Banks-Zaks Fixed Points and Critical Exponents in Momentum Subtraction Schemes

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Based on: J.A. Gracey and R.M. Simms, Phys. Rev. D 91, 085037 (2015)

Overview

1. Renormalization Group and Fixed Points
   - Concept of Renormalization Group
   - Renormalization Group Flows
   - Critical Exponents
2. Banks-Zaks Fixed Points and Critical Exponents
   - Results: Critical Exponents ω and ρ
3. Conclusions

Concept of Renormalization Group

In perturbative QFT we have to regularise the divergences that occur in loop diagrams.

Dimensional regularisation $\Rightarrow$ Analytically continue the space-time dimension to d-dimensions.
However, still need the action to be dimensionless $[S] = 0$

Rescale coupling constant in such a way that it is dimensionless in d-dimensions $g \rightarrow g\mu^{\frac{\epsilon}{2}}$

To do this we need to introduce an arbitrary energy scale μ

Concept of Renormalization Group

The theory of RG postulates that one can change the arbitrary scale of the theory in such a way that the physics on energy scales $< \mu$ remains constant.

Action at a particular energy scale is known as the Wilsonian Effective Action $S[\phi; \mu, g_i]$

Key RG Equation:

$$S\left[Z(\mu)^{\frac{1}{2}}\phi; \mu, g_i(\mu)\right] = S\left[Z(\mu')^{\frac{1}{2}}\phi; \mu', g_i(\mu')\right] \quad (1)$$

Concept of Renormalization Group

Summary: QFT is now a regularised theory but depends on an arbitrary scale μ. This is a problem as physical quantities *cannot* depend on arbitrary scales.

Resolution is via renormalization group equation which requires and is deduced from

$$\frac{\mu d\Gamma_{0(n)}}{d\mu} = 0 \qquad \text{(as } \Gamma_{0(n)} \text{ is independent of } \mu\text{)}$$

But $\Gamma_{(n)}$ are not unconnected as $\quad \phi_0 = \sqrt{Z_\phi}\phi \quad \Rightarrow \quad \Gamma_{0(n)} = Z_\phi^{\frac{n}{2}} \Gamma_{(n)}$

$$\Rightarrow \quad \boxed{\mu \frac{\partial}{\partial \mu}\left(Z_\phi^{\frac{n}{2}} \Gamma_{(n)}\right) = 0}$$

Concept of Renormalization Group

From this we can deduce

Callan-Symanzik Equation

$$0 = \left[\mu\frac{\partial}{\partial\mu} + \mu\frac{\partial g}{\partial\mu}\frac{\partial}{\partial g} + \mu\frac{\partial m}{\partial\mu}\frac{\partial}{\partial m} + \frac{n}{2}\frac{\mu}{Z_\phi}\frac{\partial Z_\phi}{\partial\mu}\right]\Gamma_{(n)} \qquad (2)$$

$$\underbrace{\beta(g) = \mu\frac{\partial g}{\partial\mu}}_{\beta-\text{function}} \qquad \underbrace{\gamma_m(g) = \frac{\mu}{m}\frac{\partial m}{\partial\mu}}_{\text{mass anomalous dimension}} \qquad \underbrace{\gamma_\phi(g) = \mu\frac{\partial(ln(Z_\phi))}{\partial\mu}}_{\text{wavefunction anomalous dimension}}$$

All renormalization group functions are renormalization scheme dependent.

But, at two loops the coefficients of the $\beta-$function are scheme independent in single coupling theories.

Renormalization Group Flows

In perturbation theory (In 4−dimensions),

$$\beta(g) = (d-4)g + Ag^2 + Bg^3 + Cg^4 + \dots \qquad (3)$$

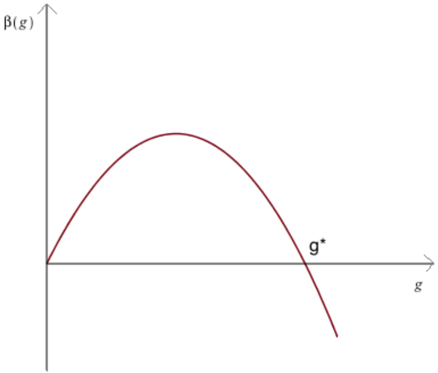
$(A < 0 \Leftarrow \text{ QCD})$

General property: There exists a value g^* for which $\beta(g^*) = 0$, which are known as *fixed points.* These underline phase transitions.

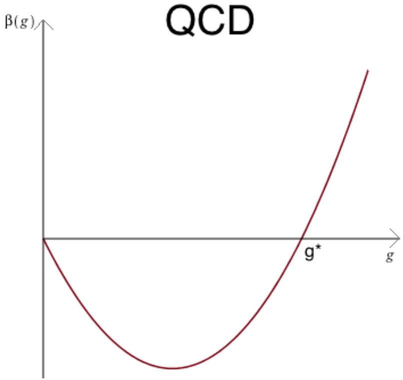
Example

If we have $\quad \beta(g) = Ag^2 + Bg^3, \quad g^* = -\frac{A}{B} > 0$
In QCD, A and B have opposite signs for $9 \leq N_f \leq 16$
(Conformal window)

Renormalization Group Flows



Wilson-Fisher fixed point in d-dimensions

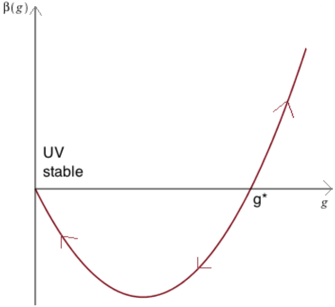


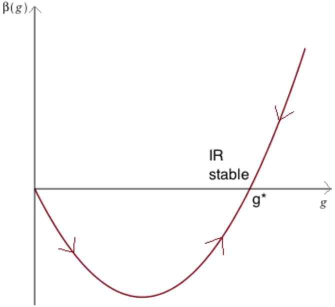
Banks-Zaks fixed point in 4-dimensions

Conformal window: $9 \leq N_f \leq 16$ The range of N_f values for which the nontrivial fixed point exists.

Renormalization Group Flows

$\beta(g)$ can have many forms,





As μ increases, flow is away from g^* and to 0 if $g < g^*$. This is called ultraviolet flow.

Reversing flow direction, the infrared flow is to $g^* \neq 0$, which is therefore infrared stable.

Critical Exponents

If the $\beta-$function has a nontrivial fixed point at the value g^*, then the renormalization group functions evaluated at g^* are termed **critical exponents** which are thought to be renormalization group invariants.

Critical exponents can also be found using scaling relations.

Critical exponents describe the behaviour of physical quantities near continuous phase transitions.

$$\underbrace{\omega = \beta'(g^*)}_{\text{measure of corrections to scaling}} \qquad \eta = \gamma_\phi(g^*)$$

$$\underbrace{\rho = \gamma_{\bar{\psi}\psi}(g^*)}$$

Quark mass anomalous dimension exponent
is of primary interest because of its relation to the definition of conformal theory

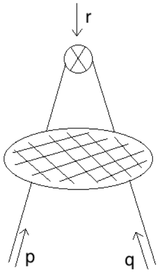
Universality

A feature of fixed points is that there can be more than one theory giving the same critical exponents. If two theories have a common fixed point with the same critical exponents they are said to be in the same **universality class**.

Quark Mass Anomalous Dimension

The quark mass anomalous dimension is deduced from the renormalization of the associated quark mass operator $\bar{\psi}\psi$.

This is renormalized by inserting it into a quark 2-point function and ensuring that the Green's function is rendered finite with respect to the particular renormalisation scheme of interest.



$$= \langle \psi(p)\bar{\psi}(q)[\bar{\psi}\psi](r)\rangle \Big|_{p^2=q^2=-\mu^2}$$

The restriction above indicates evaluation at the symmetric point which is defined as $p^2 = q^2 = r^2 = -\mu^2$, implying $pq = \frac{1}{2}\mu^2$

$$\boxed{\gamma_{\bar{\psi}\psi}(g) = \mu \frac{\partial}{\partial \mu} \ln Z_{\bar{\psi}\psi}}$$

Motivation

If one computes the critical exponents in different renormalization schemes, say $\overline{\mathrm{MS}}$ and *MOMh*, at the Banks-Zaks fixed point then both expressions ought to be the same. This is because ultimately the critical exponent is a physical quantity and hence a renormalisation group invariant.

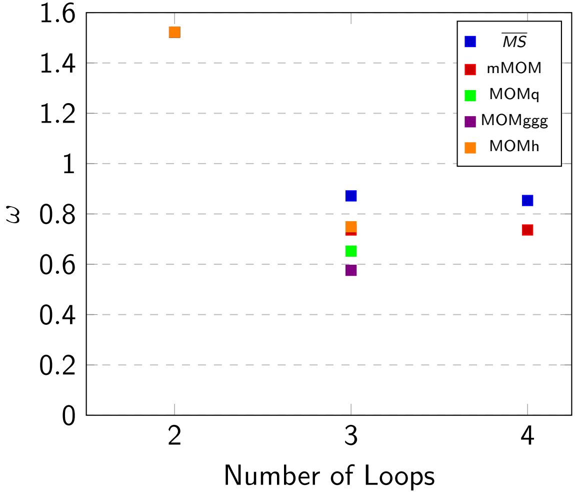
The aim is to see if the numerical values for the exponents in various schemes show the consistency which would indicate renormalisation group invariance.

Also want to look at the specific range of values of the number of quark flavours, N_f, where the exponents appear to be scheme independent.

Critical exponent ω at the Banks-Zaks fixed point

$$\omega_L = 2\beta_L'(g^*, 0)$$

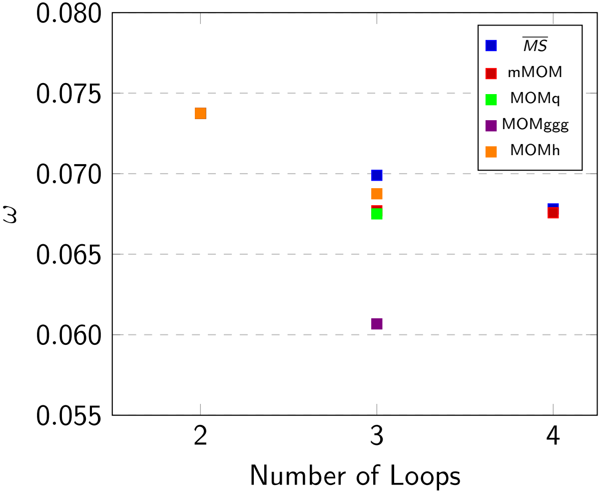
Value of the critical exponent ω in $SU(3)$ for $N_f = 10$



Critical exponent ω at the Banks-Zaks fixed point

$$\omega_L = 2\beta_L'(g^*, 0)$$

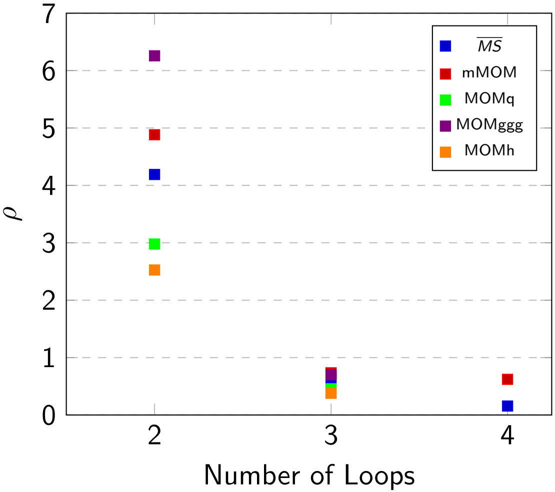
Value of the critical exponent ω in $SU(3)$ for $N_f = 14$



Critical exponent ρ at the Banks-Zaks fixed point

$$\rho_L = -2\gamma_{\bar{\psi}\psi L}(g^*, 0)$$

Value of the critical exponent ρ in $SU(3)$ for $N_f = 10$



Critical exponent ρ at the Banks-Zaks fixed point

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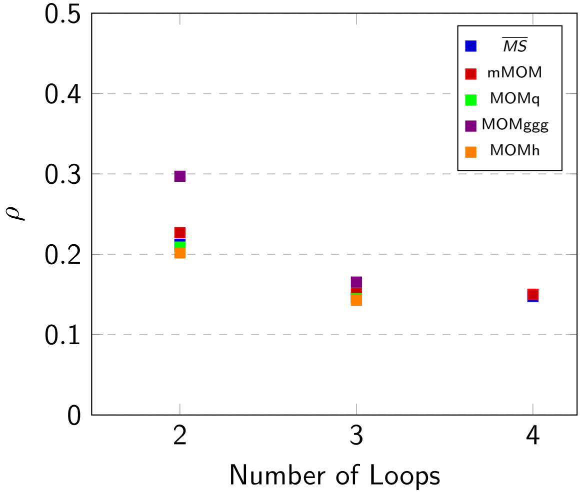
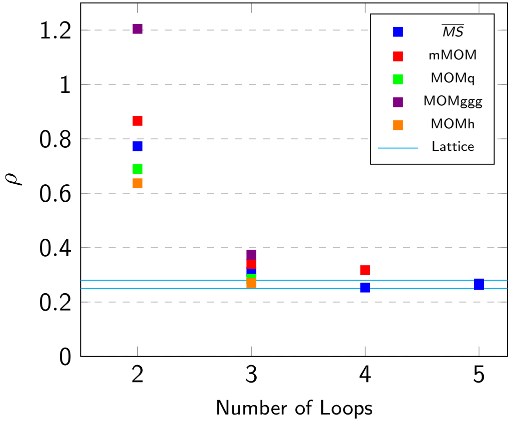


Critical exponent ρ at the Banks-Zaks fixed point

$$\rho_L = -2\gamma_{\bar{\psi}\psi L}(g^*, 0)$$

Value of the critical exponent ρ in $SU(3)$ for $N_f = 12$



Five loop: P.A. Baikov, K.G. Chetyrkin & J.H. Kühn (2014)

Lattice: Cheng, Hasenfratz, Liu, Petropoulos & Schaich (2014). Lombardo, Miura, Nunes da Silva & Pallante (2014)

Conclusions

Scheme dependence appears to disappear for values of N_f near the upper end of the conformal window. This is where perturbation theory is at its most reliable.

In the main the *MOMggg* scheme appears mostly to be the outlier class. This is not unreasonable due to the nature of the scheme. It is based on ensuring that the triple gluon vertex has no $O(a)$ corrections at the completely symmetric point. Therefore, with the associated renormalization group functions their content is necessarily weighted by gluonic rather than quark contributions.

Therefore, for the quark mass anomalous dimension the quark content is not dominant.

Thank you!