Aspects of Four and Two Dimensional Supersymmetric Gauge Theories

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Introduction

4d-2d Connection

- Couple the current supermultiplet to an external vector superfield

$$
\begin{gathered}
(A_\mu, \lambda_\alpha, D) \\
\Re^{(1,1)} \times T^2 \\
(x^0, x^3) \epsilon \Re^{(1,1)} \\
(x^1, x^2) \epsilon T^2 \\
F_{12} = B
\end{gathered} \tag{1}
$$

Through an analysis of the variation of the gaugino field, this breaks supersymmetry completely.

$$\delta\lambda = F_{\mu\nu}\sigma^{\mu\nu}\epsilon \tag{2}$$

$$\delta\lambda = (F_{\mu\nu}\sigma^{\mu\nu} + iD)\epsilon$$

$$\delta\lambda = i\begin{pmatrix} D-B & 0 \\ 0 & D+B \end{pmatrix}\begin{pmatrix} \epsilon_- \\ \epsilon_+ \end{pmatrix}$$

(0,2) Field Theory in 2 Dimensions

$$Q_+ = \frac{\partial}{\partial\theta^+} + i\bar{\theta}^+(\partial_0 + \partial_3), \bar{Q}_+ = -\frac{\partial}{\partial\bar{\theta}^+} - i\theta^+(\partial_0 + \partial_3) \quad (3)$$

And satisfy:

$$Q_+^2 = \bar{Q}_+^2 = 0 \quad (4)$$

$$\{Q_+, \bar{Q}_+\} = -2i(\partial_0 + \partial_3)$$

The covariant derivatives are given by:

$$D_+ = \frac{\partial}{\partial\theta^+} - i\bar{\theta}^+(\partial_0 + \partial_3), \bar{D}_+ = -\frac{\partial}{\partial\bar{\theta}^+} + i\theta^+(\partial_0 + \partial_3)$$

(0,2) Field Theory in Two Dimensions

We now come to the task of constructing $(0,2)$ supersymmetric theory and to aid in this endeavour, the superspace derivatives are extended to gauge covariant derivatives written in a [wess zumino type gauge];

$$\hat{D}_0 + \hat{D}_3 = \partial_0 + \partial_3 + i(A_0 + A_3) \tag{5}$$

$$\hat{D}_+ = \frac{\partial}{\partial\theta^+} - i\bar{\theta}^+(\hat{D}_0 + \hat{D}_3)$$

$$\hat{\bar{D}}_+ = -\frac{\partial}{\partial\bar{\theta}^+} + i\theta^+(\hat{D}_0 + \hat{D}_3)$$

$$\hat{D}_0 - \hat{D}_3 = \partial_0 - \partial_3 + iV$$

$$V = A_0 + A_3 - 2i\theta^+\bar{\lambda}_- - 2i\bar{\theta}^+\lambda_- + 2\theta^+\bar{\theta}^+ D$$

(0,2) Field Theory in Two Dimensions

V is a vector superfield transforming in the adjoint representation of the gauge group G. λ_- is the left moving gaugino and D is a non propagating auxiliary field. Another object that is pertinent to $(0,2)$ field theory is an object known as the field strength, which is the combination of the aforementioned fields and the gauge field:

$$\Upsilon = [\hat{\bar{D}}_+, \hat{D}_0 - \hat{D}_3] = -2(\lambda_- - i\theta^+(D - iF_{03}) \tag{6}$$

$$-i\theta^+\bar{\theta}^+(D_0 + D_3)\lambda_-)$$

$$\hat{\bar{D}}_+\Upsilon = 0$$

(0,2) Field Theory in Two Dimensions

Such a field transforms in the adjoint representation of the gauge group and satisfies the chirality constraint as shown in the last equation of (18). There are two types of matter superfields in a representation r of the gauge group G that are important. These are the bosonic chiral superfield Φ and the Fermi superfield Λ which have the following component expansions and constraints:

$$\Phi = \phi + \sqrt{2}\theta^+\psi_+ - i\theta^+\bar{\theta}^+(D_0 + D_3)\phi, \quad (7)$$

$$\Lambda = \psi_- - \sqrt{2}\theta^+ F - i\theta^+\bar{\theta}^+(D_0 + D_3)\psi_- - \sqrt{2}\bar{\theta}^+ E$$

$$\hat{\bar{D}}_+\Phi = 0$$

$$\hat{\bar{D}}_+\Lambda = \sqrt{2}E$$

$$\hat{\bar{D}}_+ E = 0$$

In the equations (19), Φ is a complex scalar field, ψ_+ a complex

Classical Theory For Single U(1)

We have the Klein Gordon equation

$$(-\partial_0^2 + \partial_3^2 + \partial_1^2 + \tilde{D}_2^2)\phi = 0 \tag{8}$$

We obtain the 1+1 dimensional spectrum as follows:

$$\phi(x^0, x^1, x^2, x^3) = \varphi(x^0, x^3)\chi(x^1, x^2)$$

χ is taken to be an eigenfunction of

$$H = -(\partial_1^2 + \tilde{D}_2^2) = p_1^2 + (p_2 + eBx_1)^2$$

$$H\chi = m^2\chi$$

The spectrum is as follows:

$$m_n^2 = (2n+1)|eB| \tag{9}$$

To preserve supersymmetry the D component in addition to the magnetic field of the corresponding vector multiplet is turned on. The spectrum is as follows,

$$m_n^2 = (2n+1)|eB| - eD$$

$$eB > 0$$

$$eB < 0$$

For free fermions, the folowing relation applies,

$$m_+^2 = (2n+1)|eB| - eB$$

$$m_-^2 = (2n+1)|eB| + eB$$

Classical Theory For Multiple U(1)'s

For the case of a $U(1) \times U(1)$ product group, we require the following coupling

$$\tilde{D}_2 = \partial_2 + ie_1 B_1 x_1 + ie_2 B_2 x_1 \tag{10}$$

The Klein Gordon equation for ϕ takes the form;

$$(-\partial_0^2 + \partial_3^2 + \partial_1^2 + \tilde{D}_2^2)\phi = 0 \tag{11}$$

The 1+1 dimensional spectrum is obtained by writing

$$\phi(x^0, x^1, x^2, x^3) = \varphi(x^0, x^3)\chi(x^1, x^2) \tag{12}$$

χ is subsequently taken to be an eigenfunction of

$$H = -(\partial_1^2 + \tilde{D}_2^2) \tag{13}$$

Explicitly, the hamiltonian, after multiplying out the factors takes the form, The hamiltonian only depends on the x_1 coordinate therefore the P_2 and P_3 coordiantes of the momentum are conserved . seek a wavefunction given by the following

$$\psi = e^{i(P_2X_2+P_3X_3)}\chi(x_1) \tag{15}$$

After computation, the Schrodinger equation assumes the following form given by

$$\chi''(x_1)+[E-P_2^2-(e_1B_1+e_2B_2)^2x_1^2-2(e_1B_1+e_2B_2)x_1]=0 \tag{16}$$

And finally,

$$\chi''(x_1)+[E-\frac{1}{2}(e_1B_1+e_2B_2)^2(x_1-x_0)^2x_1^2]=0 \tag{17}$$

The Hamiltonian (17) is that of the Landau problem of a particle in a magentic field, whose spectrumis given in a shifted version of that given by;

$$m_n^2 = (2n+1)|e_1 B_1 + e_2 B_2| \tag{18}$$

For the case $B = D$

The above discussion can be repeated for spin $\frac{1}{2}$ fields. A four dimensional charged weyl fermion in a magnetic field satisfies the wave equation

$$\begin{pmatrix} -\partial_0 - \partial_3 & -\partial_1 + i\tilde{D}_2 \\ -\partial_1 - i\tilde{D}_2 & -\partial_0 + \partial_3 \end{pmatrix} \begin{pmatrix} \psi_- \\ \psi_+ \end{pmatrix} = 0$$

The top component of the spinor (ψ_-) is a left moving fermion in the two dimensions (x^0, x^3); the bottom component, (ψ_+), is right moving. squaring (3.6) yields decoupled equations for $\psi_\pm$,

$$(-\partial_0^2 + \partial_3^3 + \partial_1^2 + \tilde{D}_2^2 \mp i[\partial_1, \tilde{D}_2])\psi_\pm = 0 \tag{19}$$

Using the fact that $[\partial_1, \tilde{D_2}] = ieB$, (10) is essentially identical to (2), so the left and right moving fermions have the spectrum,

$$m_+^2 = (2n+1)|e_1 B_1 + e_2 B_2| - e_1 B_1 - e_2 B_2 \tag{20}$$

$$m_-^2 = (2n+1)|e_1 B_1 + e_2 B_2| + e_1 B_1 + e_2 B_2 \tag{21}$$

N=1 SQCD

Following constraints,

$$\sum_i e_i T(r_i) = 0 \tag{22}$$

$$\sum_i e_i^2 TrT^a(r_i) = 0 \tag{23}$$

Supersymmetric qcd with gauge group $G = U(N_c)$ and N_f flavors of chiral superfields in the fundamental representation of the gauge grouo, $Q^i, \tilde{Q}, i = 1, 2, ..., N_f$.
The global (non-R) symmetry group of this theory is

$$SU(N_f) \times SU(N_f) \tag{24}$$

The anomaly freedom constraints imply that

$$\sum_i e_i + \sum_i \tilde{e}_i = 0 \tag{25}$$

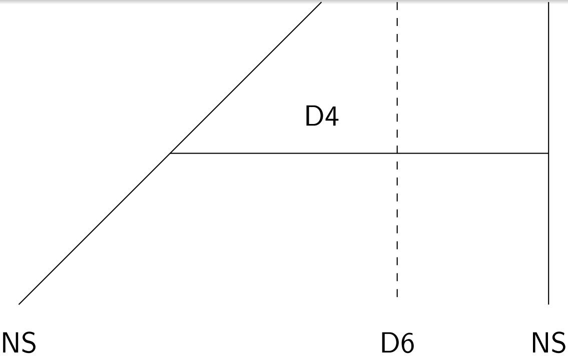
$$\sum_i e_i^2 - \sum_i \tilde{e}_i{}^2 = 0 \tag{26}$$

$$\sum_i e_i = \sum_i \tilde{e}_i = 0 \tag{27}$$

An example of a solution is to take (27),which exists for all even N_f, is to take $\frac{N_f}{2}$ of the e_i to be equal to $+1$ and the rest equal to -1 and similarly for $\tilde{e}_i$. this breaks the symmetry (24) to

$$SU(N_f)^4 \times U(1) \tag{28}$$

The N_f fundamentals Q^i give rise to $\frac{N_f}{2}$ chiral superfields and $\frac{N_f}{2}$ fermi superfields in the fundamental, and similarly for $\tilde{Q}$.



N=1 sqcd has a handy interpretation in terms of branes. The brane content is given as follows; • N_c D4 branes |01236|

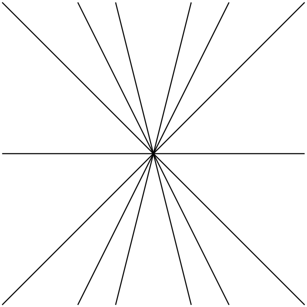
• NS and and NS' branes respectively |012345| and |012389|

• N_c $D6$ branes |0123789| are placed between fivebranes

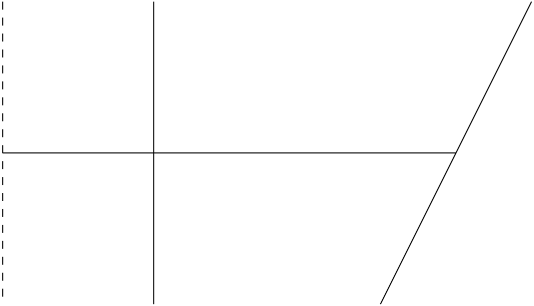
To implement the construction of (24), compactify $x^1, X^2)$ on a two torus, pick up a $U(1)$ inside the global symetry group (24), and turn on the magnetic field on the torus and D field for it. In the brane construction it isslightly simpler to deal with the diagonal $SU(N_f)$. Can generalise this discussion to other $U(1)$ symmetries, by placing the $D6$-branes at the location of the NS' brane and using the results of [12].

The procedure for turning on the external fields are as follows; Each of the N_f $D6$ branes gives rise to one hypermultiplet, which is charged under the $U(1)$ gauge field living on the sixbrane. We turn on a magnetic field B_i for this $U(1)$ field, and accompany it by a suitable rotation of the $D6$ brane from the x^7 to the x^6 direction.

This is the brane analogue of the D-term in the low energy gauge theory, and it preserves susy if we tune the rotation angle to correspond to the magnetic field that we turned on. The resulting



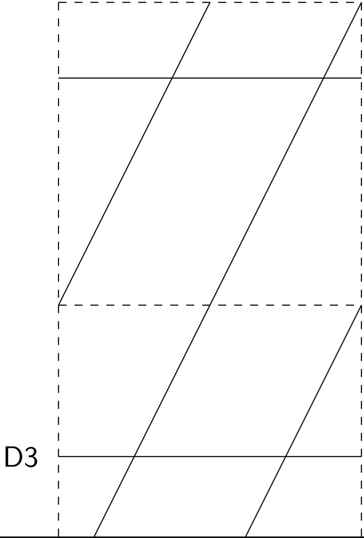
The brane system that realises the seiberg dual of $N = 1$ SQCD includes N_f-N_c colour $D4$ branes connecting two NS5 branes , and N_f flavour $D4$-branes, each connecting the NS'-brane to one of N_f $D6$-branes.



Seiberg duality in the compactified theory with the background B and D fields. The seiberg dual theory is depicted above. The $N_f - N_c$ colour $D4 brames$ connecting the $NS5$ branes give rise to

- Magnetic quarks $q,\tilde{q}$ come from strings stretched between the colour and flavour branes and are localised near the NS' brane.

Tdualise the n=1 sqcd in the x^2 direction.



An example of a solution is to take (27),which exists for all even N_f, is to take $\frac{N_f}{2}$ of the e_i to be equal to $+1$ and the rest equal to -1 and similarly for $\tilde{e}_i$. this breaks the symmetry (24) to

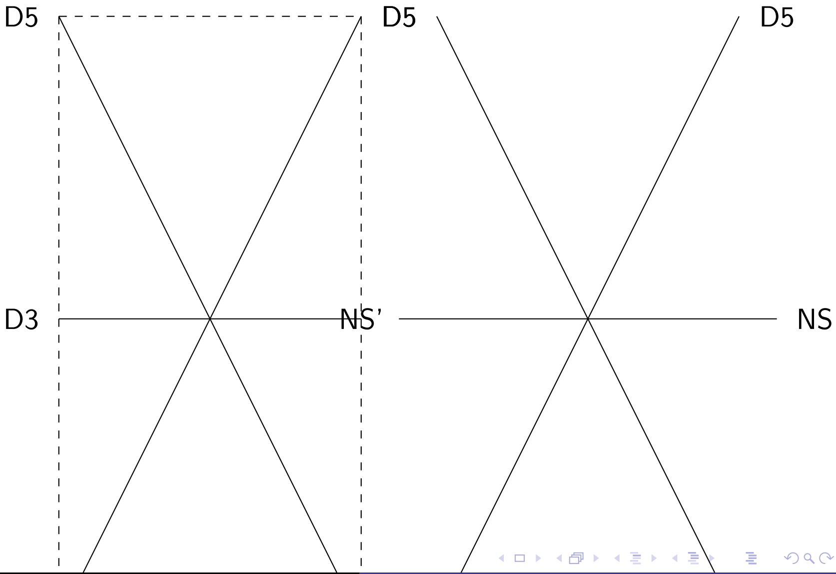
$$SU(N_f)^4 \times U(1) \tag{30}$$

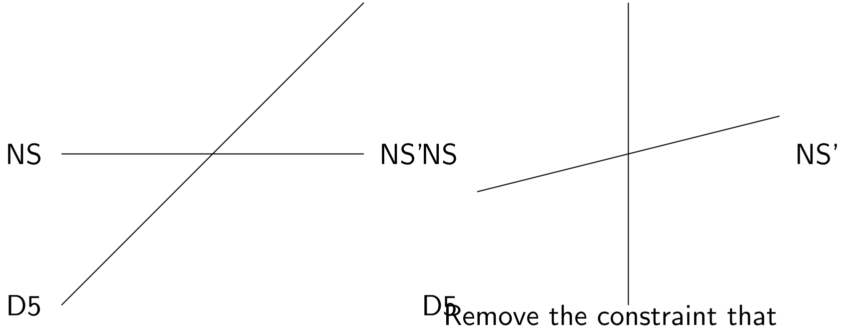
The N_f fundamentals Q^i give rise to $\frac{N_f}{2}$ chiral superfields and $\frac{N_f}{2}$ fermi superfields in the fundamental, and similarly for $\tilde{Q}$.

T-dualise the systerm in the X^2 direction, which changes $D4$ to $D3$
• compactify x^1 on a circle and turn on background fields. in the four dimensions, B corresponds to a vector potential $A_2 = Bx^1$ for global $U(1)$, which in the brane description as a $U(1)$ gauge field on a $D6 - brane$.
• $3d$ description, A_2 becomes a scalar field in the vector multiplet on a $D-5$ brane. Expectation value corresponds to rotation.
• Quantisation of the Bfield on the two torus corresponds in the IIB language to quantisation of the angle that the $D5$-brane makes with the x^1 axis. as the $D5$ brane wraps the x^1 ciecle once, it wraps the x^2 circle twice.
• moving a $D5$ in x^2 corresponds to giving equal and opposite real masses to the corresponding chiral superfields. Rotation in the (x^1, x^2) plane coresponds to a real mass that depends linearlyon x^1, which breaks Lorentz symmetry to $SO(1,1)$ and localises these fields at the minimum of the resulting potential.

• can now combine the above elements to describe the elementsof section 4 in terms of branes.

• consider the model in which we give $\frac{N_f}{2}$ of the $Q's$ $\tilde{Q}$'s charge $e = +1(-1)$, and to the other $\frac{N_f}{2}$ the opposite charge. The corresponding brane configuration is depicted in figue 6. Each intersection of the N_c $D3$ branes with one of the N_f rotated $D5$ branes supports either a $(0,2)$ chiral superfield coming from Q and a fermi superfield from $\tilde{Q}$, or the other way around, depending on the sign of the rotation angle.


D5
D5
D5
D3
NS'
NS



Remove the constraint that the $U(1)$, which is used for the construction is not part of the gauge group. can consider taking the charges to be $e_i = e$ $\tilde{e}_i = -e$ for all $i = 1, ..., N_F$. This corresponds to turning on a magnetic field for the $U(1)$ factor in the gauge group $U(1)_B$. in the brane picture this corresponds to rotating all N_f $D5$ branes by the same angle, leading to the brane configuration.

An equaivalent brane configuration is obtained by turning on an FI term for the $U(1)$ factor in the guage group. Corresponds to the relative displacement of the two $NS5$ branes in x^7. Both of the figures are related by an overall rotation in the (67) plane, corresponding to turning on a D term for a $U(1)$ symmetry.

Geometric transitions are used to discuss strongly coupled field theories. A conifold is defined by the following equation

$$xy - uv = 0 \tag{31}$$

A resolution of the conifold involves replacing the singularity point by P^1. The conifold is defomed as follows

$$xy - uv = \mu \tag{32}$$

where μ gives the size of an s^3 There are two types of P^1 Rigid and non rigid. The size of the P^1 comes from the existence of an NS 2-form

$$\int_a^b B_{NS} \neq 0 \tag{33}$$

We have an $SU(3)$ structure manifold

Put $D5$ branes on an non-rigid P^1 central charge

$$Z = \int iB_{NS} = ib_{NS} \tag{34}$$

In the case of a rigid P^1 The central charge is given by

$$z = \int (J + iB_{NS} = j + ib_{NS} \tag{35}$$

Turning on the J corresponds to a D-term

$$\Delta L = \sqrt{2}\xi TrD \tag{36}$$

$$U(1) \subset U(N) \tag{37}$$

For many gauge groups

$$U(N_1) \times U(N_2) \tag{38}$$

$$\int_{P^1} B_{NS} \neq 0 \tag{39}$$

For one rigid and one non rigid case, supersymmetry is broken. Now try and put some $D5$ branes on a two torus T^2 Put m units of magnetic flux on T^2