Calculating the Hadronic Vacuum Polarisation Contribution to the Anomalous Magnetic Moment of the Muon, $a_\mu^{had,VP}$

Alex Keshavarzi

University of Liverpool

a.i.keshavarzi@liverpool.ac.uk

May 20, 2015

Overview

1. Muon g-2: The Basics
   - Motivation
   - The Hadronic LOVP Contribution
2. Computing $a_\mu^{had,VP\ LO}$
   - Data Processing
   - Data Combination
   - Minimisation
   - Integration
3. Updating the Current Value for $a_\mu^{had,VP\ LO}$
   - Results
   - The Future...

The Standard Model (SM): An Incomplete Theory

- a_μ is one of the most precisely measured quantities in particle physics, accurate to 0.54ppm.
- $a_\mu^{SM} < a_\mu^{exp}$ by approximately 3 standard deviations.
- Any deviation could herald the existence of as-yet-unknown new physics beyond the SM.
- Experiment will be four times more accurate after the completion of Muon g-2 experiment at Fermilab!

By improving the precision of a_μ, we can determine whether the SM is an incomplete theory!!

The Anomalous Magnetic Moment

- A charged elementary particle generates an angular momentum, $\vec{\mu}$

$$\vec{\mu} = g\frac{e}{2m}\vec{s}$$

- In 1928, Dirac predicted that $g = 2$ for spin-1/2 particles.
- However, due to radiative corrections

$$a_l = \frac{(g-2)}{2} \quad \text{where } (l = e, \mu, \tau)$$

- In 1948, Schwinger calculated the leading order (LO) contribution as

$$a_l^{QED,LO} = \frac{\alpha}{2\pi} \ .$$

Contributions from the SM

$$a_\mu = \frac{(g-2)}{2} = a_\mu^{QED} + a_\mu^{EW} + a_\mu^{had} + a_\mu^{NewPhysics?}$$

$$a_\mu^{QED} = (11658471.808 \pm 0.015) \times 10^{-10}$$ [Kinoshita et al.]

$$a_\mu^{EW} = (15.4 \pm 0.2) \times 10^{-10}$$ [Czarnecki et al., Knecht et al.]

$$a_\mu^{had} = (695.6 \pm 4.9) \times 10^{-10}$$ [HLMNT 2012]

$$a_\mu^{SM} = a_\mu^{QED} + a_\mu^{EW} + a_\mu^{had} = (11659182.8 \pm 4.9) \times 10^{-10}$$

$$a_\mu^{exp} = (11659208.9 \pm 6.3) \times 10^{-10}$$ [arXiv:1001.2898]

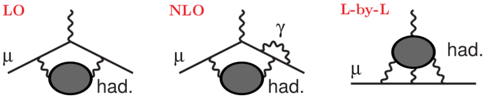
$$\therefore \mathbf{a_\mu^{exp} - a_\mu^{SM} = (26.1 \pm 8.0) \times 10^{-10}}$$

corresponding to a 3.3σ discrepancy.

The Hadronic Contribution

Uncertainties from the hadronic sector completely dominate Δa_μ^{SM}!

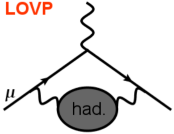
$$a_\mu^{\text{had}} = a_\mu^{\text{had,VP LO}} + a_\mu^{\text{had,VP NLO}} + a_\mu^{\text{had,Light-by-Light}}$$



Of these, $a_\mu^{\text{had, VP LO}}$ has the largest uncertainty.

The internal hadronic "blob" contains the contributions from all the possible hadronic states!!

Calculating a_μ^{had}



- By means of unitarity,

$$\sigma_{e^+e^-\to hadrons}(s) = \left(\frac{4\pi\alpha}{s}\right)\mathrm{Im}\Pi^\gamma_{had}(s)$$

- Using analyticity, we get the dispersion integral

$$a_\mu = \frac{\alpha}{\pi}\int_0^\infty \frac{ds}{s}\frac{1}{\pi}\mathrm{Im}\Pi^\gamma_{had}(s)K(s)$$

$$\text{and} \quad a_\mu^{had,LOVP} = \left(\frac{\alpha m_\mu}{3\pi}\right)^2\int_{s^{th}}^\infty ds\ \frac{R^{data}_{had}(s)K(s)}{s^2}$$

$$\text{where} \quad R_{had}(s) \equiv \sigma^0_{e^+e^-\to hadrons}/\tfrac{4\pi\alpha^2}{3s} \quad \text{and} \quad K(s) = \tfrac{m_\mu^2}{3s}(0.4\ ...\ 1).$$

- Integration limits go from $s = 4m_\pi^2 \to \infty$ as $K(s) = 0.4 \to 1$.

Combining Data Sets

Large number of available data sets for a given hadronic channel.

```
Z 75 !KLOE10
Z 337 !BaBar, PRL103 231801 (2009); 337pts, 0.30 .. 3.0
Z 60 !KLOE08, 0809.3950v2 (2008); 0.5916 .. 0.9747
y 10 !CMD-2, Akhmetshin06, hep-ex/0610016; 10 pts, 0.37 .. 0.52
y 29 !CMD-2, Akhmetshin06, hep-ex/0610021; 29 pts, 0.6 .. 0.97
y 45 !SND, Achasov04, re-analysis, hep-ex/0605013; 45 pts, 0.39 .. 0.97
y 36 !CMD-2, Aulchenko06,hep-ex/0603021,JETP Lett82(2005)743, 36pts, 1.04-1.38
n 60 !KLOE05, Rad.Ret., KLOE Note n.192, 60 pts, 0.35 .. 0.94 GeV^2 (OLD KLOE Data, exclude)
y 2  !OLYA-VEPP2, Vasserman79, SJNP30(1979)519; 2 pts, 0.4 and 0.44
y 4  !TOF-VEPP_2M, Vasserman81, SJNP33(1981)368; 4 pts, 0.4 .. 0.46
y 4  !NA7-CERN, Amendolia84, PL138B(1984)454; 4 pts, 0.32 .. 0.422
y 77 !OLYA-VEPP_2M, Barkov85, NPB256(1985)365; 77 pts, 0.6426 .. 1.397
y 24 !CMD-VEPP_2M, Barkov85, NPB256(1985)365; 24 pts, 0.36 .. 0.82
y 16 !DM1-ACO, Quenzer78, PL76B(1978)512; 16 pts, 0.483 .. 1.096
y 17 !DM2-DCI, Bisello89, PLB220(1989)321; 17 pts, 1.35 .. 2.125
y 13 !BCF-ADONE, Bollini75, Lett.NC14(1975)418; 13 pts, 1.2 .. 3.
y 1  !MEA-ADONE-Frascati, Esposito77, PLB67(1977)239; 1pt at 1.6
y 1  !MEA-ADONE-Frascati, Esposito80, NCL28(1980)337; 1pt at 1.45..1.52
y 8  !BCF-ADONE, Bernardini73, PL46B(1973)261; 8 pts, 1.2 .. 3.
y 28  !NOVOSIBIRSK-CMD, Dolinsky91, PRep202(1991)99; 28 pts, 0.81 .. 1.39
y 29  !NOVOSIBIRSK-VEPP-2M, OLYA, Bukin78, PL73B(1978)226; 29 pts, 1.33 .. 0.78
y 17  !NOVOSIBIRSK-VEPP-2M, OLYA, Koop79, INP-79-67 (1979); 17 pts, 1.06 .. 1.40
y 3   !Cosme85, LAL-1287 (1985) and NPB256(1985)365; 3 pts, 0.915, 0.99, 1.076 !should be from 1976
y 43  !CMD-2, REANALYSIS, hep-ex/0308008; 43 pts, 0.6105 .. 0.96152
y 60  !KLOE, Babusci 2013, PL 720B,336; 60 pts, 0.5958 .. 0.9721
```

We must make sure that for each data set we,

- correctly remove the photon VP effects.
- calculate accurate errors considering our VP corrections
- account for any final state radiative (FSR) corrections.

How do we use data from different experiments in a given channel to compute its contribution to a_μ^{had}?

- Combine the data in a given channel before integrating.
- Re-bin the data points into *energy clusters*.

$$R_m = \left[\sum_k \sum_{i=1}^{N^{(k,m)}} \frac{R_i^{(k,m)}}{(d\tilde{R}_i^{(k,m)})^2}\right]\left[\sum_k \sum_{i=1}^{N^{(k,m)}} \frac{1}{(d\tilde{R}_i^{(k,m)})^2}\right]^{-1}$$

$$E_m = \left[\sum_k \sum_{i=1}^{N^{(k,m)}} \frac{E_i^{(k,m)}}{(d\tilde{R}_i^{(k,m)})^2}\right]\left[\sum_k \sum_{i=1}^{N^{(k,m)}} \frac{1}{(d\tilde{R}_i^{(k,m)})^2}\right]^{-1}$$

$N^{(k,m)}$ = total no. of data points within cluster m.

- Use adaptive clustering algorithm to produce *target clusters*.
  $\rightarrow$ too small a cluster = precise data overwhelmed
  $\rightarrow$ too large a cluster = data missed about resonance peaks

Fitting The Data

Use a non-linear non-linear χ^2-function [HLMNT, 2012]

$$\chi^2(R_m, f_k) = \sum_{k=1}^{N_{exp}} \left(\frac{1-f_k}{df_k}\right)^2 + \left\{ \sum_{m=1}^{N_{clu}} \sum_{i=1}^{N^{(k,m)}} \left(\frac{R_i^{(k,m)} - f_k R_m}{d\tilde{R}_i^{(k,m)}}\right)^2 \right\}_{\text{w/o cov. mat}}$$
$$+ \left\{ \sum_{m=1}^{N_{clu}} \sum_{i=1}^{N^{(k,m)}} \sum_{j=1}^{N^{(k,n)}} (R_i^{(k,m)} - f_k R_m) C^{-1}(m_i, n_j) (R_j^{(k,n)} - f_k R_n) \right\}$$

where N_{clu}, N_{exp} are total number of clusters, experiments.

- Treat the statistical/systematic errors according to experimental data.
- Input covariance matrices where provided (last term!).

Goodness-of-Fit

Minimise $\chi^2(R_m, f_k)$ to give

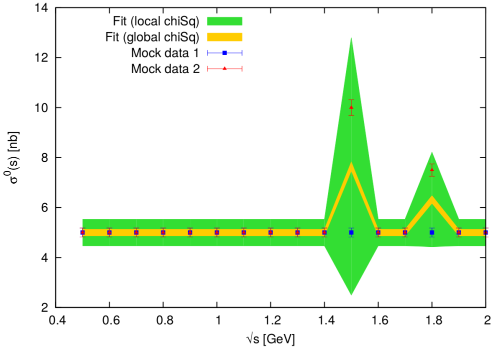
- Fitted values $\bar{R}_m$ and $\bar{f}_k$.
- Minimised global χ^2_{min}
- Covariance matrix $V(m,n)$ that defines the correlation of errors between the clusters.

$$V(m,n) = (dR_m)(dR_n)\rho_{\text{corr}}(m,n)$$

$\rightarrow$ Determine a 'goodness' of our overall fit from

$$\frac{\chi^2_{\text{min}}}{\text{d.o.f.}} = \frac{\chi^2_{\text{min}}}{N_{\text{tot}} - N_{\text{clu}} - N_{\text{exp}}}$$

Local χ^2_m [HLMNT, 2012]



Channels	Global χ^2/dof	Global inf err	Local inf err	Global - Local
2π	1.3971	3.0560	3.0896	-0.0336
3π	2.5669	1.1865	0.8045	$+0.3820$
$4\pi(2n)$	1.2922	1.1915	1.2593	-0.0678
4π	1.6926	0.4902	0.4738	$+0.0164$
K^+K^-	1.8423	0.5910	0.4781	$+0.1129$
$K^0_S K^0_L$	0.8055	0.1602	0.1627	-0.0025
$6\pi(2n)$	4.0304	0.3945	0.2383	$+0.1562$
$5\pi(1n)$	1.3831	0.0903	0.0857	$+0.0046$
$kk\pi\pi$	N/A	1.3234	1.3225	$+0.0009$

Using the Trapezoid Rule

Between two arbitrary energies E_a and E_b

$$I = \int_{E_b^2}^{E_a^2} \frac{ds}{s}\ R_{had}(s)K(s) = 2\int_{E_b^2}^{E_a^2} \frac{dE}{E^2}\ E\ R_{had}(E^2)K(E^2) = \bar{I} \qquad \text{with error} \pm \Delta\bar{I}$$

Then, suppose that $E_m < E_a < E_{m+1}$ which is less than $E_{n-1} < E_b < E_n$ and estimate using linear interpolation,

$$\bar{I} = 2\left(\frac{E_{m+1}-E_a}{2E_a}\bar{R}_a K_a + \frac{E_{m+2}-E_a}{2E_{m+1}}\bar{R}_{m+1}K_{m+1}\right) + 2\left(\sum_{l=m+2}^{n-2}\frac{E_{l+1}-E_{l-1}}{2E_l}\bar{R}_l K_l\right)$$
$$+ 2\left(\frac{E_b-E_{n-2}}{2E_{n-1}}\bar{R}_{n-1}K_{n-1} + \frac{E_b-E_{n-1}}{2E_b}\bar{R}_b K_b\right)$$

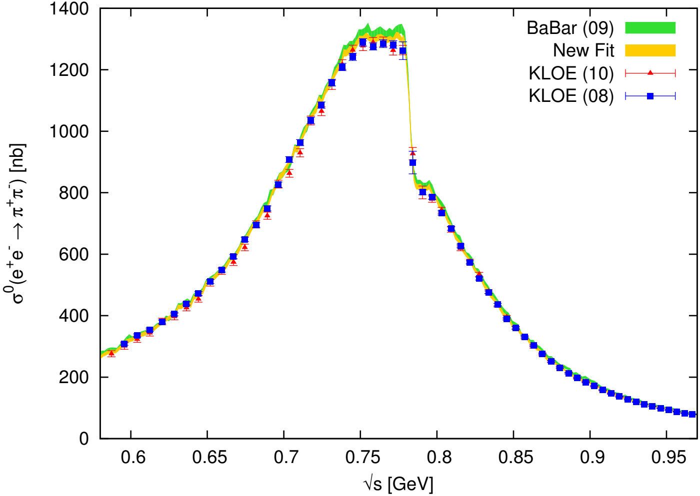
- We find the integral error $\Delta\bar{I}$ via the inflated covariance matrix $\tilde{V}$.

$$(\Delta\bar{I})^2 = \sum_{p=m}^{n}\sum_{q=m}^{n}\frac{\partial\bar{I}}{\partial\bar{R}_p}\tilde{V}(p,q)\frac{\partial\bar{I}}{\partial\bar{R}_q}$$

where we must be careful that the border terms are well defined.

$e^+e^- \to \pi^+\pi^-$, [HLMNT, 2012]

$\rightarrow$ Most important channel ($> 70\%$)



$e^+e^- \to K^+K^-$, [PhysRevD.88.032013]

- 159 New Points (14 sets, 364 points in total)
  $\to$ Improved precision in tails and around resonance.
- Systematic covariance matrix required.
- First analysis indicates definite reconsideration of data treatment to improve fit.

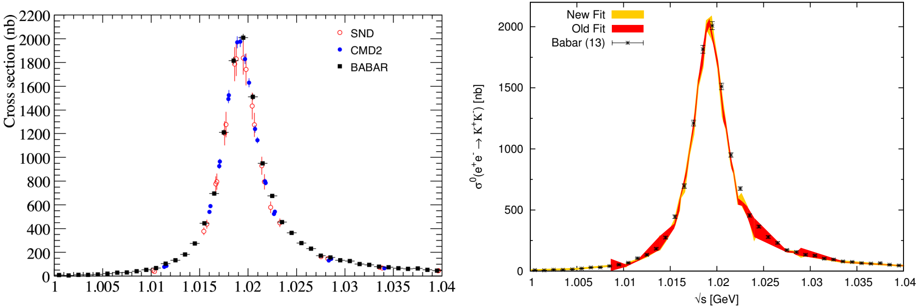


Figure : BABAR plot vs. New/Old fit

Conclusions

- Improving accuracy of a_μ provides stringent test of the SM and the possibly of new physics.
- Hadronic leading-order VP sector provides largest uncertainty.
- Using unitarity and analyticity, we can calculate a_μ^{had} via a dispersion relation.
- Combine data sets through locally inflated target clusters.
- Employ a χ^2-function to minimise and fit data.
- Once integrated, we can produce a new value for a_μ^{had} and compare.
- Imminent emergence of new data promises for exciting years to come...

Thank You