The Hunt for the QCD Axion

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“Birds born in a cage think flying is an illness.”
Alejandro Jodorowsky

Motivation

Quantum chromodynamics (QCD) is a remarkable theory and is almost universally believed to be the theory of strong interactions. However, it suffers from one serious blemish.

The Strong-CP Problem.

The CP Violating Interaction Term

The $SU(3)$ gauge theory allows a CPV interaction term of the form

$$\mathcal{L}_{CP} = \frac{\overline{\theta}\alpha_s}{32\pi^2}\tilde{G}_{\mu\nu}G^{\mu\nu}$$

to be added to the QCD Lagrangian which contributes to the neutron electric dipole moment (nEDM).

Note.

$$\overline{\theta} \equiv \theta_0 + \theta_{weak}$$

with θ_0 being the angle given above the electroweak scale and θ_{weak} is the value introduced by the electroweak CP violation.

What Is The Strong CP Problem?

The current bound on the nEDM is

$$|d_N| < 2.9 \times 10^{-26} e\ cm$$

so that

$$|\overline{\theta}| < 10^{-10}\ \mathtt{rad}$$

which is a strikingly small value for a dimensionless natural constant given that the CP violating phase, θ, in the CKM mixing matrix is of order one.

This smallness of $\overline{\theta}$ despite the large amount of CP violation in the weak sector is known as the strong CP problem.

Solutions To The Strong CP Problem

Axions are important because they are the most promising solution to the Strong CP problem.

Other solutions are ruled out or disfavoured by phenomenology:

- Calculable θ - The Nelson-Barr Mechanism (mimics CKM-type CP violation)
- Up Quark Mass Vanishing - Weinberg's famous up-down quark mass ratio

$$Z = \frac{m_u}{m_d} = 0.5 \text{ (current value)}$$

What Are Axions?

Axions are the quanta of the axion field, $a(x)$, which is the phase of the PQ complex scalar field after the spontaneously breaking of the PQ symmetry gives it an absolute value f_a.

Simply put axions are pseudo-Nambu-Goldstone bosons related to the spontaneous breaking of the anomalous $U(1)$ global symmetry.

Axions in String Theory

It is well-known that axions arise in string compactifications. Axions enjoy PQ shift symmetries of the form

$$a \mapsto a + \alpha f_a$$

where a is the axion field, f_a is the decay constant with α being an arbitrary constant.

Cosmological Bound & The Axion Decay Constant

It is a well known fact that large f_a especially

$$f_a > 10^{12} \texttt{ GeV}$$

means axion energy density

$$\rho_a > \rho_{critical}$$

and therefore is unacceptable.

The **Axion Decay Constant Window** is

$$10^{9-10} \texttt{ GeV} < f_a < 10^{12} \texttt{ GeV}.$$

f_a smaller than 10^{9-10} GeV, will couple very weakly and f_a greater than 10^{12} GeV, will couple too strongly.

There is always an anti-symmetric tensor field

$$B_{\mu\nu}, \quad \mu, \nu = 0, \ldots 3,$$

which is crucial for anomaly cancellation, the gauge-invariant field strength for which is given by

$$H = \mathrm{d}B + \omega_{3L} - \omega_{3YM}$$

giving rise to a single scalar field in four dimensions with axion-like couplings.

The Model-Independent Axion Is Present In All Superstring Models.

$$M_a' = 8\pi^2 M_a \Rightarrow M_a = \frac{M_a'}{8\pi^2}$$

and

$$M_a' = \frac{M_{Pl}}{12\sqrt{\pi}} \simeq 5.64 \times 10^{17}$$

leading to

$$M_a'^2 = \frac{M_{Pl}^2}{144\pi} \Rightarrow M_a = \frac{1}{8\pi^2} \cdot \frac{M_{Pl}}{12\sqrt{\pi}} \Rightarrow M_a \simeq 7.15 \times 10^{15} \text{ GeV}$$

clearly violating the cosmological energy density upper bound on f_a.

The Invisible Axions

- Peccei-Quinn-Weinberg-Wilzcek (PCWW) axion was experimentally ruled out.
- The invisible axion resides mostly in the phase(s) of a complex standard model singlet field σ.
- How σ couples to the quarks distinguishes between the two types of models.

The KSVZ (Kim-Shifman-Vainshtein-Zakharov) Hadronic Model

The axion field is a component of the SM singlet scalar field σ.

$$\mathcal{L} = f\overline{Q}_L Q_R \sigma + h.c.$$

where Q are the heavy quarks.

The DFSZ (Dine-Fischler-Srednicki-Zhitnitsky) Axion Model

The axion is predominantly a part of the SM singlet scalar field σ.

$$\mathcal{L} = \lambda\sigma\sigma H_1 H_2 + \sum_{ij}(f_d^{ij}\overline{q}_L^i d_R^j H_1 + f_u^{ij}\overline{q}_L^i u_R^j H_2) + h.c.$$

where H_1 and H_2 are the two Higgs doublets of the Standard Model.

The Axion-Photon-Photon Coupling

- Calculation is performed in two stages
  above and below the chiral symmetry breaking scale

$$c_{a\gamma\gamma} = \overline{c}_{a\gamma\gamma} - \frac{2}{3}\left(\frac{4 + 1.05Z}{1 + 1.05Z}\right)$$

- $\overline{c}_{a\gamma\gamma}$ is given in terms of PQ charges of fermions

$$\overline{c}_{a\gamma\gamma} = \frac{E}{C}$$

where

$$E = \mathrm{Tr}\, Q_{PQ} Q_{em}^2, \quad C\delta_{ab} = \mathrm{Tr}\, \lambda_a \lambda_b Q_{PQ}$$

$$\text{Obs} : SU(5)_{\text{flip}} \times U(1)_1 \times U(1)_2 \times U(1)_3$$
$$\text{Hid} : SU(5)' \times SU(2)' \times U(1)'_4 \times U(1)'_5 \times U(1)'_6$$

$$Q_{anom} = 84Q_1 + 147Q_2 - 42Q_3 - 63Q_5 - 9Q_6$$

where Q_4 is anomaly free.

The Forsaken Free Fermions

A general boundary condition basis vector is of the form

$$\alpha = \left\{ \psi^{1,2}, \chi^i, y^i, \omega^i | \overline{y}^i, \overline{\omega}^i, \overline{\psi}^{1,..,5}, \overline{\eta}^{1,2,3}, \overline{\phi}^{1,..,8} \right\}$$

where $i = 1, ..., 6$

- $\overline{\psi}^{1,..,5}$ - $SO(10)$ gauge group
- $\overline{\phi}^{1,..,8}$ - $SO(16)$ gauge group

The Observable $SO(10)$

- Edi Halyo (EH): The Standard-Like Model

$$SO(10) \to SU(3)_C \times SU(2)_L \times U(1)_Y \times U(1)_{Z'}$$

where

$$\begin{aligned} U(1)_Y &= \frac{1}{3}U(1)_C + \frac{1}{2}U(1)_L \\ U(1)_{Z'} &= U(1)_C - U(1)_L \end{aligned}$$

- Antoniadis, Leontaris, Rizos (ALR): The Pati-Salam Model

$$SO(10) \to SO(6) \times SO(4)$$

The Hidden $SO(16)$

- EH: The Standard-Like Model

$$SO(16) \to SU(5) \times SU(3) \times U(1)^2$$

- ALR: The Pati-Salam Model

$$SO(16) \to SU(8) \times U(1)'$$

The Global Anomalous $U(1)$

- EH: The Standard-Like Model
  $U(1)_A = 2(U(1)_1 + U(1)_2 + U(1)_3) - (U(1)_4 + U(1)_5 + U(1)_6)$
  with
  Tr $U(1)_A = 180$

  Note. In this case the $U(1)_A$ is color-anomalous. That is

$$\mathrm{Tr}[SU(3)^2_{Obs} U(1)_A] \neq 0.$$

- ALR: The Pati-Salam Model

$$U(1)_A = U(1)_1 - U(1)_2 - U(1)_3, \ \mathrm{Tr}\, U(1)_A = 72.$$

Aside

EH went on to show that the it is also a **harmful**

$$\begin{aligned} \mathrm{Tr}[SU(5)^2_{Hid} U(1)_A] &\neq 0, \\ \mathrm{Tr}[SU(3)^2_{Hid} U(1)_A] &\neq 0. \end{aligned}$$

The Dine-Seiberg-Witten Mechanism

The cancellation mechanism generates a large Fayet-Iliopoulos D-term for the anomalous $U(1)_A$ which would break supersymmetry and destabilize the vacuum. However, in all known instances one can give VEVs to scalar fields charged under $U(1)_A$ along the F- and D- flat directions to cancel the Fayet-Iliopoulos D-term and restore supersymmetry.
Basically, we want

$$\boxed{\sum_i Q_A^i |\langle \phi_i \rangle|^2 < 0.}$$

Note. In general, all the local and global $U(1)$s will be spontaneously broken by the DSW mechanism.

The Fayet-Iliopoulos D-Terms

The general form of the anomalous D-term is

$$D_A = \sum_i Q_A^i |\phi_i|^2 + \frac{g^2 e^{\Phi_D}}{192\pi^2} \,\mathrm{Tr}\, Q_A$$

- EH: The Standard-Like Model
  $\Rightarrow \sum_i Q_A^i |\langle \phi_i \rangle|^2 (= -\frac{15g^2}{16\pi^2}) < 0$
- ALR: The Pati-Salam Model
  $\Rightarrow \sum_i Q_A^i |\langle \phi_i \rangle|^2 (= -\frac{3g^2}{8\pi^2}) < 0$

The scalar VEVs resulting from these are at the scale

$$\frac{M}{10} \sim 10^{17} \mathtt{GeV}.$$

- Orbifold Models (J. E. Kim et. al.)
- CICYs (A. Lukas et. al.)
- Misalignment Mechanism
- Non-Abelian Hidden Gauge Theory (pions or glueballs)

"With wisdom comes humility."
Jauhar

THANK YOU!!!