Nernst Branes

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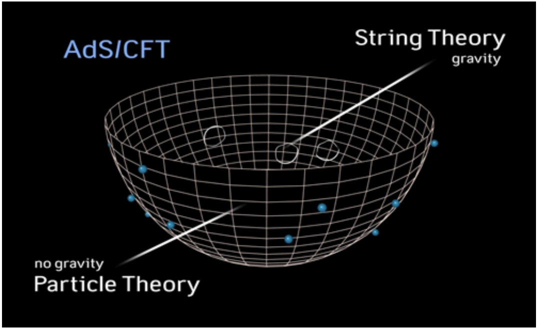
AdS_{d+1}/CFT_d



- asymptotically AdS gravity in bulk $\longleftrightarrow$ CFT on boundary
- strong/weak coupling duality
- explore previously inaccessible systems e.g. AdS/CMT

AdS/CMT

black objects in bulk $\longleftrightarrow$ thermal ensemble in field theory with same thermodynamic properties $(T, S, \mu, \dots)$

- CMT obeys all thermodynamic laws.
- There is a well established correspondence between laws of thermodynamics and laws of black hole mechanics.
- We need to build black objects that satisfy **all** of these.

Nernst Law/3rd law of thermodynamics

- All black objects seem to satisfy the $0^{\text{th}}, 1^{\text{st}}$ and 2^{nd} laws.
- There are several different forms of third law.
- We follow strictest definition (unique ground state):

$$S \xrightarrow{T \longrightarrow 0} 0 \quad \text{holding other parameters fixed}$$

  Said to have vanishing entropy in *extremal limit*.
- Easy to find black objects with $S(T=0) \neq 0$ indicating no unique ground state (there are ways to account for this!)
- Are there gravitational systems with $S \xrightarrow{T \longrightarrow 0} 0$ making them suitable duals to CMT systems?

Aims

Cardoso *et al.* [2011]: 4d "Nernst" brane with $s(T=0)=0$.

Goal: Systematically construct a family of non-extremal black branes in 4d, $\mathcal{N}=2$ gSUGRA s.t. $s \xrightarrow{T\longrightarrow 0} 0$ i.e. *Nernst branes*.

Why non-extremal?

- Need non-extremal to see limiting behaviour of Nernst Law.
- Extremal Nernst branes turn out to not be completely regular suggesting breakdown of effective theory - examine non-extremal solns in near extremal limit to study this.

gSUGRA Lagrangian

4d, $\mathcal{N} = 2$ $U(1) \subset SU(2)_R$ gSUGRA with n VMs coupled to GM:

$$e_4^{-1}\mathcal{L}_4 = -\frac{1}{2\kappa^2}R_4 - g_{IJ}\partial_{\hat{\mu}}X^I\partial^{\hat{\mu}}\bar{X}^J + \frac{1}{4}\mathcal{I}_{IJ}F^I_{\hat{\mu}\hat{\nu}}F^{J|\hat{\mu}\hat{\nu}} + \frac{1}{4}\mathcal{R}_{IJ}F^I_{\hat{\mu}\hat{\nu}}\tilde{F}^{J|\hat{\mu}\hat{\nu}} - V(X,\bar{X}).$$

$V(X,\bar{X})$ describes the gauging.

$\hat{\mu} = 0, \ldots, 3, \qquad I, J = 0, \ldots, n.$

3d Lagrangian

- Seek stationary brane soln $\Rightarrow$ dim red all fields over timelike S^1.
- e.g. KK ansatz: $ds_4^2 = -e^{\phi}\left(dt + V_\mu dx^\mu\right)^2 + e^{-\phi} ds_3^2$
  with ϕ, V the KK scalar and vector resp.
- Repackage d.o.f. of 3d Euclidean theory using $4n+5$ real scalars: $\{q^a, \hat{q}^a, \tilde{\phi}\}$.
- Restrict to **static** and **purely imaginary** field config to find:

$$e_3^{-1}\mathcal{L}_3 = -\frac{1}{2}R_3 - \tilde{H}_{ab}\left(\partial_\mu q^a \partial_\mu q^b - \partial_\mu \hat{q}^a \partial_\mu \hat{q}^b - g^a g^b\right) + 4\left(g^a q_a\right)^2$$

EoMs

Scalar equations of motion:

$$\nabla^2 \hat{q}_a = 0$$

$$\nabla^2 q_a + \frac{1}{2}\partial_a \tilde{H}^{bc}\left(\partial_\mu q_b \partial^\mu q_c - \partial_\mu \hat{q}_b \partial^\mu \hat{q}_c\right) - \frac{1}{2}\partial_a \tilde{H}_{bc} g^b g^c + 4\tilde{H}_{ab} g^b \left(g^c q_c\right) = 0$$

$$-\frac{1}{2}R_{3|\mu\nu} - \tilde{H}^{ab}\left(\partial_\mu q_a \partial_\nu q_b - \partial_\mu \hat{q}_a \partial_\nu \hat{q}_b\right) + g_{\mu\nu}\left(-\tilde{H}_{ab} g^a g^b + 4\left(g^a q_a\right)^2\right) = 0$$

Goal: solve these EoMs to find 3d instantons that we can lift back to regular 4d black branes.

We want Nernst brane solutions supported by:

- single electric charge, Q_0
- electric fluxes $g_1, \ldots, g_n$

Regular Black Brane Solution

Black brane solution has metric

$$ds_4^2 = -e^{\phi} dt^2 + e^{-\phi+4\psi} d\tau^2 + e^{-\phi+2\psi}\left(dx^2 + dy^2\right)$$

$$\text{Recall } ds_4^2 = -e^{\phi} dt^2 + e^{-\phi} ds_3^2, \quad ds_3^2 = e^{4\psi} d\tau^2 + e^{2\psi}\left(dx^2 + dy^2\right)$$

N.B. $\tau \to 0$ represents the asymptotic regime and $\tau = \infty$ is the event horizon.

The metric d.o.f. are

$$e^{-4\psi} = \left(\frac{1}{B_0}\right)^3 \sinh^3(B_0\tau) e^{B_0\tau},$$

$$e^{\phi} = -2H = \frac{1}{2}(-q_0)^{\frac{1}{2}}\left(f(q_1,\ldots,q_n)\right)^{-\frac{1}{2}},$$

with scalar fields given by

$$\dot{q}_0 = -Q_0, \quad q_0 = \pm\frac{-Q_0}{B_0}\sinh\left(B_0\tau + B_0\frac{h_0}{Q_0}\right), \quad q_A = \pm\frac{3}{8ng_A}\left(\frac{1}{B_0}\right)^{\frac{1}{2}} e^{\frac{1}{2}B_0\tau}\left(\sinh(B_0\tau)\right)^{\frac{1}{2}}$$

Leaves a family of solns parameterised by B_0 and h_0.

Change of Coordinates

It's convenient to change to the radial coordinate ρ given by

$$e^{-2B_0\tau} = 1 - \frac{2B_0}{\rho} = W(\rho)$$

The scalars become

$$q_0 = \pm\frac{\mathcal{H}_0}{W^{\frac{1}{2}}}, \qquad q_A = \pm\frac{3}{8ng_A}(\rho W)^{-\frac{1}{2}} \quad \text{with } \mathcal{H}_0(\rho) \text{ a harmonic fn.}$$

The general expression for the 4d line element is

$$ds_4^2 = -\mathcal{H}^{-\frac{1}{2}}\rho^{\frac{3}{4}}dt^2 + \mathcal{H}^{\frac{1}{2}}\rho^{-\frac{7}{4}}\frac{d\rho^2}{W} + \mathcal{H}^{\frac{1}{2}}\rho^{\frac{3}{4}}\left(dx^2 + dy^2\right)$$

where $\mathcal{H}$ is a fn of $\mathcal{H}_0, g_A$.

- This change of coordinates makes limits more transparent. Horizon now at $\rho = 2B_0$ and asymptotic region at $\rho \to \infty$.
- $B_0 \to 0$ reproduces extremal soln in literature ($\therefore B_0$ is non-ext parameter).

Thermodynamics

- Zooming in on near-horizon geometry, one can compute the horizon temperature and entropy density of the black brane. These are related by

$$B_0 = 2\pi s T_H$$

- We can also look at the asymptotic values of the 4d gauge fields to find the chemical potential

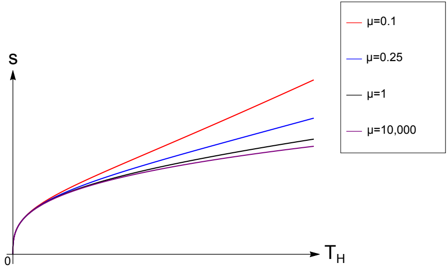
$$\mu = \frac{1}{2}\left(\frac{B_0}{Q_0}\right)\left[\coth\left(\frac{B_0 h_0}{Q_0}\right) - 1\right]$$

- Have a 2 parameter family with (B_0, h_0) controlling:
  - brane geometry on gravity side
  - thermodynamic quantities s, T_H and μ on CMT side

Equation of State

- Can combine above expressions to find the equation of state

$$s^3 = 4\pi Z^2 T_H \left(1 + \frac{2\pi s T_H}{Q_0 \mu}\right) \qquad Z \text{ is fn of charges and fluxes.}$$



We see that $s \to 0$ as we send $T_H \to 0$ so we are justified in calling our solutions **Nernst branes**.

- Smooth crossover in behaviour from:
  - $s \sim T_H^{\frac{1}{3}}$ regime when $T_H/\mu \ll 1$
  - $s \sim T_H$ regime when $T_H/\mu \gg 1$.

hvLif Spacetimes

- Want to look at the geometry of our 2 parameter family.
- $\exists$ 4 cases to consider depending on whether B_0, h_0 are zero or not.
- We consider the near-horizon and asymptotic geometries (IR and UV of field theory) of all 4 cases.
- In each case, find a hyperscaling-violating Lifshitz geometry:

$$ds^2_{d+2} = r^{-\frac{2(d-\theta)}{d}} \left(-r^{-2(z-1)} dt^2 + dr^2 + dx_i^2 \right)$$

Under $t \to \lambda^z t, r \to \lambda r, x_i \to \lambda x_i$ we find $ds \to \lambda^{\frac{\theta}{d}} ds$.
z is Lifshitz exponent, θ is hyperscaling violating exponent
$(z, \theta) = (1, 0)$ returns AdS$_{d+2}$

- Don't worry about not having asymptotically AdS solns.
- Recently there has been much work on hvLif holography.
- CMTs are inherently non-relativistic.

hvLif Geometries

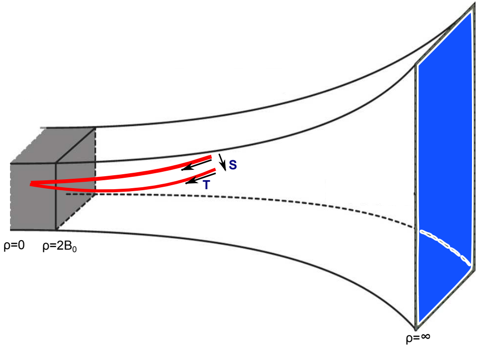
hvLif Geometries			
B_0	h_0	Horizon (z, θ)	Asymptotic (z, θ)
$B_0 = 0$ ($T_H = 0$)	$h_0 = 0$ ($\mu \to \infty$)	$(3, 1)$	$(3, 1)$
$B_0 = 0$ ($T_H = 0$)	$h_0 \neq 0$ (μ finite)	$(3, 1)$	$(1, -1)$
$B_0 \neq 0$ ($T_H \neq 0$)	$h_0 = 0$ ($\mu \to \infty$)	$(0, 2)$	$(3, 1)$
$B_0 \neq 0$ ($T_H \neq 0$)	$h_0 \neq 0$ (μ finite)	$(0, 2)$	$(1, -1)$

Some comments:

- $B_0 = 0, h_0 = 0$ ($T_H = 0, \mu \to \infty$) is a global solution $\Rightarrow$ interpret as gravitational ground state.
- $B_0 = 0, h_0 \neq 0$ ($T_H = 0, \mu$ finite) is extremal Nernst brane soln of Cardoso *et al*.
- B_0 (resp. h_0) controls near horizon (resp. asymptotic) geometry.

Infinite Tidal Forces

ρ coordinate $\longleftrightarrow$ energy scale in field theory.
$\exists$ some bad behaviour for small ρ (IR of field theory)
All curvature invariants finite as $\rho \to 0 \Rightarrow$ no curvature singularity.
The singular behaviour in question is less severe:



In hvLif spacetime, geodesic acceleration is

$$\nabla_T \nabla_T S = R(S,T)T$$

with

$$R(S,T) \sim \frac{z-1}{\rho^{2z}}$$

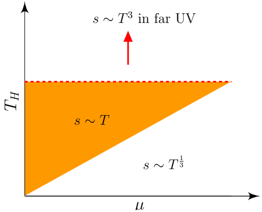
Extremal ($B_0 = 0$): $z = 3$ near horizon so $R(S,T) \to \infty$ as $\rho \to 0$.

Deep IR Spaghettification

Infinite tidal forces result in **"spaghettification"** of infalling observers.

- **Extremal ($B_0 = 0$):** 4d scalars $\sim \rho^{-1/4}$ and blow up on horizon.
- **Non-extremal ($B_0 \neq 0$):** Horizon at $\rho = 2B_0$ providing protection. Singular behaviour occurs behind the horizon.
- Tidal forces still large for low temp (low B_0) case.
- Field theory can't be trusted in deep IR (very low temps).

Is There a UV Duality?



Eqn of state for gravity soln,
$s^3 = 4\pi Z^2 T_H \left(1 + \frac{2\pi s T_H}{Q_0 \mu}\right)$,
gives the following phase diagram.

Scaling argument $\Rightarrow$ $s \sim T^{\frac{d-\theta}{z}}$ for field theory

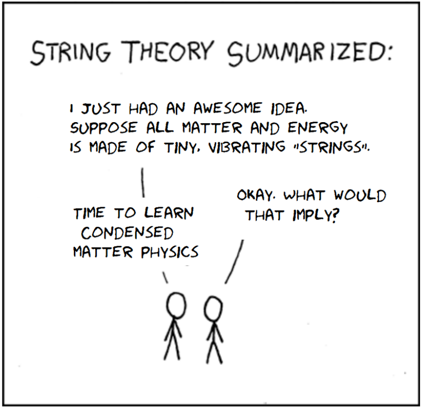
Do the predictions match?

- $B_0 \neq 0, h_0 = 0$ ($T_H \neq 0, \mu \to \infty$):
  - Gravity $\Rightarrow s \sim T^{\frac{1}{3}}$
  - Field theory has $(z, \theta) = (3, 1) \Rightarrow s \sim T^{\frac{1}{3}}$
- $B_0 \neq 0, h_0 \neq 0$ ($T_H \neq 0, \mu$ finite):
  - Gravity $\Rightarrow s \sim T^{\frac{1}{3}}$ or $s \sim T$ depending on μ
  - Field theory has $(z, \theta) = (1, -1) \Rightarrow s \sim T^3$
- Don't trust duality in deep UV - extra d.o.f.s become relevant.

Conclusion

- Family of non-extremal black branes whose entropy density vanishes in extremal limit. These are **Nernst branes**.
- Should be holographically useful.
- $\exists$ holographic duality in finite region of parameter space: excluding deep IR/UV.
- Future work (ask for more details):
  1. V. special lift to 5d - fix IR and UV problems
  2. Dyonic configurations - phase transitions
  3. Entanglement entropy as an a-function?
  4. hidden Fermi surfaces?

Thank You!


STRING THEORY SUMMARIZED:
I JUST HAD AN AWESOME IDEA. SUPPOSE ALL MATTER AND ENERGY IS MADE OF TINY, VIBRATING "STRINGS".
OKAY. WHAT WOULD THAT IMPLY?
TIME TO LEARN CONDENSED MATTER PHYSICS