The correspondence between free fermionic models and orbifolds

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Motivation

- String theory provides the most promising framework for a fundamental theory of physics.
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- There have been extensive computer scans towards that goal both in the (bosonic) orbifold and in the free fermionic formulation.
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- –Which formalism is better?
  –"Ours!"

Motivation

- It would be very useful to have a dictionary from the orbifold formalism (OF) to the free fermionic formalsim (FFF) that would allow us to compare the previous results.

Bonus:

- Equivalent formulations of particular models allow us to use tools from one formalism to solve difficult problems in the other. For example:
  - It is much easier to construct asymmetric orbifold actions in the FFF than in the OF.
  - It is much easier to move in the Narain moduli space in the OF but not in the FFF.
  - many more examples!

Bosonization and fermionization

In a 2d CFT bosons and fermions are equivalent and we can convert from one to the other using

$$y + iw = :e^{iX}:$$

which is know as the **bosonization/fermionization formula**.

The relation above assumes that the bosons are compactified on a circle with a specific radius (or on a specific lattice in the general case). This is known as the **fermionic point** in the moduli space of lattice compactifications.

Orbifold models

We are interested in **toroidal orbifolds**. Such models are specified by:

1) A **Narain lattice** on which the internal 6 dimensions (and the gauge degrees of freedom) are compactified.

2) An **orbifold action** compatible with the lattice.

3) A choice of the relative phases when we have more than one action (**discrete torsion**).

Orbifold models - The 6d lattice

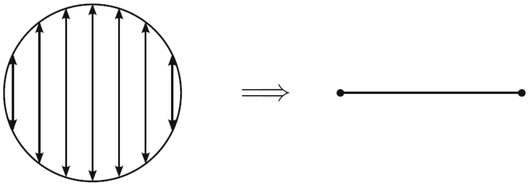
Bosonic string theory is formulated in 10d. In order to recover the 4d-spacetime we observe, we need to compactify six of these dimensions:

$$X^J \equiv X^J + L^J, \quad J = 5, \ldots 10$$

The vectors $\{L^J\}_{J=5,\ldots10}$ then form a lattice which “uniquely” characterizes this compactification. They also define a compactification manifold which in this case is $(S^1)^6$.

A 1d example

Orbifold models - An orbifold action example



A 1d orbifold example

The action above turns the circle (smooth manifold) into a line segment which is not a manifold because of the endpoints. Such operations create **orbifolds**. Orbifolds are preferable to manifolds because they break SUSY from $N = 4$ to something smaller.

Orbifold models - Discrete torsion

- If there is only one orbifold action, then the description of the orbifold model is complete.
- However, there are multiple ways to combine more than one orbifold actions. Introducing a relative phase between them will change the model.
- These **discrete torsions** form an integral part of the definition of an orbifold model and they need to be specified.

Free fermionic models

Free fermionic models are specified by (cf talk by Johar and Glyn):

1) A set of **basis vectors** that describe the boundary conditions of the worldsheet fermions around the cycles of the worldsheet torus.

2) A choice of the relative phases between different basis vectors (**discrete torsion**).

Converting from one to the other

To convert a free fermionic model to an orbifold we must then know how to implement the following steps:

1) Choose how to bosonize, *ie.* which fermions to combine.
2) Extract the Narain lattice from the basis vectors.
3) Extract the orbifold action from the basis vectors.
4) Extract the orbifold phases from the free fermionic phases.

In this talk, I will only briefly comment on steps 2 and 3.

2) orbifold action from the basis vectors

The geometric data of the orbifold model can be read from the partition function of the free fermionic model. As an example, the free fermionic model with basis vectors $\{\mathbb{1}, \mathbf{S}\}$ corresponds to a bosonic model with:

$$G = \frac{1}{2}\mathbb{1}_6$$

$$B = \frac{1}{2}\begin{pmatrix} 0 & -1 & \cdots & -1 \\ 1 & \ddots & \ddots & \vdots \\ \vdots & & \ddots & -1 \\ 1 & \cdots & 1 & 0 \end{pmatrix}_{6\times 6} \quad , \quad A = -\begin{pmatrix} 1 & \cdots & 1 \\ 2 & \cdots & 2 \\ \vdots & \vdots & \vdots \\ 13 & \cdots & 13 \\ 13/2 & \cdots & 13/2 \\ 15/2 & \cdots & 15/2 \\ 2 & \cdots & 2 \end{pmatrix}_{16\times 6}$$

2) orbifold action from the basis vectors

$$
g = \begin{pmatrix}
2 & -1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
-1 & 2 & -1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & -1 & 2 & -1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & -1 & 2 & -1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & -1 & 2 & -1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & -1 & 2 & -1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & -1 & 2 & -1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & -1 & 2 & -1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & -1 & 2 & -1 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & -1 & 2 & -1 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & -1 & 2 & -1 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & -1 & 2 & -1 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & -1 & 2 & -1 & -1 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & -1 & 2 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & -1 & 0 & 2 & 1 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 & 4
\end{pmatrix}
$$

3) orbifold action from the basis vectors

Using

$$y + iw = :e^{iX}:$$

we see that:

- When

$$y + iw \rightarrow -(y + iw) \quad \Rightarrow \quad X \rightarrow X + \pi$$

(shift action)

- When

$$y + iw \rightarrow y - iw \quad \Rightarrow \quad X \rightarrow -X$$

(twist action)

- When

$$y + iw \rightarrow -y + iw \quad \Rightarrow \quad X \rightarrow -X + \pi$$

(roto-translational action)

Summary and outlook

1. The heterotic string provides a nice framework to construct (semi-)realistic models. Understanding the **moduli space** of heterotic models is of great importance.
2. Free fermionic and orbifold models are related and we can translate from one to the other.
3. Such a dictionary also allows us to address difficult problems in one formalism using tools from the other.
4. Please have a look at the upcoming paper for a description in exquisite detail!

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Thank you very much!