Charged-Current Deep Inelastic Scattering at α_s^3

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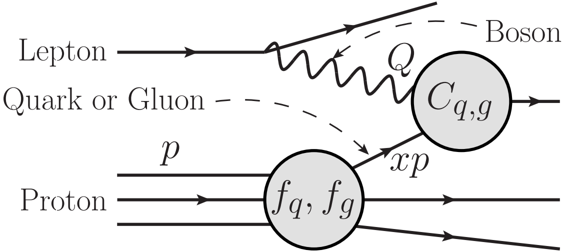
INTRODUCTION

Deep Inelastic Scattering – lepton scatters from a proton:



The Boson can be γ, H, Z^0 (NC) or $W^\pm$ (CC).

Cross-section $\sigma \sim \sum_a F_a = \sum_a \left[C_{a,q} \otimes f_q + C_{a,g} \otimes f_g\right]$.

$C_{a,j}$ – *"Coefficient Function"*
f_j – *"Parton Distribution Function"*
F_a – *"Structure Function"*

Known so Far

Natural to consider *combinations* $F_a^{\nu P + \bar{\nu} P}$ and $F_a^{\nu P - \bar{\nu} P}$.

Known, to third-order (massless DIS):

- γ probe: $C_{2,q}, C_{2,g}, C_{L,q}, C_{L,g}$. [hep-ph/0504.242]
- H probe: $C_{\phi,q}, C_{\phi,g}$. [hep-ph/0912.0369]
- $W^{\pm}$ probe (+): $C_{2,q}$, $C_{2,g}$, $C_{L,q}$, $C_{L,g}$, $C_{3,q}$. [hep-ph/0812.4168]

Unknown/Incomplete:

- Z^0 probe: $C_{2,q}, C_{2,g}, C_{L,q}, C_{L,g}, C_{3,q}$.
- $W^{\pm}$ probe ($-$): $\boxed{C_{2,q}, C_{L,q}, C_{3,q}}$. (No gluon dependence!)

Useful in neutrino detection, for e.g., IceCube.
Probes at small-x. NLO bad here. [hep-ph/1505.06583]

NuTeV anomaly, $\sin^2 \theta_W$.

Charged-Current DIS

Previous work at α_s^3. Moch, Rogal: [hep-ph/0704.1740]

- $N = 3, 5, ..., 11$ of $C_{2,q}$, $C_{L,q}$.
- $N = 2, 4, ..., 10$ of $C_{3,q}$.

Used to produce approximations of analytic result.

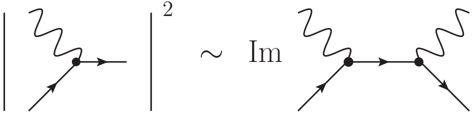
(We compute in Mellin space. Nth Mellin Moment:

$$F_a(N, Q^2) = \int_0^1 x^{N-1} F_a(x, Q^2) \mathrm{d}x.$$

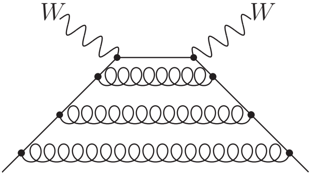
For $F_a^{\nu P - \bar{\nu} P}$: F_2, F_L live on odd moments, F_3 on even moments.)

THE OPTICAL THEOREM

Relates structure functions to the *forward scattering amplitude*:



Calculations proceed via loop integrals rather than phase-space integrals.



+ 208 more (F_2, F_L) + 196 more (F_3), after applying symmetries.

Big improvement on diagram database of Moch, Rogal (1300+)

Check: gauge parameter in propagators, ensure cancellation.

RENORMALIZATION

Forward scattering amplitude T_a will contain poles in Dimensional-Regularization parameter ε (where $D = 4 - 2\varepsilon$), due to loop integrals (UV) and initial state collinear momenta.

Structure functions F_a are physically measurable.

- Poles are *unphysical*: renormalize.
- UV Poles: coupling constant renormalization.

T_a can then be *mass factorized*. That is, we can write

$$F_a = T_a\left(\frac{1}{\varepsilon}, \varepsilon\right)\hat{f}_q = C_a(\varepsilon)\, Z\left(\frac{1}{\varepsilon}\right)\hat{f}_q = C_a(\varepsilon)\, f_q$$

The poles can then be absorbed into a redefinition of the non-perturbative partion distribution function $\hat{f}_q$.

Z is a function of the (already known) anomalous dimension γ_{qq}. Recovering these is a strong consistency check.

TWO WAYS TO PROCEED...

- Extend moment-based calculation of SM, MR and reconstruct all-N expressions in the style of Polarized Splitting Function calculation of Moch, Vogt, Vermaseren.
  [hep-ph/1409.5131]
- Direct all-N calculation. `FORM` code to do integral reduction exists, but did not run with modern versions of `FORM`.

MOMENT-BASED RECONSTRUCTION

Compute moments:

- $N = 3, 5, ..., 29$ of $C_{2,q}$, $C_{L,q}$.
- $N = 2, 6, ..., 30$ of $C_{3,q}$.

(Not easy! Total 14 years CPU time.
Hardest diagram: 7TB intermediate expressions.
Each additional N takes $\sim$2x runtime and diskspace of previous.)

Choose a basis of functions:

- All-N results in (massless) DIS can be written in terms of *Harmonic Sums* combined with simple denominators $1/(N \pm c)$.

$$S_m(N) = \sum_{i=1}^{N} \frac{1}{i^m}, \quad S_{-m}(N) = \sum_{i=1}^{N} \frac{(-1)^i}{i^m}, \quad S_{n,m}(N) = \sum_{i=1}^{N} \frac{1}{i^n} S_m(i)$$

Solve! (LLL Lattice Basis Reduction)

MOMENT-BASED RECONSTRUCTION

Eg: α_s^1 contribution to $C_{2,q}$. Choose basis (up to weight 2):

$$\left(A_1+\frac{A_2}{N}+\frac{A_3}{N+1}+\frac{A_4}{N^2}+\frac{A_5}{(N+1)^2}\right)+\left(A_6+\frac{A_7}{N}+\frac{A_8}{N+1}\right)S_1(N)+A_9S_{1,1}(N)+A_{10}S_2(N)$$

Compute moments: $(N=3,5)$

$$A_1+\frac{A_2}{3}+\frac{A_3}{4}+\frac{A_4}{9}+\frac{A_5}{16}+\frac{11A_6}{6}+\frac{11A_7}{18}+\frac{11A_8}{24}+\frac{85A_9}{36}+\frac{49A_{10}}{36}=\frac{29}{12}$$

$$A_1+\frac{A_2}{5}+\frac{A_3}{6}+\frac{A_4}{25}+\frac{A_5}{36}+\frac{137A_6}{60}+\frac{137A_7}{300}+\frac{137A_8}{360}+\frac{12019A_9}{3600}+\frac{5269A_{10}}{3600}=\frac{589}{90}$$

+ 2 more, *Diophantine* system yields the solution:

$$\{A_1,A_2,A_3,A_4,A_5,A_6,A_7,A_8,A_9,A_{10}\}=\{-9,3,4,2,0,3,-2,2,4,-4\}\ \checkmark$$

MOMENT-BASED RECONSTRUCTION

For terms proportional to n_f, this method worked. Basis is not too big (n_f reduces weight).

${n_f}^0$ terms: have $\sim$200 unknowns

- smaller than the maximal set – have applied constraints

System of 13 (+1) Diophantine equations.

Tried various implementations, no success.

- Basis is too big!

(Andreas used 8 equations for 41 unknowns in Polarized Splitting Function calculation.)

DIRECT ALL-N CALCULATION

Meanwhile... Andreas was speaking to Jos Vermaseren regarding `FORM 4.1` and running the all-N code.

Success!

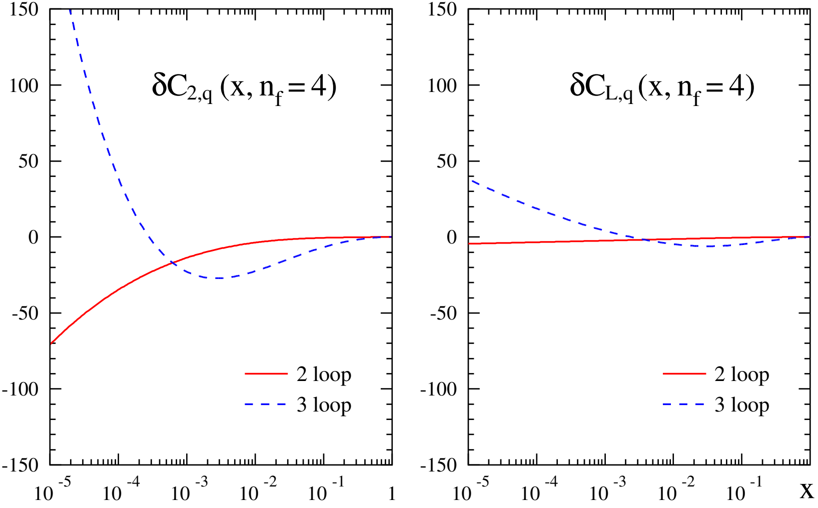
- After various bug-fixes/tweaks, the code runs.
- We have a result for ${n_f}^0$ terms!

The computed moments have not been a waste of time/resources

- Provide a verification that the code is running *correctly*.
- Techniques developed will be used in future calculations where a direct all-N calculation will *not* be possible.

RESULTS

Difference between $F_{2,L}^{\gamma}$ and $F_{2,L}^{\nu P-\bar{\nu}P}$ Coefficient Functions:



Paschos-Wolfenstein Relation

Ratio between NC and CC cross-sections:

$$R^- = \frac{\sigma(\nu P \to \nu X) - \sigma(\bar{\nu} P \to \bar{\nu} X)}{\sigma(\nu P \to \mu^- X) - \sigma(\bar{\nu} P \to \mu^+ X)}$$

Determines $\sin^2 \theta_W$.

Can now provide an α_s^3 correction (uses 2nd moments of $\delta C_{2,q}$, $\delta C_{L,q}$):

$$R^- = \left(\frac{1}{2} - \sin^2 \theta_W\right) + \frac{u^- - d^- + c^- - s^-}{u^- + d^-} \left\{1 - \frac{7}{3} \sin^2 \theta_W \right. \\ \left. + \left(\frac{1}{2} - \sin^2 \theta_W\right) \frac{8\alpha_s}{9\pi} \left[1 + 1.689\alpha_s + 3.661\alpha_s^2\right]\right\}$$

16% correction of [ ], 1% correction of { } over α_s^2 contribution.

CONCLUSION AND FUTURE WORK

Remaining α_s^3 corrections to CC-DIS Coefficient Functions computed.

Future work:

- Large-x, small-x resummation of leading logarithms in the new Coefficient Functions (last year's talk)
- Polarized CC-DIS
- Complete NC-DIS (Z^0 exchange)
- Photon structure with polarization (reconstruction from moments)