Climbing NNLL of Weak Decays

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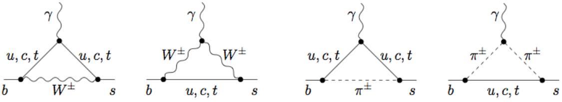
Rare B decays

Rare leptonic decays of the neutral B mesons are highly suppressed in the SM, and provide important constraints on models of NP.

The $\mathbf{b} \to \mathbf{s}$ transition is mediated by FCNCs and induced at one loop level.



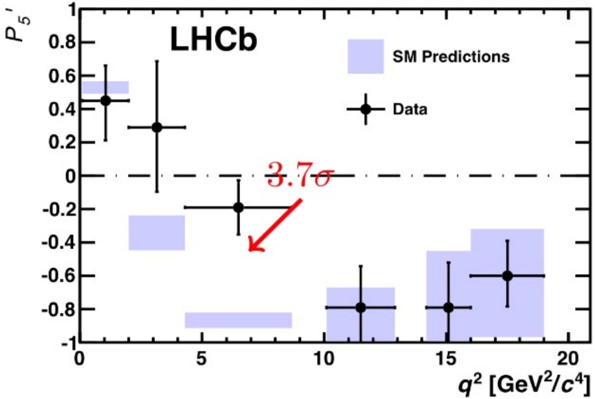
$$\underbrace{V_{tb}V_{ts}^*}_{\lambda^2} = -\underbrace{V_{cb}V_{cs}^*}_{\lambda^2} - \underbrace{V_{ub}V_{us}^*}_{\lambda^4}$$

Flavour-changing processes that we are interested in at the LHCb occur at low energies, at scales $\mu \ll M_W$.

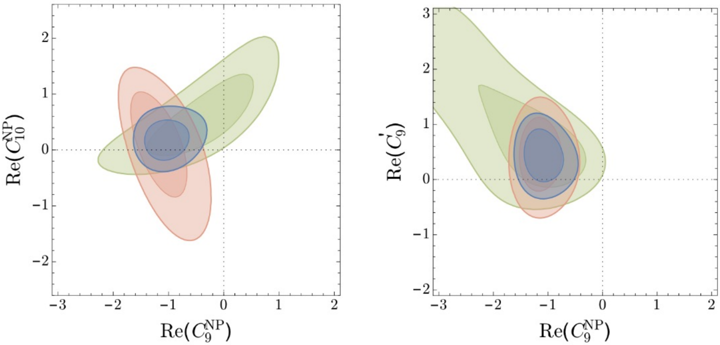
Electroweak interactions are described by an effective theory.

Why $b \to s$ transition? [PRL 111, 191801 (2013)]

In the recent updated angular analysis of the $B \to K^* \mu^+ \mu^-$ decay by the LHCb collaboration a significant tension with Standard Model expectations has been found.



New Physics?? [arXiv: 1503.06199]

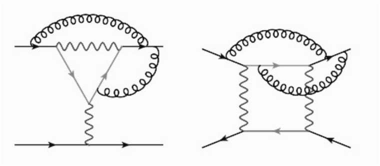


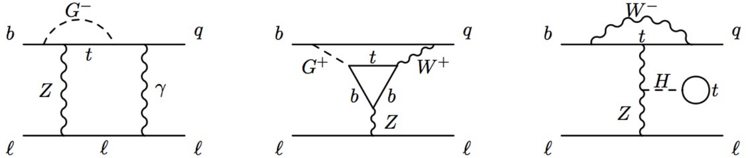
A solution with new physics modifying the Wilson Coefficient $\mathcal{C}_9$ is preferred over the Standard Model by 3.7σ.

$\rightarrow$ Accuracy in the SM for the Wilson Coefficient $\mathcal{C}_9$ is required.

Status of the Wilson Coefficients in the SM

For the decay $B_s \to l^+ l^-$, the three-loop QCD and two-loop EW matching conditions have been studied. **arXiv:1311.1347,arXiv:1311.1348**

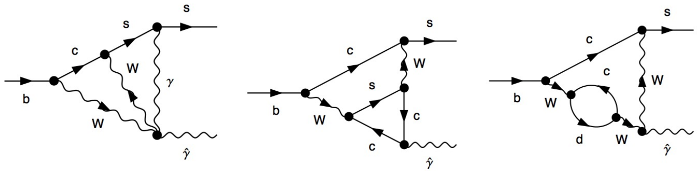




The NNLL EW corrections for $\mathcal{C}_9$ have been employed in the limit of large top-quark mass, which is known to be insufficient.

What are we looking for?

We are interested in the calculation of the two-loop EW corrections to the specific semileptonic operator $\mathcal{O}_9$.



We expand in external momentum prior to loop-momentum integration. And we take light particles to be massless.

This introduces Infrared Divergences (Spurious) in the full theory amplitude, which have to be cancelled by the Ultra-Violet divergences in the effective field theory side.

In addition, we compute everything in the Background Field Method.

Why BFM? Nuclear Physics B185 (1981) 189-203

The Background-field approach makes the structure of gauge theories more transparent and easier to understand.

The gauge and the Higgs fields are expressed as follow,

$$W^a_\mu \to \hat{W}^a_\mu + W^a_\mu \quad B_\mu \to \hat{B}_\mu + B_\mu \quad \phi \to \hat{\phi} + \phi$$

Only the quantum fluctuations appear as internal particles. Therefore, it is not necessary to renormalize the quantum fields. However, one have to understand the mass renormalization for the gauge bosons and goldstone bosons.

We work on the off-shell matching: $b \to s\hat{\gamma}$. One can use the Ward identities for the background photon and deal just with the gauge invariant operators.

Calculations of the top and charm quarks are done independently.

Theoretical Framework for $\mathrm{B_s} \to \mathrm{l^+l^-}$

The basis for any serious phenomenology of weak decays of hadrons is the **Operator Product Expansion**.

$$\mathcal{H}_{\text{eff}} = \frac{G_F}{\sqrt{2}} \sum_i V^i_{\text{CKM}} \mathcal{C}_i(\mu) \mathcal{Q}_i(\mu).$$

The relevant operators for this decay are,

$$\mathcal{O}_7^\gamma = \frac{e}{16\pi^2} m_b (\overline{s}_L \sigma^{\mu\nu} b_R) F_{\mu\nu}$$

$$\mathcal{O}_\mathbf{9} = \frac{e^2}{g^2} (\overline{s}_L \gamma_\mu b_L) \sum_l (\bar{l}\gamma^\mu l) \qquad \mathcal{O}_{10} = \frac{e^2}{g^2} (\overline{s}_L \gamma_\mu b_L) \sum_l (\bar{l}\gamma^\mu \gamma_5 l)$$

Howard Georgi, HUTP 93 /A003 1/93

Matching and Mixing

The matching conditions are most easily found by requiring the equality of the full SM and the effective theory 1PI off-shell Green's functions, ($\mathcal{A}_{\text{full}} = \mathcal{A}_{\text{eff}}$).

The full and the effective Lagrangians have to be renormalised.

We express everything in terms of renormalisable couplings, masses, fields:

$$g_0 \to Z_g g, \quad m_0 \to Z_m m, \quad \psi_0 \to Z_\psi^{1/2} \psi, \quad \mathcal{C}_{j0} \to \mathcal{C}_i Z_{ij} ...$$

In the effective sector, the $\mathcal{O}_i$ must also be renormalised.
The renormalization group equation for $\mathcal{C}_i$ is given by,

$$\frac{d\bar{\mathcal{C}}(\mu)}{d\ln\mu} = \gamma^T(g)\bar{\mathcal{C}}$$

which describes the mixing among the operators.

Effective Theory Calculation

After renormalization, the effective amplitude projected onto $\mathcal{O}_9$ reads,

$$\mathcal{A}_9^Q|_{\text{eff}} = Z_d Z_l Z_g^{-2} \sum_i \mathcal{C}_i^Q Z_{i9} \qquad Q = t, c$$

Expanding up to two-loop level,

$$\begin{aligned}\mathcal{A}_9^{c,(2)}|_{\text{eff}} = V_{cb} V_{cs}^* \tilde{\alpha}_e^2 \; (& G_\mu^{(0)} C_2^{(0)} Z_{29}^{(1)} (\delta Z_l^{(1)} + \delta Z_d^{(1)}) + G_\mu^{(0)} C_2^{(0)} Z_{29}^{(2)} \\ & + G_\mu^{(0)} C_9^{(1)} (\delta Z_l^{(1)} + \delta Z_d^{(1)}) + G_\mu^{(0)} C_9^{(2)} \\ & + G_\mu^{(0)} \sum_i Z_{i9}^{(1)} C_i^{(1)} + G_\mu^{(1)} C_2^{(0)} Z_{29}^{(1)} + C_9^{(1)} G_\mu^{(1)})\end{aligned}$$

where G_F has been identified with the measured muon lifetime and its theory predictions, $G_\mu = G_\mu^{(0)} + G_\mu^{(1)} + \ldots$

What is done?/What is missing?

Loop Level	$\mathcal{A}_{\text{full}}$	$\mathcal{A}_{\text{eff}}$	$\mathcal{C}_i$
1	Done	Done	in progress
2	Except triangle diagrams	Expression	in progress

+ counterterms in the Background field Method.

Outlook

- At the LHCb, several tensions with SM predictions have shown up in the data. (In particular, in the angular observable $\mathbf{P}_5'$).
- These anomalies could be explained by introducing New Physics in the Wilson coefficient $\mathcal{C}_9$.
- The accuracy of the $\mathcal{C}_i$ in the SM is particularly important to constrain New Phyisics.
- We should have an analysis of higher order electroweak corrections to $\mathcal{C}_9$.