalization

- Choose an action *e.g.*

$$S[\phi] = \int d^D x \left[\frac{1}{2}\partial_\mu\phi\partial_\mu\phi + \frac{1}{2}m^2\phi^2 + \frac{\lambda}{4!}\phi^4\right]$$

- Choose a UV regulator
- Start computing the correlation functions

$$\langle\phi(x_1)\cdots\phi(x_n)\rangle = \frac{1}{Z}\int \mathcal{D}\phi\,\phi(x_1)\cdots\phi(x_n)e^{-S[\phi]}$$

- Adjust the action to absorb UV divergences:

$$S[\phi] \to S[\phi] + \delta S[\phi]$$

- If δS has the same form as S, the theory is renormalizable

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$$S[\phi] \to S[\phi] + \delta S[\phi]$$

- If δS has the same form as S, the theory is renormalizable

Textbook renormalization

- Choose an action *e.g.*

$$S[\phi] = \int d^D x \left[\frac{1}{2}\partial_\mu \phi \partial_\mu \phi + \frac{1}{2} m^2 \phi^2 + \frac{\lambda}{4!}\phi^4\right]$$

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The Question

Are there effective actions $S_{\Lambda,\Lambda_0}[\phi]$ for which we can safely send $\Lambda_0 \to \infty$?

The Simplest Answer

- Rescale all quantities, using Λ
- Only dimensionless variables appear
- Fixed-points of the ERG correspond to continuum limits!

- S_* is independent of all scales, including Λ_0
- Trivially, we can send $\Lambda_0 \to \infty$

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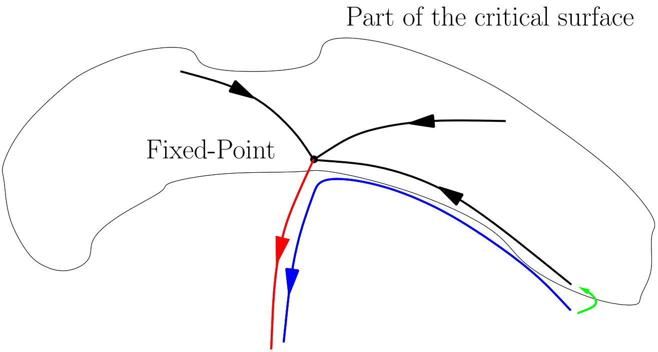

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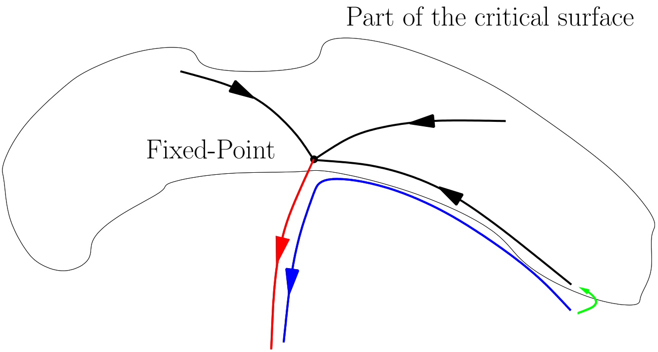

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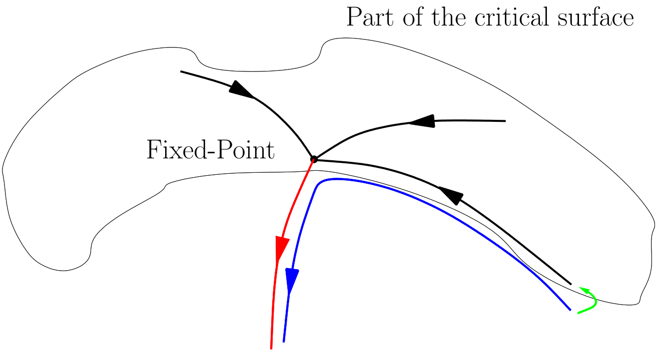

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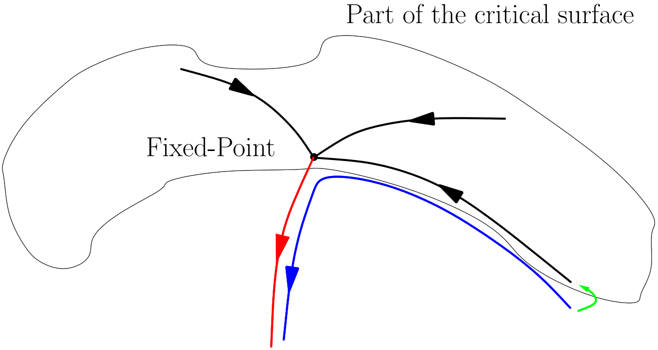

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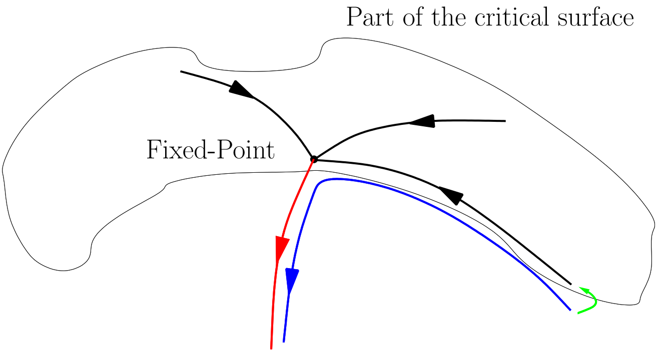

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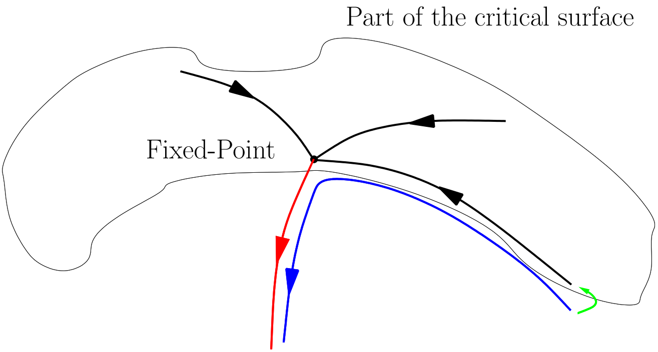

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- Actions on the RT are renormalizable

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Nonperturbatively renormalizable theories follow from fixed-points

- Either directly
- Or from the renormalized trajectories emanating from them

Theory Space

- QFTs should be understood in terms of 'theory space'
- Renormalizable QFTs follow from the solution to an equation

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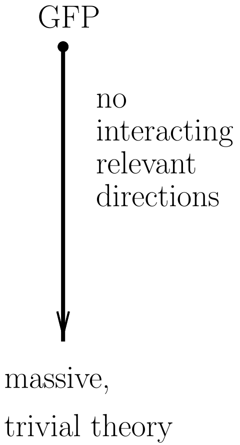

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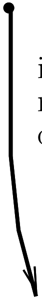

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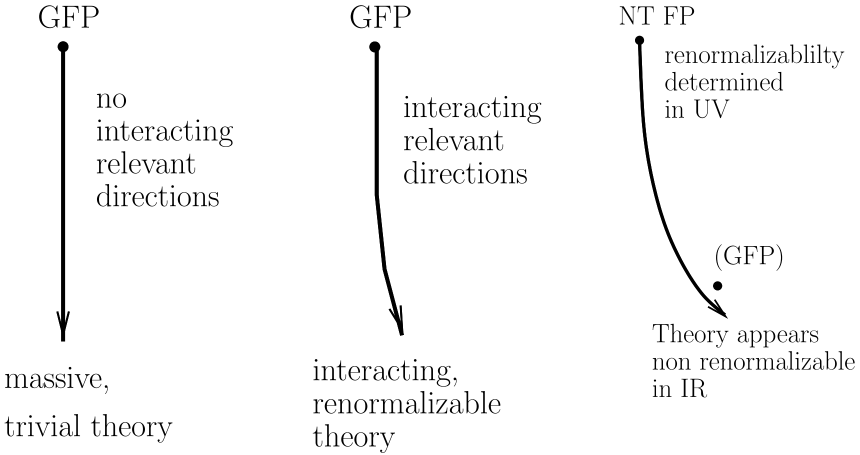

Scalar Field Theory: Four Dimensions

Nonperturbative

- Only the Gaussian FP exists
- The mass is relevant
- The four point coupling is marginally irrelevant
- All other couplings are irrelevant
- The only nonperturbatively renormalizable scalar field theories in four dimensions are trivial!

Perturbative

- Order by order in perturbation theory, $\lambda \varphi^4$ is renormalizable
- $\lambda_{\mathrm{bare}} = \sum a_n \lambda^n_{\mathrm{renorm}} + f(\Lambda_{\mathrm{renorm}}/\Lambda_0)$
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- The four-point coupling is relevant
- Non-trivial renormalizable theories exist along the $\lambda\varphi^4$ direction!

Wilson-Fisher Fixed-point

- In addition to the Gaussian FP, there is a non-trivial FP
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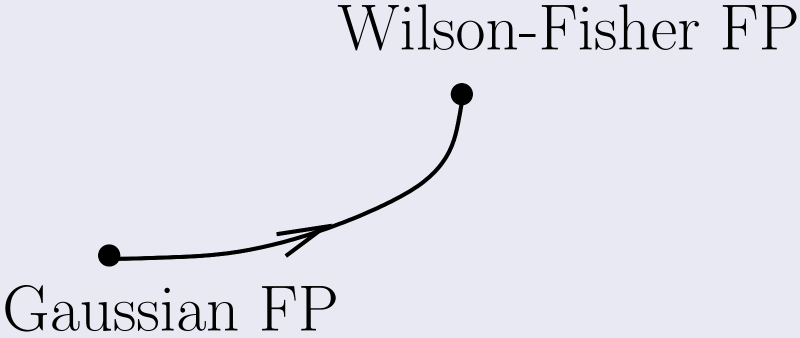
Example of a Continuum Limit in D=3



Textbook versus Wilsonian

Question: What is the link?

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- Wilsonian formulation is in terms of the WEA

My aims in the rest of this talk

- To convince you that the question is profound

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1 Qualitative Aspects of the ERG

2 Renormalizability

3 Correlation Functions in the ERG

Polchinski's Equation

- Polchinski made a particular choice

$$\Psi = \Psi_{\mathrm{Pol}}$$

Pros

- The flow equation is simple
- The correlation functions can be extracted from $S_{\Lambda \to 0}$
- Renormalizability of $S \Rightarrow$ renormalizability of $\langle \varphi(x_1) \cdots \varphi(x_n) \rangle$

Cons

- It is inconvenient for finding fixed points

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$$\Psi = \Psi_{\mathrm{Pol}}$$

Pros

- The flow equation is simple
- The correlation functions can be extracted from $S_{\Lambda=0}$
- Renormalizability of S $\Rightarrow$ renormalizability of $\langle \phi(x_1) \cdots \phi(x_n) \rangle$

Cons

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A modified version of Polchinski's equation

- Allow for an extra field redefinition along the flow

$$\Psi = \Psi_{\mathrm{Pol}} + \psi$$

- Choose

$$\psi = -\frac{1}{2}\eta\phi, \qquad \eta \equiv \Lambda\frac{d\ln Z}{d\Lambda}$$

- Since ψ is a field redefinition, this choice ensures canonical normalization of the kinetic term
- The redundant coupling, Z, is removed from the action

Pros

- Easy to find fixed-points with $\eta_\star \neq 0$

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- The link between S and $\langle\varphi(x_1)\cdots\varphi(x_n)\rangle$ changes

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The Standard Correlation Functions

- Introduce a source term in the bare action

$$Z[J] = \int D\phi \, e^{-S_\Lambda[\phi]+J\cdot\phi}$$

- Extract the connected correlation functions from $W[J] = \ln Z$

$$\langle \phi(x_1) \cdots \phi(x_n) \rangle_{\mathrm{conn}} = \frac{\delta}{\delta J(x_1)} \cdots \frac{\delta}{\delta J(x_n)} W[J] \bigg|_{J=0}$$

Composite Operators

- Add additional source terms e.g. $J_2 \cdot \phi^2$
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New ERG Approach

- Introduce an external field, J, with undetermined scaling dimension, d_J
- Allow for J-dependence of the action

$$S_\Lambda[\phi] \to T_\Lambda[\phi, J]$$

- The flow equation follows as before

$$-\Lambda\partial_\Lambda e^{-T_\Lambda[\phi,J]} = \int d^D x \frac{\delta}{\delta\phi(x)} \left\{ \Psi(x) e^{-T_\Lambda[\phi,J]} \right\}$$

- A sensible boundary condition would be

$$\lim_{\Lambda\to\Lambda_0} T_\Lambda[\phi, J] - S_\Lambda[\phi] = -J \cdot \phi$$

- But we will not implement the bc in this way

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The game

- Search for renormalizable source-dependent solutions

The strategy

Nonperturbatively renormalizable solutions follow from fixed-points

- Either directly: $\partial_t T_\star[\varphi, j] = 0$
- Or from relevant (source-dependent) perturbations

Notation

- ϕ, J dimensionful, φ, j dimensionless

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A Source-Dependent Fixed-Point

- Suppose that we have found a critical fixed-point

$$\partial_t S_\star[\varphi] = 0$$

- Then there is always a source-dependent f-p

$$T_\star[\varphi, j] = S_\star[\varphi] + \left[e^{-\bar{j}\cdot\varrho\cdot\delta/\delta\varphi} - 1\right]\left[S_\star[\varphi] + \frac{1}{2}\varphi\cdot f\cdot\varphi\right]$$

  - $\bar{j}(p) = j(p)/p^2$
  - $\varrho = \varrho(p^2)$, $f = f(p^2)$

Two crucial points

- The solution only works if $d_j = (D+2-\eta_\star)/2$
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And more. . .

- For each critical f-p, we can find the eigenperturbations

$$S_t[\varphi] = S_\star[\varphi] + \sum_i \alpha_i e^{\lambda_i t} \mathcal{O}_i[\varphi]$$

- Every eigenperturbation, $\mathcal{O}_i$ has a source-dependent extension

$$\tilde{\mathcal{O}}_i[\varphi, j] = e^{\bar{j}\cdot\varrho\cdot\delta/\delta\varphi} \mathcal{O}_i$$

- At the linear level

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- Every eigenperturbation, $\mathcal{O}_i$ has a source-dependent extension

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- Every critical f-p has a particular source-dependent extension
- Every renormalized trajectory has a source-dependent extension
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$$\lim_{\Lambda \to \infty} T_\Lambda[\phi, J] - S_\Lambda[\phi] = -J \cdot \phi$$

Conclusion

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A new perspective on QFT?

- Decide which correlation functions to compute
- Introduce appropriate source term *e.g.* $J \cdot \phi$
- Analyse renormalizability of correlation functions
- Allow arbitrary source dependence
- Search for fixed-point solutions
- Deduce the correlation functions to which the solution(s) correspond

Philosophy

- The Wilsonian effective action is fundamental
- QFT determines which quantities we should compute

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Modified Polchinski Equation $\psi = -\eta\varphi/2$

- What other renormalizable source dependent solutions exist?
- How does the OPE play a role?
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Other flow equations

- What happens for other flow equations?
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  - Covariant higher derivatives
  - Pauli-Villars fields
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- No gauge fixing is required at any stage!
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- Throw in sources and let the ERG tell us!

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