Model Building in Grand Unified Theories

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Outline

Motivation

Review of Grand Unified Theories

Overview of Group Theory

Model Building
- Groups and Representations
- Theories and Models

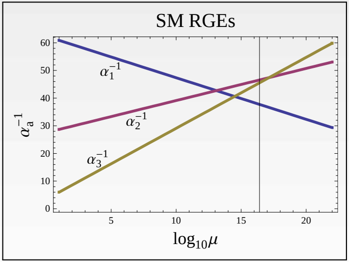
Conclusions and Applications

Motivation

- The Standard Model of Particle Physics is not the ultimate theory
- Among its shortcomings it fails to explain the several phenomena, such as gravity, neutrino masses, dark matter, dark energy, etc
- There must be an extension of the Standard Model that can explain some of these observations
- We expect to see something new at the LHC in the next run

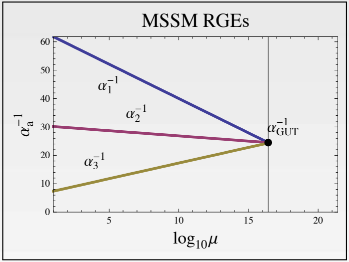
Motivation

- Grand Unified Theories are among the best ways to extended the Standard Model, by enhancing its internal symmetries
- The partial unification of gauge couplings in the SM is a hint to a model such as this



Motivation

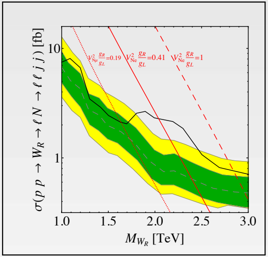
- If one includes low energy Supersymmetry, at the TeV scale, for example, the running gauge couplings is modified in such a way that the unification is even more evident



- Modulo some threshold corrections, Supersymmetric predicts the unification scale to be at $M_G \sim 2 \times 10^{16}$, which incidentally is high enough to be consistent with current bounds on proton decay.

Motivation

- Grand Unified Theories are even motivated from the preliminary results from the LHC experiments



- CMS has found a peak on the $pp \to lljj$ cross section, maybe corresponding to a W_R of around 2.2 GeV. The signal is only about 2.8σ as of today, but it turns out to be confirmed, it would be the first evidence for a GUT, in particular a Left-Right symmetric model.

Motivation

- However, the vast amount of different GUT models, with different representations and breaking paths makes it hard to match the phenomenology with the theory
- We argue that a tool that may take care of most of the model building chaos, discriminating among models and identifying those that are viable representations of reality, will be quite useful.
- The goal will be to construct such a tool, in order to automatise the model building process, with a minimum set of inputs, providing different scenarios and models to choose from.

Motivation

- In Model Building the ultimate goal is to build a theory that is consistent mathematically and physically.
- The starting point will be **Group Theory**
- We begin with a minimal set of inputs at high energies: the Lie Group of internal symmetries and the field content.

$$\{\mathcal{G}, \quad \mathcal{R}_1, \mathcal{R}_2, \dots\}$$

- We will use group theoretical methods to build viable models

Motivation

- The tool will generate all possible models from that set of inputs
  1. Breaking paths from $\mathcal{G}$ to the Standard Model
  2. Set of fields/representations at every scale
- Models will be discarded if they don't satisfy some constraints, e.g., reproduce the SM at low energies

$$Q \to (\mathbf{3}, \mathbf{2})_{\frac{1}{6}}, \quad \bar{u} \to (\bar{\mathbf{3}}, \mathbf{1})_{-\frac{2}{3}}, \quad \bar{d} \to (\bar{\mathbf{3}}, \mathbf{1})_{\frac{1}{3}},$$
$$L \to (\mathbf{1}, \mathbf{2})_{-\frac{1}{2}}, \quad \bar{e} \to (\mathbf{1}, \mathbf{1})_{1}, \quad (\times 3)$$
$$H \to (\mathbf{1}, \mathbf{2})_{-\frac{1}{2}}$$

Review of Grand Unified Theories

Review of GUTs

- Extend the symmetries of the Standard Model, whose gauge group is:

$$\mathcal{G}_{SM} \equiv SU(3)_c \otimes SU(2)_L \otimes U(1)_Y.$$

- One needs a Lie Group, of rank ≥ 4, that contains the SM group as subgroup, $\mathcal{G} \supset \mathcal{G}_{SM}$.
- The SM field content should be contained in representations of $\mathcal{G}$ that satisfy the chiral structure and don't generate anomalies.

$$\sum_R \mathcal{A}(R) = 0 \qquad (1)$$

Review of GUTs

- H. Georgi and S. Glashow proposed in 1974 the first unified model, using the simple group $SU(5)$.
- The SM matter field content is embedded univocally in two representations of $SU(5)$, $\mathbf{10}_F$ and $\bar{\mathbf{5}}_F$, in the following way:

$$\mathbf{10}_F \equiv \begin{pmatrix} 0 & u_3^c & -u_2^c & u_1 & d_1 \\ -u_3^c & 0 & u_1^c & u_2 & d_2 \\ u_2^c & -u_1^c & 0 & u_3 & d_3 \\ -u_1 & -u_2 & -u_3 & 0 & e^c \\ -d_1 & -d_2 & -d_3 & -e^c & 0 \end{pmatrix}, \quad \bar{\mathbf{5}}_F \equiv \begin{pmatrix} d_1^c \\ d_2^c \\ d_3^c \\ e \\ -\nu \end{pmatrix}$$

- And the Higgs field falls into the representation $\mathbf{5}_H$, together with a colour triplet.

Review of GUTs

- The $SU(5)$ model is that predicts the precise charge quantisation present in the Standard Model.

$$\frac{Y(Q)}{Y(e^c)} = \frac{1}{6},\ \frac{Y(u^c)}{Y(e^c)} = -\frac{2}{3},\ \frac{Y(d^c)}{Y(e^c)} = \frac{1}{3},\ \frac{Y(L)}{Y(e^c)} = -\frac{1}{2}.$$

- Breaking of $SU(5) \to \mathcal{G}_{SM}$ happens when the **24**-dimensional representation acquires a vacuum expectation value.
- It requires precise gauge coupling unification, $g_3 = g_2 = g_1$, at a scale M_G, which does not happen exactly in the SM.
- Yukawa coupling unification is needed as well, but it does not predict the right fermion masses at the renormalizable level.

Review of GUTs

- Non-SUSY $SU(5)$ predicts rapid proton decay, which happens through the off-diagonal gauge bosons, X,

$$\Gamma(p \to \pi^0 e^+) \sim \frac{\alpha^2 m_p^5}{M_X^4}, \quad \tau_{exp} > 10^{34} \text{ years.}$$

- Supersymmetric $SU(5)$ improves the unification of gauge couplings, to happen precisely at $M_G = 2 \times 10^{16}$, and requires an extra Higgs representation, $\bar{\mathbf{5}}_H$. It is also compatible with proton decay.
- A successful non-supersymmetric model for $SU(5)$ can be built, by enhancing the symmetry to $SU(5) \otimes U(1)$, and taken the "flipped" embedding.

$$u_i^c \leftrightarrow d_i^c, \quad e^c \leftrightarrow \nu^c, \quad \mathbf{1}_F \equiv (e^c).$$

Review of GUTs

- Next attempt for an unified model was by J. Pati and A. Salam, shortly after. It involved the semi-simple group $SU(4)_c \otimes SU(2)_L \otimes SU(2)_R$.
- The SM field content is embedded in $(\mathbf{4}, \mathbf{2}, \mathbf{1})$ and $(\bar{\mathbf{4}}, \mathbf{1}, \mathbf{2})$.

$$(\mathbf{4}, \mathbf{2}, \mathbf{1}) \equiv \begin{pmatrix} u_1 & u_2 & u_3 & \nu \\ d_1 & d_2 & d_3 & e \end{pmatrix},$$

$$(\bar{\mathbf{4}}, \mathbf{1}, \mathbf{2}) \equiv \begin{pmatrix} d_1^c & d_2^c & d_3^c & e^c \\ -u_1^c & -u_2^c & -u_3^c & -\nu^c \end{pmatrix}.$$

- And the SM Higgs is a bi-doublet $(\mathbf{1}, \mathbf{2}, \mathbf{2})$.

Review of GUTs

- Breaking to the SM can happen in different steps, through one or more intermediate groups

$$SU(4)_c \otimes SU(2)_L \otimes U(1)_R,$$
$$SU(3)_c \otimes SU(2)_L \otimes SU(2)_R \otimes U(1)_{B-L}.$$
$$SU(3)_c \otimes SU(2)_L \otimes U(1)_R \otimes U(1)_{B-L}.$$

- The Higgs sector includes fields in the representations $(\mathbf{\overline{10}}, \mathbf{3}, \mathbf{1})$ and $(\mathbf{\overline{10}}, \mathbf{1}, \mathbf{3})$, and the order in which the acquire v.e.v.s determines the breaking path.

Review of GUTs

- This model naturally includes the right-handed neutrino in the content, which requires some sort of Seesaw Mechanism to explain the hierarchy.

$$\mathbf{M}_\nu = \begin{pmatrix} 0 & m_D \\ m_D & M_R \end{pmatrix} \rightarrow \begin{cases} m_\nu \sim \frac{m_D^2}{M_R} \\ m_{\nu^c} \sim M_R \end{cases}$$

- There are three (two) different gauge couplings, so strict unification is not required, and thus this model can be satisfied in the non-supersymmetric scenario.
- Neither the gauge or scalar sectors induce proton decay, so it is possible to have some light states ($\gtrsim$ TeV), maybe within reach of the LHC.

Review of GUTs

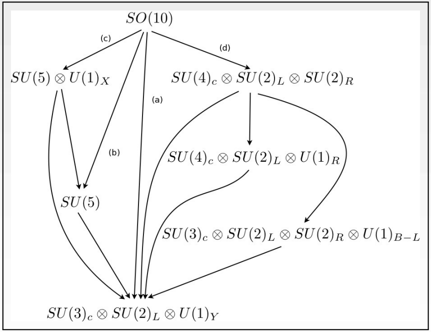
- The first model to have all the SM fermions unified in a single representation is $SO(10)$ unification (H. Fritsch and P. Minkowski, 1975).
- The spinor representation, **16** is not self conjugate, so it respects the SM chiral structure. A particular choice for the Clifford algebra gives the embedding

$$\mathbf{16}_F \equiv \{u_1, \nu, u_2, u_3, \nu^c, u_1^c, u_3^c, u_2^c, d_1, e, d_2, d_3, e^c, d_1^c, d_3^c, d_2^c\}$$

- The SM Higgs doublet (or both MSSM Higgs doublets) can be embedded in the $\mathbf{10}_H$ representation, although an accurate prediction for fermion masses requires the addition of higher dimensional representations such as $\mathbf{120}_H$ or $\overline{\mathbf{126}}_H$.

Review of GUTs

- $SO(10)$ contains maximally the subgroups $SU(5) \otimes U(1)$ and $SU(4) \otimes SU(2) \otimes SU(2)$, so it favour from the advantages of both previous models.
- It can break directly to the SM, or through either of the maximal subgroups as intermediate steps.



Review of GUTs

- Another family unified group is E_6, which contains in it fundamental representation, **27**, all the SM matter content, plus some Higgs multiplets and a singlet
- E_6 has the maximal subgroup $SO(10) \times U(1)$, under which the **27** representation decomposes as

$$\mathbf{27} \to \mathbf{16}_1 \oplus \mathbf{10}_{-2} \oplus \mathbf{1}_4$$

- There is an alternative, and also quite interesting, embedding of the SM into E_6, which is through the subgroup $SU(3)_c \times SU(3) \times SU(3)_w$. And **27** decomposes as

$$\mathbf{27} \to (\mathbf{3}, \mathbf{1}, \mathbf{3}) \oplus (\bar{\mathbf{3}}, \bar{\mathbf{3}}, \mathbf{1}) \oplus (\mathbf{1}, \mathbf{3}, \bar{\mathbf{3}})$$

Overview of Group Theory

- The Cartan Classification of (compact) Lie Groups:

$$A_n \leftrightarrow SU(n+1), \quad B_n \leftrightarrow SO(2n+1),$$
$$C_n \leftrightarrow Sp(2n), \quad D_n \leftrightarrow SO(2n),$$
$$G_2, \quad F_4, \quad E_6, \quad E_7, \quad E_8.$$

- Let t_a be the generators of the Lie algebra associated with the Lie group. Then the Lie algebra is univocally defined by the structure constants f_{abc}.

$$[t_a, t_b] = f_{abc}\, t_c$$

- A_n has $n(n+2)$ generators, B_n and C_n have $n(2n+1)$, D_n has $n(2n-1)$ and the exceptional algebras, G_2, F_4, E_6, E_7 and E_8 have 14, 52, 78, 133 and 248 respectively.

Overview of Group Theory

- If we call h_i the maximal set of commuting generators, called the Cartan subalgebra, of size n, the rank of the group, such that

$$[h_i, h_j] = 0, \ \forall i, j$$

- Let e_α be the other generators, with $e_{-\alpha} \equiv e_\alpha^\dagger$ and

$$[h_i, e_\alpha] = \alpha_i \ e_\alpha$$

- The roots α define the algebra. The minimum set of linearly independent roots, known as the simple roots, has size n and contains only positive roots.
- The last commutation relations are

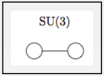
$$[e_\alpha, e_{-\alpha}] = \alpha_i h_i, \quad [e_\alpha, e_\beta] = c_{\alpha,\beta} \ e_{\alpha+\beta} \ \text{ if } \alpha + \beta \neq 0$$

Overview of Group Theory

- The Cartan matrix (standard normalisation, $\alpha \cdot \alpha = 2$), e.g., A_2

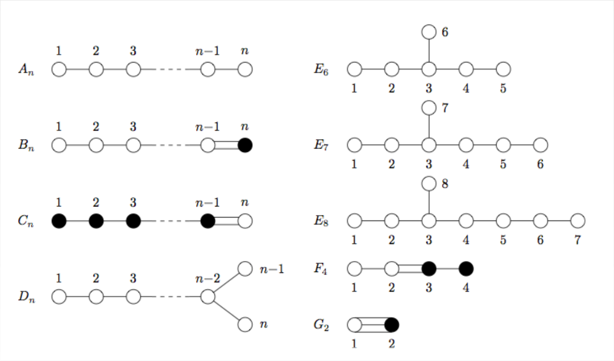
$$K(A_2) = \begin{pmatrix} 2 & -1 \\ -1 & 2 \end{pmatrix}$$

- The simple roots can be represented using **Dynkin diagrams**



- The dots represent the roots, black dots are shorter roots
- The links represent the angle between roots
  - 0 links $\rightarrow \angle\{\alpha, \beta\} = \frac{\pi}{2}$
  - 1 link $\rightarrow \angle\{\alpha, \beta\} = \frac{2\pi}{3}$
  - 2 links $\rightarrow \angle\{\alpha, \beta\} = \frac{3\pi}{4}$
  - 3 links $\rightarrow \angle\{\alpha, \beta\} = \frac{5\pi}{6}$

- The Dynkin diagrams for all simple groups are



Overview of Group Theory

- An n-dimensional **Representation** of the group is a set of $n \times n$ matrices that act on an n-dimensional Hilbert space
- They satisfy the same commutation relations as the generators t_a.

$$[\mathcal{R}(t_a), \mathcal{R}(t_b)] = f_{abc}\mathcal{R}(t_c)$$

- The **weights** of the representation are the eigenvalues of the generators of the Cartan subalgebra on such Hilbert space

$$\mathcal{R}(h_i)|\lambda\rangle = w_i|\lambda\rangle$$

Overview of Group Theory

- There is a $|\lambda\rangle$ such that $\mathcal{R}(e_\alpha)|\lambda\rangle = 0$ for all the simple roots α. Its weight is the **highest weight** and defines the representation, e.g.

$$w = (1,1) \leftrightarrow \mathbf{8} \in SU(3)$$

- From the highest weight all weights can be obtain using $\mathcal{R}(e_{-\alpha})$, we obtain the **weight diagram**, e.g.

$$(1,1) \quad \begin{matrix} (2,-1) & (0,0) & (-2,1) \\ (-1,2) & (0,0) & (1,-2) \end{matrix} \quad (-1,-1)$$

Overview of Group Theory

- **Roots** define Simple Groups
  - A root system can be represented by a **Dynkin Diagram**
  - Non-simple groups are defined by the roots of its factors

- **Weights** define Representations
  - From the highest weight the **Weight Diagram** can be obtained

Model Building: Groups and Representations

Model Building: Groups and Reps

- Three main concepts from group theory:
  1. **Direct products of representations** → invariants
     e.g. $SU(5)$,
     $$\mathbf{5} \otimes \bar{\mathbf{5}} = \mathbf{24} \oplus \mathbf{1}$$
  2. **Subgroups of a group** → breaking chains
     e.g.
     $$E_6 \to SO(10) \to SU(5) \to SU(3)_c \times SU(2)_L \times U(1)_Y$$
  3. **Decomposition of the representations** → field content
     e.g. $SO(10) \to SU(5) \times U(1)$
     $$\mathbf{16} \to \mathbf{10}_{-1} \oplus \bar{\mathbf{5}}_3 \oplus \mathbf{1}_{-5}$$

Model Building: Groups and Reps

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Model Building: Groups and Reps

- The first case, the direct product of representations,

$$\mathcal{R}_1 \otimes \mathcal{R}_2 = \bigoplus_i \mathcal{R}_i$$

- Take each weight w_i from $\mathcal{R}_1$ and each v_j from $\mathcal{R}_2$
- The reducible representation $\mathcal{R}_1 \otimes \mathcal{R}_2$ has weights $w_i + v_j$.
- Pick the highest weight from $w_H \in w_i + v_j$ (most positive) that identifies a irrep
- Construct the weight diagram for w_H and take out those weights from $w_i + v_j$
- Repeat until there is no more positive weights, leftovers will be $(0, 0)$, i.e., singlets

Model Building: Groups and Reps

- An example, in $SU(3)$, $\mathbf{3} \otimes \bar{\mathbf{3}}$

$$\begin{array}{ccccc} (1,0) & & (0,1) & & (1,1)(2,-1)(0,0) \\ (-1,1) & \otimes & (1,-1) & = & (-1,2)(0,0)(-2,1) \\ (0,-1) & & (-1,0) & & (0,0)(1,-2)(-1,-1) \end{array}$$

Model Building: Groups and Reps

- An example, in $SU(3)$, $\mathbf{3} \otimes \bar{\mathbf{3}}$

$$\begin{matrix} (1,0) \\ (-1,1) \\ (0,-1) \end{matrix} \quad \otimes \quad \begin{matrix} (0,1) \\ (1,-1) \\ (-1,0) \end{matrix} \quad = \quad \begin{matrix} {\color{red}(1,1)}(2,-1)(0,0) \\ (-1,2)(0,0)(-2,1) \\ (0,0)(1,-2)(-1,-1) \end{matrix}$$

Model Building: Groups and Reps

- An example, in $SU(3)$, $\mathbf{3} \otimes \bar{\mathbf{3}}$

$$\begin{matrix} (1,0) \\ (-1,1) \\ (0,-1) \end{matrix} \otimes \begin{matrix} (0,1) \\ (1,-1) \\ (-1,0) \end{matrix} = \begin{matrix} \color{red}{(1,1)}(2,-1)(0,0) \\ (-1,2)(0,0)(-2,1) \\ (0,0)(1,-2)(-1,-1) \end{matrix}$$

- The weight diagram obtained from $\color{red}{(1,1)}$, of dimension 8, is

$$\color{red}{(1,1)} \quad \begin{matrix} (2,-1) \\ (-1,2) \end{matrix} \quad \begin{matrix} (0,0) \\ (0,0) \end{matrix} \quad \begin{matrix} (-2,1) \\ (1,-2) \end{matrix} \quad (-1,-1)$$

Model Building: Groups and Reps

- An example, in $SU(3)$, $\mathbf{3} \otimes \bar{\mathbf{3}}$

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- The weight diagram obtained from $(1,1)$, of dimension 8, is

$$(1,1) \quad \begin{matrix} (2,-1) \\ (-1,2) \end{matrix} \quad \begin{matrix} (0,0) \\ (0,0) \end{matrix} \quad \begin{matrix} (-2,1) \\ (1,-2) \end{matrix} \quad (-1,-1)$$

Model Building: Groups and Reps

- An example, in $SU(3)$, $\mathbf{3} \otimes \bar{\mathbf{3}}$

$$\begin{array}{c} (1,0) \\ (-1,1) \\ (0,-1) \end{array} \otimes \begin{array}{c} (0,1) \\ (1,-1) \\ (-1,0) \end{array} = \begin{array}{c} (1,1)(2,-1)(0,0) \\ (-1,2)(0,0)(-2,1) \\ (0,0)(1,-2)(-1,-1) \end{array}$$

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- The weight $(0,0)$ is just the singlet in $SU(3)$, $\mathbf{1}$

Model Building: Groups and Reps

- An example, in $SU(3)$, $\mathbf{3} \otimes \bar{\mathbf{3}}$

$$\begin{matrix} (1,0) \\ (-1,1) \\ (0,-1) \end{matrix} \otimes \begin{matrix} (0,1) \\ (1,-1) \\ (-1,0) \end{matrix} = \begin{matrix} (1,1)(2,-1)(0,0) \\ (-1,2)(0,0)(-2,1) \\ (0,0)(1,-2)(-1,-1) \end{matrix}$$

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- The weight $(0,0)$ is just the singlet in $SU(3)$, $\mathbf{1}$
- So the result is

$$\mathbf{3} \otimes \bar{\mathbf{3}} = \mathbf{8} \oplus \mathbf{1}$$

Model Building: Groups and Reps

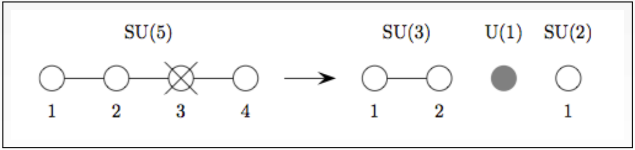
- Three main concepts from group theory:
  1. **Direct products of representations** $\rightarrow$ invariants
     e.g. $SU(5)$,
     $$\mathbf{5} \otimes \bar{\mathbf{5}} = \mathbf{24} \oplus \mathbf{1}$$
  2. **Subgroups of a group** $\rightarrow$ breaking chains
     e.g.
     $$E_6 \rightarrow SO(10) \rightarrow SU(5) \rightarrow SU(3)_c \times SU(2)_L \times U(1)_Y$$
  3. **Decomposition of the representations** $\rightarrow$ field content
     e.g. $SO(10) \rightarrow SU(5) \times U(1)$
     $$\mathbf{16} \rightarrow \mathbf{10}_{-1} \oplus \bar{\mathbf{5}}_3 \oplus \mathbf{1}_{-5}$$

Model Building: Groups and Reps

- The maximal subgroups of a given group are of two types: **Regular Subgroups** and **Special Subgroups**.
- The **Regular maximal subgroups** can be calculated simply by removing a dot from the Dynkin diagram or the Extended Dynkin diagram.
  e.g.
  $$SU(5) \to SU(3) \times SU(2) \times U(1)$$
- The **Special maximal subgroups** must be obtained in a more heuristic way, by finding a group $\mathcal{F} < \mathcal{G}$ for which there exists the decomposition $\mathcal{R}(\mathcal{G}) \to \mathcal{R}(\mathcal{F})$, e.g.,
  $$\mathbf{7}(SO(7)) \to \mathbf{7}(G_2)$$

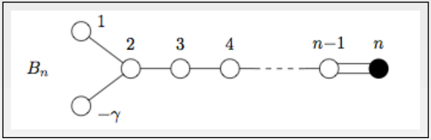
Model Building: Groups and Reps

- The **Regular maximal subgroups** can be either semisimple or not, and the way of obtaining either is different
- Given the Dynkin diagram for a group, a **non-semisimple** subgroup can be obtained by simply eliminating a dot from the diagram
- The resulting disconnect diagrams correspond to the semi-simple part of the subgroup and the eliminated dot becomes the $U(1)$ generator.
  e.g. $SU(5) \to SU(3) \times SU(2) \times U(1)$

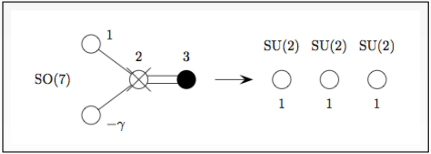


Model Building: Groups and Reps

- The **semisimple** groups are obtained by adding a root to the Dynkin diagram, to form the **Extended or Affine Dynkin Diagram**. This root, $-\gamma$, is the most negative root of the group, e.g. for B_n.



- For the example case $B_3 \rightarrow A_1 \times A_1 \times A_1$, eliminating the dot in the middle



Model Building: Groups and Reps

- Through this procedure one can obtain all maximal subgroups of a given Lie Group
- To obtain all subgroups, one needs to iterate the procedure for the subgroups. This way the subgroups of $SU(5)$ are

$$
\begin{aligned}
SU(5) \supset & SU(4) \times U(1), \\
& SU(3) \times SU(2) \times U(1), \\
& SU(3) \times U(1) \times U(1)^*, \\
& SU(2) \times SU(2) \times U(1) \times U(1)^*, \\
& SU(2) \times U(1) \times U(1) \times U(1)^*, \\
& U(1) \times U(1) \times U(1) \times U(1)^*.
\end{aligned}
$$

* this subgroup is embedded into $SU(5)$ in more than one way

Model Building: Groups and Reps

- The final step while calculating the subgroups is the breaking of the **abelian factors**.
- Whenever there is more than one copy of $U(1)$ the broken generator is a linear combination of the both generators, e.g. $Y = I_{3R} + \frac{B-L}{2}$

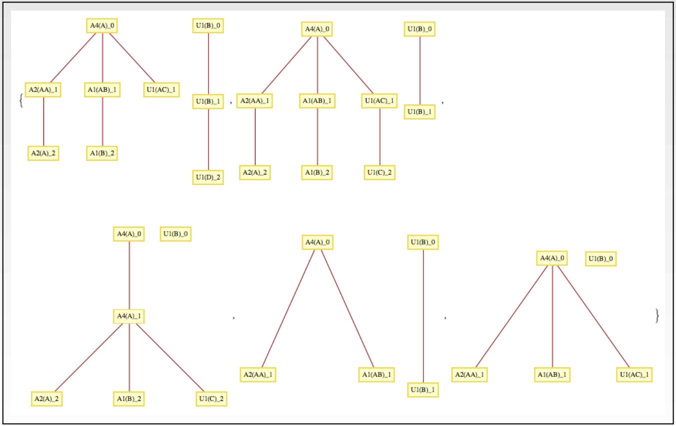
$$SU(3)_c \times SU(2)_L \times U(1)_R \times U(1)_{B-L} \to SU(3)_c \times SU(2)_L \times U(1)_Y$$

- Then, for the $SU(5)$ example above, include the subgroups

$$\begin{aligned} SU(5) \supset & SU(4),\ SU(3) \times SU(2)^*,\ SU(3) \times U(1)^*, \\ & SU(3)^*,\ SU(2) \times SU(2) \times U(1)^*, \\ & SU(2) \times SU(2)^*,\ SU(2) \times U(1)^*,\ SU(2)^* \\ & U(1) \times U(1) \times U(1)^*,\ U(1) \times U(1)^*,\ U(1)^* \end{aligned}$$

Model Building: Groups and Reps

- Breaking chains
  e.g., $SU(5) \times U(1) \rightarrow SU(3) \times SU(2) \times U(1)$



Model Building: Groups and Reps

- Three main concepts from group theory:
  1. **Direct products of representations** → invariants
     e.g. $SU(5)$,
     $$\mathbf{5} \otimes \bar{\mathbf{5}} = \mathbf{24} \oplus \mathbf{1}$$
  2. **Subgroups of a group** → breaking chains
     e.g.
     $$E_6 \to SO(10) \to SU(5) \to SU(3)_c \times SU(2)_L \times U(1)_Y$$
  3. **Decomposition of the representations** → field content
     e.g. $SO(10) \to SU(5) \times U(1)$
     $$\mathbf{16} \to \mathbf{10}_{-1} \oplus \bar{\mathbf{5}}_3 \oplus \mathbf{1}_{-5}$$

Model Building: Groups and Reps

- The **Projection Matrix** projects the weights of a representation into weights of representations of the subgroup
$$P \cdot W = W'$$
- e.g. the decomposition of the $\mathbf{5} \in SU(5)$ into irreps of $SU(3) \times SU(2) \times U(1)$

$$\begin{pmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 1 \\ \frac{1}{3} & \frac{2}{3} & 1 & \frac{1}{2} \end{pmatrix} \cdot \begin{pmatrix} 1 & -1 & 0 & 0 & 0 \\ 0 & 1 & -1 & 0 & 0 \\ 0 & 0 & 1 & -1 & 0 \\ 0 & 0 & 0 & 1 & -1 \end{pmatrix} = \begin{pmatrix} 1 & -1 & 0 & 0 & 0 \\ 0 & 1 & -1 & 0 & 0 \\ 0 & 0 & 0 & 1 & -1 \\ \frac{1}{3} & \frac{1}{3} & \frac{1}{3} & -\frac{1}{2} & -\frac{1}{2} \end{pmatrix}$$

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- So the decomposition goes: $\mathbf{5} \rightarrow (\mathbf{3}, \mathbf{1})_{\frac{1}{3}}$

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- So the decomposition goes: $\mathbf{5} \to (\mathbf{3}, \mathbf{1})_{\frac{1}{3}} \oplus (\mathbf{1}, \mathbf{2})_{-\frac{1}{2}}$

Model Building: Groups and Reps

- The projection matrices are calculated at the time of obtaining the subgroups
- For **non-semisimple** subgroups, simply move the element of weights corresponding to the eliminated dot to the end and substitute every element by the dual of the weight it belongs to

$$W = \begin{pmatrix} 1 & -1 & 0 & 0 & 0 \\ 0 & 1 & -1 & 0 & 0 \\ 0 & 0 & 1 & -1 & 0 \\ 0 & 0 & 0 & 1 & -1 \end{pmatrix}, \quad W' = \begin{pmatrix} 1 & -1 & 0 & 0 & 0 \\ 0 & 1 & -1 & 0 & 0 \\ 0 & 0 & 0 & 1 & -1 \\ \frac{1}{3} & \frac{1}{3} & \frac{1}{3} & -\frac{1}{2} & -\frac{1}{2} \end{pmatrix}$$

- And thus $P = W' \cdot W^{-1}$, where W^{-1} is the *pseudoinverse* of W, and we obtain the projection matrix from before

$$P = \begin{pmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 1 \\ \frac{1}{3} & \frac{2}{3} & 1 & \frac{1}{2} \end{pmatrix}$$

Model Building: Groups and Reps

- In the case of **semisimple** subgroups, add an element to every weight corresponding to the product $\alpha \cdot \gamma$, and then remove an element of every weight corresponding to the eliminated dot in the diagram
- e.g, for the case of $SO(7)$, the generating rep is the $\mathbf{8}$, whose weight matrix is

$$W = \begin{pmatrix} 0 & 0 & 1 & -1 & 1 & -1 & 0 & 0 \\ 0 & 1 & -1 & 0 & 0 & 1 & -1 & 0 \\ 1 & -1 & 1 & 1 & -1 & -1 & 1 & -1 \end{pmatrix}$$

- Now, adding the extended root, $-\gamma$, and dropping the second dot, to give $SU(2) \times SU(2) \times SU(2)$

$$W' = \begin{pmatrix} 0 & 0 & 1 & -1 & 1 & -1 & 0 & 0 \\ -1 & -1 & 0 & 0 & 0 & 0 & 1 & 1 \\ 1 & -1 & 1 & 1 & -1 & -1 & 1 & -1 \end{pmatrix}$$

- Which can be identified as $\mathbf{8} = (\mathbf{2}, \mathbf{1}, \mathbf{2}) \oplus (\mathbf{1}, \mathbf{2}, \mathbf{2})$, and

$$P = W' \cdot W^{-1} = \begin{pmatrix} 1 & 0 & 0 \\ -1 & -2 & -1 \\ 0 & 0 & 1 \end{pmatrix}$$

Model Building: Theories and Models

Model Building: Theories and Models

- In order to build a model, we start with the minimal inputs

$$\{\mathcal{G}, \quad \mathcal{R}_1, \mathcal{R}_2 \ldots\}$$

- We obtain the breaking chains of $\mathcal{G}$ to the SM

$$\mathcal{G} \to \mathcal{G}_1 \to \mathcal{G}_2 \to \cdots \to \mathcal{G}_n \to \mathcal{G}_{SM}$$

- For all possible chains, we choose one path and we build all possible model that spawn from it and check their viability.
- One can iterate over all possible breaking chains to consider all models given by the pair Group + Reps.

Model Building: Theories and Models

- For a particular path, we can define a **Theory** as a set containing a *Lie Group*, a list of *Reps* of the group and a *Breaking Chain*, e.g.

$$\{\mathcal{G} = SU(5) \times U(1),$$
$$\mathcal{R} = \mathbf{10}_{-1} \oplus \bar{\mathbf{5}}_3 \oplus \mathbf{5}_2,$$
$$SU(5) \times U(1) \to SU(3) \times SU(2) \times U(1)\}$$

- We define then a **Model** as a list of Theories, one per step on the breaking chain, e.g.

$$\left\{ \begin{array}{l|l} \mathcal{G} = SU(5) \times U(1), & \mathcal{G} = SU(3) \times SU(2) \times U(1), \\ \mathcal{R} = \mathbf{10}_{-1} \oplus \bar{\mathbf{5}}_3 \oplus \mathbf{5}_2, & \mathcal{R} = (\mathbf{3},\mathbf{2})_{\frac{1}{6}} \oplus (\bar{\mathbf{3}},\mathbf{1})_{-\frac{2}{3}} \oplus (\bar{\mathbf{3}},\mathbf{1})_{\frac{1}{3}} \oplus \\ SU(5) \times U(1) \to & \qquad (\mathbf{1},\mathbf{2})_{-\frac{1}{2}} \oplus (\mathbf{1},\mathbf{1})_1 \oplus (\mathbf{1},\mathbf{2})_{\frac{1}{2}} \oplus (\mathbf{3},\mathbf{1})_{-\frac{1}{3}} \\ \qquad \to SU(3) \times SU(2) \times U(1) & SU(3) \times SU(2) \times U(1) \to \{\} \end{array} \right\}$$

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Model Building: Theories and Models

- Not every model will be a successful model
- One needs to impose a set of constraints
  1. **Anomaly free** and must satisfy **charge conservation**

$$\sum_i \mathcal{A}(\mathcal{R}_i) = 0, \quad \sum_i \mathcal{Q}(\mathcal{R}_i) = 0$$

  2. **Symmetry breaking** required by the chain must happen

$$H \to (\mathbf{1},\mathbf{2})_{-\frac{1}{2}} \left(H_2 \to (\mathbf{1},\mathbf{2})_{\frac{1}{2}} \right)$$

  3. The field content at the lowest step should be the **Standard Model field content** (singlets)

$$Q \to (\mathbf{3},\mathbf{2})_{\frac{1}{6}}, \quad \bar{u} \to (\bar{\mathbf{3}},\mathbf{1})_{-\frac{2}{3}}, \quad \bar{d} \to (\bar{\mathbf{3}},\mathbf{1})_{\frac{1}{3}},$$
$$L \to (\mathbf{1},\mathbf{2})_{-\frac{1}{2}}, \quad \bar{e} \to (\mathbf{1},\mathbf{1})_{1}, \quad (\times 3)$$

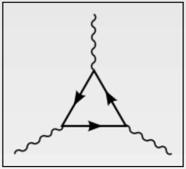
  4. **Chirality** must be satisfied

Model Building: Theories and Models

- To make sure that **only the SM survives at the EW scale**, one needs to integrate out any exotic field content at higher energies
- As a requirement for symmetry breaking, all gauge boson are assumed to acquire masses of the order of the symmetry at which they decouple $M_X \sim v$
- Keeping the SM field content aside, we will generate all the possible models where the exotic fields are integrated out at the different scales of the model
- For every such model, we will check for the constraints above to classify it as valid or not

Model Building: Theories and Models

- Gauge **Anomalies** arise whenever one-loop triangle diagrams do not cancel



$$\mathrm{Tr}\{t^a_{\mathcal{R}}, t^b_{\mathcal{R}}\}t^c_{\mathcal{R}} = \mathcal{A}(\mathcal{R})d^{abc}$$

- In general, only $SU(N)$, $N \geq 3$ and E_6 suffer from this anomalies. For those cases, the field content must be such that makes the theory anomaly free, using the properties

$$\mathcal{A}(\mathcal{R}_1 \oplus \mathcal{R}_2) = \mathcal{A}(\mathcal{R}_1) + \mathcal{A}(\mathcal{R}_2), \quad \mathcal{A}(\bar{\mathcal{R}}) = -\mathcal{A}(\mathcal{R}), \quad \mathcal{A}(\mathbf{1}) = 0$$

- e.g. for the case of $SU(5)$, it turns out that $\mathcal{A}(\mathbf{10} \oplus \bar{\mathbf{5}}) = 0$, so the matter field content is anomaly free

Model Building: Theories and Models

- Anomaly cancellation in the case of $U(1)$ implies **charge conservation**, which means that for every abelian factor $U(1)_j$, one needs that

$$\sum_i \mathcal{Q}_j(\mathcal{R}_i) = 0$$

  where $\mathcal{Q}$ are the $U(1)$ charges, weighted by $d(\mathcal{R}_i)$.
- Thus, for the $SU(5) \times U(1)$ model above, one needs to add an extra $\mathbf{1}_{-5}$, for this to happen
- The last anomaly is the **Witten anomaly**, with has to do with the topology of $SU(2)$, and its avoided whenever there is an even number of $SU(2)$ fermion doublets, as in the SM

Model Building: Theories and Models

- **Spontaneous symmetry breaking** from one step of the chain to another must happen whenever a scalar field gets a vacuum expectation value
- At this stage we do not worry about the scalar potential, we assume that if such field exists, there is a suitable potential that is unstable at $\phi = 0$ at some breaking scale

$$\left.\frac{\partial V}{\partial \phi}\right|_{\phi=v} = 0, \quad \left.\frac{\partial^2 V}{\partial \phi^2}\right|_{\phi=v} > 0, \quad \langle \phi \rangle = v \neq 0$$

- We then impose that in order to break $\mathcal{G} \to \mathcal{F}$, there must be a non-singlet field $\phi \in \mathcal{G}$ that contains a singlet when decomposed under $\mathcal{F}$, $\mathbf{1} \in \phi|_{\mathcal{F}}$

Model Building: Theories and Models

- At every step, we check that there is one such field ϕ, and if so, we only keep the model that integrates out the singlet component at that step
- The rest of the components of ϕ may be integrated out or not, there could be mass splitting among components
- For the case of $SU(5) \times U(1) \to SU(3) \times SU(2) \times U(1)$, one could add $\phi \to \mathbf{24}_X$, with $X \neq 0$, which decomposes

$$\mathbf{24}_X \to (\mathbf{8}, \mathbf{1})_X \oplus (\mathbf{1}, \mathbf{3})_X \oplus (\mathbf{3}, \mathbf{2})_{X+1} \oplus (\bar{\mathbf{3}}, \mathbf{2})_{X-1} \oplus (\mathbf{1}, \mathbf{1})_X$$

- So adding that representation to the field content and giving a v.e.v. to the singlet component would trigger the symmetry breaking

Model Building: Theories and Models

- With all this, one can make a realistic model from the example above

$$\left\{ \begin{array}{l|l} \mathcal{G} = SU(5) \times U(1), & \mathcal{G} = SU(3) \times SU(2) \times U(1), \\ \mathcal{R} = \mathbf{10}_{-1} \oplus \bar{\mathbf{5}}_3 \oplus \mathbf{5}_2, \quad \times 3 & \mathcal{R} = (\mathbf{3},\mathbf{2})_{\frac{1}{6}} \oplus (\bar{\mathbf{3}},\mathbf{1})_{-\frac{2}{3}} \oplus (\bar{\mathbf{3}},\mathbf{1})_{\frac{1}{3}} \oplus \quad \times 3 \\ & \qquad (\mathbf{1},\mathbf{2})_{-\frac{1}{2}} \oplus (\mathbf{1},\mathbf{1})_1 \oplus (\mathbf{1},\mathbf{2})_{\frac{1}{2}} \oplus (\mathbf{3},\mathbf{1})_{-\frac{1}{3}} \\ SU(5) \times U(1) \rightarrow & \\ \quad \rightarrow SU(3) \times SU(2) \times U(1) & SU(3) \times SU(2) \times U(1) \rightarrow \{\} \end{array} \right\}$$

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- Include 3 generations of matter fields to reproduce the SM field content

Model Building: Theories and Models

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- Include 3 generations of matter fields to reproduce the SM field content
- Add a singlet, $\mathbf{1}_{-5}$ to ensure charge quantisation, and a fiveplet, $\bar{\mathbf{5}}_{-2}$ for anomaly cancellation

Model Building: Theories and Models

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- Include 3 generations of matter fields to reproduce the SM field content
- Add a singlet, $\mathbf{1}_{-5}$ to ensure charge quantisation, and a fiveplet, $\bar{\mathbf{5}}_{-2}$ for anomaly cancellation
- Add a scalar $\mathbf{24}_X$ to ensure symmetry breaking
- Integrate out all exotic fields (except maybe the singlet)

Model Building: Theories and Models

- To summarise, the process of generating models goes like this
  1. First, given a **theory**, calculate the **full model**, with all irreps at all scales
  2. Start at the **second-to-highest scale**
  3. Generate of **possible combinations** of non-SM representations, including the case with **all** of them and the case with **none**
  4. For every combination, create the corresponding **model**
  5. Check if the model is **valid** with respect to the constraints
  6. If the model is valid, move to the **next scale**, and go back to step **3**
  7. If at the low scale any of the constraints are **not satisfied** it will feed back to the high scale and exclude that model
- **In the end, we will have a list of models that satisfy all the imposed constraints**

Conclusions and Applications

Conclusions and Applications

- A result of the model building tool is the **RGE running of the gauge couplings** (at one-loop level)
- At every step they only depend on group parameters, such as the **Casimir** of the group and the **Dynkin Index** of the representations involved,

$$\beta_{g_a} = (\textstyle\sum_i \mathcal{I}(\mathcal{R}_i) - 3\mathcal{C}(\mathcal{G}_a))g_a^3, \quad \text{SUSY}$$
$$\beta_{g_a} = (\tfrac{2}{3}\textstyle\sum_{i\in F} \mathcal{I}(\mathcal{F}_i) + \tfrac{1}{3}\sum_{i\in S}\mathcal{I}(\mathcal{S}_i) - \tfrac{11}{3}\mathcal{C}(\mathcal{G}_a))g_a^3 \quad \text{Non-SUSY}$$

- With the SM gauge couplings as the low energy fixed points, the running of the couplings and the intermediate scales can be obtained by satisfying the relevant boundary conditions

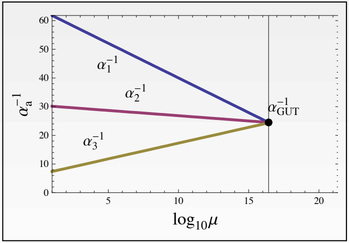
Conclusions and Applications

- Applications of this include both **Supersymmetric** and **Non-Supersymmetric** Grand Unified Models
- It can potentially deal with models in which **Supersymmetry breaking** happens at any scale, since the effect would be to integrate out the Supersymmetric partners at the scale of SUSY breaking
- Three example cases of model are given
  1. **A minimal Supersymmetric $SO(10)$ model, with minimal Higgs content and direct breaking to the Standard Model**
  2. **A non-supersymmetric, $SO(10)$ inspired, left-right symmetry model**
  3. **A model of GUT scale, hybrid inflation, with an $SU(5) \times U(1)$ intermediate waterfall breaking**

Conclusions and Applications

- **Minimal Supersymmetric $SO(10)$ model**

$$\left\{ \begin{array}{l|ll} \mathcal{G} = SO(10), & \mathcal{G} = SU(3)\times SU(2)\times U(1), & \\ \mathcal{R} = \mathbf{16}\oplus\mathbf{16}\oplus\mathbf{10}\oplus\mathbf{144} & \mathcal{R} = (\mathbf{3},\mathbf{2})_{\frac{1}{6}}\oplus(\bar{\mathbf{3}},\mathbf{1})_{-\frac{2}{3}}\oplus(\bar{\mathbf{3}},\mathbf{1})_{\frac{1}{3}}\oplus & \}\times 3 \\ & \quad (\mathbf{1},\mathbf{2})_{-\frac{1}{2}}\oplus(\mathbf{1},\mathbf{1})_1\oplus & \}\times 3 \\ & \quad (\mathbf{1},\mathbf{2})_{\frac{1}{2}}\oplus(\mathbf{1},\mathbf{2})_{-\frac{1}{2}} & \\ SO(10)\to SU(3)\times SU(2)\times U(1) & SU(3)\times SU(2)\times U(1)\to\{\} & \end{array} \right\}$$

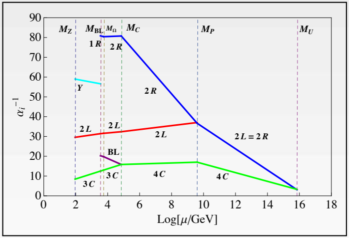


F.Deppisch, N.Desai, T.G. [Front.Phys. 2 (2014) 00027]

Conclusions and Applications

- **Non-SUSY Left-Right Symmetry model**

$$\begin{aligned} SO(10) &\to SU(4)\times SU(2)\times SU(2)\times D \quad \to SU(4)\times SU(2)\times SU(2) \\ &\to SU(3)\times SU(2)\times SU(2)\times U(1) \to SU(3)\times SU(2)\times U(1)\times U(1) \\ &\to SU(3)\times SU(2)\times U(1) \end{aligned}$$



F.Deppisch, T.G., S.Patra, N.Sahu, U.Sarkar [Phys. Rev. D 90, 053014]
F.Deppisch, T.G., S.Patra, N.Sahu, U.Sarkar [pending publication]

Conclusions and Applications

- **GUT scale, hybrid inflation, with an** $SU(5)\times U(1)$ **intermediate waterfall breaking**

$$\mathbf{16}_F^3, \mathbf{16}_H, \overline{\mathbf{16}}_H, \mathbf{45}_H, \mathbf{45}_H, \mathbf{10}_H$$

$$SO(10)\times U(1) \to SU(5)\times U(1) \to SU(3)\times SU(2)\times U(1)$$

J.Ellis, T.G., J. Harz, W-C.Huang [in progress]

Thank you!

Backup

- Properties of groups
- Metric of the group

$$G_{ij} = K_{ij}^{-1} \frac{(\alpha_j, \alpha_j)}{2}$$

- Product of roots

$$(\alpha, \beta) = \sum_{i,j} \alpha_i G_{ij} \beta_j$$

- Dual of a root

$$\alpha_i^* = G_{ij} \alpha_j$$

Backup

- Properties of representations
- Dimension of an irrep

$$d(\mathcal{R}) = \prod_{\alpha} \frac{\alpha \cdot (\Lambda + \delta)}{\alpha \cdot \delta}$$

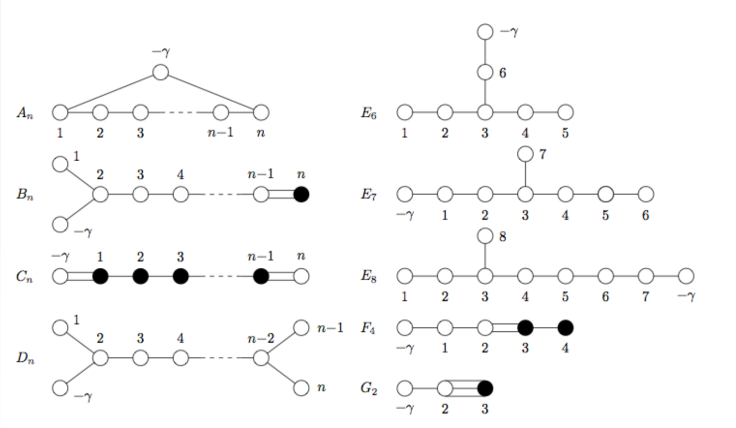
  where Λ is the highest weight of the irrep.
- The Casimir of a representation is defined as

$$\mathcal{C}(\mathcal{R}) = \mathrm{Tr} t_a t_a = \Lambda \cdot (\Lambda + 2\delta)$$

- And the Dynkin Index of the representation

$$\mathcal{I}(\mathcal{R}) = \frac{d(\mathcal{R})}{d(\mathcal{G})} \mathcal{C}(\mathcal{R})$$

- Extended Dynkin diagrams



Backup

- Definition of pseudo inverse
- For a non-square matrix, $n \times m$, A the pseudo inverse can be define such that

$$\tilde{A}^{-1} \equiv A^T \cdot (A \cdot A^T)^{-1}$$

so that

$$A \cdot \tilde{A}^{-1} = I_n$$