NNLO QCD Corrections to the Neutral Kaon Oscillation Amplitude

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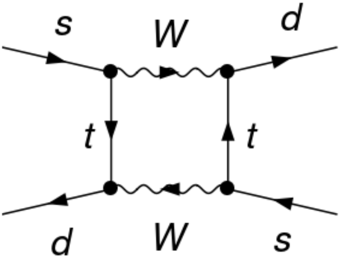
Contents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Kaon Physics

Kaon physics rich seam for new physics

- K^0-$\bar{K}^0$ oscillations
    - First detection of CP violation

Neutral Kaon flavour eigenstates $K^0(d\bar{s}), \bar{K}^0(\bar{d}s)$ are not mass eigenstates $\implies$ mass eigenstates are superposition of flavour states

Small Kaon mass $m(K^0) \approx 498 MeV \implies K^0, \bar{K}^0$ only decay weakly

- Hadronic decay to either $\pi\pi$ or $\pi\pi\pi$

CP States

Flavour eigenstates are not CP eigenstates:

$$CP\,|K^0\rangle = |\bar{K}^0\rangle \qquad CP\,|\bar{K}^0\rangle = |K^0\rangle \tag{1}$$

Build CP eigenstates satisfying $CP\,|K_1\rangle = |K_1\rangle\,, CP\,|K_2\rangle = -\,|K_2\rangle$ via

$$K_1 = \frac{1}{\sqrt{2}}\left(K^0 + \bar{K}^0\right) \qquad K_2 = \frac{1}{\sqrt{2}}\left(K^0 - \bar{K}^0\right) \tag{2}$$

Also know that

$$CP\,|\pi\pi\rangle = |\pi\pi\rangle \qquad CP\,|\pi\pi\pi\rangle = -\,|\pi\pi\pi\rangle \tag{3}$$

CP Violation

If weak interactions conserve CP

- $K_1 \to \pi\pi$ (large phase space $\to$ fast)
- $K_2 \to \pi\pi\pi$ (small phase space $\to$ slow)

$\Longrightarrow$ far from source only observe $\pi\pi\pi$. Not true! Observe CP violation.

CP violation weak, so write physical (indirectly CP-violating states) as

$$|K_S\rangle = \frac{1}{\sqrt{1+|\epsilon|^2}}(|K_1\rangle + \epsilon\,|K_2\rangle) \tag{4}$$

$$|K_L\rangle = \frac{1}{\sqrt{1+|\epsilon|^2}}(|K_2\rangle + \epsilon K_1) \tag{5}$$

Calculation and Assumptions

Want to calculate 1-loop electroweak diagram for K^0-$\bar{K}^0$ mixing, denoted $Box(\Delta S = 2)$.

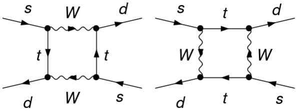
Assume:

- External momenta $p^\mu = 0$ for all external particles
- $m_u = m_c = 0$ (only massive quark is t)
- Feynman gauge $\xi = 1$ (implies Goldstone bosons)
- Neglect neutral exchange bosons (want FCNC)
- Unitarity of CKM matrix

Diagrams

Using assumptions, generate 72 FCNC diagrams. Can reduce this number for our purposes.

- Colour structure $\rightarrow$ for structure, only consider half of diagrams



- $m_u = m_c = 0$ implies

$$A(u,u) = A(c,u) = A(u,c) = \underline{A(c,c)} \tag{6}$$

$$A(u,t) = A(t,u) = A(t,c) = \underline{A(c,t)} \tag{7}$$

Unitarity of CKM Matrix

With assumptions, and $\lambda_i \equiv V_{id}V_{is}^*$

$$\begin{aligned} Box(\Delta S = 2) = &\ A(c,c)\left[\lambda_u^2 + 2\lambda_u\lambda_c + \lambda_c^2\right] \\ &+ A(c,t)\left[2\lambda_u\lambda_t + 2\lambda_c\lambda_t\right] + A(t,t)\lambda_t^2 \\ &+ \lambda_t^2\left[F_1(t,t) + F_2(t,t) + F_3(t,t)\right] \end{aligned} \tag{8}$$

Unitarity of CKM implies

$$\lambda_u + \lambda_c + \lambda_t = 0 \tag{9}$$

$$\begin{aligned} \therefore Box(\Delta S = 2) = &\ \lambda_t^2\Big[A(c,c) + A(t,t) - 2A(c,t) \\ &+ F_1(t,t) + F_2(t,t) + F_3(t,t)\Big] \end{aligned} \tag{10}$$

1-loop Feynman Integrals

Get integrals like

$$A(t,t) \propto \int_{-\infty}^{\infty} \frac{d^4q}{(2\pi)^4} \frac{q^2}{(q^2 - m_t^2)^2(q^2 - M_W^2)^2} \tag{11}$$

Identities like

$$1 = \frac{[(q^2 - m_t^2) - (q^2 - M_W^2)]}{M_W^2 - m_t^2} \tag{12}$$

break into smaller *divergent* integrals. Dimensionally regularize with $d = 4 - 2\epsilon$.

Solution of Integrals

All integrals can be broken down into sums of integrals of the form

$$F(a) = \int d^d q \frac{1}{(q^2 - m^2)^a} \tag{13}$$

Solve using integration by parts method (IBP) to find

$$F(a) = \frac{(-1)^{a-1}(1-\frac{d}{2})_{a-1}}{(a-1)!(m^2)^{a-1}} \left(-i\pi^{\frac{d}{2}} (m^2)^{\frac{d}{2}-1} \Gamma\left(1 - \frac{d}{2}\right)\right) \tag{14}$$

where

$$(t)_n \equiv t(t+1)(t+2)\dots(t+n-1) \tag{15}$$

Solution and $S_0(x_t)$

Bringing it all together,

$$Box(\Delta S = 2) = \lambda_t^2 \frac{G_F^2}{16\pi^2} M_W^2 S_0(x_t)(\bar{s}d)_{V-A}(\bar{s}d)_{V-A} \tag{16}$$

where $x_t = m_t^2/M_W^2, G_F$ is the Fermi coupling and

$$S_0(x_t) = \frac{4x_t - 11x_t^2 + x_t^3}{4(1-x_t)^3} - \frac{3x_t^3 \ln x_t}{4(1-x_t)^2} \tag{17}$$

Previous Work on QCD Corrections

QCD corrections of $\mathcal{O}(\alpha_s)$ considered by Buras *et al.* (1990) with effective Hamiltonian

$$\mathcal{H}_{eff} = \frac{G_F^2}{16\pi^2} M_W^2 \left[\lambda_c^2 \eta_1 S_0(x_c) + \lambda_t^2 \eta_2 S_0(x_t) + 2\lambda_c \lambda_t \eta_3 S_0(x_c, x_t)\right] \left[\alpha_3(\mu)\right]^{2/9} \hat{O}_{LL}(\mu) \quad (18)$$

QCD corrections encoded in coefficients η_1, η_2, η_3. To $\mathcal{O}(\alpha_s)$,

$$\eta_1 = 1.38 \pm 0.20 \qquad \eta_2 = 0.57 \pm 0.01 \qquad \eta_3 = 0.47 \pm 0.04 \quad (19)$$

η_1, η_3 has been calculated to $\mathcal{O}(\alpha_s^2)$ by Brod, Gorbahn (2010). We are now trying to calculate η_2 to $\mathcal{O}(\alpha_s^2)$. Especially important for B^0-$\bar{B}^0$ oscillations.

Expected Integrals

At three loops, with no external momenta, get integrals of the form

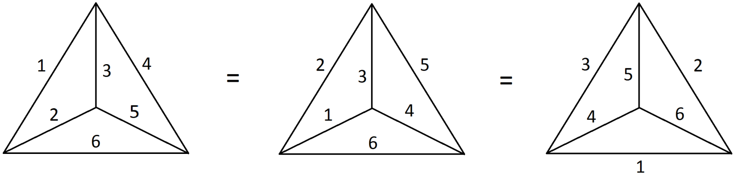
$$\int \left\{ \frac{d^d q_1 d^d q_2 d^d q_3}{(q_1^2 - m_1^2)^{i_1} (q_2^2 - m_2^2)^{i_2} (q_3^2 - m_3^2)^{i_3} ((q_1 - q_2)^2 - m_4^2)^{i_4}} \right. \\ \left. \times \frac{1}{((q_2 - q_3)^2 - m_5^2)^{i_5} ((q_1 - q_3)^2 - m_6)^{i_6}} \right\}$$

Masses $m_i \in \{0, M_W, m_t\}$. Expect maximum of 4 massive propagators.

3-loop integral has known solution for the case of 1 mass $m_i \in \{0, m\}$ (Broadhurst (1999))

Symmetries of the 3-loop Integral

Considering case that all propagators have equal mass, can be shown that integral has tetrahedral symmetry. Total of 24 equal permutations.



Have written code that identifies equal integrals that superficially look different, then writing in terms of chosen "standard" integrals.

Reducing the Number of Scales

Know solution to 3-loop integrals of 1 scale (but we have two - M_W and m_t). $\therefore$ if we remove one scale from integrals, we are (largely) done.

Can remove a scale by Taylor expanding integral in ${}^{M_W}/_{m_t}$. Experimentally, we know that:

- $\frac{M_W}{m_t} \sim \frac{1}{2}$
- There are no particles with mass $m_?$ in the range $M_W < m_? < m_t$

Naively expanding about $M_W/m_t = 0$ will generate IR divergences - need to be careful about this point.

Expanding about $M_W/m_t = 1$ is safe.

Expansion by Subdiagrams

For expanding about $M_W/m_t = 0$, follow approach of Smirnov's expansion by subdiagrams.

- Expand diagram in small parameter (M_W)
- Identify *asymptotically irreducible* (AI) subdiagrams
- Expand AI subdiagrams in their small parameters
- Sum expanded diagram and AI subdiagrams

Approach has a similar structure to the forest formula used in renormalization.

After expanding about $M_W/m_t = \{0, 1\}$, can interpolate results to find behaviour around experimental value.

Conclusion and Aims

- K^0-$\bar{K}^0$ oscillations are an important probe into CP violation
- Pure electroweak diagrams have been calculated
- $\mathcal{O}(\alpha_s)$ corrections η_1, η_2, η_3, known
- $\mathcal{O}(\alpha_s^2)$ correction η_3 known - now calculating η_2
- Can use tetrahedral symmetries to relate superficially different integrals
- Will (carefully) use Taylor expansions to reduce integrals to a single scale

The End!

Thank you for listening!

"Beauty is truth, truth beauty - that is all ye know on earth and all ye need to know"
(John Keats 1795-1821)

Earliest reference to $SU(2)$ symmetry for heavy flavours?
(Peter Renton)