

# DETERMINING THE STRONG COUPLING CONSTANT AT NNLO FROM JET OBSERVABLES

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# Outline

## • Motivation

- Why study jet observables at NNLO?

- Jet observables in experiments

  - jet rates

  - event-shape observables

- Jet observables in theory

## • Determinations of $\alpha_s$

- $\alpha_s$  from event-shape distributions at NLLA+NNLO

- $\alpha_s$  from event-shape moments at NNLO

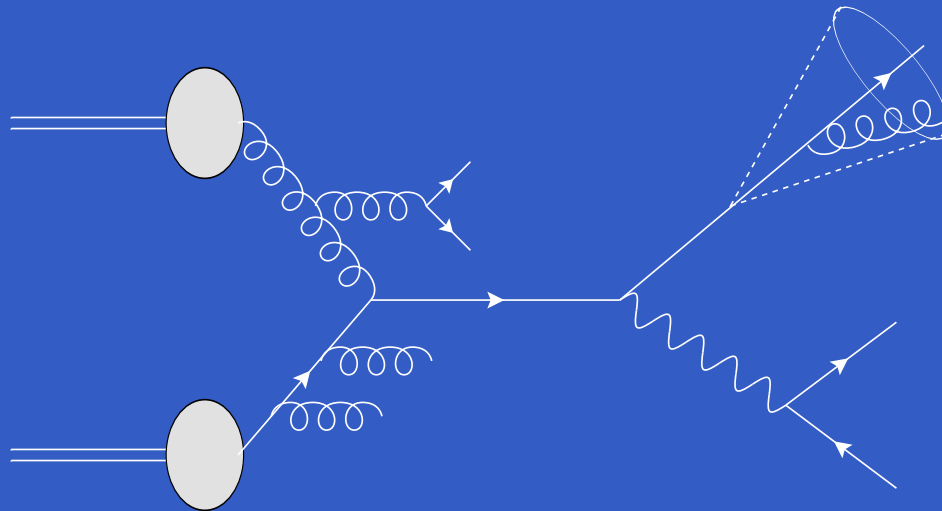
- $\alpha_s$  from three-jet rates at NNLO

- Hadronization corrections

## • Conclusions

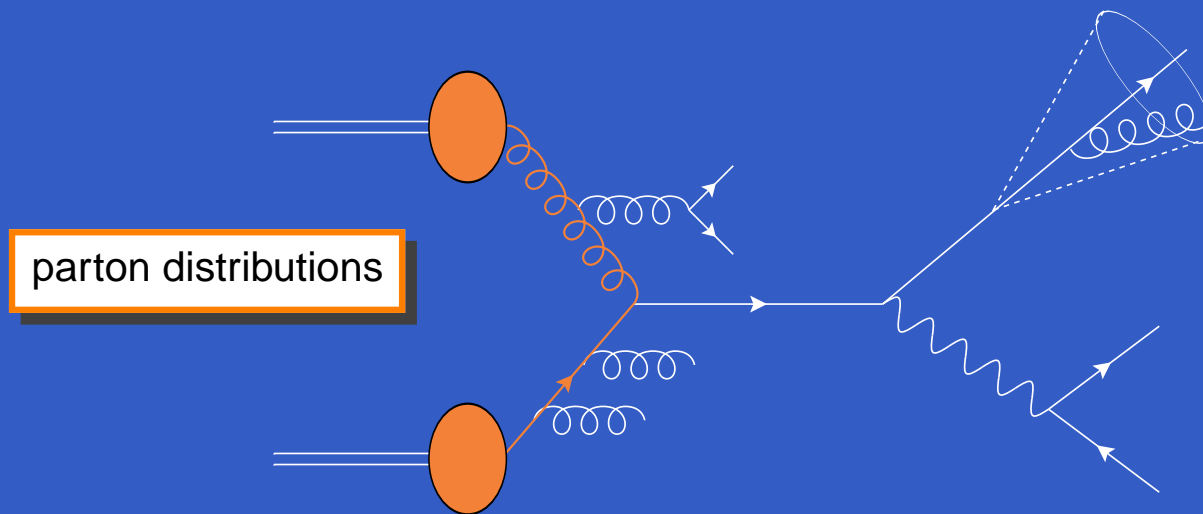
# QCD & Jet Observables: why NNLO?

- QCD: very successful theory of strong interactions
- QCD is omnipresent in high energy collisions at many different stages:



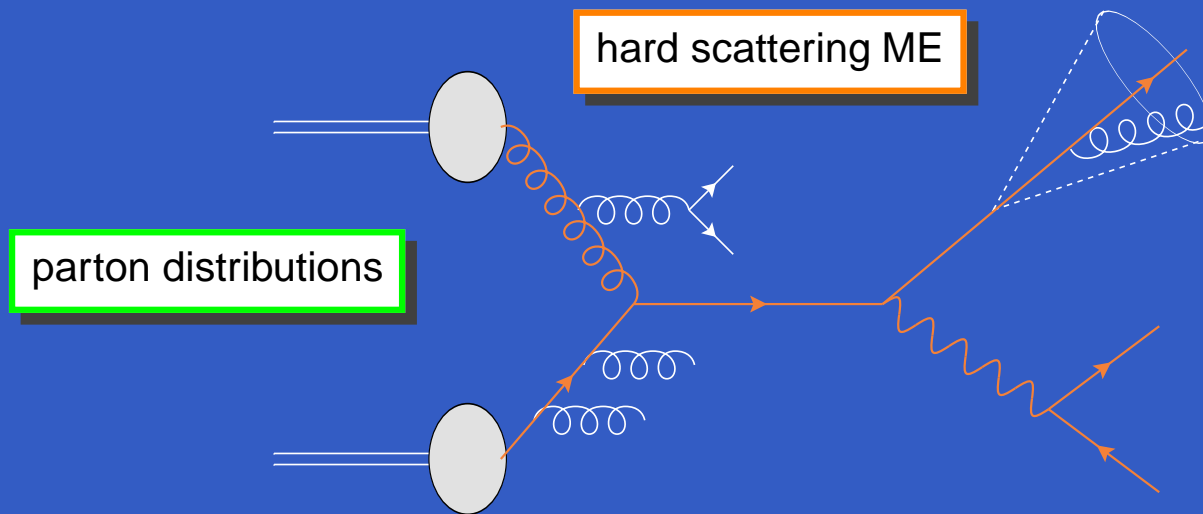
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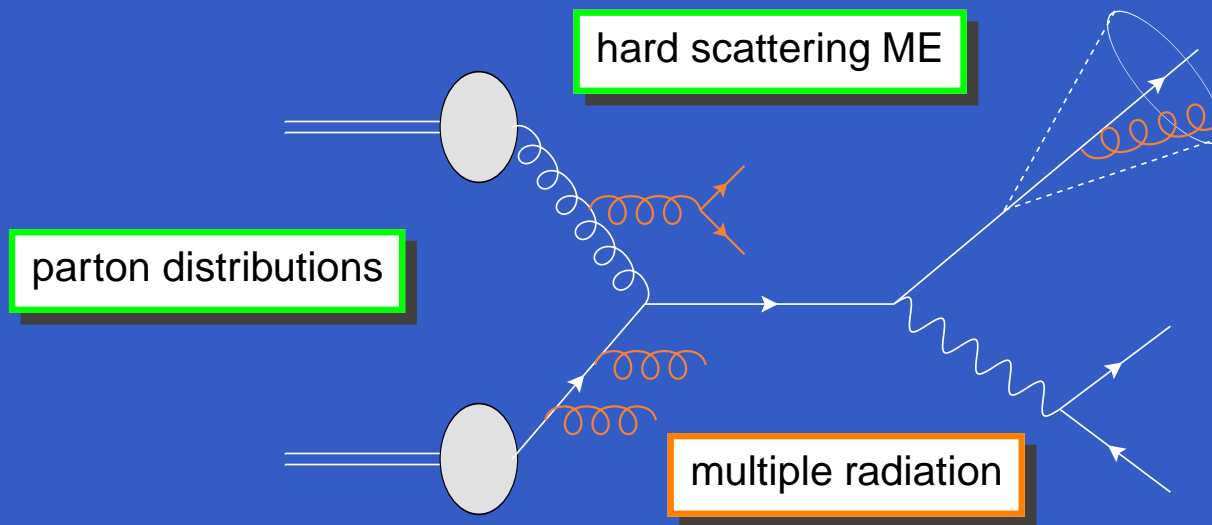
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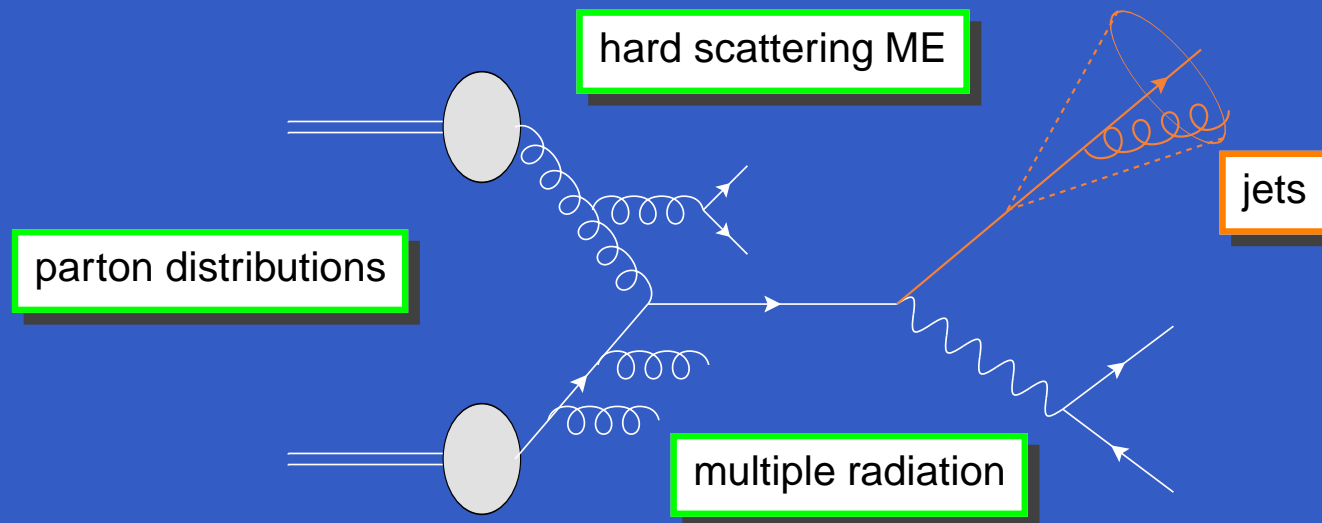
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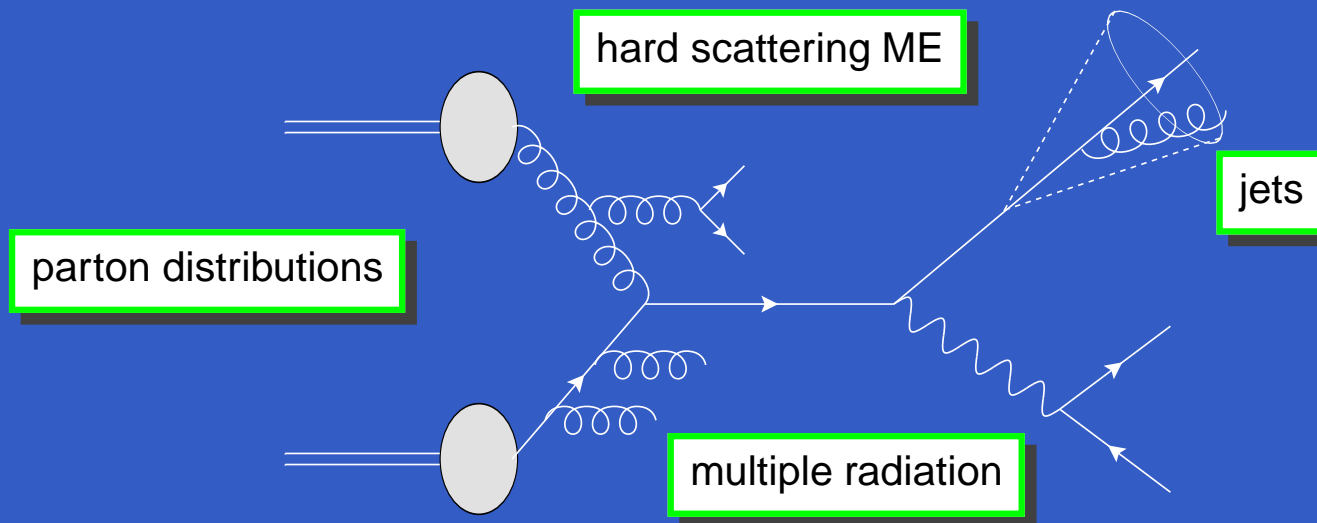
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# QCD & Jet Observables: why NNLO?

- QCD: very successful theory of strong interactions
- QCD is omnipresent in high energy collisions at many different stages:



- Need to understand QCD to the highest level of accuracy for
  - interpretation of collider data
  - precision studies: coupling, masses, ...
  - enhance new discovery potential of present collider experiments

Need higher order corrections!



# Massless QCD: 1-Parameter Theory

- Apart the quark masses, there is only one free parameter in the QCD lagrangian,

$$L_{\text{QCD}} = \left( \begin{array}{c} \text{diagram 1} \\ \delta^{ab} \end{array} + \begin{array}{c} \text{diagram 2} \\ g_s f^{abc} \end{array} + \begin{array}{c} \text{diagram 3} \\ g_s^2 f^{abe} f^{cde} \end{array} \right) + \sum_{\text{flavours}} \left( \begin{array}{c} \text{diagram 4} \\ \delta^{ij} \end{array} + \begin{array}{c} \text{diagram 5} \\ g_s T_{ij}^a \end{array} \right)$$

$$\alpha_s = g_s^2 / (4\pi)$$

- Can be extracted with good accuracy from  $e^+e^-$  data, however
- the value of  $\alpha_s$  from LEP data suffers mainly from theoretical scale uncertainty:

$$\alpha_s(M_Z) = 0.1202 \pm 0.0003(\text{stat}) \pm 0.0009(\text{sys}) \pm 0.0013(\text{had}) \pm 0.0047(\text{scale})$$

[LEPQCDWG]

# Massless QCD: 1-Parameter Theory

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$$L_{\text{QCD}} = \left( \begin{array}{c} a \text{-----} b \\ \delta^{ab} \end{array} + \begin{array}{c} a \text{-----} b \\ \text{-----} \\ \text{-----} \\ c \end{array} + \begin{array}{c} a \text{-----} d \\ \text{-----} \\ \text{-----} \\ b \text{-----} c \end{array} \right) + \sum_{\text{flavours}} \left( \begin{array}{c} i \text{-----} j \\ \delta^{ij} \end{array} + \begin{array}{c} i \text{-----} a \\ \text{-----} \\ \text{-----} \\ j \end{array} \right)$$

$\delta^{ab}$        $g_s f^{abc}$        $g_s^2 f^{abe} f^{cde}$        $\delta^{ij}$        $g_s T_{ij}^a$

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[LEPOCDWG]

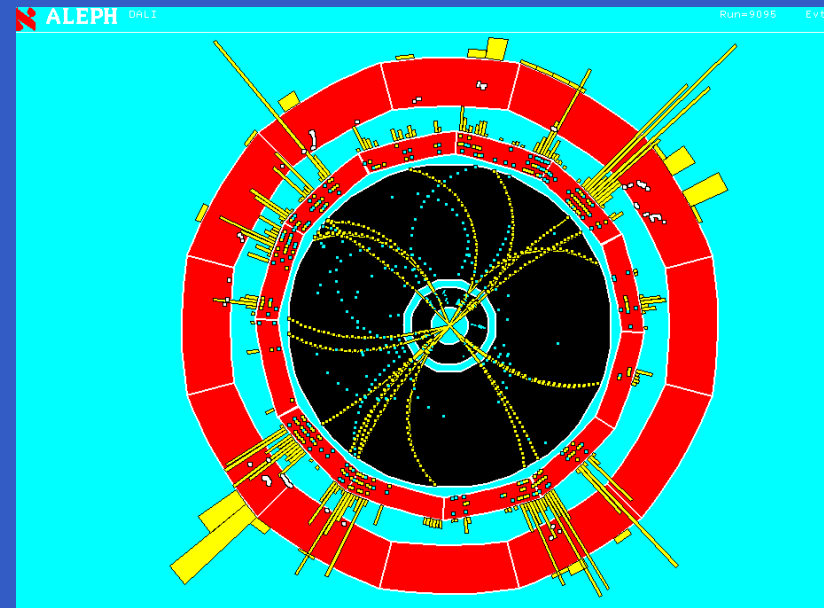
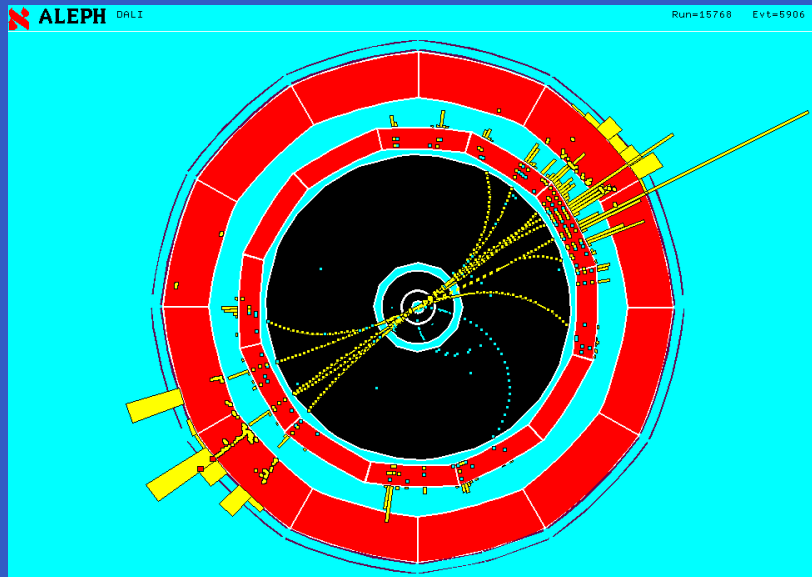
# Jet Observables in Experiments

● QCD final states visible in form of **JETS**

● bundle of final-state particles

● cluster of Hadrons

⇒ Study phenomenology with **JET OBSERVABLES**



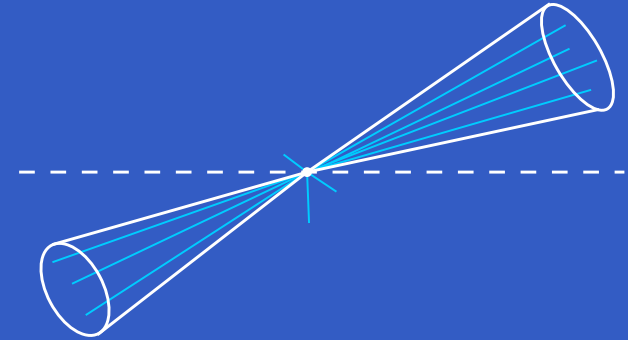
● Possible jet observables

● number of jets in an event: **jet rates**

● spatial distribution of particles in an event: **event-shape observables**

# Jet Observables in Experiments

- Definition of a jet relies on a **jet algorithm**,
- cone algorithms:
  - construct **stable** cones of particles,



# Jet Observables in Experiments

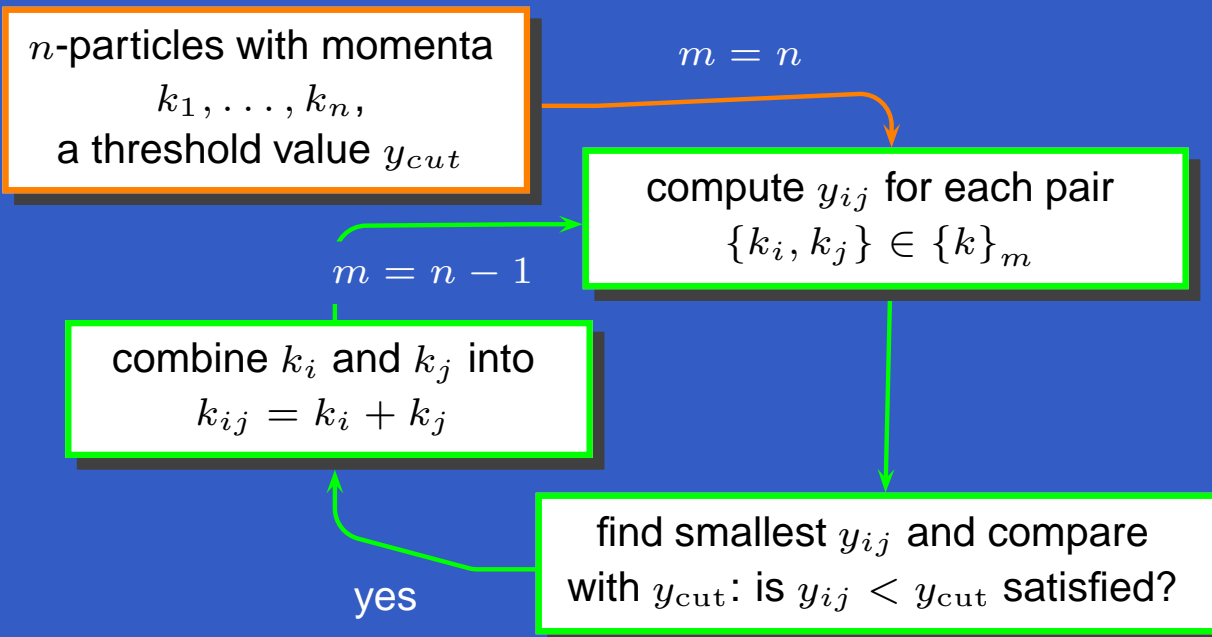
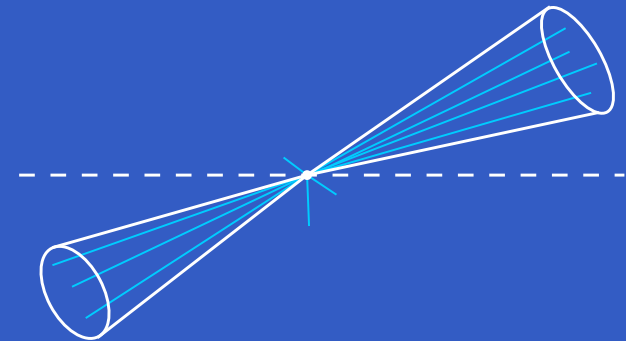
● Definition of a jet relies on a **jet algorithm**,

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→ construct **stable** cones of particles,

● recombination algorithms:

→ recombination according to **distance measure**  $y_{ij}$  and **rec. scheme**,

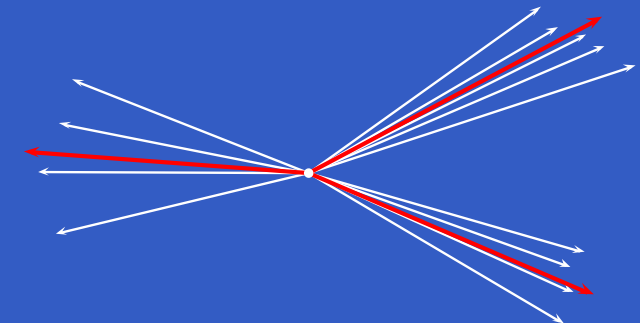


Jade algorithm:

$$y_{ij,J} = \frac{2E_i E_j (1 - \cos \theta_{ij})}{E_{\text{CM}}^2}$$

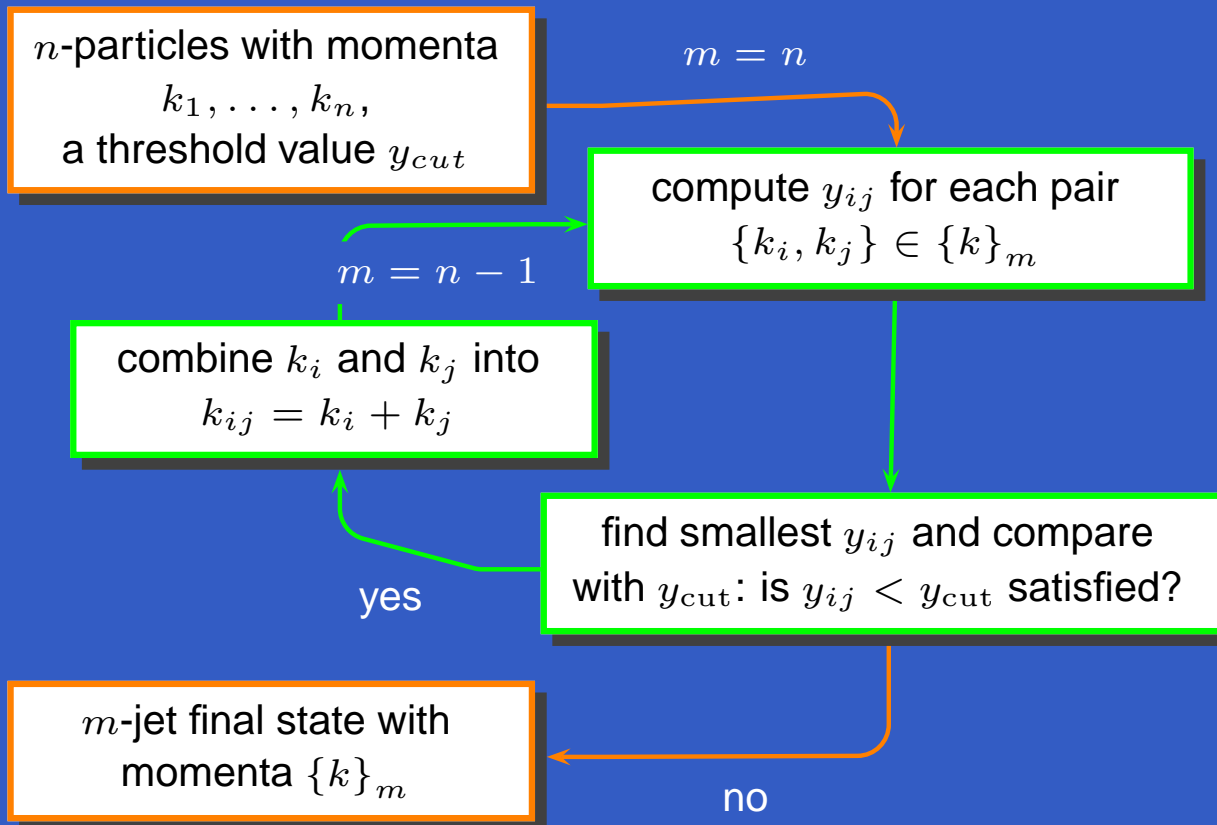
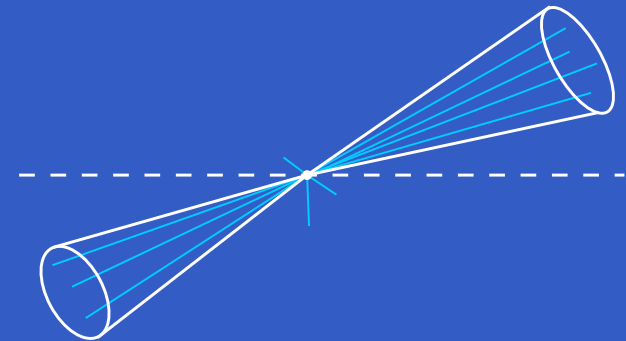
Durham algorithm:

$$y_{ij,D} = \frac{2 \min(E_i^2, E_j^2) (1 - \cos \theta_{ij})}{E_{\text{vis}}^2}$$



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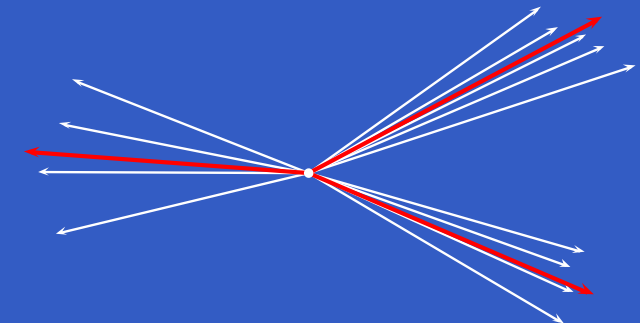


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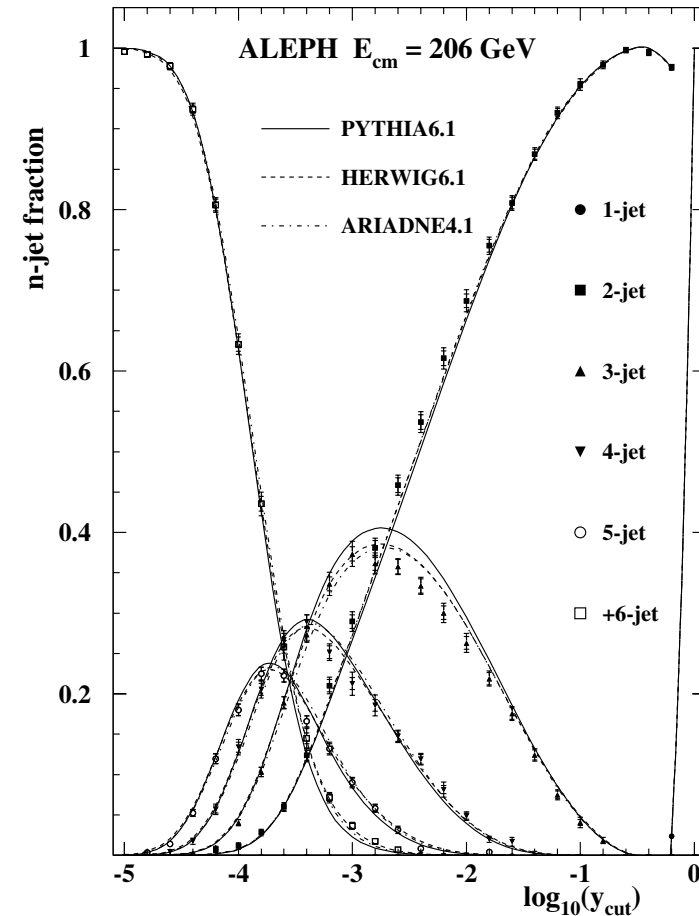
# Jet Rates

- Experimental studies based on jet rates:

$$R_n = \frac{n - \text{jet cross section}}{\text{total hadronic cross section}}$$

with  $n = 2, 3, 4, 5$ .

[ALEPH Collaboration, 2004]

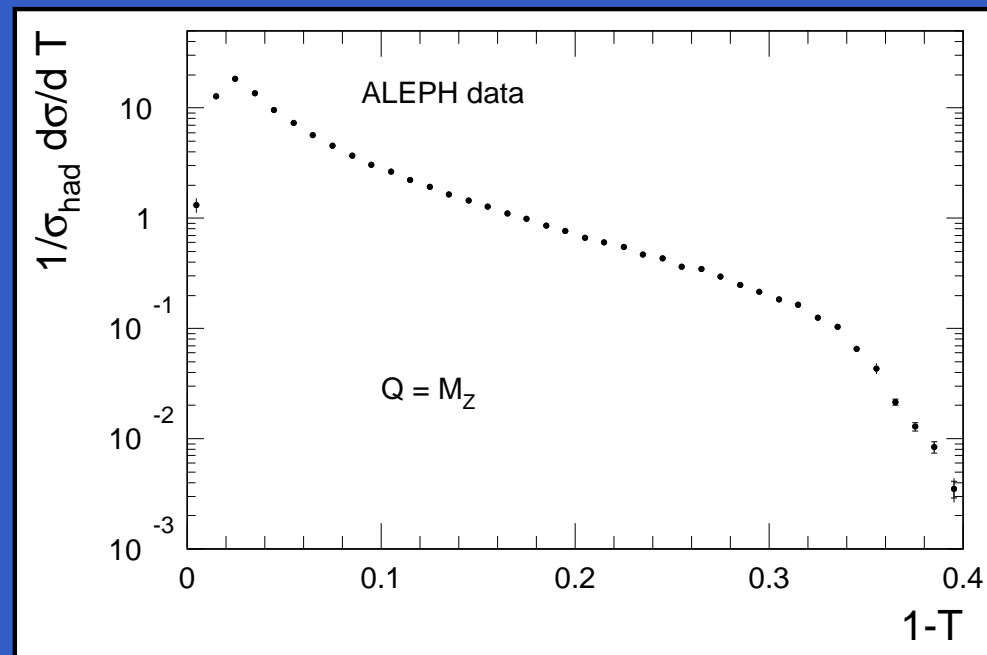
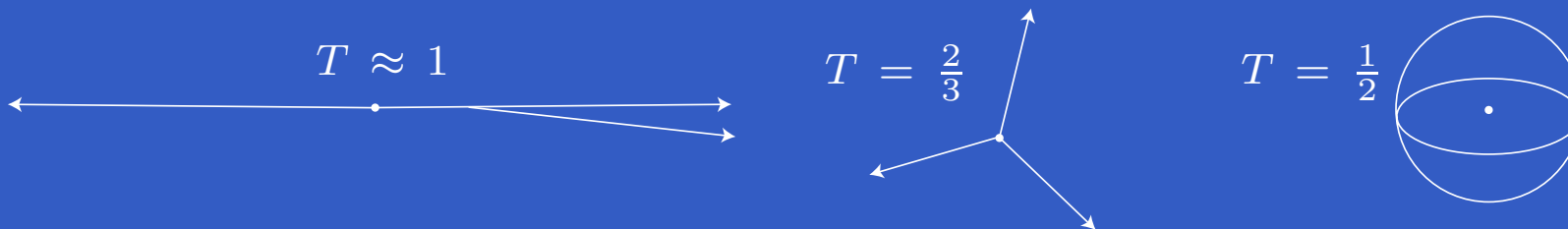


**Fig. 7.** Measured  $n$ -jet fractions for  $n = 1, 2, 3, 4, 5$  and  $n \geq 6$  and the predictions of Monte Carlo models, at a centre-of-mass energy of 206 GeV

# Event-Shape Observables

- Parametrize geometrical properties of an event,
- canonical example: **Thrust**

$$T = \max_{\vec{n}} \frac{\sum_i |\vec{p}_i \cdot \vec{n}|}{\sum_i |\vec{p}_i|}$$



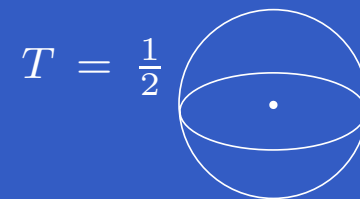
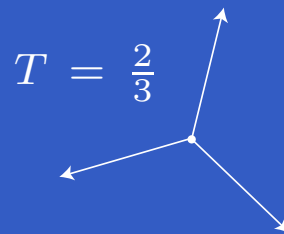


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- LEP standard set:

- Thrust: [Brandt,Fahri]

$$T = \max_{\vec{n}} \frac{\sum_i |\vec{p}_i \cdot \vec{n}|}{\sum_i |\vec{p}_i|}$$

- Heavy jet mass: [Clavelli,Wyler]

$$\rho = \max_i \frac{\left( \sum_{n \in H_i} |\vec{p}_n| \right)^2}{E_{\text{tot-vis.}}^2}$$

- C-parameter: EV of tensor [Parisi]

$$\Theta^{\alpha\beta} = \frac{1}{\sum_i |\vec{p}_i|} \sum_i \frac{p_i^\alpha p_i^\beta}{|\vec{p}_i|}$$

- Jet Broadenings: [Rakow,Webber]

$$B_i = \frac{\sum_{k \in H_i} |\vec{p}_k \times \vec{n}_T|}{2 \sum_j |\vec{p}_j|}$$

$$B_T = B_1 + B_2$$

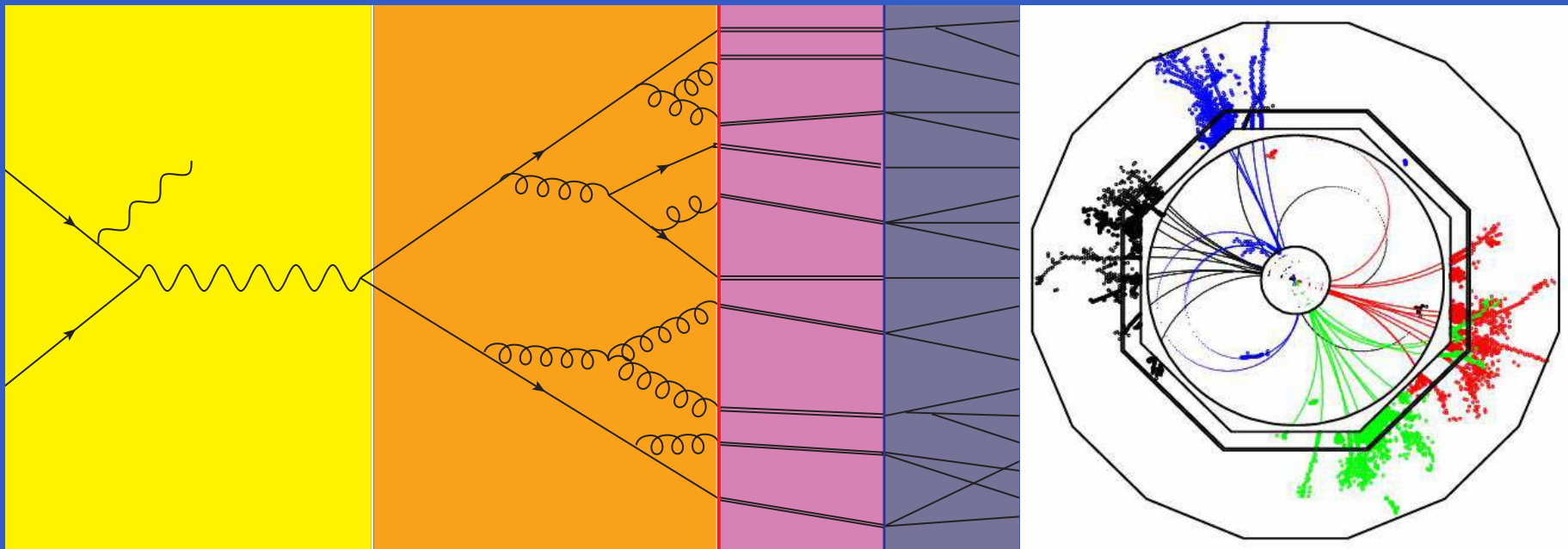
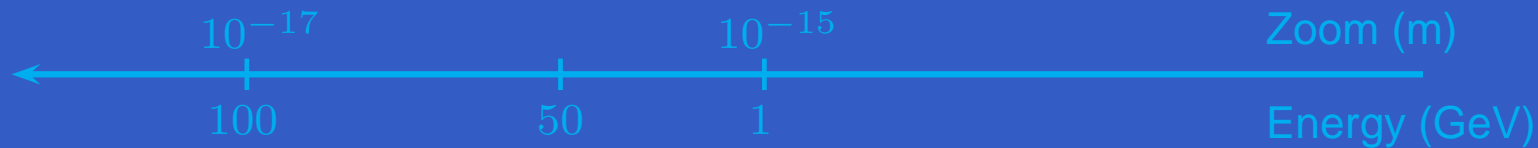
$$B_W = \max(B_1, B_2)$$

- Durham 2  $\rightarrow$  3 jet parameter:  $Y_3$

[Catani,Dokshitzer,Olsson,Turnock,Webber]

# QCD and Jets

- How are Jets of final state particles related to QCD?



Dynamics described by: **parton level**  
**QUARKS AND GLUONS**

**hadron level**  
**HADRONS**

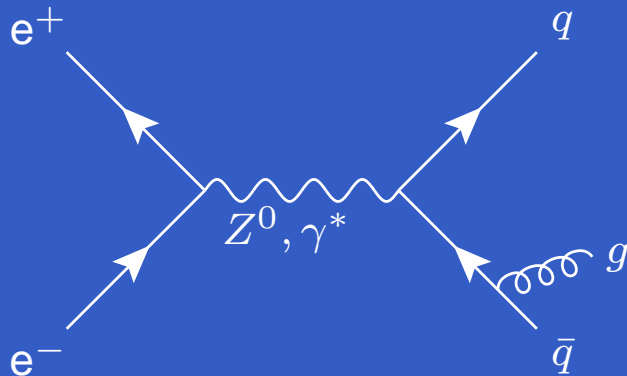
**detector level**  
**HADRONS**

Regime:

PERTURBATIVE QCD

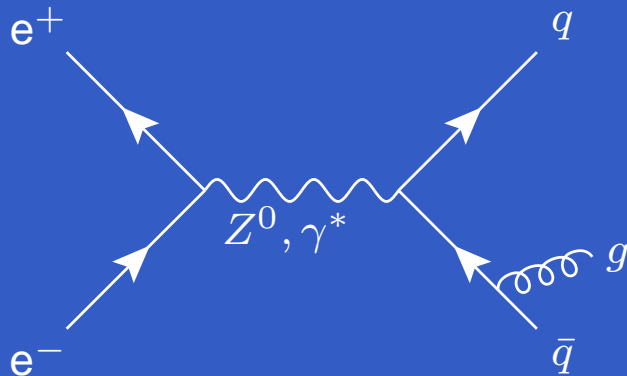
NON-PERTURBATIVE QCD

# 3 Jets at Leading Order



$$\frac{d\sigma}{dE_g d\cos\theta_{\bar{q}g}} \propto \sigma_0 \frac{\alpha_s}{2\pi} \frac{1}{E_g(1-\cos\theta_{\bar{q}g})}$$

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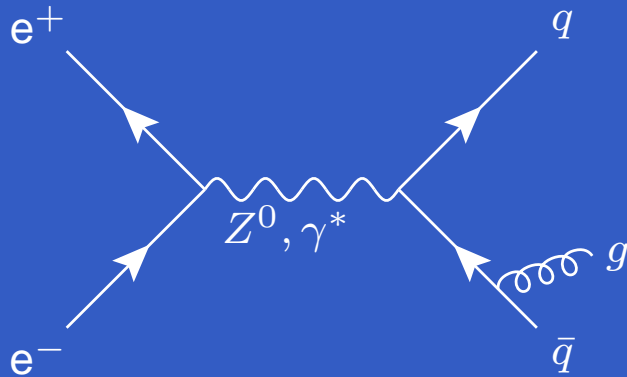


Born cross section for  $Z, \gamma \rightarrow q\bar{q}$

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Bremsstrahlung:  
enhancement for  $E_g \rightarrow 0$   
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● Jet rates and event-shape observables:

- deviation from 2-jet configuration is proportional to  $\alpha_s$
- enhancement in 2-jet region due to soft and collinear emissions
- suited also for theoretical pQCD since many are **IR and collinear safe**

# 3 Jets at NNLO

- Cross section for event shape  $y$  at NNLO described by:

( $y$  : event-shape variable,  $\bar{\alpha}_s = \frac{\alpha_s}{2\pi}$ ,  $x_\mu = \frac{\mu}{Q}$ ):

$$\frac{1}{\sigma_{\text{had}}} \frac{d\sigma}{dy}(y, Q, \mu) = \underbrace{\bar{\alpha}_s(\mu) \frac{dA}{dy}(y)}_{LO} + \underbrace{\bar{\alpha}_s^2(\mu) \frac{dB}{dy}(y, x_\mu)}_{NLO} + \underbrace{\bar{\alpha}_s^3(\mu) \frac{dC}{dy}(y, x_\mu)}_{NNLO} + \mathcal{O}(\bar{\alpha}_s^4).$$

<b>LO</b>	$\gamma^* \rightarrow q\bar{q}g$	<b>tree level</b>	<b>NNLO</b>	$\gamma^* \rightarrow q\bar{q}g$	<b>two loop</b>
				$\gamma^* \rightarrow q\bar{q}gg$	<b>one loop</b>
<b>NLO</b>	$\gamma^* \rightarrow q\bar{q}g$	<b>one loop</b>		$\gamma^* \rightarrow q\bar{q}q\bar{q}$	<b>one loop</b>
	$\gamma^* \rightarrow q\bar{q}gg$	<b>tree level</b>		$\gamma^* \rightarrow q\bar{q}q\bar{q}g$	<b>tree level</b>
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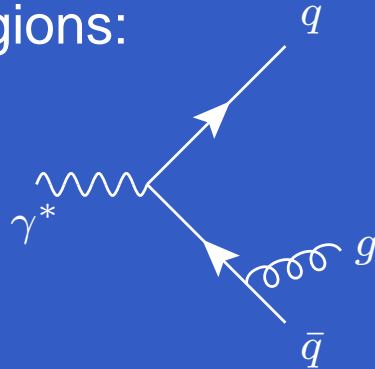
- Need subtraction scheme at NNLO: antenna subtraction

[Gehrmann, Gehrmann-De Ridder, Glover]

- Coefficient functions  $\frac{dA}{dy}$ ,  $\frac{dB}{dy}$ ,  $\frac{dC}{dy}$  are functions of  $\log\left(\frac{1}{y}\right)$ ,

# Fixed Order Calculations

- Logarithms are originated from integration over soft and collinear regions:



$$\frac{1}{\sigma_{\text{had}}} \frac{d\sigma}{dy} \propto \sigma_0 \frac{\alpha_s}{2\pi} \frac{1}{E_g (1 - \cos \theta_{\bar{q}g})}$$

Integral over phase space gives:

$$\frac{d\sigma}{dy} \propto \int \frac{dE_g}{E_g} \frac{d\theta_{\bar{q}g}^2}{\theta_{\bar{q}g}^2} \delta(y - y(E_g, \theta_{\bar{q}g})) \propto \frac{1}{y} \log\left(\frac{1}{y}\right)$$

- They describe the enhancement due to **soft** and **collinear** emissions.
- In phase space regions where  $\alpha_s \log(1/y) \approx 1 \rightarrow$  need **resummation**.
- Matching** of fixed order and resummed calculation can be performed.

# Recent Theoretical Progress

- State-of-the-art at LEP times (until 2007):
  - fixed NLO calculations, [Ellis, Ross, Terrano; Kunszt, Nason; Giele, Glover; Catani, Seymour]
  - NLL resummation, [Catani, Trentadue, Turnock, Webber; Banfi, Salam, Zanderighi].



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- Very important progress in the last three years:
  - for all event-shape observables (in particular LEP standard set):
    - fixed NNLO computation of jet rates and event-shape observables [Gehrmann, Gehrmann-De Ridder, Glover, Heinrich, Weinzierl]
    - matching of NNLO+NLLA [Gehrmann, Stenzel, G.L.]
    - non-perturbative corrections to moments at NNLO [Gehrmann, Jaquier, G.L.]
  - for single event shapes:
    - T:  $N^3$ LL resummation in SCET and matching with NNLO, [Becher, Schwartz]
    - T: non-perturbative corrections to NNLO+NLLA distribution, [Davison, Webber]
    - T: power corrections to  $N^3$ LL+NNLO, [Abbate, Fickinger, Hoang, Mateu, Steward]
    - MH:  $N^3$ LL resummation in SCET and matching with NNLO, [Chien, Schwartz]

# Determination of $\alpha_S$

## Recent works:

- $\alpha_S$  fit from NNLO and NNLO+NLLA distributions (ALEPH),  
[Dissertori, Gehrmann-De Ridder, Gehrmann, Glover, Heinrich, G. L., Stenzel.]
- $\alpha_S$  fit from NNLO and NNLO+NLLA distributions (JADE),  
[Bethke, Kluth, Pahl, Schieck and JADE Collaboration.]
- $\alpha_S$  fit using NNLO for moments (JADE/OPAL), [Gehrmann, Jaquier, G. L.]
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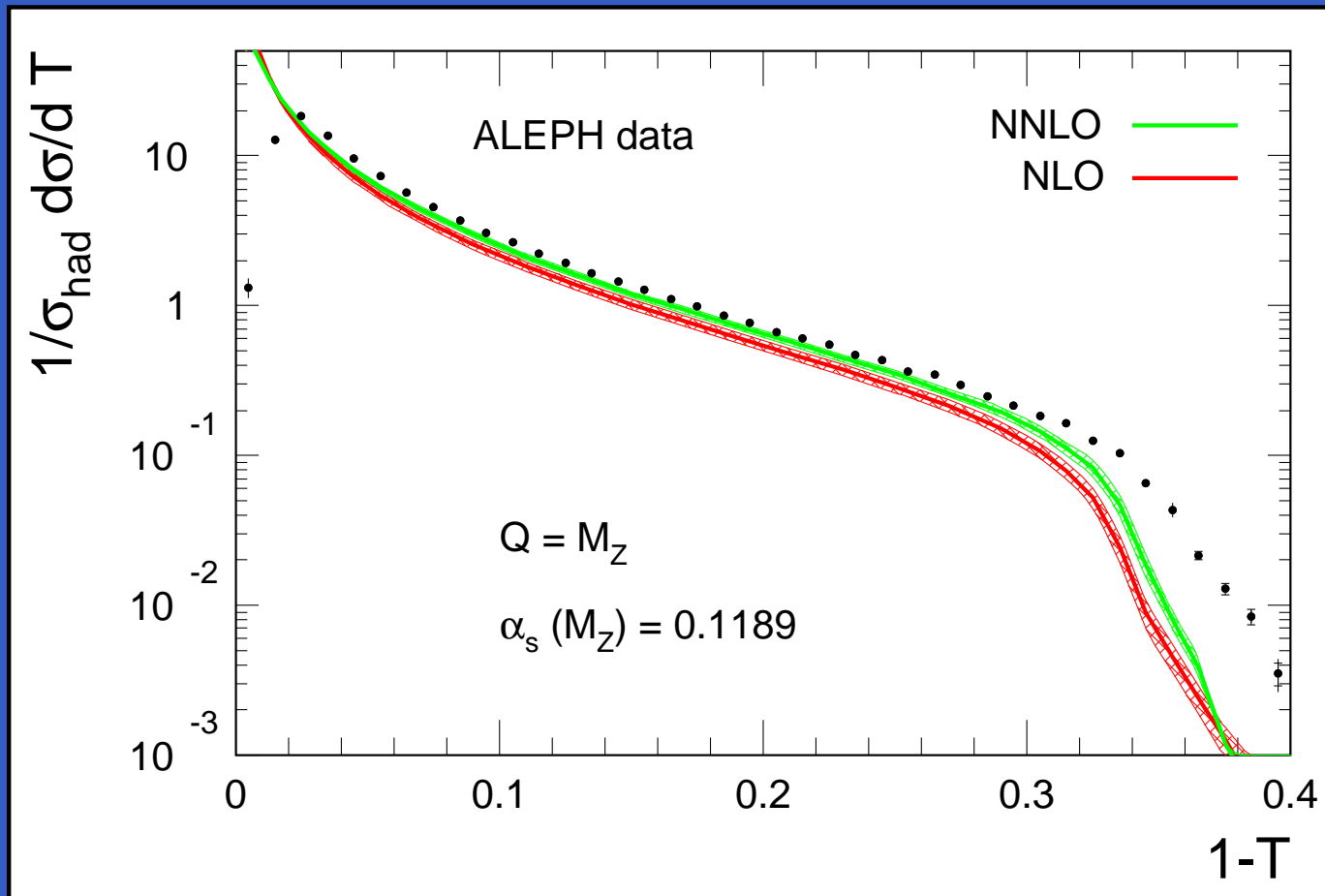
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$\alpha_s$  from

# Event-Shape Distributions

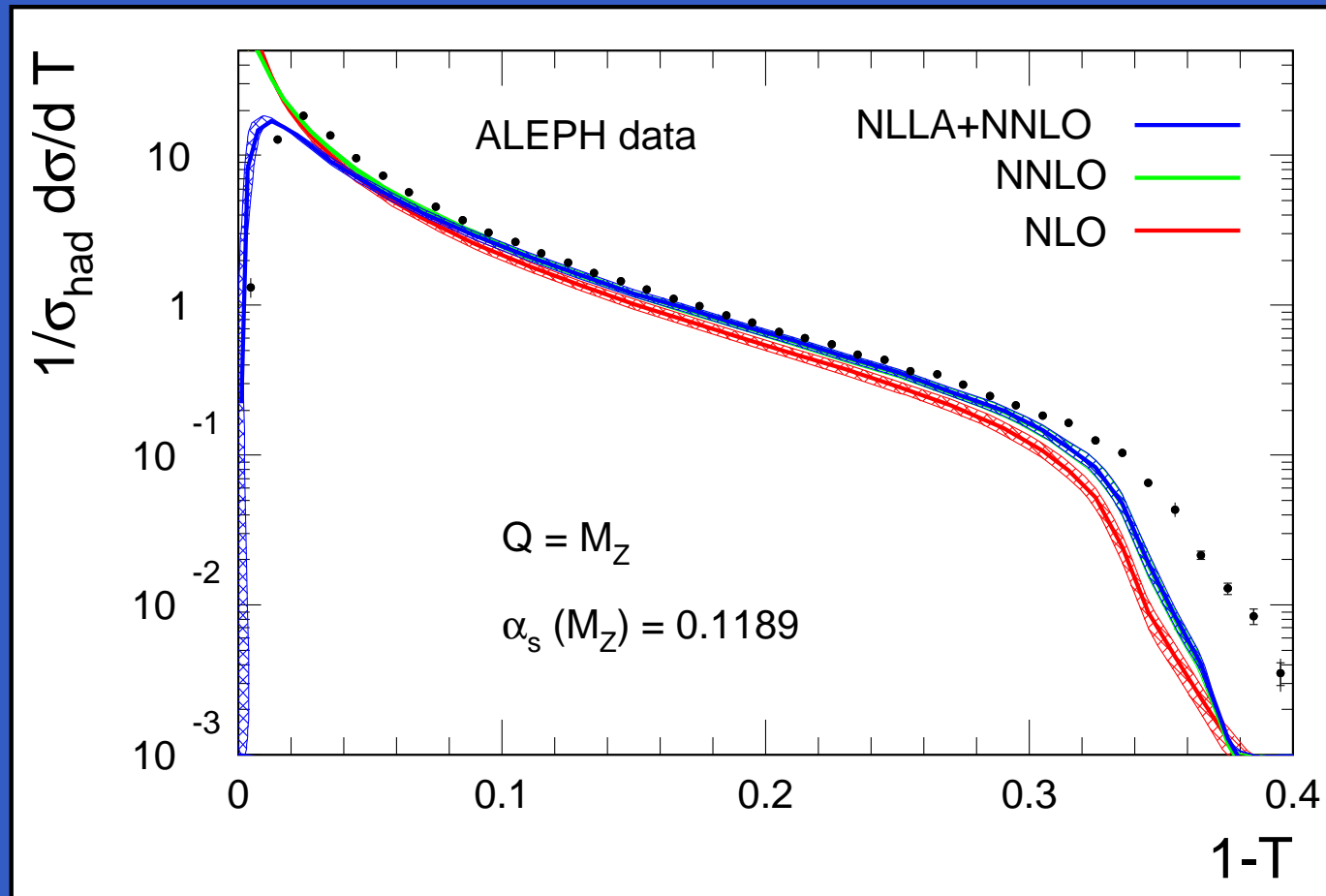
# Thrust: Theory vs Data

- NNLO result describes data better than NLO



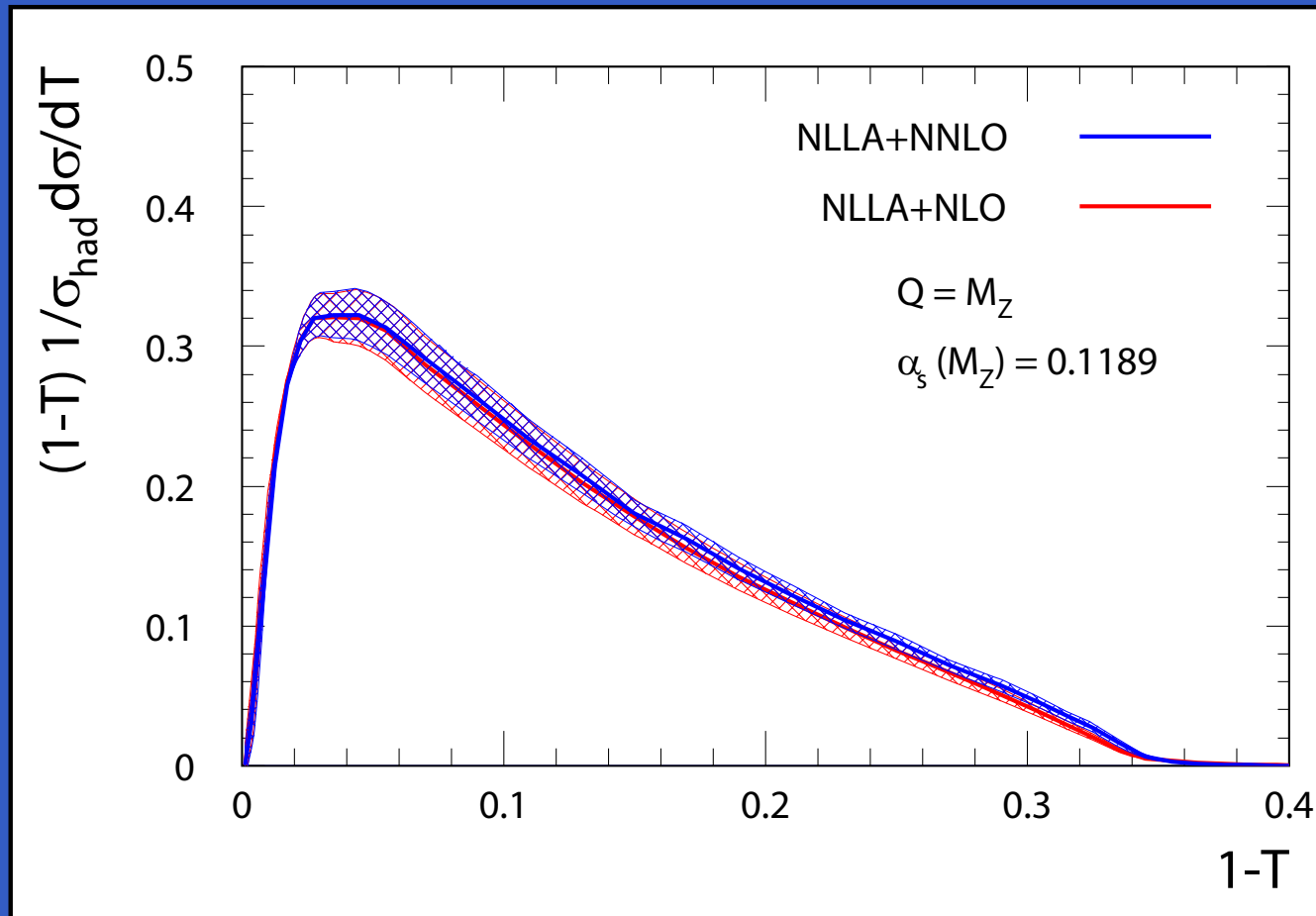
# Thrust: Theory vs Data

- Addition of resummation improves description in 2-jet region



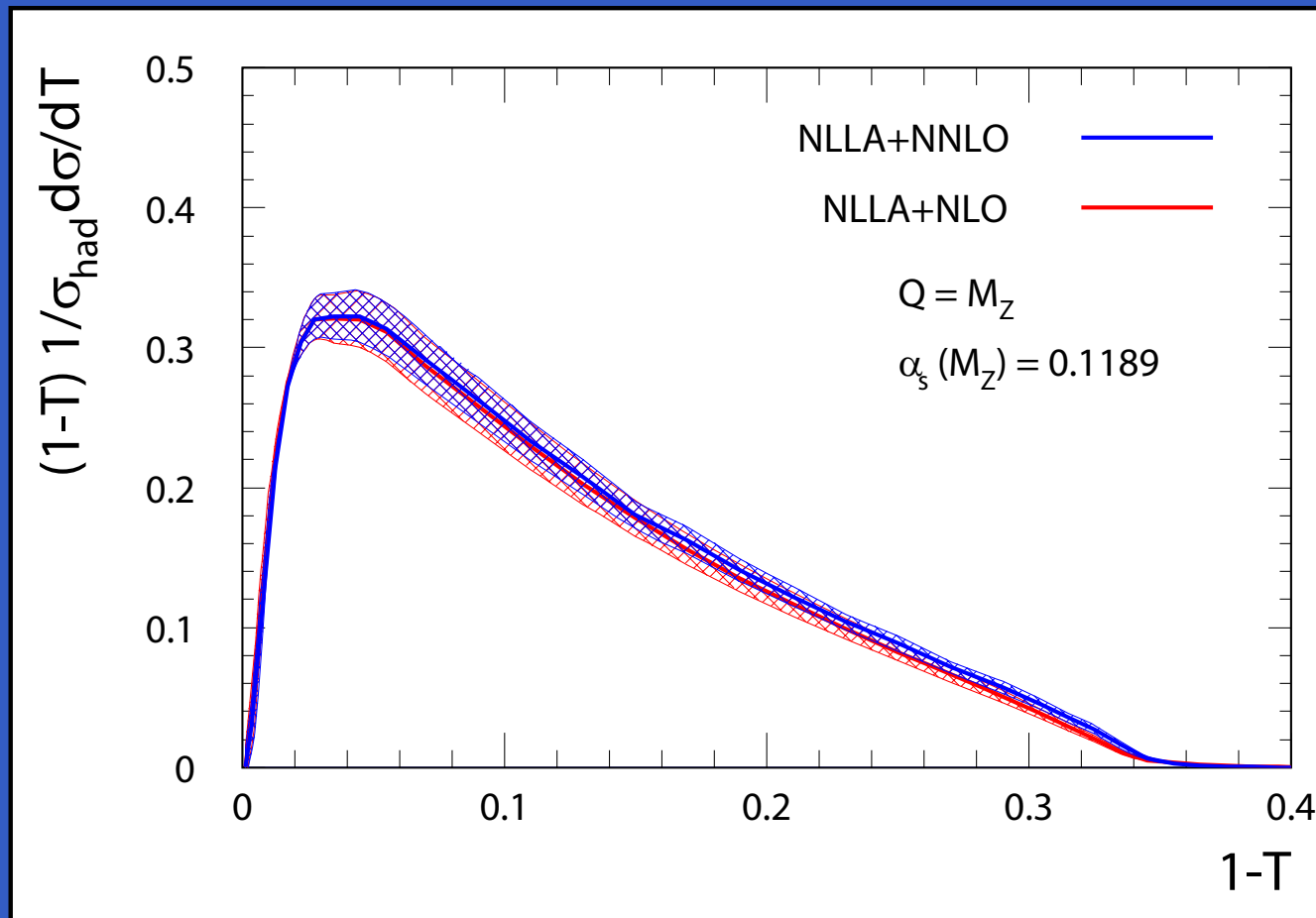
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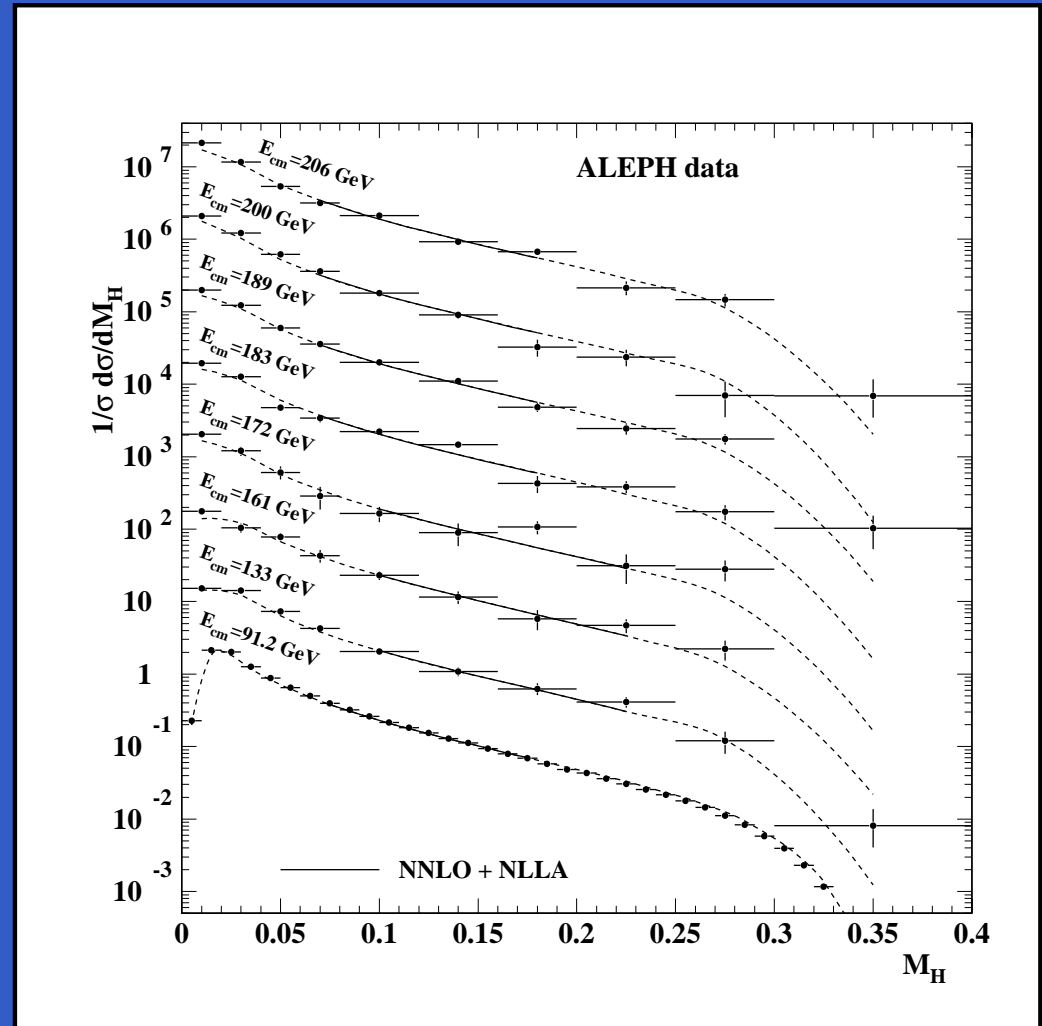


Can perform fit: 6 event-shape observables at 8 different energies



# Determination of $\alpha_S$ : NLLA+NNLO fits

- data are fit in the central part of the event shape distribution,
- only statistical uncertainties are included in the  $\chi^2$ .
- good fit quality (but includes still large statistical uncertainties of C-coefficient)



# Uncertainties in $\alpha_S$ from distributions

## ● Experimental uncertainties:

- track reconstr., event selection, detector corrections: cut variations or MC
- background and ISR (LEP2),  $\sim 1\%$

## ● Hadronization uncertainties:

- difference between various models for hadronization:  $\sim 0.7 - 1.5\%$

Pythia (String frag.), Herwig (Cluster frag.), Ariadne (Dipole + String frag.),

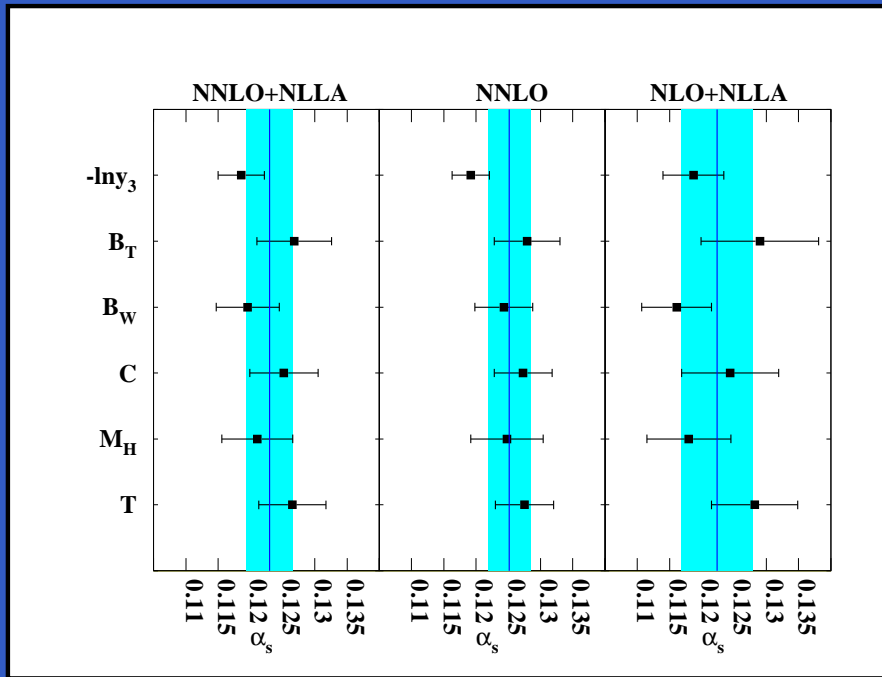
## ● Theoretical uncertainties (pQCD and resummation):

- variation of theoretical parameters:  $x_\mu, \dots$   $\sim 3.5 - 5\%$
- uncertainty for b-quark mass correction.

## ● Uncertainty band method to estimate missing higher orders

[Ford, Jones, Salam, Stenzel, Wicke.]

# Extraction of $\alpha_s$ : NNLO+NLLA

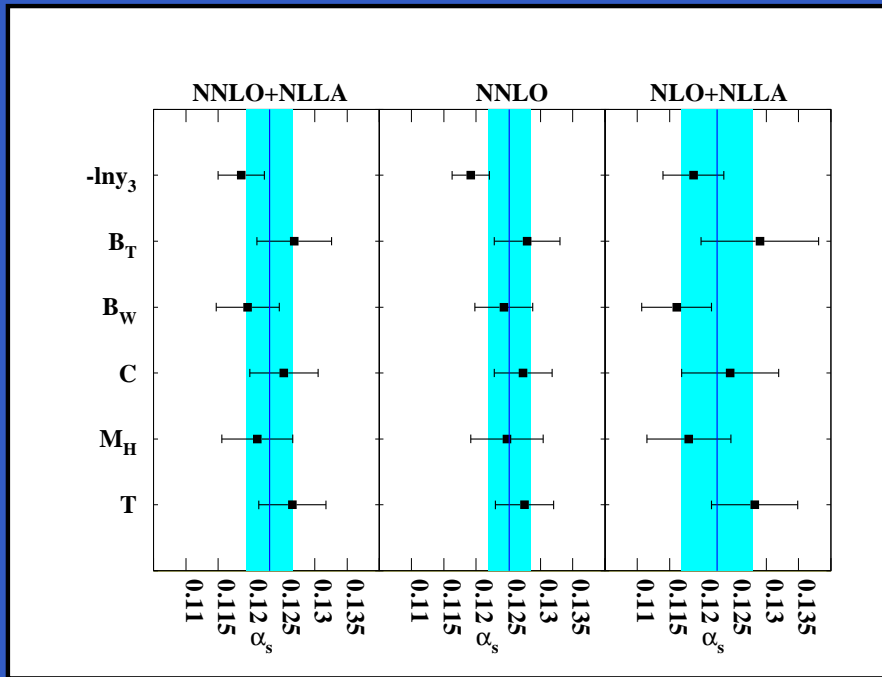


- reduced scatter among variables at NNLO
- reduced scale uncertainty compared to NLO+NLLA
- scale uncertainty increase from NNLO to NNLO+NLLA  $\rightarrow$  two-loop running not compensated in resummation

$$\alpha_s (M_Z) = 0.1224 \pm 0.0009(\text{stat.}) \pm 0.0009(\text{exp.}) \pm 0.0012(\text{had.}) \pm 0.0035(\text{theo.})$$

- Two classes of observables:
  - T, C-par.,  $B_{\text{tot}}$ : higher fit result  $\rightarrow$  sizeable missing higher order
  - $-\ln y_3$ ,  $B_w$ ,  $M_H$ : lower fit result,  $\rightarrow$  good convergence of pert. expansion

# Extraction of $\alpha_s$ : NNLO+NLLA



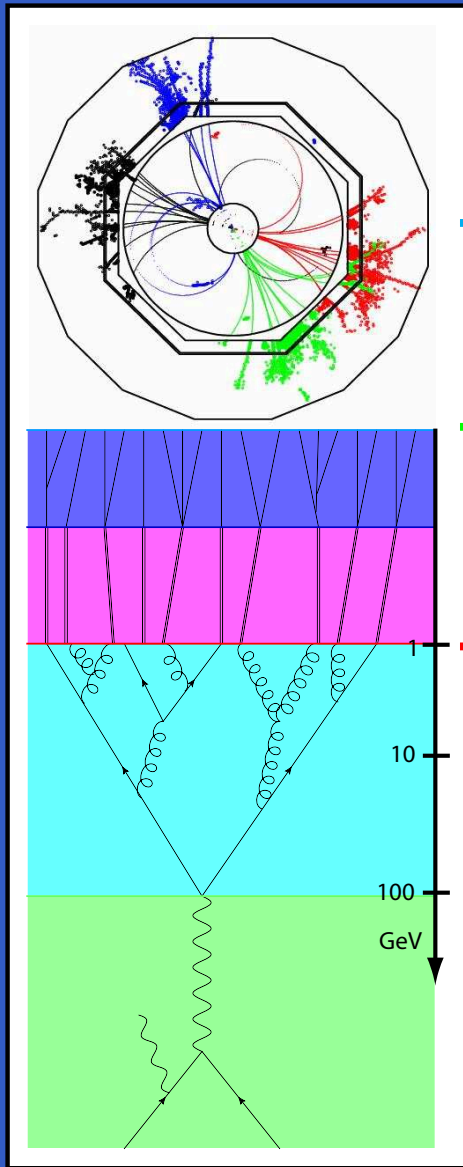
- reduced scatter among variables at NNLO
- reduced scale uncertainty compared to NLO+NLLA
- scale uncertainty increase from NNLO to NNLO+NLLA  $\rightarrow$  two-loop running not compensated in resummation

$$\alpha_s (M_Z) = 0.1224 \pm 0.0009(\text{stat.}) \pm 0.0009(\text{exp.}) \pm 0.0012(\text{had.}) \pm 0.0035(\text{theo.})$$

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What about hadronization corrections?

# Determination of $\alpha_s$ : Hadronization

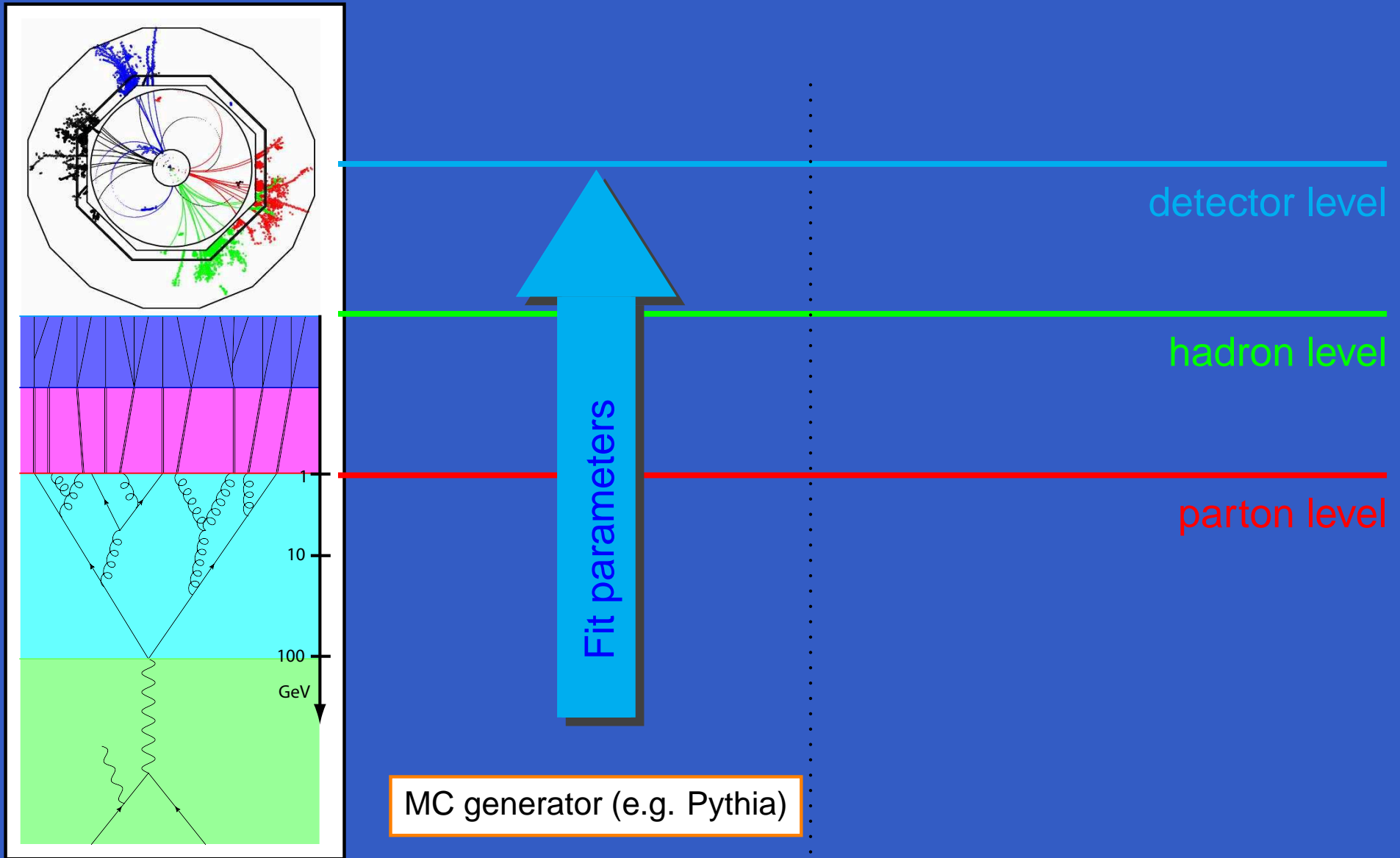


detector level

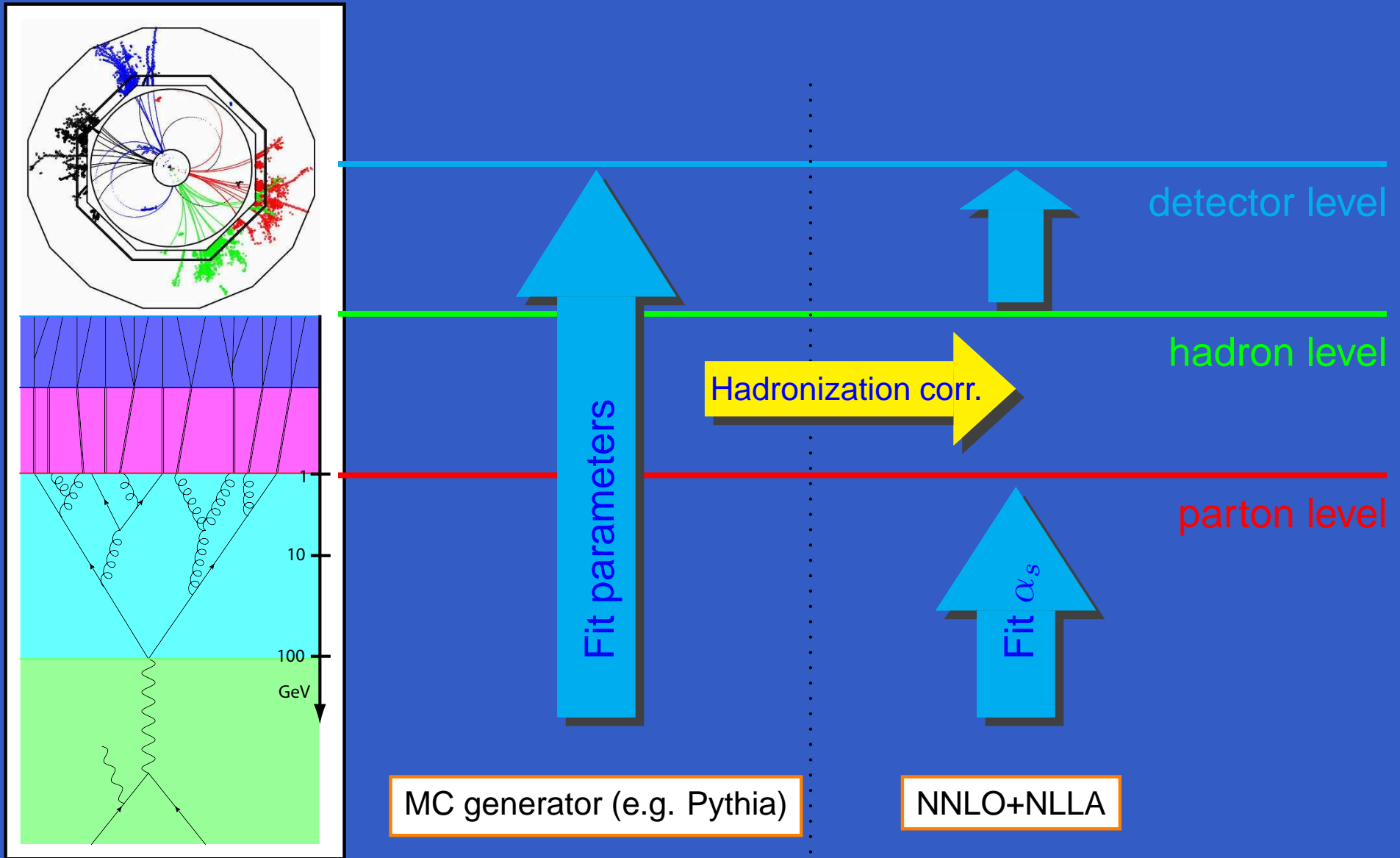
hadron level

parton level

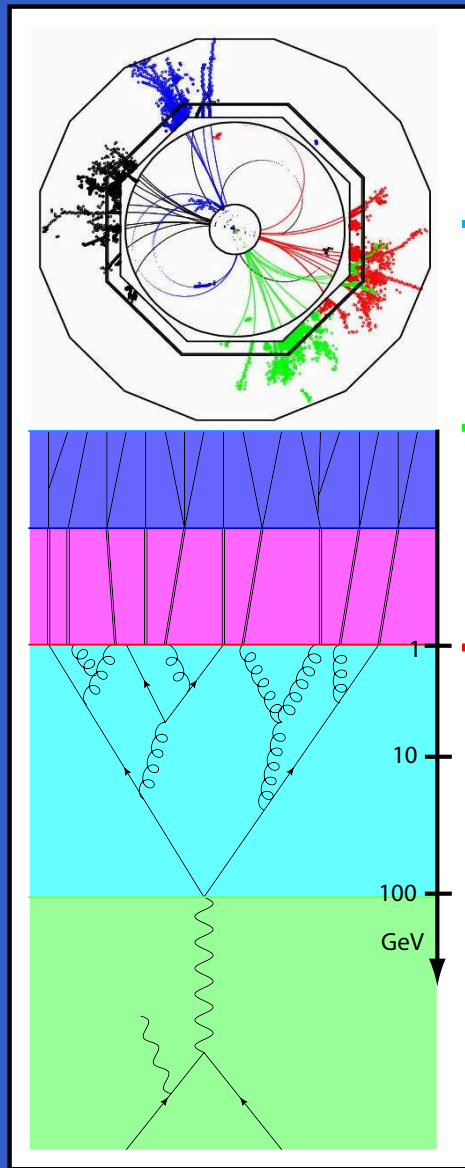
# Determination of $\alpha_s$ : Hadronization



# Determination of $\alpha_s$ : Hadronization



# Determination of $\alpha_s$ : Hadronization



Problematic if parton level in MC and pQCD predictions are very different

MC generator (e.g. Pythia)

NNLO+NLLA

Fit parameters

Fit  $\alpha_s$

Hadronization corr.

detector level

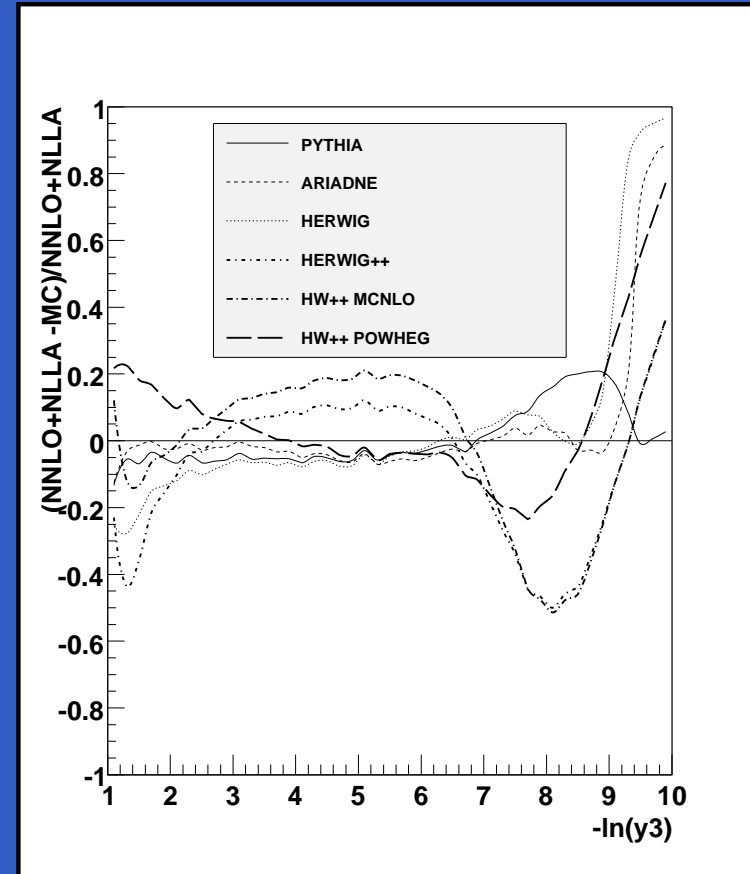
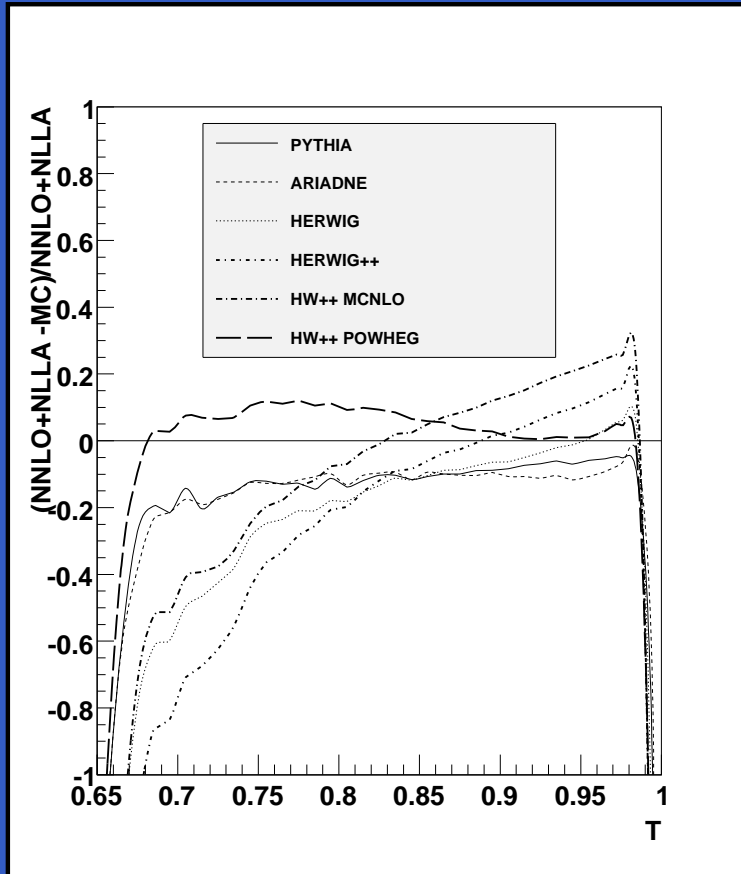
hadron level

parton level



# Determination of $\alpha_s$ : Hadronization

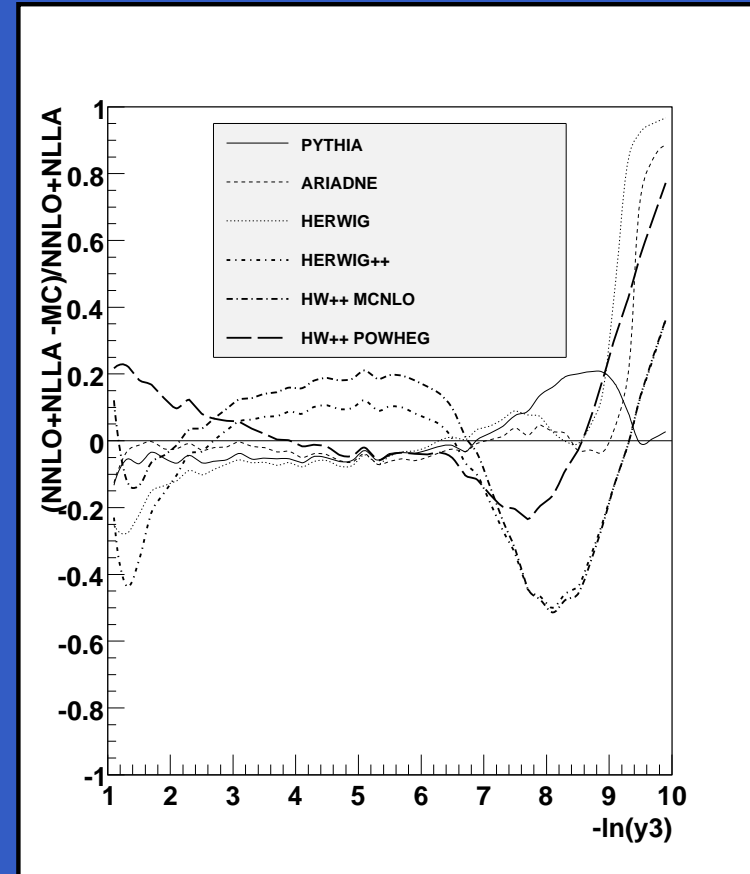
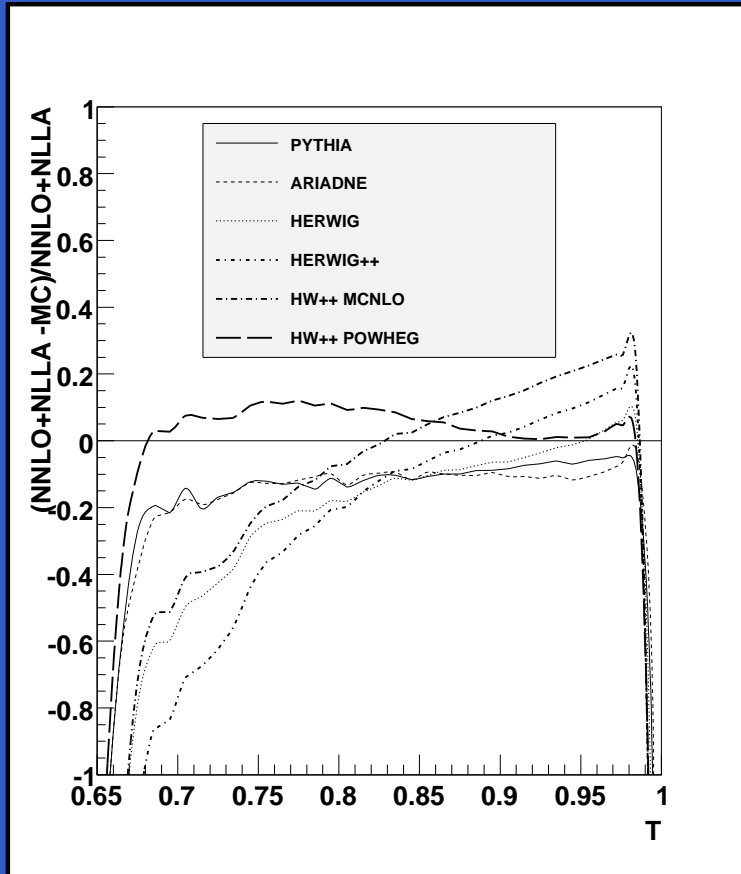
- Comparison with modern MC generators:



$\alpha_s (M_z)$	$T$	$C$	$M_H$	$B_W$	$B_T$	$-\ln y_3$
PYTHIA	0.1266	0.1252	0.1211	0.1196	0.1268	0.1186
$\chi^2/N_{dof}$	0.16	0.47	4.4	4.4	0.84	1.89
HW++ POWHEG	0.1189	0.1179	0.1236	0.1169	0.1224	0.1142
$\chi^2/N_{dof}$	1.46	2.55	3.8	3.9	1.54	0.56

# Determination of $\alpha_s$ : Hadronization

- Comparison with modern MC generators:

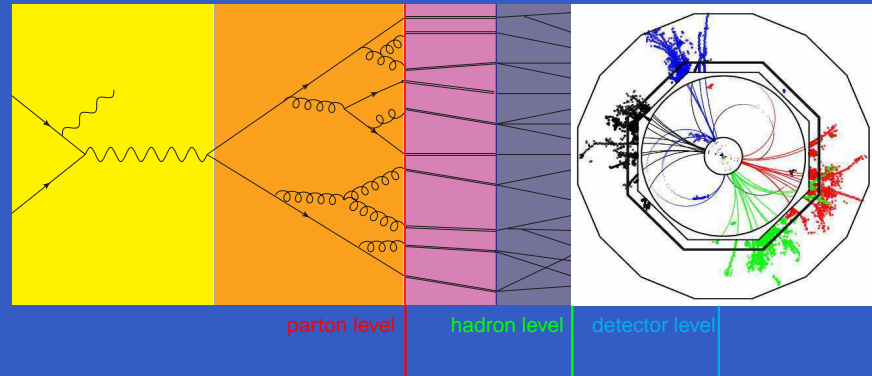
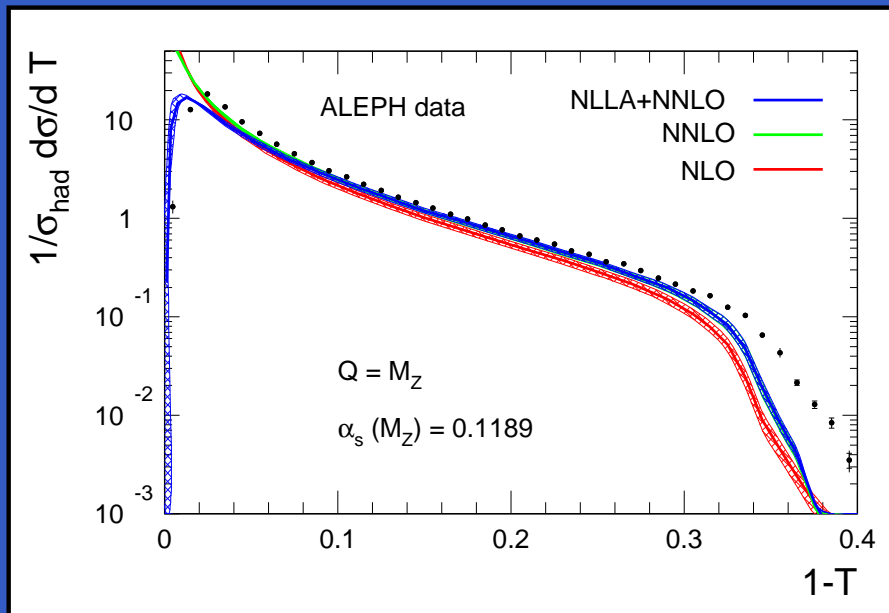


- Thrust: MC parton level prediction larger than in NNLO+NLLA
- Pythia parameters tuned such that missing HO terms are (over-)compensated and hadronization corrections are effectively too small

# Hadronization corrections

- NLLA+NNLO results are big improvement in description of data...

... however still important differences for  $(1 - T) \rightarrow 0$ .

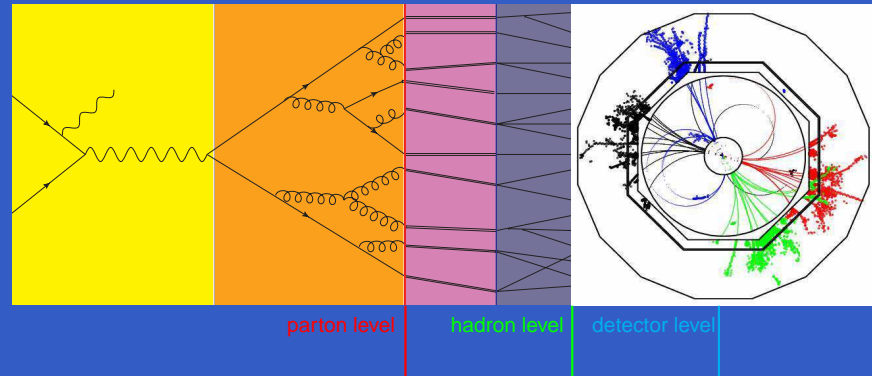
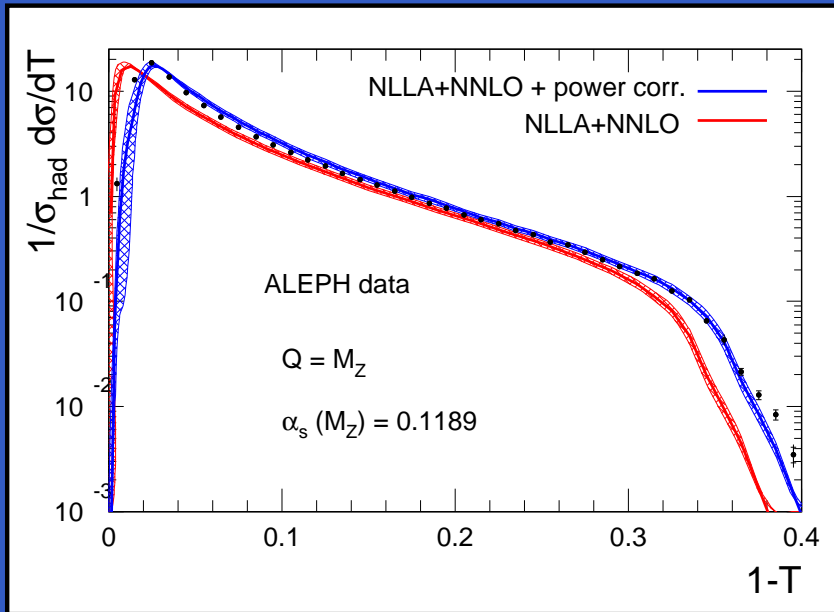


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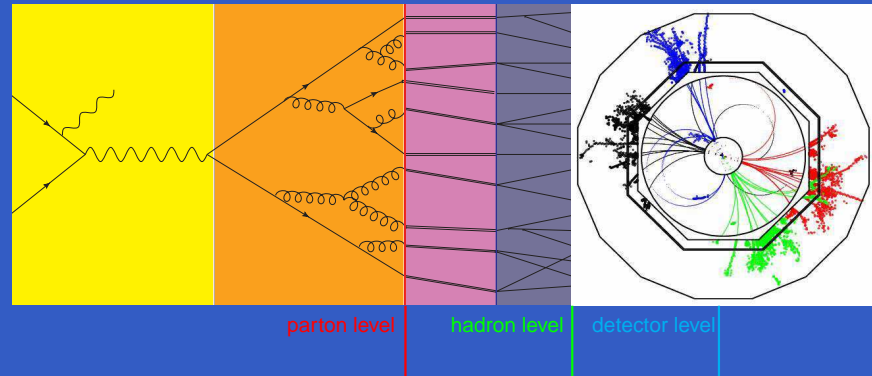
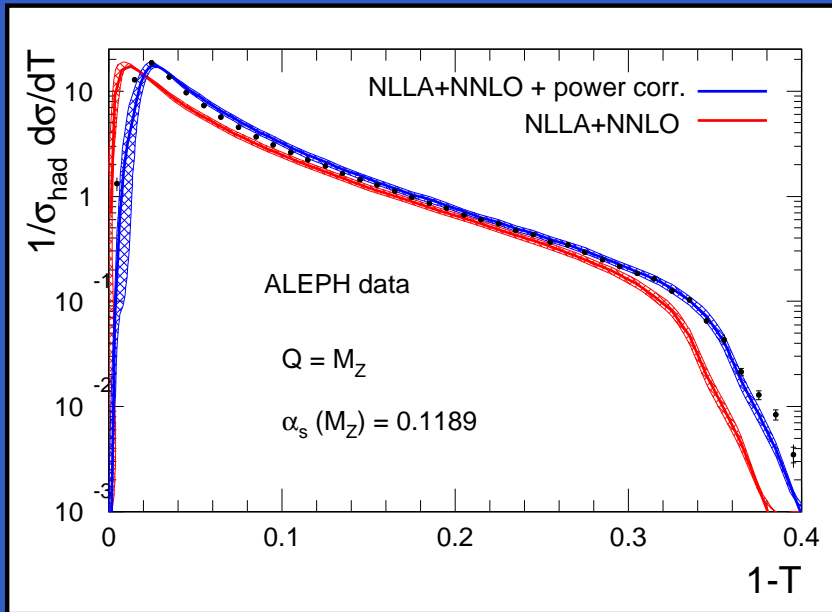
PRELIMINARY



# Hadronization corrections

- NLLA+NNLO results are big improvement in description of data...
- ... however still important differences for  $(1 - T) \rightarrow 0$ .

PRELIMINARY



- Distortion caused by hadronization corrections can account for part of the difference,
- however effects of hadronization corrections can be analyzed better by studying **moments**.

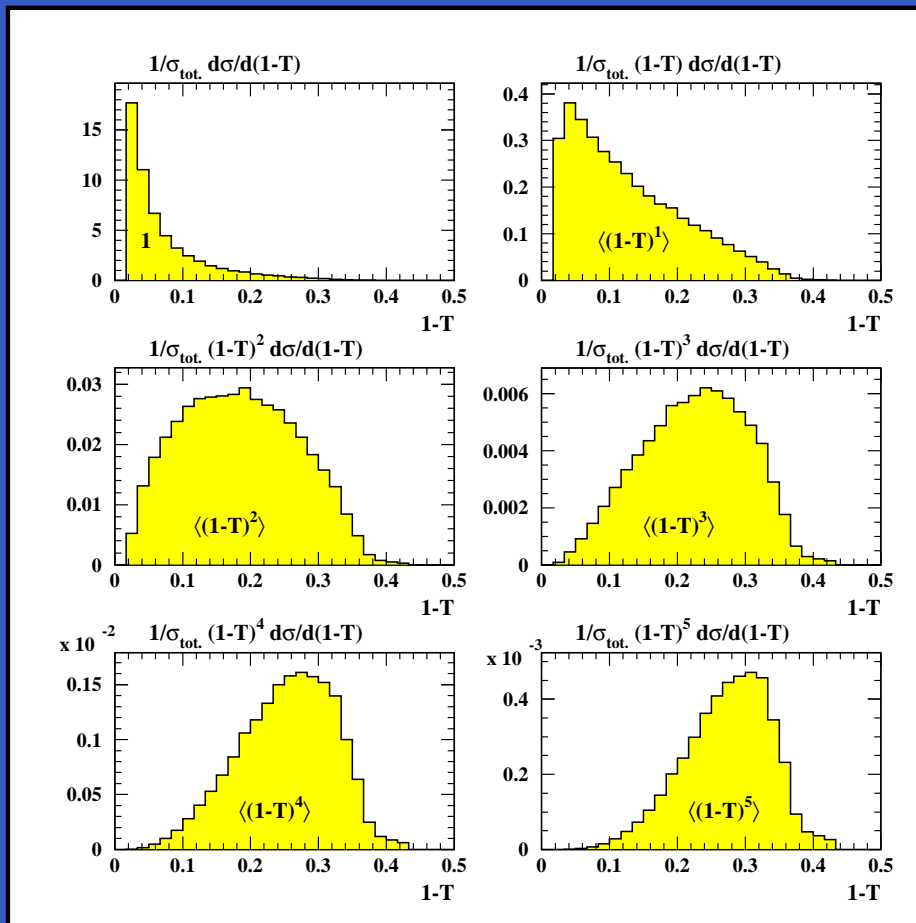
$\alpha_s$  from

# Event-Shape Moments

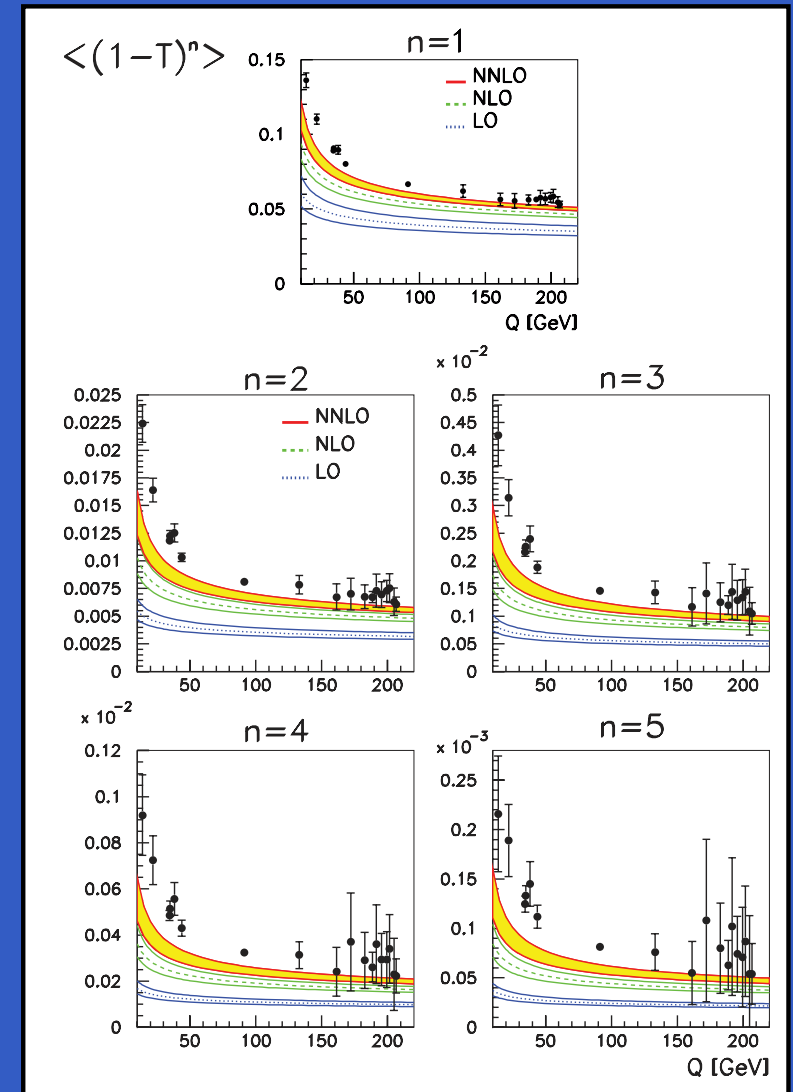
# Moments of Event Shapes

- $n$ -th moment of event-shape observable  $y$  defined by:

$$\langle y^n \rangle = \frac{1}{\sigma_{\text{had}}} \int_0^{y_{\text{max}}} y^n \frac{d\sigma}{dy} dy$$



[C. Pasca] (Note: The image shows 'Pani' in the original, likely a typo for Pasca)



[A. Gehrmann-De Ridder, T. Gehrmann, N. Glover, G. Heinrich]

# Moments of Event Shapes

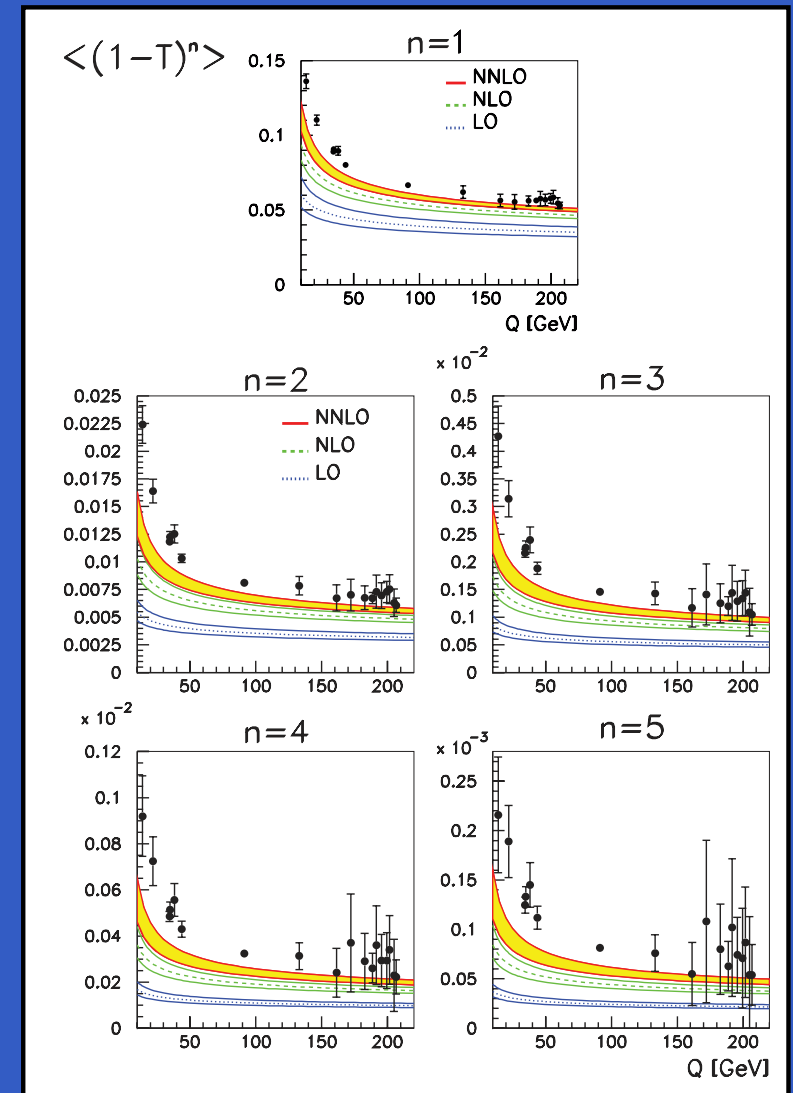
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$$\langle y^n \rangle = \frac{1}{\sigma_{\text{had}}} \int_0^{y_{\text{max}}} y^n \frac{d\sigma}{dy} dy$$

- Divide perturbative and non-perturbative contributions:

$$\langle y^n \rangle = \langle y^n \rangle_{\text{pt}} + \langle y^n \rangle_{\text{np}},$$

- discrepancy between parton level predictions and data increase with decreasing energy,
- can add non-perturbative corrections either using an analytical model or a Monte Carlo event generator.



[A. Gehrmann-De Ridder, T. Gehrmann, N. Glover, G. Heinrich]



# The Dispersive Model

- Idea:  
replace the strong coupling constant below a cut-off scale  $\mu_I \approx 2 \text{ GeV}$  by an effective coupling: [J. Dokshitzer, G. Marchesini B. Webber]

$$\frac{1}{\mu_I} \int_0^{\mu_I} dQ \alpha_{\text{eff}}(Q^2) = \alpha_0(\mu_I).$$

- Non-perturbative corrections result in a shift of the distribution:

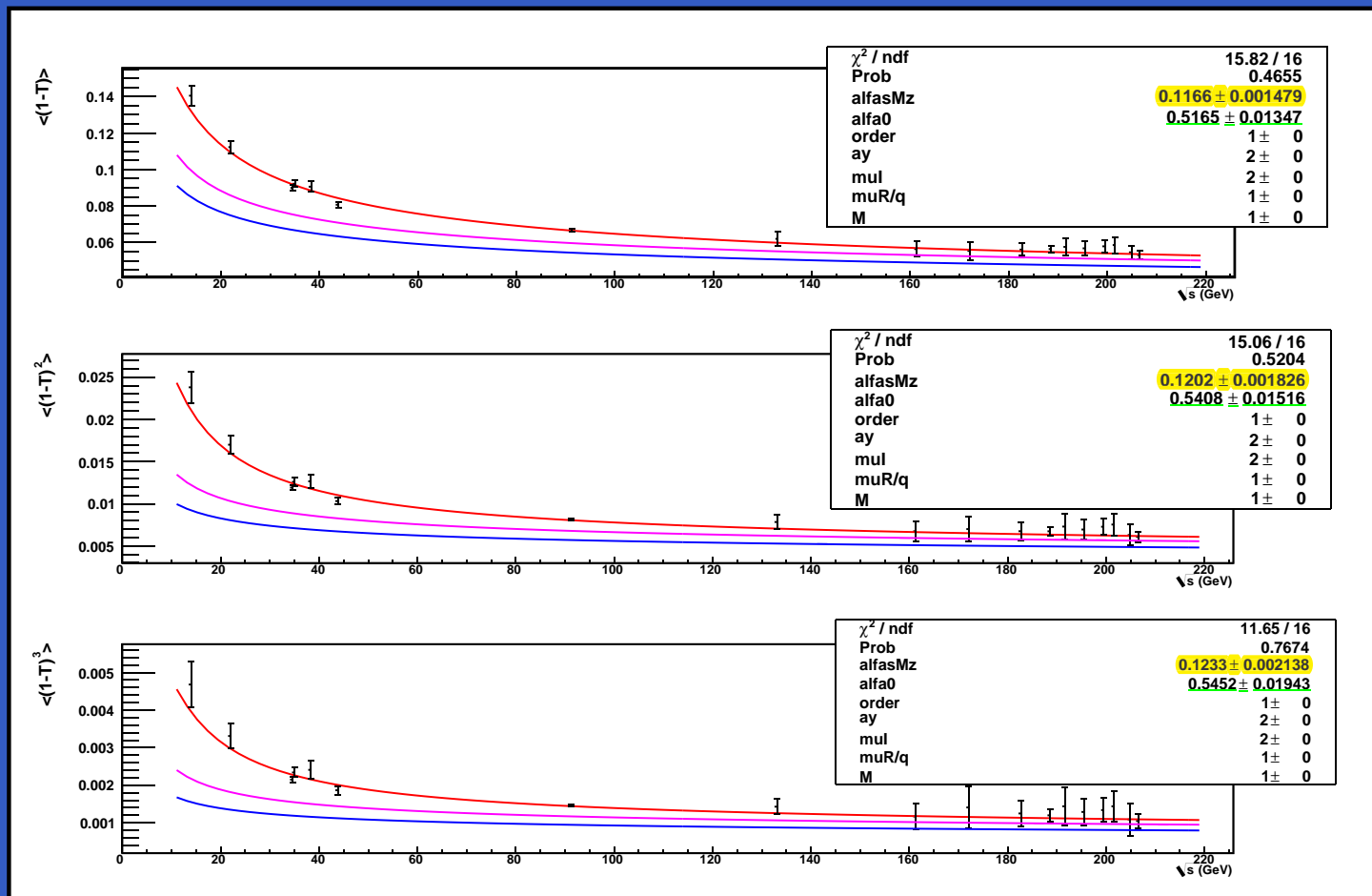
$$\frac{d\sigma}{dy} = \frac{d\sigma_{\text{pt}}}{dy}(y - a_y P) \Rightarrow \langle y^n \rangle = \int_0^{y_{\text{max}}} dy (y + a_y P)^n \frac{1}{\sigma_{\text{tot}}} \frac{d\sigma_{\text{pt}}}{dy}(y).$$

- Analytical power correction  $P = P(\mu_R)$  extended to NNLO, [T. Gehrmann, M. Jaquier, G.L.]

$$P = \frac{4C_F}{\pi^2} \mathcal{M} \left\{ \alpha_0 - \left[ \alpha_s(\mu_R) + \frac{\beta_0}{\pi} \left( 1 + \ln \left( \frac{\mu_R}{\mu_I} \right) + \frac{K}{2\beta_0} \right) \alpha_s^2(\mu_R) + \right. \right. \\ \left. \left( 2\beta_1 \left( 1 + \ln \left( \frac{\mu_R}{\mu_I} \right) + \frac{L}{2\beta_1} \right) + 8\beta_0^2 \left( 1 + \ln \left( \frac{\mu_R}{\mu_I} \right) + \frac{K}{2\beta_0} \right) \right. \right. \\ \left. \left. + 4\beta_0^2 \ln \left( \frac{\mu_R}{\mu_I} \right) \left( \ln \left( \frac{\mu_R}{\mu_I} \right) + \frac{K}{\beta_0} \right) \right] \frac{\alpha_s^3(\mu_R)}{4\pi^2} \right\} \times \frac{\mu_I}{Q}.$$

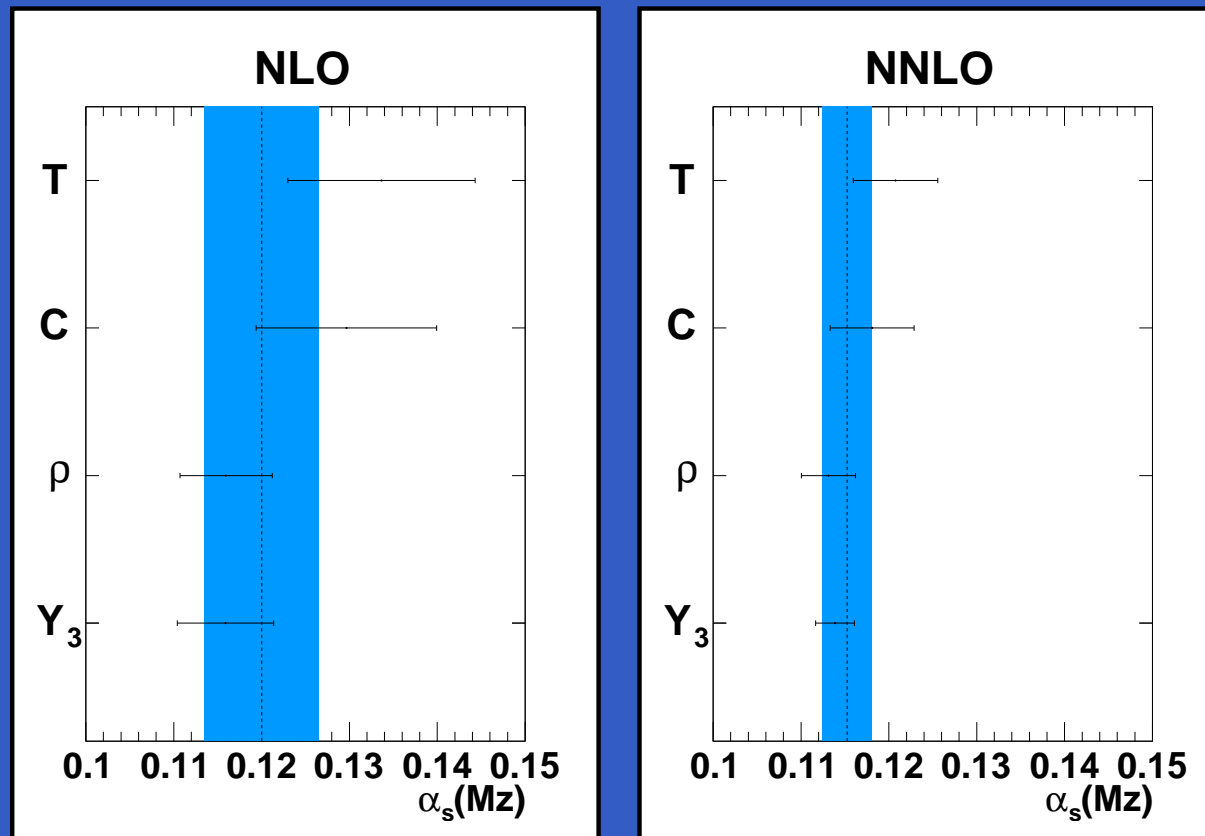
# The Dispersive Model

- Fit of  $\alpha_s$  and  $\alpha_0$  to JADE and OPAL data for  $n = 1, \dots, 5$ :
- total experimental error used in  $\chi^2$ ,
- theoretical uncertainty determined by varying  $\mu_R, \mu_I$  and  $\mathcal{M}$ .



# Determination of $\alpha_s$ Using Moments

Result from analytical power corrections:



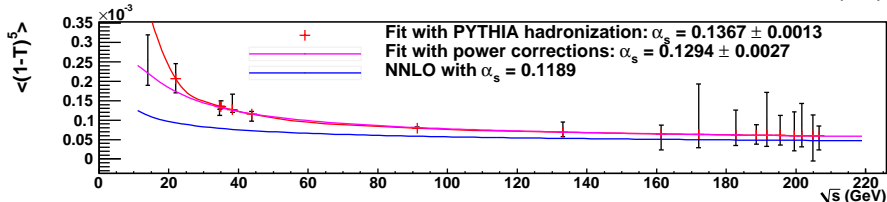
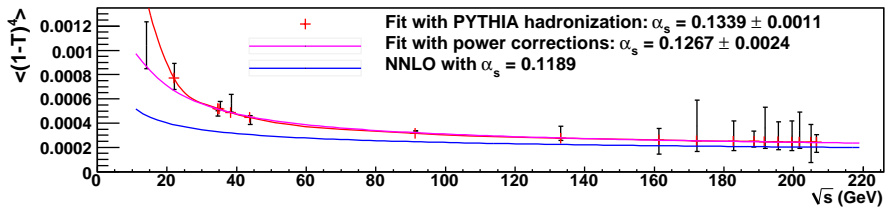
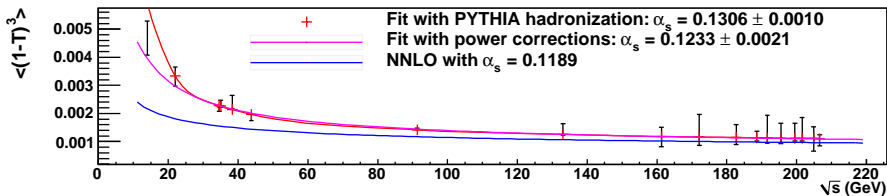
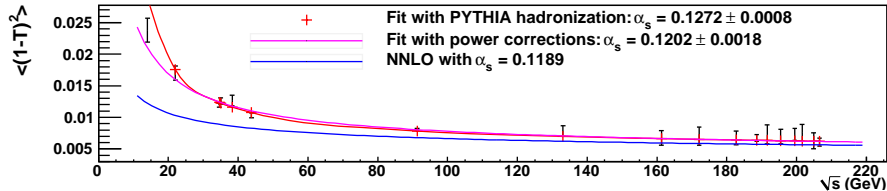
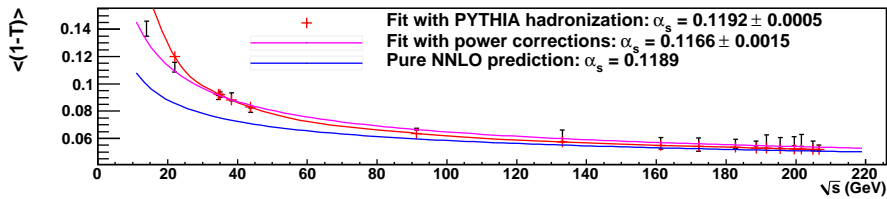
Combined result:

$$\alpha_s(M_Z) = 0.1153 \pm 0.0017(\text{exp}) \pm 0.0023(\text{th}),$$

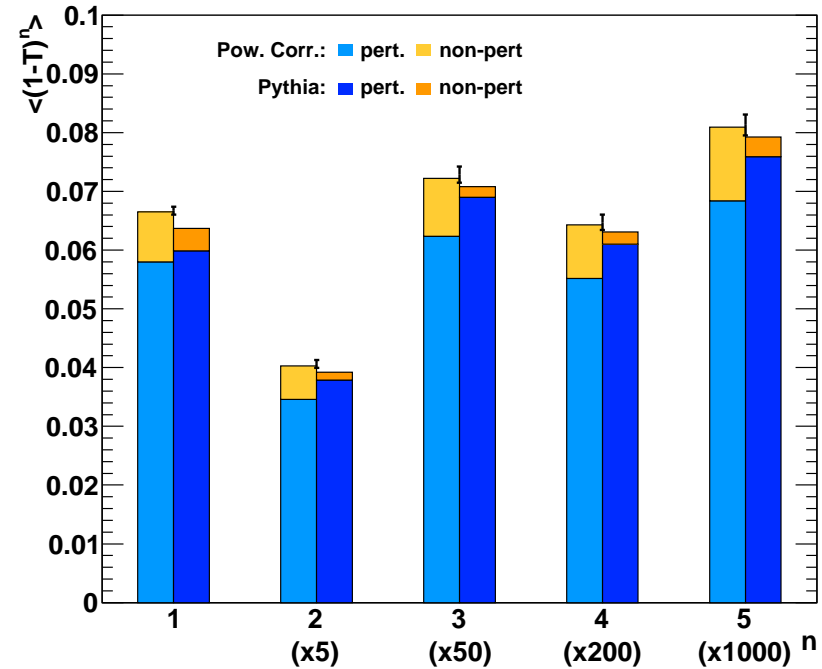
$$\alpha_0 = 0.5132 \pm 0.0115(\text{exp}) \pm 0.0381(\text{th}),$$

# Determination of $\alpha_S$ Using Moments

## Comparison with Monte Carlo:



$$\sqrt{s} = M_Z$$



With Monte Carlo:



smaller hadronization correction



higher partonic predictions

# $\alpha_s$ from 3-jet rates

# Determination of $\alpha_s$ using jet rates

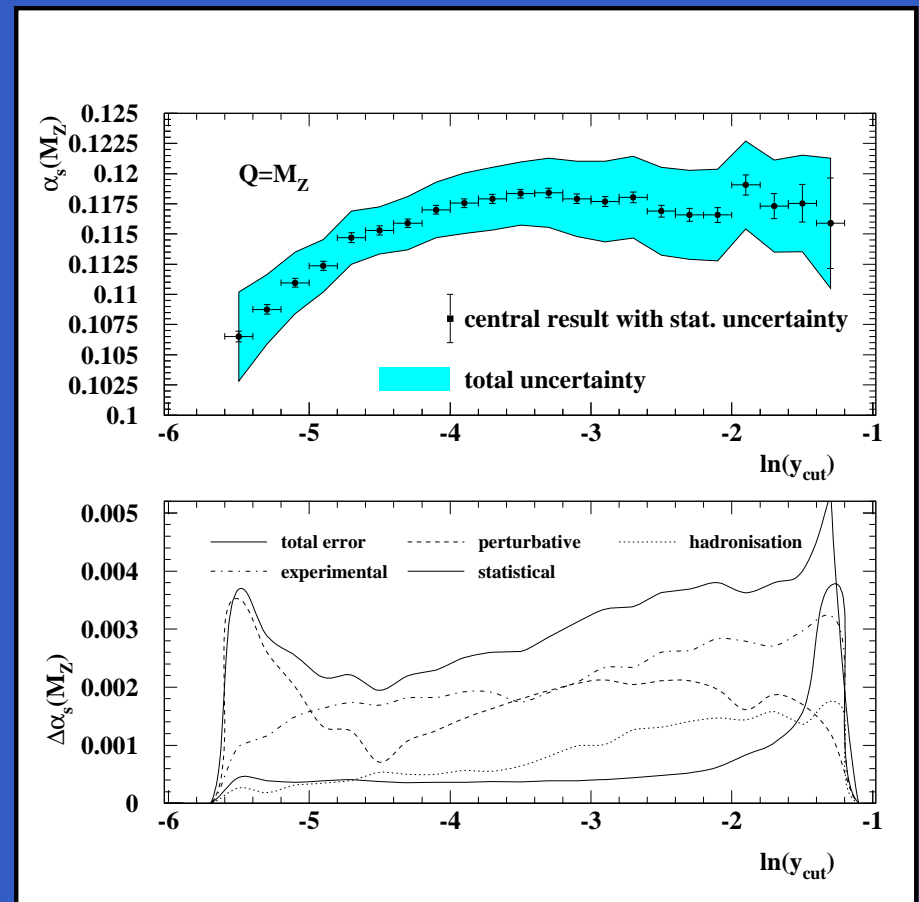
- In region  $10^{-1} > y_{\text{cut}} > 10^{-2}$  only very small hadronization corrections  $\rightarrow$  motivates a dedicated extraction of  $\alpha_s$

- Separated fits for  $-1.3 > \ln(y_{\text{cut}}) > -5.1$ ,
- stability up to  $\ln(y_{\text{cut}}) = -4.5$ ,  
(onset of large logarithms beyond),

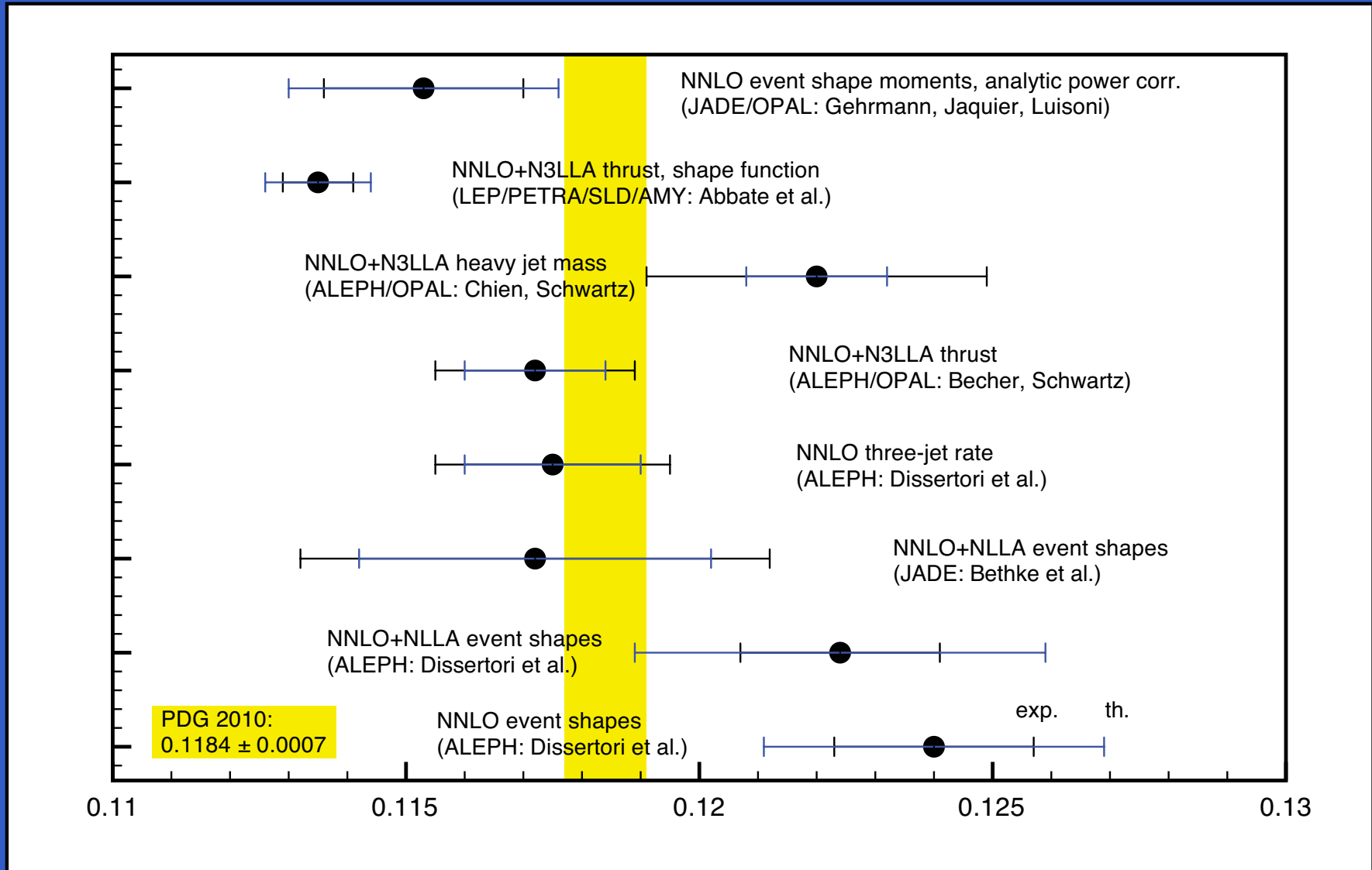
Result at  $y_{\text{cut}} = 0.02$ :

$$\alpha_s(M_Z) = 0.1175 \pm 0.0020(\text{exp}) \pm 0.0015(\text{th})$$

- more precise than extractions from event-shape distributions.



# $\alpha_s$ from NNLO Jet Observables



# Conclusions and Outlook

- New NNLO result on jet observables together with high precision data allow improved extraction of  $\alpha_s$ :

- from NLLA+NNLO event-shape distributions,:

$$\alpha_s(M_Z) = 0.1224 \pm 0.0009(\text{stat.}) \pm 0.0009(\text{exp.}) \pm 0.0012(\text{had.}) \pm 0.0035(\text{theo.})$$

- from NNLO event-shape moments with analytical power corrections:

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- from NNLO three-jet rate:

$$\alpha_s(M_Z) = 0.1175 \pm 0.0020(\text{exp}) \pm 0.0015(\text{th})$$



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- From event-shape analysis some further important observations:

- combination of NNLO results with hadronization from LO MC not reliable,
- in LO MC hadronization corrections might be underestimated,
- further studies in this direction are needed in view of the precision needed at LHC.