Precision determination of the top-quark mass

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Theoretical Physics Seminar, Liverpool, Mar 04, 2015

Introduction (I)

Classical mechanics

- Mass is defined as product of density and volume of matter
    - classical concept

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PRINCIPIA MATHEMATICA.

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AER densitate duplicata, in spatio etiam duplicato, fit quadruplus; in triplicato sextuplus. Idem intellige de nive & pulveribus per compressionem vel liquefactionem condensatis. Et par est ratio corporum omnium, quæ per causas quascunque diversimode condensantur. Medii interea, si quod fuerit, interstitia partium libere pervadentis, hic nullam rationem habeo. Hanc autem quantitatem sub nomine corporis vel massæ in sequentibus passim intelligo. Innotescit ea per corporis cujusque pondus: Nam ponderi proportionalem esse reperi per experimenta pendulorum accuratissime instituta, uti posthac docebitur.

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A

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Atomic theory

- Mass is conserved Lavoisier
- Mass of body is sum of mass of its constituents
  $M(X) = N_A m_a(X)$ Avogadro

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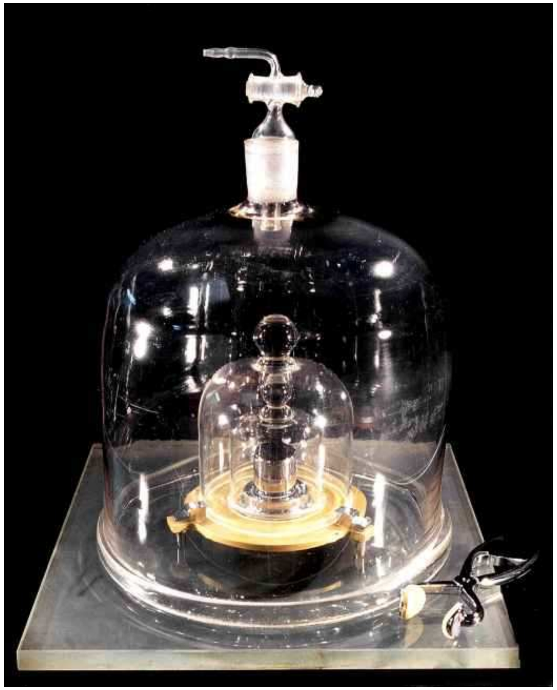
Introduction (II)

Kilogram

The kilogram is the unit of mass; it is equal to the mass of the international prototype of the kilogram.

- International prototype kilogram (IPK): made in 1889, 39 mm high, alloy of platinum and iridium

Orginal des Bureau International des Poids et Mesures



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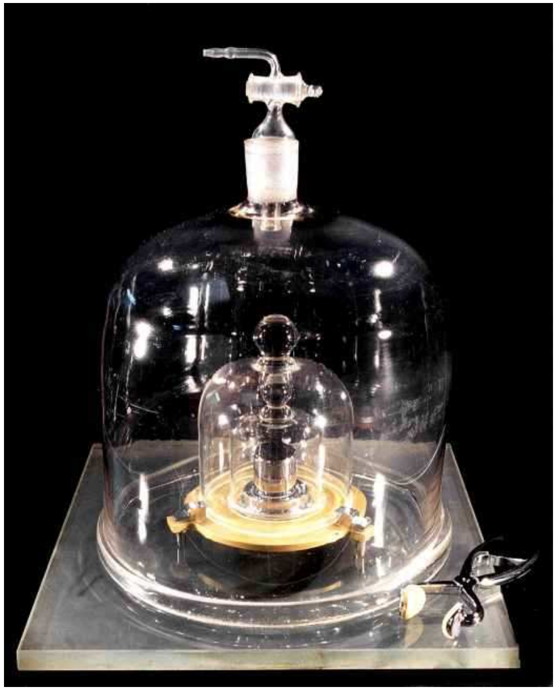

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Special relativity

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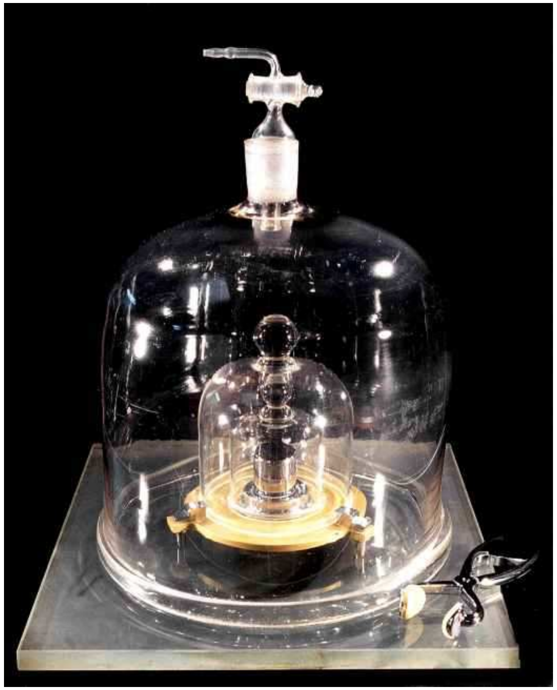

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Standard Model

- Higgs boson gives mass to matter fields via Higgs-Yukawa coupling
  - large top-quark mass m_t

Quantum field theory

QCD

- Classical part of QCD Lagrangian

$$\mathcal{L} \quad = \quad -\frac{1}{4} F^a_{\mu\nu} F_b^{\mu\nu} + \sum_{\text{flavors}} \bar{q}_i \left(\mathrm{i}\not{D} - m_q\right)_{ij} q_j$$

  - field strength tensor $F^a_{\mu\nu}$ and matter fields $q_i, \bar{q}_j$
  - covariant derivative $D_{\mu,ij} = \partial_\mu \delta_{ij} + \mathrm{i} g_s \left(t_a\right)_{ij} A^a_\mu$

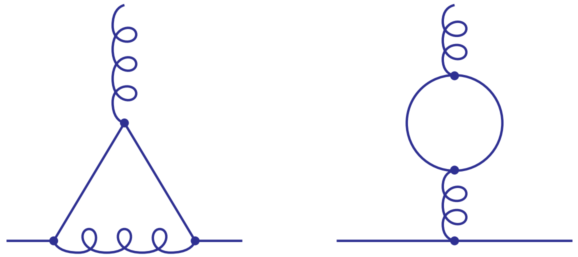
- Formal parameters of the theory (no observables)
  - strong coupling $\alpha_s = g_s^2/(4\pi)$
  - quark masses m_q
- Parameters of Lagrangian have no unique physical interpretation
  - radiative corrections require definition of renormalization scheme

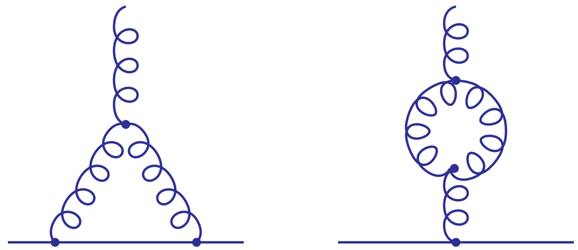
Challenge

- Suitable observables for measurements of α_s, m_q, ...
  - comparison of theory predictions and experimental data

Coupling constant renormalization

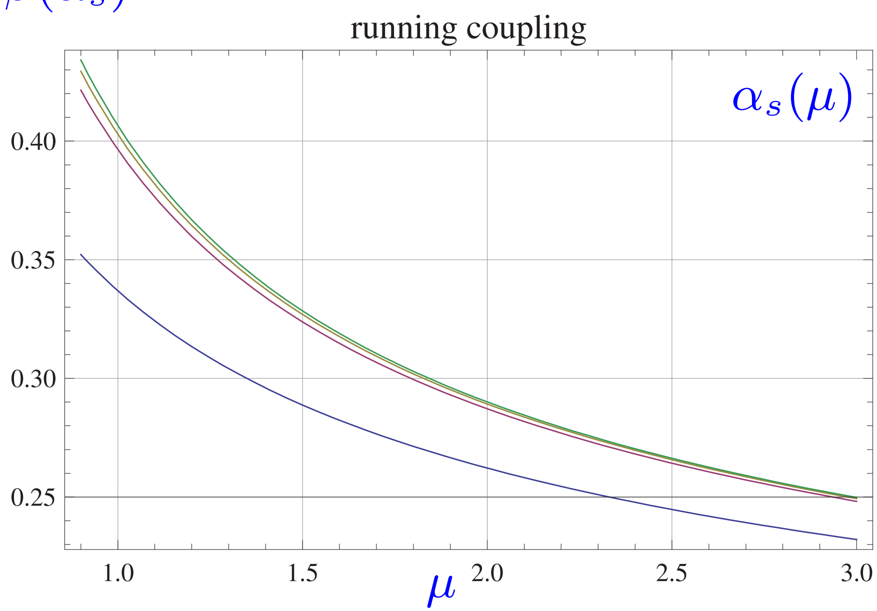
- Running coupling constant α_s from radiative corrections, e.g. one loop





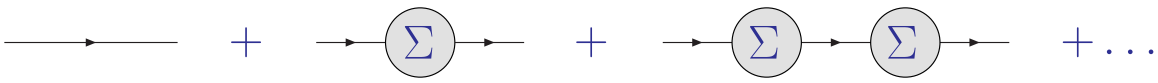
  – screening (like in QED)   – anti-screening (color charge of g)

- QCD beta function $\mu^2 \frac{d}{d\mu^2}\alpha_s(\mu) = \beta(\alpha_s)$
  - perturbative expansion to four loops
    van Ritbergen, Vermaseren, Larin '97
  - very good convergence of perturbative series even at low scales



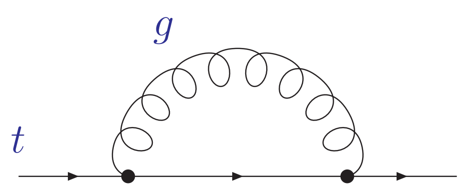
Quark mass renormalization

- Heavy-quark self-energy $\Sigma(p, m_q)$



$$= \frac{\mathrm{i}}{\not{p} - m_q - \Sigma(p, m_q)}$$

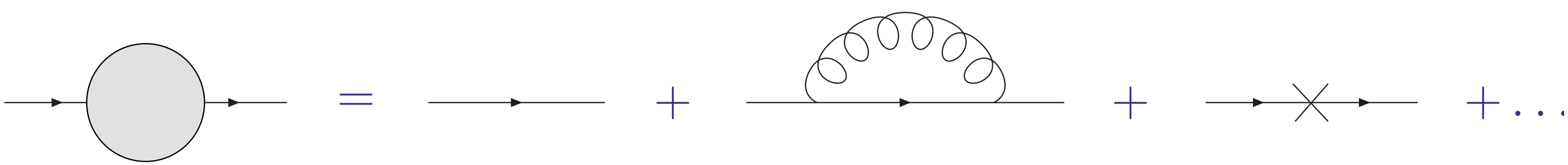
QCD



- QCD corrections to self-energy $\Sigma(p, m_q)$
  - dimensional regularization $D = 4 - 2\epsilon$
  - one-loop: UV divergence $1/\epsilon$ (Laurent expansion)

$$\Sigma^{(1),\mathrm{bare}}(p, m_q) = \frac{\alpha_s}{4\pi} \left(\frac{\mu^2}{m_q^2}\right)^{\epsilon} \left\{ (\not{p} - m_q) \left(-C_F \frac{1}{\epsilon} + \text{fin.} \right) + m_q \left(3C_F \frac{1}{\epsilon} + \text{fin.} \right) \right\}$$

- Relate bare and renormalized mass parameter $m_q^{\mathrm{bare}} = m_q^{\mathrm{ren}} + \delta m_q$



$\Sigma^{\mathrm{ren}}(p, m_q)$ $\qquad$ $(Z_\psi - 1)\not{p} - (Z_m - 1) m_q$

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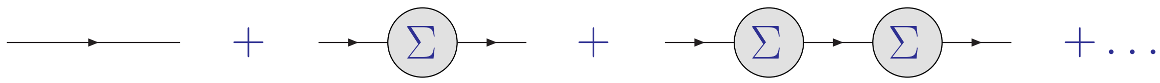

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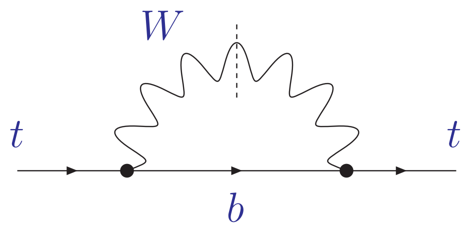
EW sector



- EW corrections to top-quark self-energy
  - on-shell intermediate (virtual) W-boson
  - m_t complex parameter with imaginary part $\Gamma_t = 2.0 \pm 0.7$ GeV
  - $\Gamma_t > 1$ GeV: top-quark decays before it hadronizes

Mass renormalization scheme

Pole mass

- Based on (unphysical) concept of top-quark being a free parton
  - $m_q^{\rm ren}$ coincides with pole of propagator at each order

$$\not{p} - m_q - \Sigma(p, m_q)\Big|_{\not{p}=m_q} \to \not{p} - m_q^{\rm pole}$$

- Definition of pole mass ambiguous up to corrections $\mathcal{O}(\Lambda_{QCD})$
  - heavy-quark self-energy $\Sigma(p, m_q)$ receives contributions from regions of all loop momenta – also from momenta of $\mathcal{O}(\Lambda_{QCD})$
  - bound from lattice QCD: $\Delta m_q \geq 0.7 \cdot \Lambda_{QCD} \simeq 200$ MeV
    Bauer, Bali, Pineda '11

$\overline{MS}$ *scheme*

- $\overline{MS}$ mass definition
  - one-loop minimal subtraction

$$\delta m_q^{(1)} \;=\; m_q \frac{\alpha_s}{4\pi} \, 3C_F \left(\frac{1}{\epsilon} - \gamma_E + \ln 4\pi \right)$$

- $\overline{MS}$ scheme induces scale dependence: $m(\mu)$

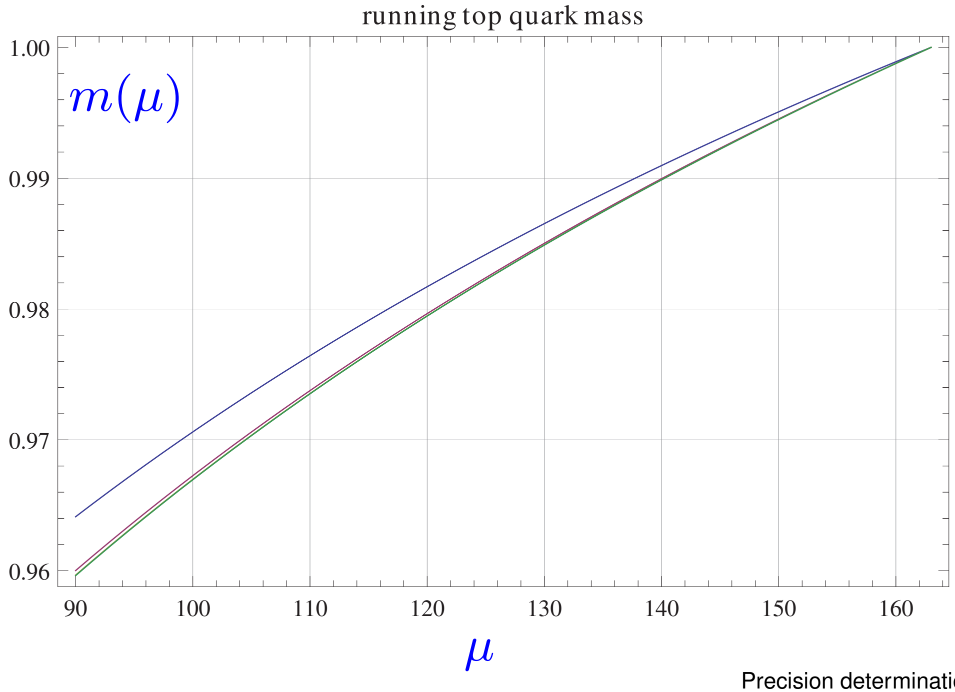
Running quark mass

Scale dependence

- Renormalization group equation for scale dependence
  - mass anomalous dimension γ known to four loops
    Chetyrkin ‘97; Larin, van Ritbergen, Vermaseren ‘97

$$\left(\mu^2 \frac{\partial}{\partial \mu^2} + \beta(\alpha_s) \frac{\partial}{\partial \alpha_s}\right) m(\mu) \;=\; \gamma(\alpha_s) m(\mu)$$

- Plot mass ratio $m_t(163\text{GeV})/m_t(\mu)$



Scheme transformations

- Conversion between different renormalization schemes possible in perturbation theory
- Relation for pole mass and $\overline{MS}$ mass
  - known to four loops in QCD Gray, Broadhurst, Gräfe, Schilcher '90; Chetyrkin, Steinhauser '99; Melnikov, v. Ritbergen '99; Marquard, Smirnov, Smirnov, Steinhauser '15
  - EW sector known to $\mathcal{O}(\alpha_{\rm EW}\alpha_{\rm s})$ Jegerlehner, Kalmykov '04; Eiras, Steinhauser '06
  - example: one-loop QCD

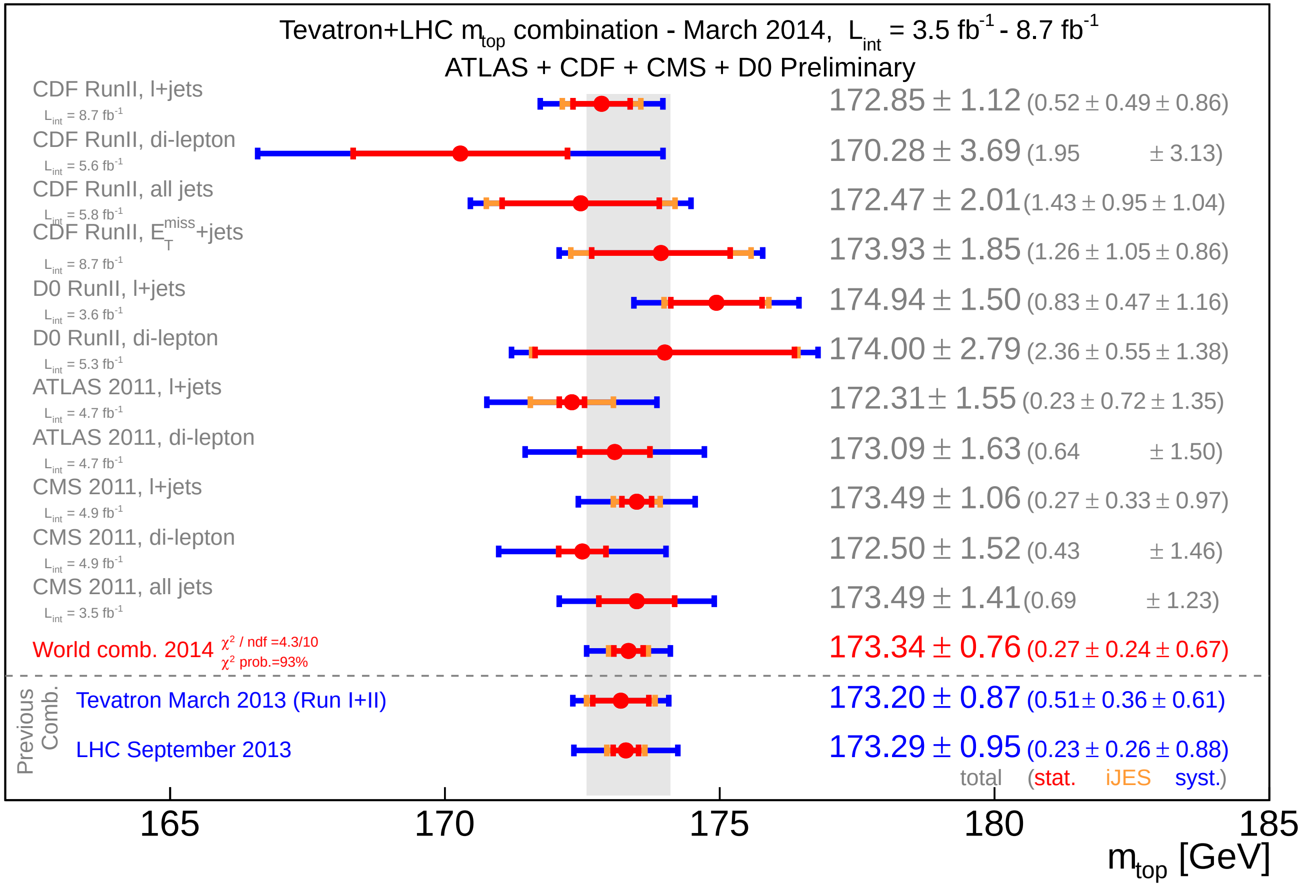
$$m^{\rm pole} = m(\mu)\left\{1 + \frac{\alpha_s(\mu)}{4\pi}\left(\frac{4}{3} + \ln\left(\frac{\mu^2}{m(\mu)^2}\right)\right) + \ldots\right\}$$

Top-quark mass

What is the value of the top-quark mass ?

$$m_t = ?$$

Some Answers



World combination

Experiment: ATLAS, CDF, CMS & D0 coll. 1403.4427

$$m_t = 173.34 \pm 0.76\,\text{GeV}$$

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Theory:

That is, we can state as the final result for the likely relation between the top-quark mass measured using a given Monte Carlo event generator ("MC") and the pole mass as

$$m_{\text{pole}} = m_{\text{MC}} + Q_0 \left[\alpha_s(Q_0) c_1 + \ldots\right]$$

where $Q_0 \sim 1$ GeV and c_1 is unknown, but presumed to be of order 1 and, according to the argument above, presumed to be positive.

A. Buckley et al. arXiv:1101.2599

Rates, shapes and peaks

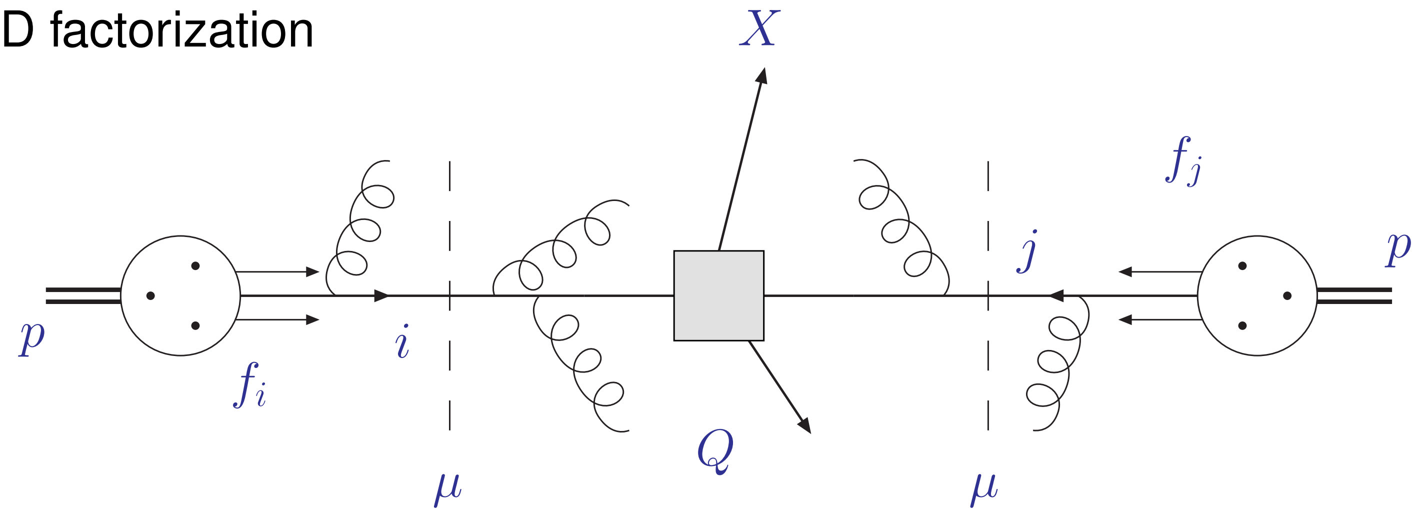
- Rates and shapes of distributions offer possibility for top mass determination with well-defined renormalization scheme
- Requirements:
  - theory predictions at least to NLO in QCD
  - sufficiently large sensitivity $\mathcal{S}$ to m_t (kinematics)

$$\left|\frac{\Delta\sigma_{t\bar{t}}}{\sigma_{t\bar{t}}}\right| \simeq \mathcal{S} \times \left|\frac{\Delta m_t}{m_t}\right|$$

- Observables (examples):
  - inclusive cross section
  - distributions for $t\bar{t} + 1\text{jet}$ samples
  - kinematic reconstruction of top mass (Monte Carlo mass)

Top mass from total cross section

- QCD factorization



$$\sigma_{pp\to X} = \sum_{ij} f_i(\mu^2) \otimes f_j(\mu^2) \otimes \hat{\sigma}_{ij\to X}\left(\alpha_s(\mu^2), Q^2, \mu^2, m_X^2\right)$$

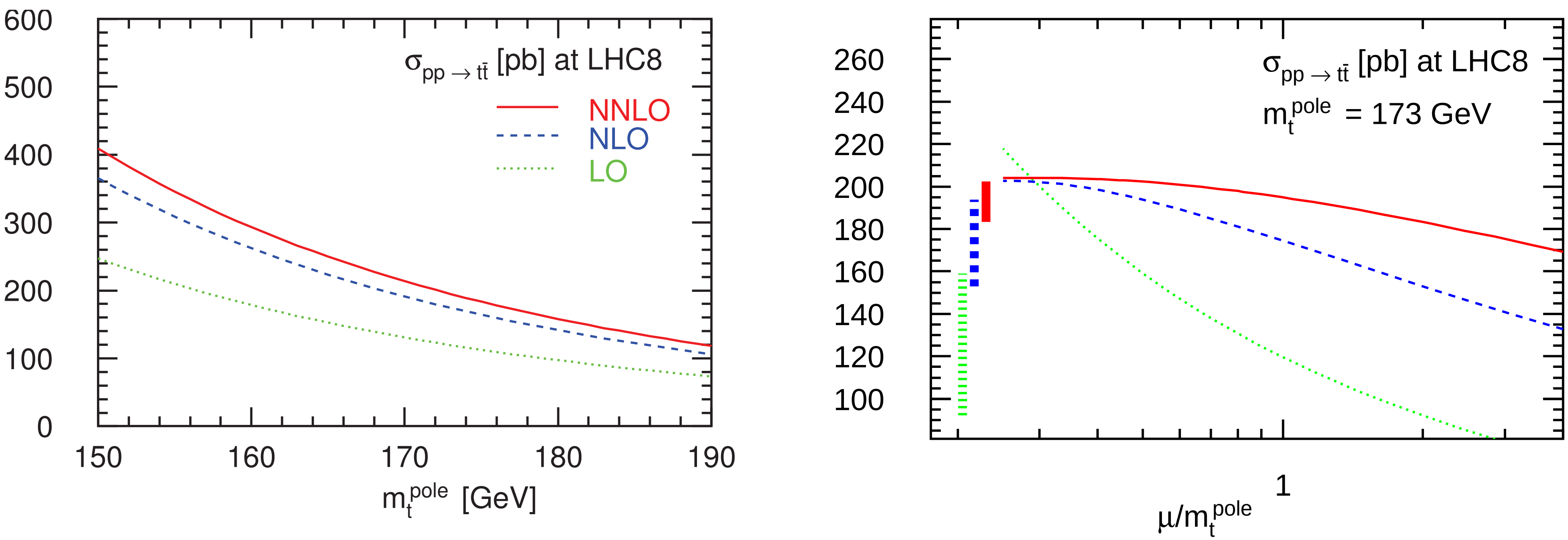
- Joint dependence on non-perturbative parameters: parton distribution functions f_i, strong coupling α_s, masses m_X
- Intrinsic limitation in total cross section through sensitivity $\mathcal{S} \simeq 5$

$$\left|\frac{\Delta\sigma_{t\bar{t}}}{\sigma_{t\bar{t}}}\right| \simeq 5 \times \left|\frac{\Delta m_t}{m_t}\right|$$

Total cross section

Exact result at NNLO in QCD

Czakon, Fiedler, Mitov ‘13

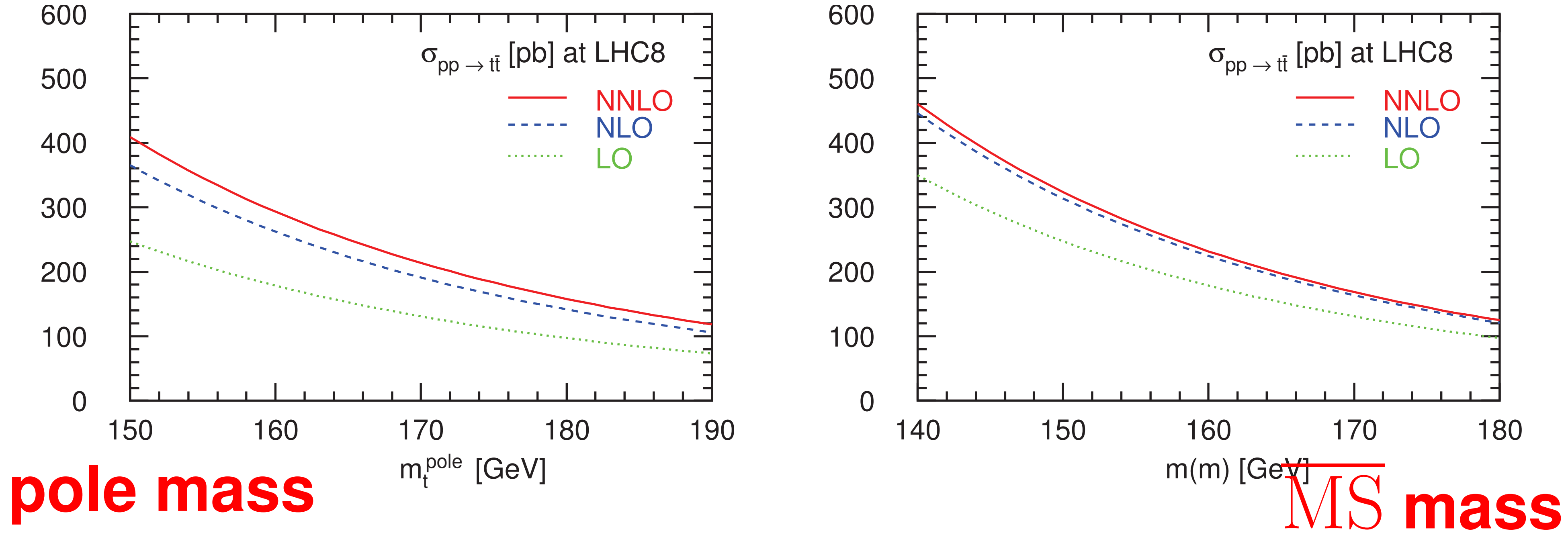


- NNLO perturbative corrections (e.g. at LHC8)
  - K-factor (NLO $\to$ NNLO) of $\mathcal{O}(10\%)$
  - scale stability at NNLO of $\mathcal{O}(\pm 5\%)$

Total cross section with running mass

Comparison pole mass vs. $\overline{\text{MS}}$ mass (I)

Dowling, S.M. '13



- NNLO cross section with running mass significantly improved
  - good apparent convergence of perturbative expansion
  - small theoretical uncertainity form scale variation

Total cross section with running mass

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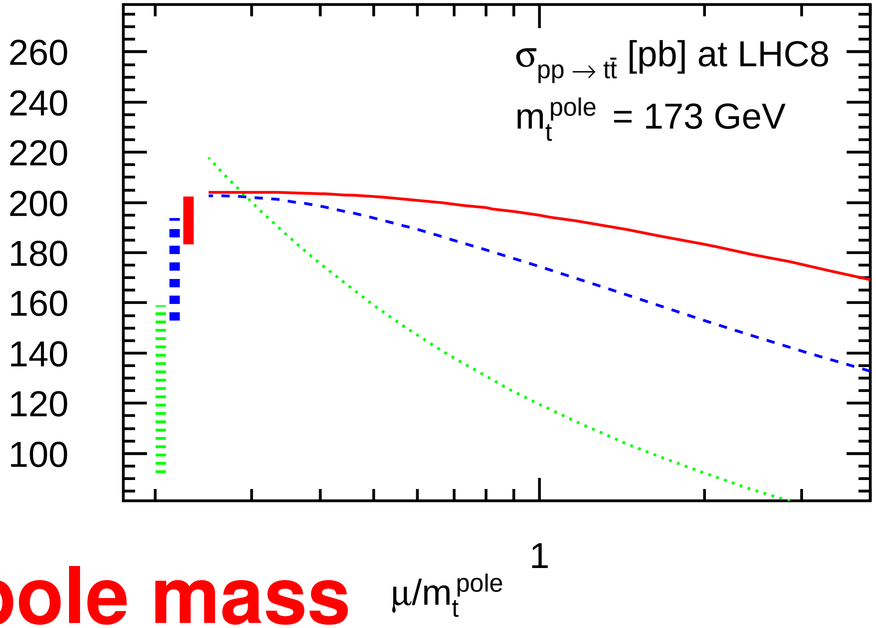
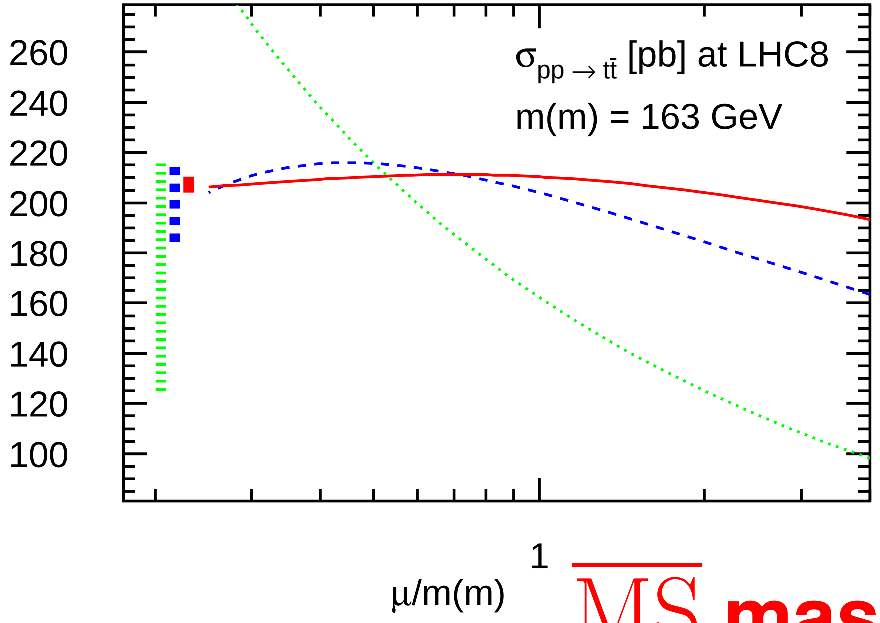


pole mass



$\overline{\mathrm{MS}}$ mass

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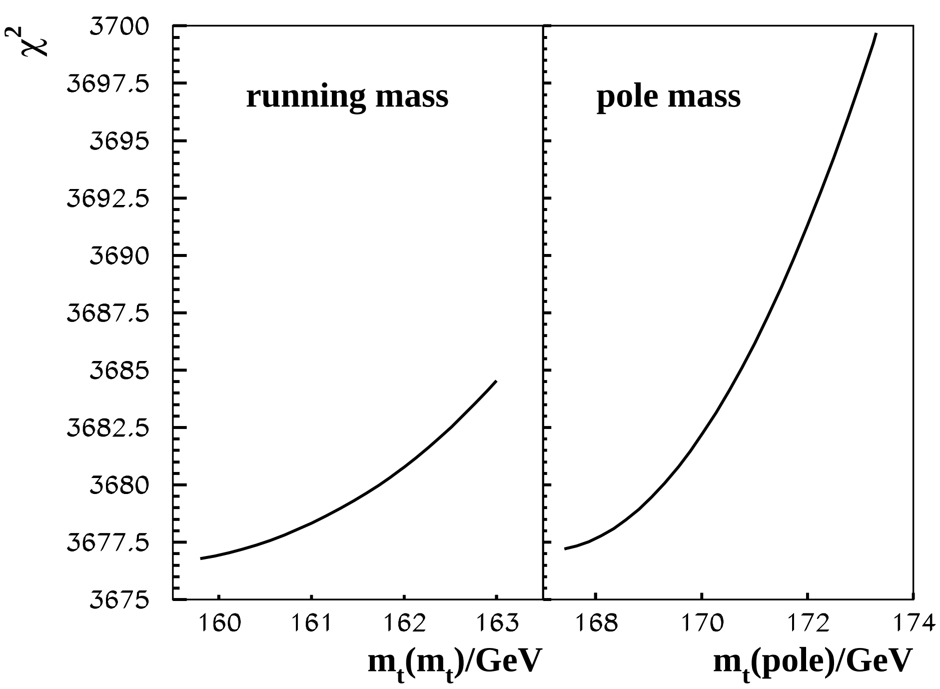
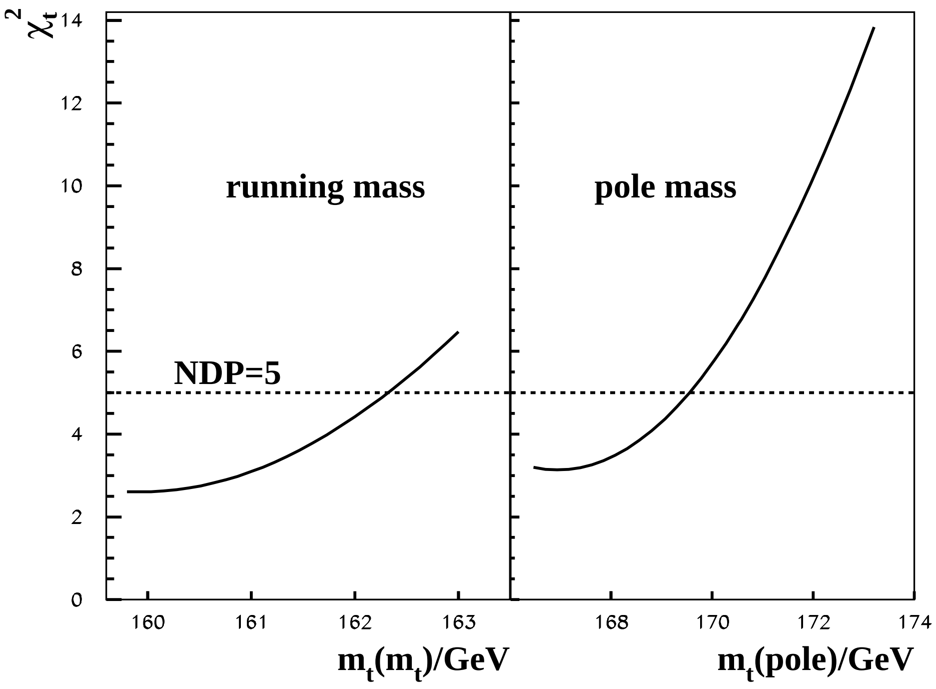
Top cross section data in ABM12 fit

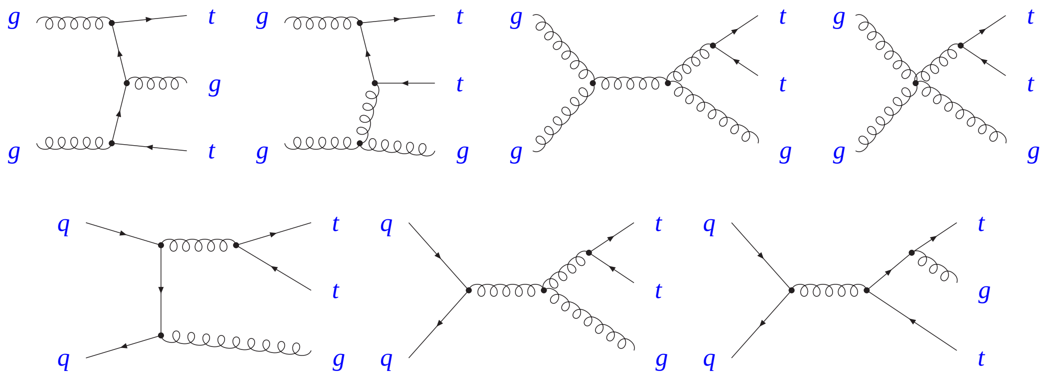
- Fit with correlations
  - $g(x)$ and $\alpha_s(M_Z)$ already well constrained by global fit (no changes)
  - for fit with $\chi^2/NDP = 5/5$ obtain value of $m_t(m_t) = 162.3 \pm 2.3$ GeV (equivalent to pole mass $m_t = 171.2 \pm 2.4$ GeV) Alekhin, Blümlein, S.M. '13
  - χ^2-profile steeper for pole mass (bigger impact of top-quark data and greater sensitivity to theoretical uncertainty at NNLO)

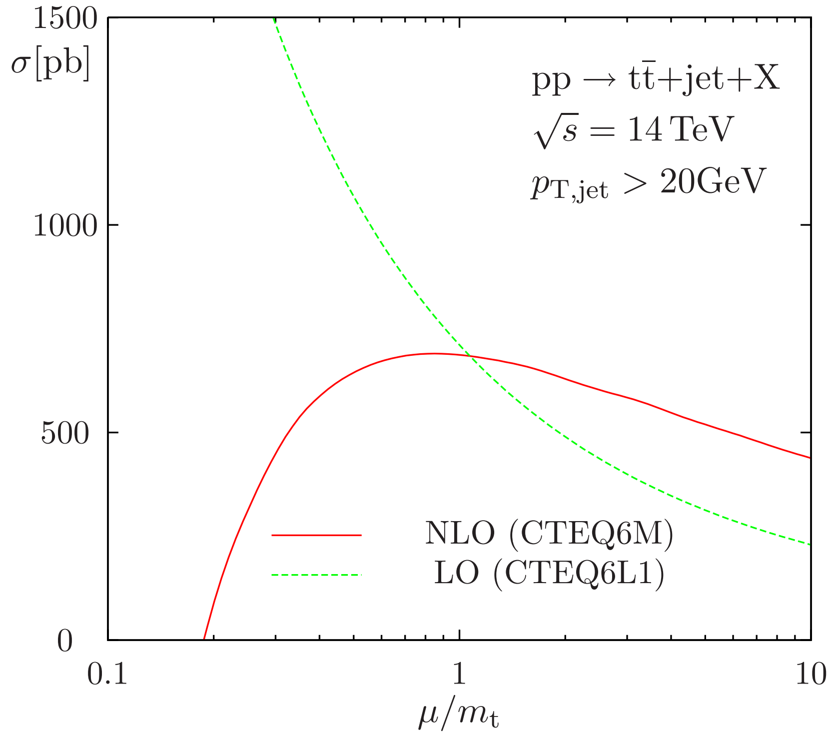




Top-quark pairs with one jet

- LHC: large rates for production of $t\bar{t}$-pairs with additional jets
- NLO QCD corrections for $t\bar{t} + 1\text{jet}$ Dittmaier, Uwer, Weinzierl '07-'08
  - scale dependence greatly reduced at NLO
  - corrections for total rate at scale $\mu_r = \mu_f = m_t$ are almost zero





- Additional jet raises kinematical threshold
  - invariant mass $\sqrt{s_{t\bar{t}+1\text{jet}}}$

Top mass with $t\bar{t} + \mathrm{jet}$*-samples*

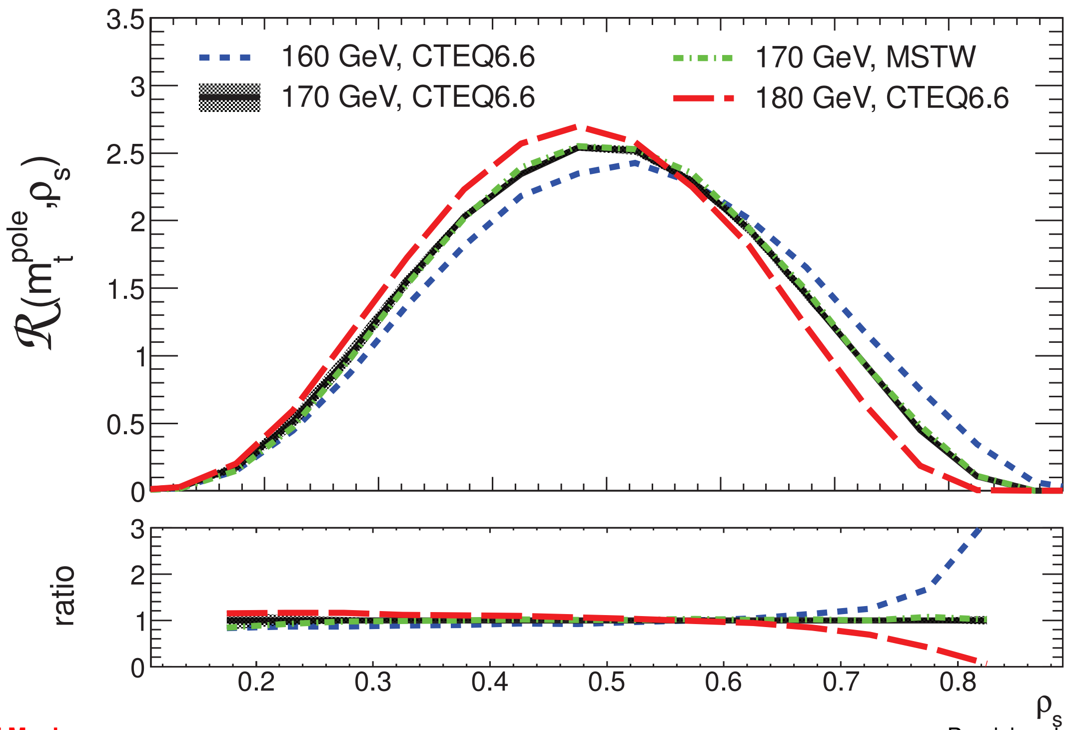
- Normalized-differential $t\bar{t} + \mathrm{jet}$ cross section

Alioli, Fernandez, Fuster, Irles, S.M., Uwer, Vos '13

$$\mathcal{R}(m_t, \rho_s) = \frac{1}{\sigma_{t\bar{t}+1\mathrm{jet}}} \frac{d\sigma_{t\bar{t}+1\mathrm{jet}}}{d\rho_s}(m_t, \rho_s)$$

  - variable $\rho_s = \frac{2 \cdot m_0}{\sqrt{s_{t\bar{t}+1\mathrm{jet}}}}$ with invariant mass of $t\bar{t} + 1\mathrm{jet}$ system and fixed scale $m_0 = 170\,\mathrm{GeV}$

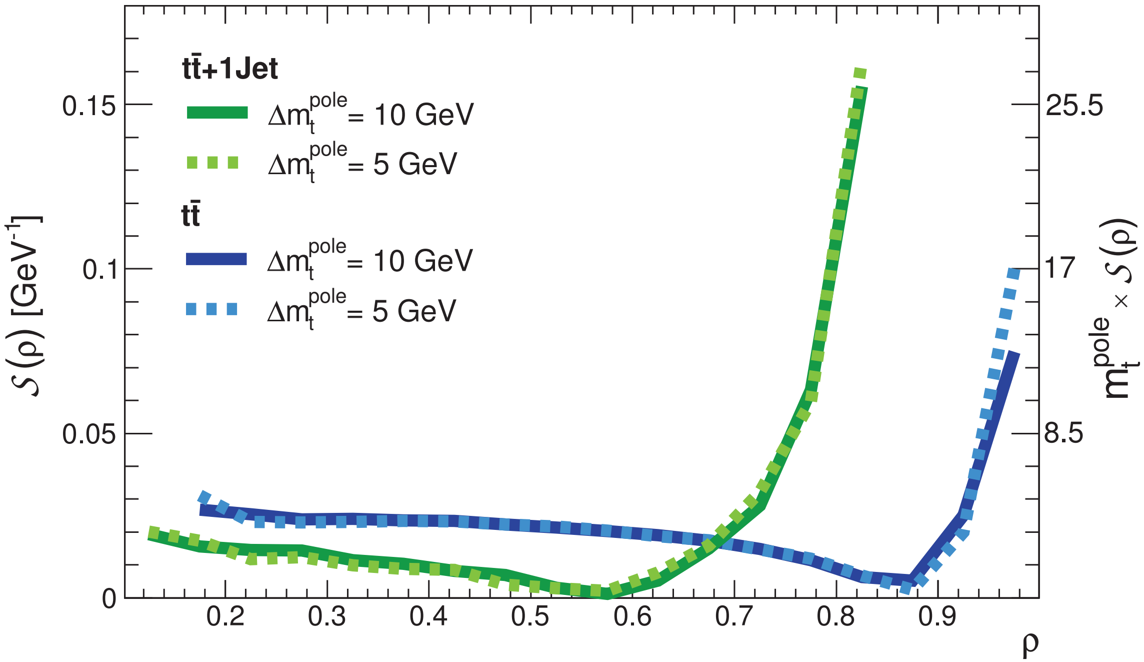
- Significant mass dependence for $0.4 \leq \rho_s \leq 0.5$ and $0.7 \leq \rho_s$



Mass sensitivity of $t\bar{t} + \mathrm{jet}$*-samples*

- Differential cross section $\mathcal{R}(m_t, \rho_s)$
  - good pertubative stability, small theory uncertainties, small dependence on experimental uncertainties, . . .
- Increased sensitivity for system $t\bar{t} + \mathrm{jet}$ compared

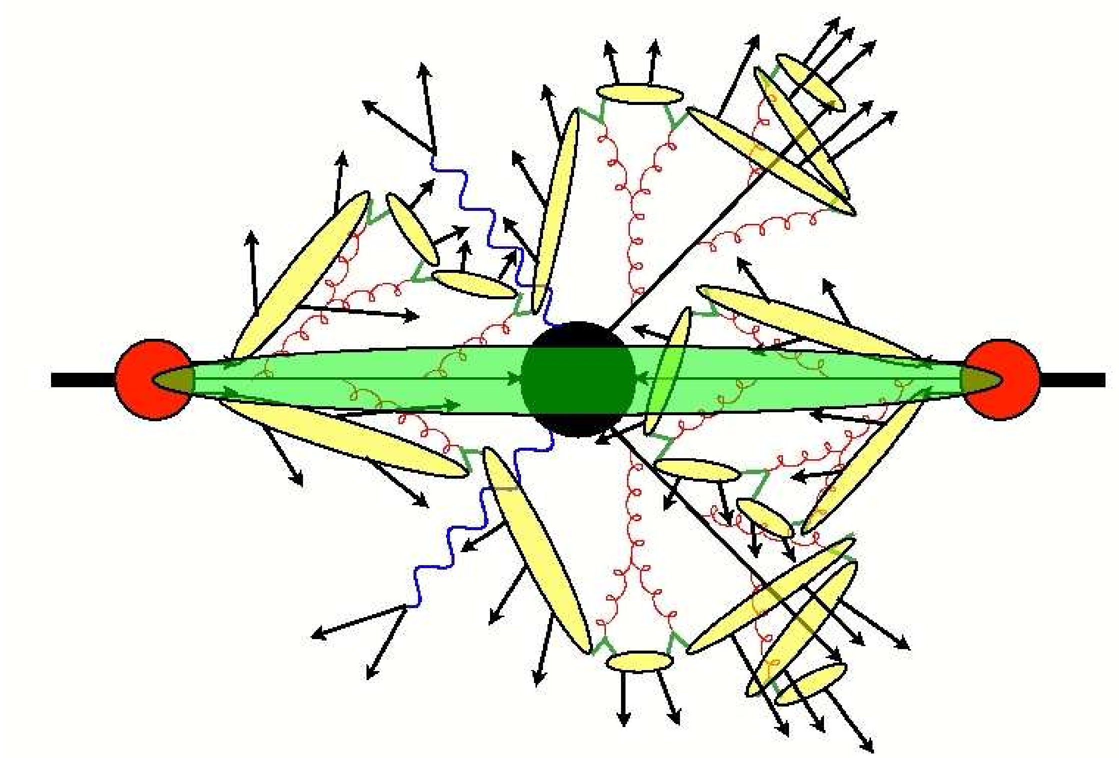
$$\left|\frac{\Delta\mathcal{R}}{\mathcal{R}}\right| \simeq (m_t\mathcal{S}) \times \left|\frac{\Delta m_t}{m_t}\right|$$



- ATLAS analysis ongoing (preliminary mass reported at Top2014)

Monte Carlo mass

- Hard interaction and parton emission in QCD followed by hadronization
- Top-quark decays on shell (e.g. leptonic decay $t \to bW \to bl\bar{\nu}_l$)



[picture by B.Webber]

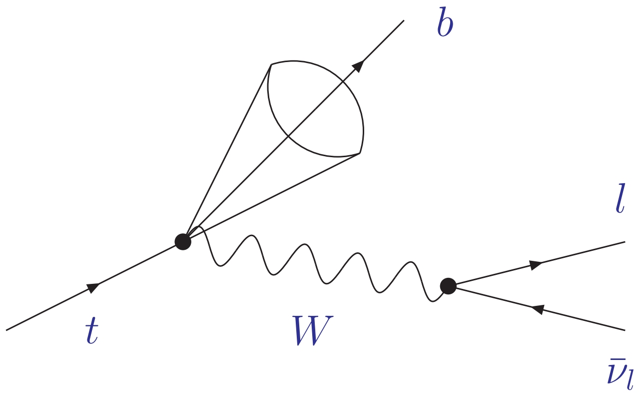
- Intuition: Monte Carlo mass identified with pole mass due to kinematics

$$m_q^2 \; = \; E_q^2 - p^2$$

- Caveat: heavy quarks in QCD interact with potential due to gluon field

Kinematic reconstruction

- Current methods based on reconstructed physics objects
  - jets, identified charged leptons, missing transverse energy
  - $m_t^2 = (p_{W-\mathrm{boson}} + p_{b-\mathrm{jet}})^2$

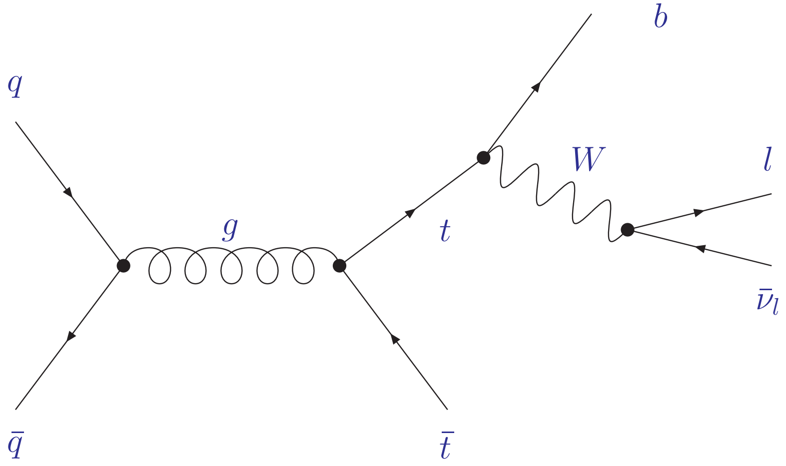


Template method

- Distributions of kinematically reconstructed top mass values compared to templates for nominal top mass values
  - distributions rely on parton shower predictions
  - uncertainties from variation of Monte Carlo parameters

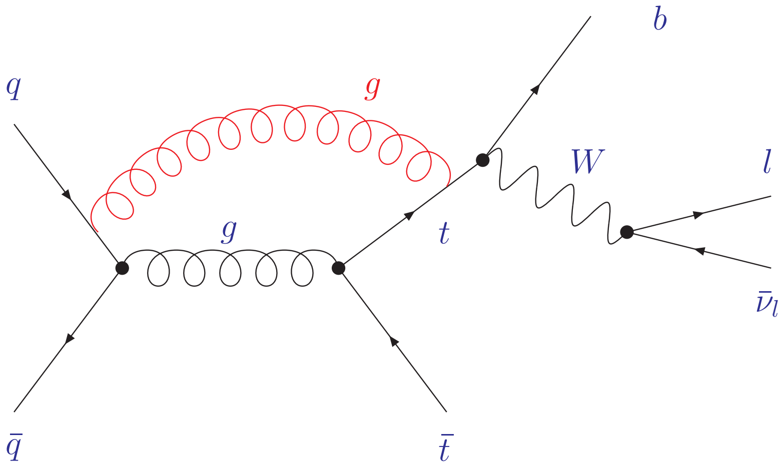
Hard scattering process

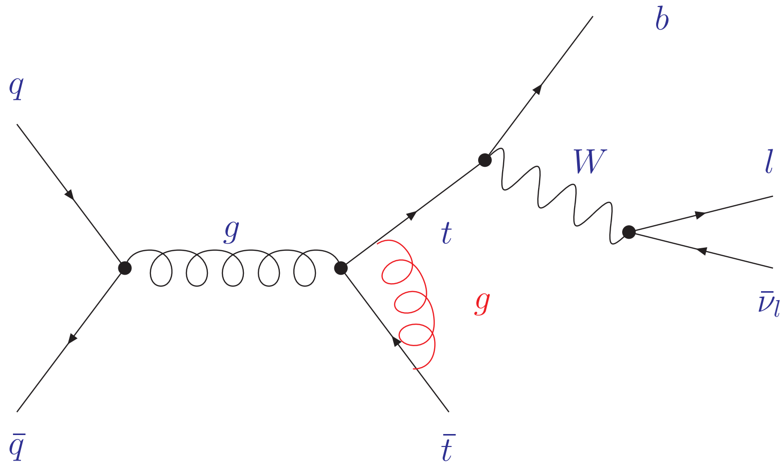
- Born process ($q\bar{q}$-channel) with leptonic decay $t \to bl\bar{\nu}_l$



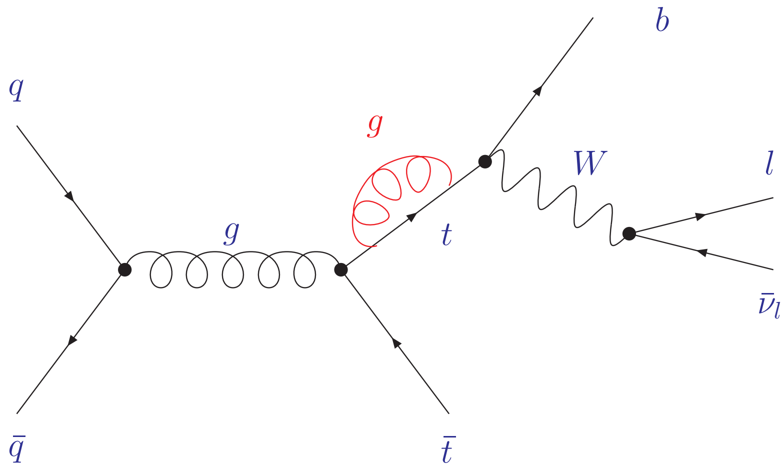
Radiative corrections

- Virtual corrections (examples): gluon exchange
  - box diagram (left) and vertex corrections (right)
  - infrared divergences cancel against real emission contributions



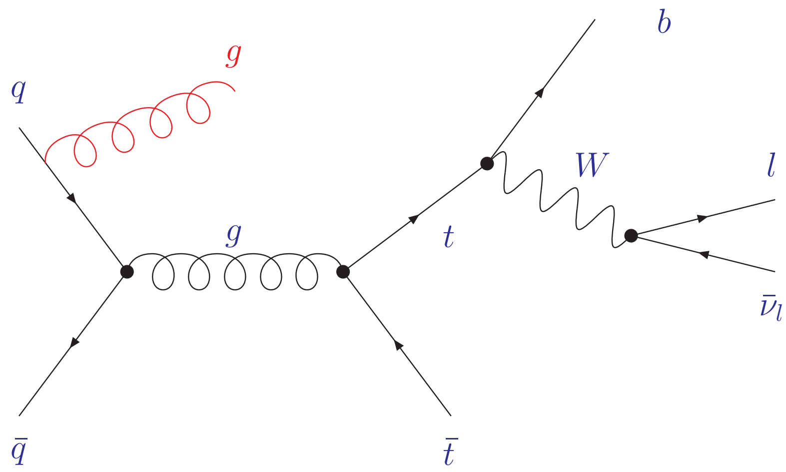


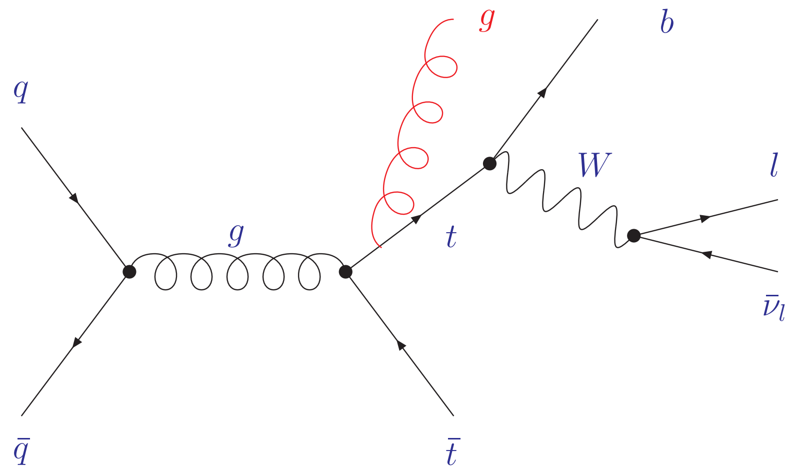
- Mass renormalization from self-energy corrections to top-quark



Radiative corrections

- Real corrections (examples): gluon emission
  - phase space integration $\rightarrow$ infrared divergences (soft/collinear singularities)





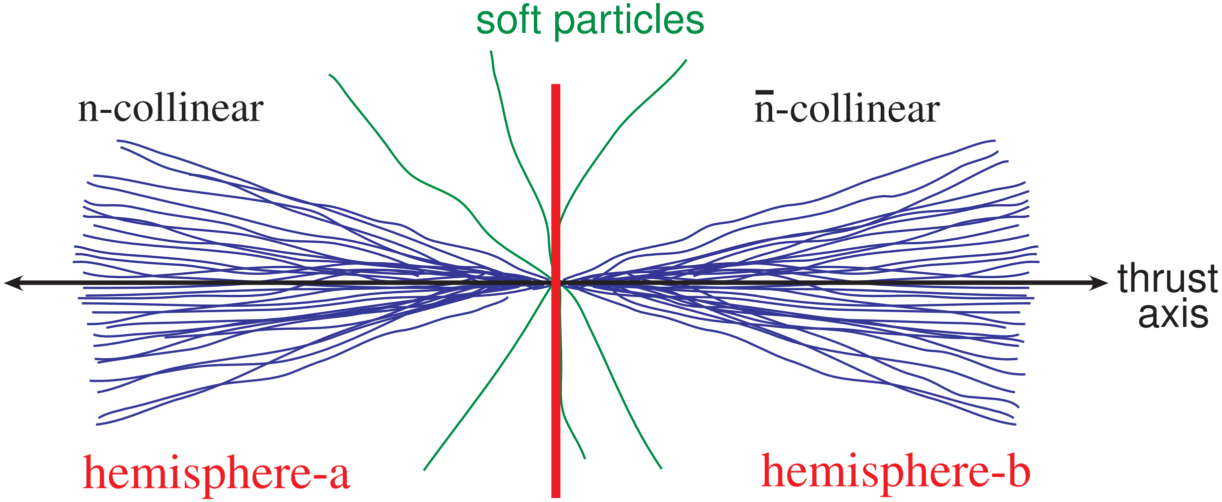
- Parton shower MC
  - emission probability modeled by Sudakov exponential with cut-off Q_0
  - leading logarithmic accuracy

$$\Delta\left(Q^2, Q_0^2\right) = \exp\left(-C_F \frac{\alpha_s}{2\pi} \ln\left(\frac{Q^2}{Q_0^2}\right)\right)$$

  - subtraction of IR contributions at hadronization scale $Q_0 \simeq \mathcal{O}(1)$GeV

Mass of heavy-quark jet (I)

- Cross section for invariant mass of jet $M_{t(\bar{t})}$ in $e^+e^- \to t\bar{t}$
- Back-to-back heavy-quark jets with collinear parton emission define hemispheres Fleming, Hoang, Mantry, Stewart '07



- Cross section factorization in effective theory (SCET)

$$\frac{d^2\sigma}{dM_t^2 dM_{\bar{t}}^2} = \sigma_0 \underbrace{H(Q,m,\mu)}_{\text{hard fct.}} \int d\ell^+ d\ell^- \underbrace{B_+(M_t,\Gamma_t,\mu)}_{\text{jet fct.}} \underbrace{B_-(M_{\bar{t}},\Gamma_t,\mu)}_{\text{jet fct.}} \underbrace{S(\ell^+,\ell^-,\mu)}_{\text{soft fct.}}$$

- hierarchy of scales $Q \gg m_t \gg \Gamma_t \gg \Lambda_{QCD}$ and $|M_t - m_t| \simeq \Gamma_t$

Mass of heavy-quark jet (II)

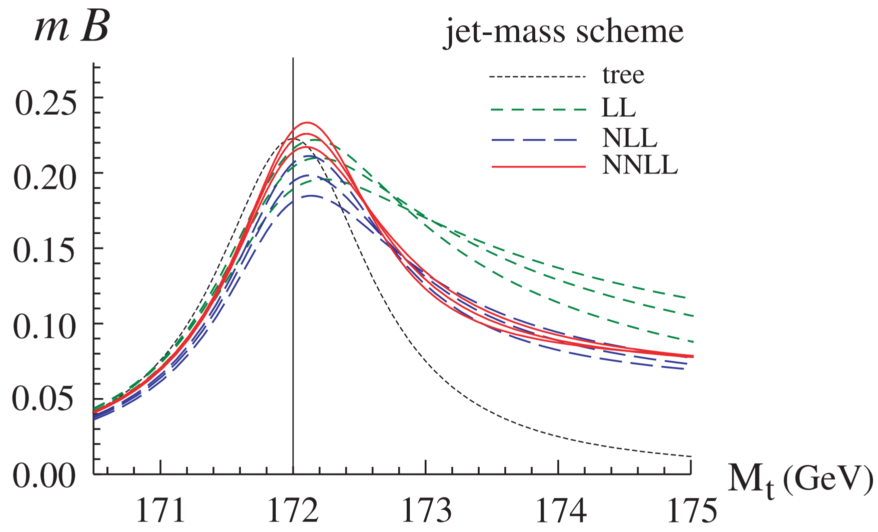
- Computation of heavy-quark jet function from discontinuity of heavy-quark propagator connected by light-like Wilson lines $W_n(0)W_n^\dagger(x)$

$$B_+(v_+ \cdot k, \Gamma_t) \;=\; \frac{-1}{8\pi N_c m} \text{Disc} \int d^4x \, e^{ik\cdot x} \langle 0|\mathrm{T}\{\bar{h}_{v_+}(0) W_n(0) W_n^\dagger(x) h_{v_+}(x)\}|0\rangle$$

- Computation of $B_\pm$ in perturbation theory with well-defined mass scheme
- Breit-Wigner resonance at tree level

$$B_\pm(\hat{s}, \Gamma_t) \;=\; \frac{1}{4\pi m} \text{Disc.} \left(\frac{i}{v_\pm \cdot k + i\Gamma_t/2} \right) \;=\; \frac{1}{\pi m} \frac{\Gamma_t}{\hat{s}^2 + \Gamma_t^2}$$

- Stable peak position at higher orders
  Hoang, Stewart '08
- Mass renormalization with short-distance mass
  $m_{\text{pole}} = m_{\text{short distance}} + \delta m$
- short-distance mass $m^{\text{MSR}}(R)$ probes scale of hard interaction: $R \simeq \Gamma_t$



Conversion Monte Carlo mass to pole mass (I)

Assumption

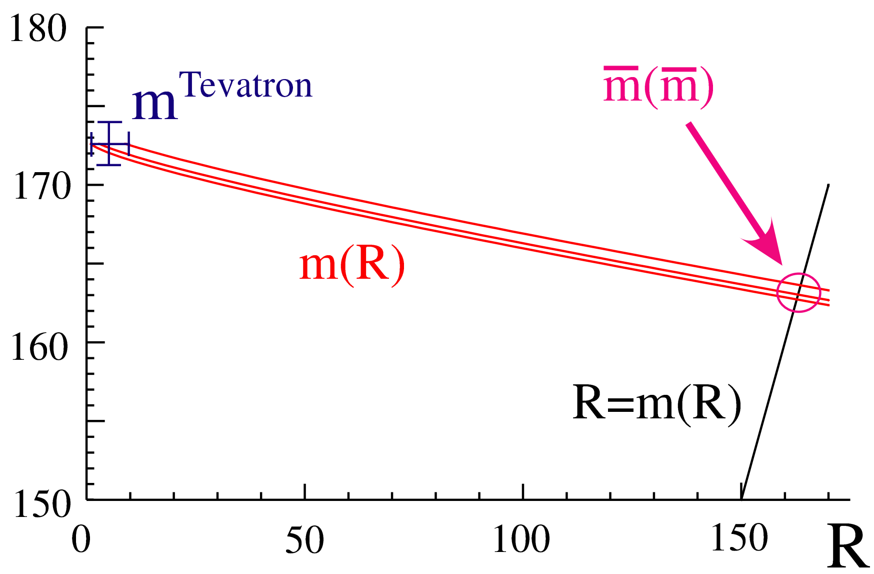
- Identify Monte Carlo mass m^{MC} with short distance mass $m^{\mathrm{MSR}}(R)$ at low scale $\mathcal{O}(1)\ \mathrm{GeV}$
  - choice for range of scale $R \simeq 1 \ldots 9\mathrm{GeV}$

$$m^{\mathrm{MC}} \quad = \quad m^{\mathrm{MSR}}(R = 3^{+6}_{-2}\text{GeV})$$

Conversion Monte Carlo mass to pole mass (II)

Strategy

- Use perturbation theory to convert $m^{\rm MSR}(R)$ to $m^{\rm pole}$
- Running of $m^{\rm MSR}(R)$ mass
  Hoang, Stewart '08



- Choice 1: run $m^{\rm MSR}(R)$ from low scale to $R = m_t$: $m^{\rm MSR}(R) \to m(m)$ and convert from $m(m)$ to pole mass [arXiv:1405.4781]

$m^{\rm MSR}(1)$	$m^{\rm MSR}(3)$	$m^{\rm MSR}(9)$	$\overline{m}(\overline{m})$	$m^{\rm pl}_{\rm 1lp}$	$m^{\rm pl}_{\rm 2lp}$	$m^{\rm pl}_{\rm 3lp}$
173.72	173.40	172.78	163.76	171.33	172.95	173.45

- Choice 2: convert from $m^{\rm MSR}(R)$ at low scale directly to pole mass

$m^{\rm MSR}(1)$	$m^{\rm MSR}(3)$	$m^{\rm MSR}(9)$	$m^{\rm pl}_{\rm 1lp}$	$m^{\rm pl}_{\rm 2lp}$	$m^{\rm pl}_{\rm 3lp}$
173.72	173.40	172.78	173.72	173.87	173.98

Conversion Monte Carlo mass to pole mass (III)

Summary

$$m_{\rm pole} \;=\; 173.34 \pm 0.76\,\text{GeV (exp)} \;+\; \Delta m(\text{th})$$

with

$$\Delta m(\text{th}) \quad = \quad {}^{+0.32}_{-0.62}\,\text{GeV}\,(m^{\rm MC} \to m^{\rm MSR}(3\text{GeV})) \;+\; 0.50\,\text{GeV}\,(m(m) \to m_{\rm pole})$$

and combined

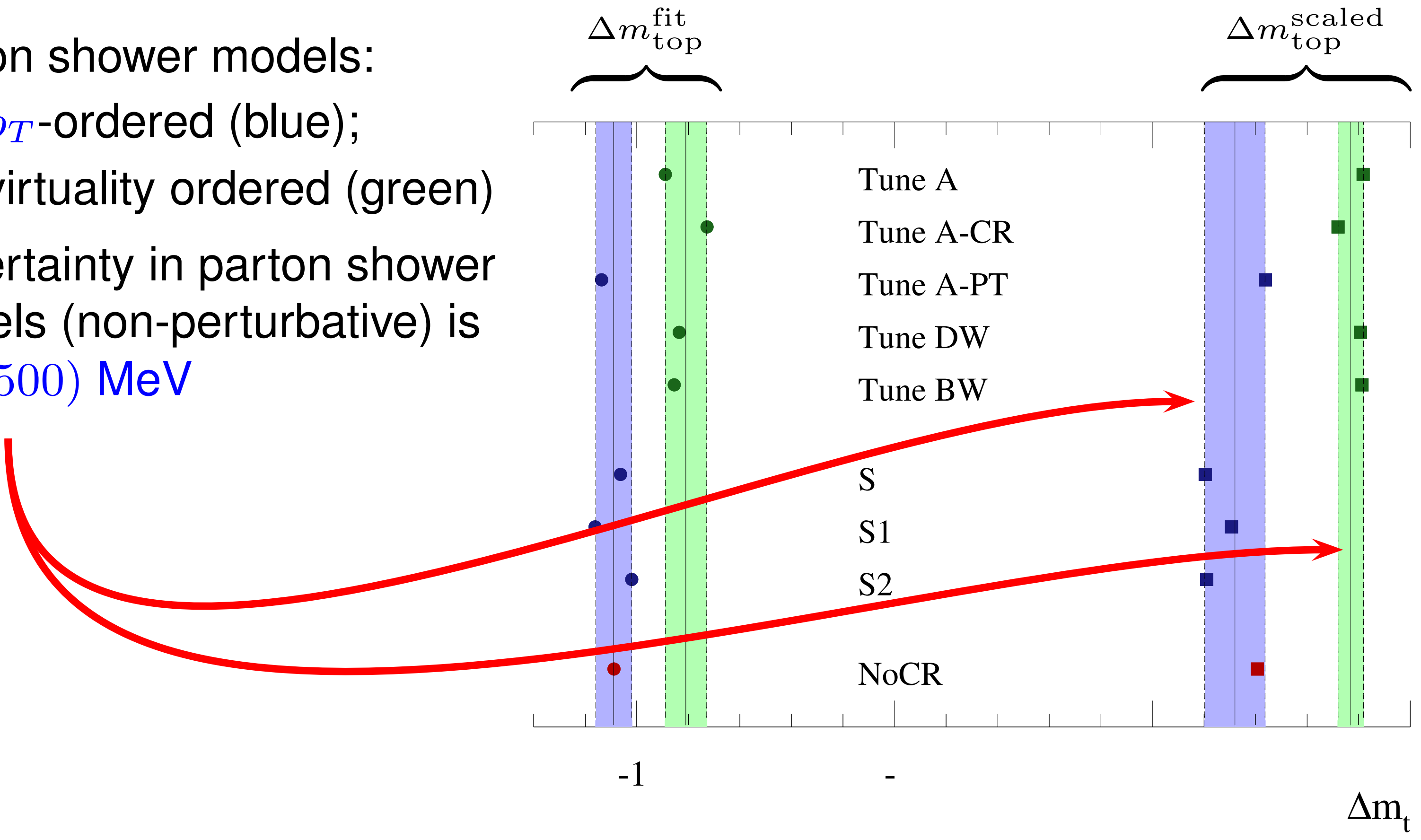
$$\Delta m(\text{th}) \quad = \quad {}^{+0.82}_{-0.62}\text{GeV}$$

In addition, unknown systematic mass shift $\mathcal{O}(1)$ GeV due to non-perturbative effects on peak position of invariant jet-mass distribution $M^{\rm peak}$ with decaying top-quark for short distance mass m_t

$$M^{\rm peak} = m_t + \Gamma_t(\alpha_s + \alpha_s^2 + \ldots) + \frac{Q\Lambda_{\rm QCD}}{m_t}$$

Non-perturbative corrections

- Simulation of top mass measurement Skands, Wicke '07
  - test of different Monte Carlo tunes for non-perturbative physics / colour reconnection
  - calibration offsets before/after scaling with jet energy scale corrections
- Parton shower models:
  - p_T-ordered (blue);
  - virtuality ordered (green)
- Uncertainty in parton shower models (non-perturbative) is $\mathcal{O}(\pm 500)$ MeV



Higgs boson mass

Experimental result

Atlas arXiv:1307.1427; CMS coll. arXiv:1312.5353; (average arXiv:1303.3570)

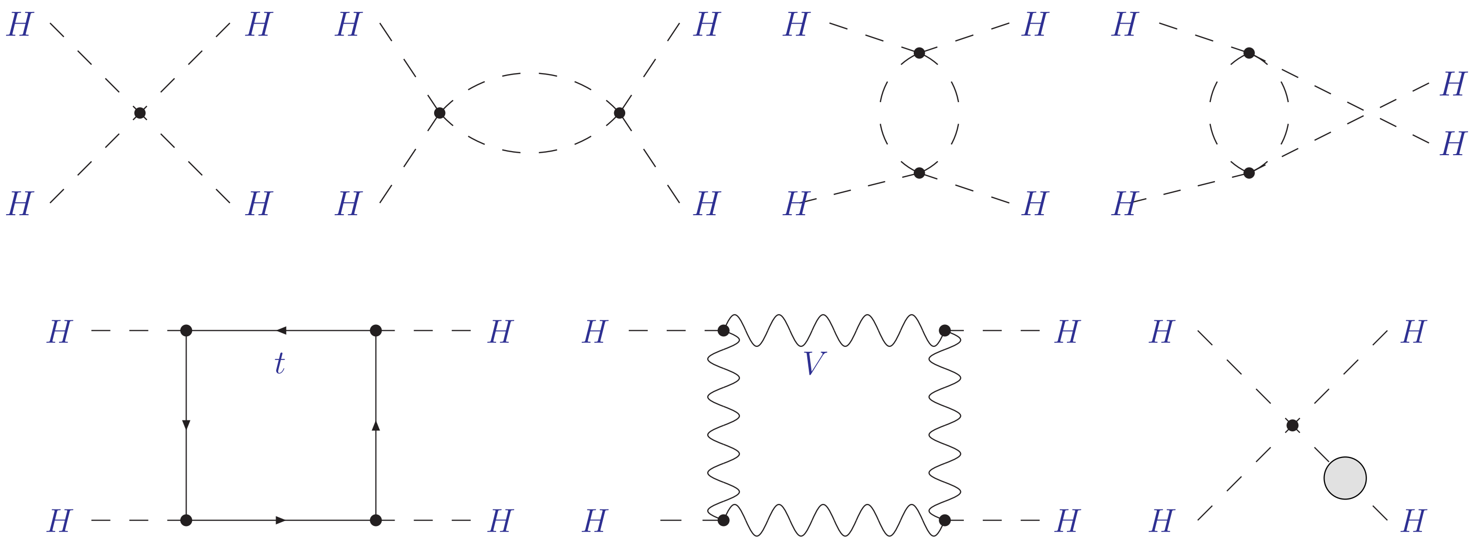
$$m_H = 125.15 \pm 0.24\,\mathrm{GeV}$$

Higgs potential

Renormalization group equation

- Quantum corrections to Higgs potential $V(\Phi) = \lambda \left| \Phi^\dagger \Phi - \frac{v}{2} \right|^2$
- Radiative corrections to Higgs self-coupling λ
  - electro-weak couplings g and g' of $SU(2)$ and $U(1)$
  - top-Yukawa coupling y_t

$$16\pi^2 \frac{d\lambda}{dQ} = 24\lambda^2 - \left(3g'^2 + 9g^2 - 12y_t^2\right)\lambda + \frac{3}{8}g'^4 + \frac{3}{4}g'^2 g^2 + \frac{9}{8}g^4 - 6y_t^4 + \ldots$$



Higgs potential

Triviality

- Large mass implies large λ
  - renormalization group equation dominated by first term

$$16\pi^2 \frac{d\lambda}{dQ} \simeq 24\lambda^2 \qquad \longrightarrow \qquad \lambda(Q) = \frac{m_H^2}{2v^2 - \frac{3}{2\pi^2} m_H^2 \ln(Q/v)}$$

- $\lambda(Q)$ increases with Q
- Landau pole implies cut-off Λ
  - scale of new physics smaller than Λ to restore stability
  - upper bound on m_H for fixed Λ

$$\Lambda \leq v \exp\left(\frac{4\pi^2 v^2}{3m_H^2}\right)$$

- Triviality for $\Lambda \to \infty$
  - vanishing self-coupling $\lambda \to 0$ (no interaction)

Higgs potential

Vacuum stability

- Small mass
  - renormalization group equation dominated by y_t

$$16\pi^2 \frac{d\lambda}{dQ} \simeq -6y_t^4 \qquad \longrightarrow \qquad \lambda(Q) = \lambda_0 - \frac{\frac{3}{8\pi^2} y_0^4 \ln(Q/Q_0)}{1 - \frac{9}{16\pi^2} y_0^2 \ln(Q/Q_0)}$$

- $\lambda(Q)$ decreases with Q
- Higgs potential unbounded from below for $\lambda < 0$
- $\lambda = 0$ for $\lambda_0 \simeq \frac{3}{8\pi^2} y_0^4 \ln(Q/Q_0)$
- Vacuum stability

$$\Lambda \leq v \exp\left(\frac{4\pi^2 m_H^2}{3y_t^4 v^2}\right)$$

  - scale of new physics smaller than Λ to ensure vacuum stability
  - lower bound on m_H for fixed Λ

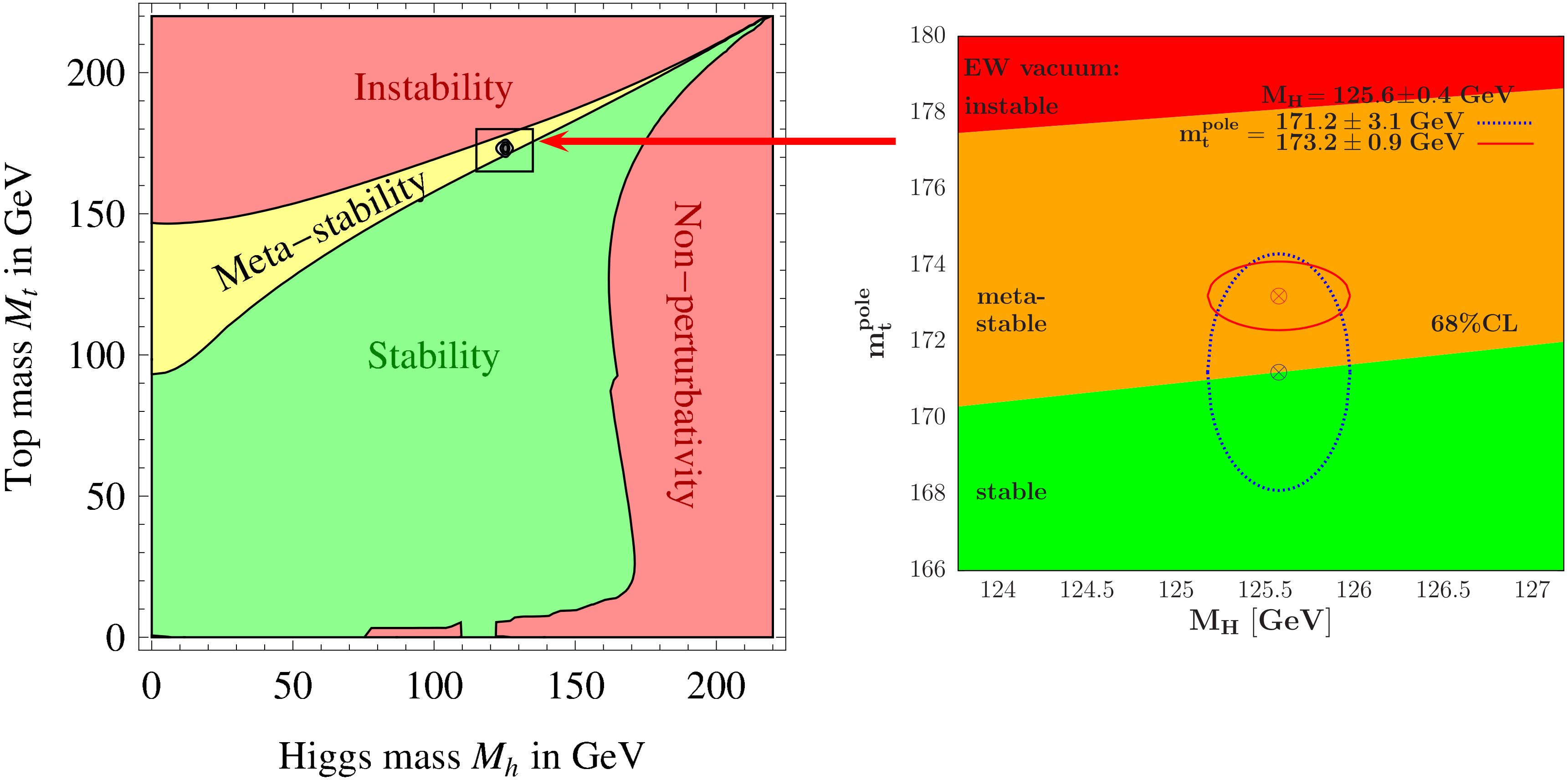
Implications on electroweak vacuum

- Relation between Higgs mass m_H and top-quark mass m_t
  - condition of absolute stability of electroweak vacuum $\lambda(\mu) \geq 0$
  - extrapolation of Standard Model up to Planck scale M_P
  - $\lambda(M_P) \geq 0$ implies lower bound on Higgs mass m_H

$$m_H \geq 129.6 + 2.0 \times \left(m_t^{\text{pole}} - 173.34 \text{ GeV} \right) - 0.5 \times \left(\frac{\alpha_s(M_Z) - 0.1184}{0.0007} \right) \pm 0.3 \text{ GeV}$$

  - recent NNLO analyses Bezrukov, Kalmykov, Kniehl, Shaposhnikov '12; Degrassi, Di Vita, Elias-Miro, Espinosa, Giudice et al. '12; Buttazzo, Degrassi, Giardino, Giudice, Sala, Salvio, Strumia '13
  - uncertainity in results due to α_s and m_t (pole mass scheme)
- Top-quark mass from total cross section (well-defined scheme)
  - $m_t^{\overline{\text{MS}}}(m_t) = 162.3 \pm 2.3 \pm 0.7 \text{ GeV}$ implies in pole mass scheme $m_t^{\text{pole}} = 171.2 \pm 2.4 \pm 0.7 \text{ GeV}$
  - mass determination accounts for correlation with gluon PDF and $\alpha_S(M_Z)$

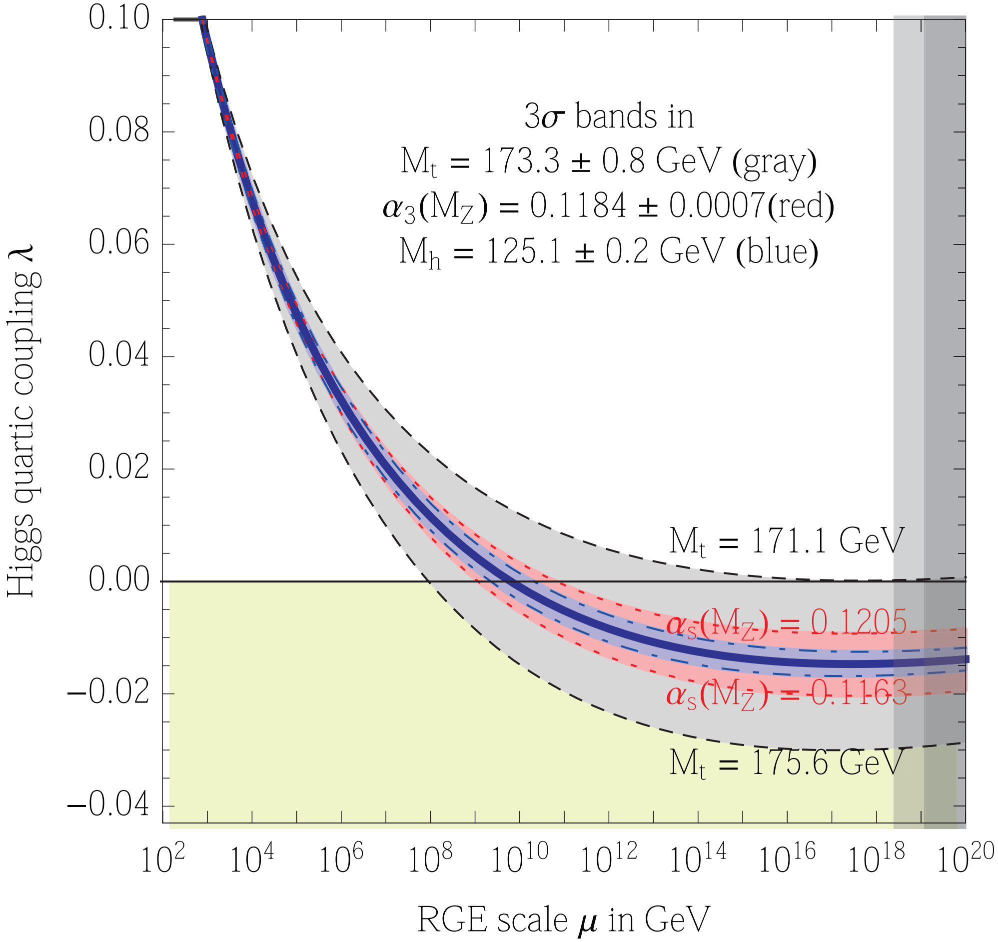
Fate of the universe



Degrassi, Di Vita, Elias-Miro, Espinosa, Giudice et al. '12; Alekhin, Djouadi, S.M. '12; Masina '12

- Uncertainty in Higgs bound due to m_t from in $\overline{MS}$ scheme
  - bound relaxes $m_H \geq 125.3 \pm 6.2$ GeV
  - "fate of universe" still undecided

Higgs self-coupling



Buttazzo, Degrassi, Giardino, Giudice, Sala, Salvio, Strumia '13

- Renormalization group evolution of λ with uncertainties in m_H, m_t and α_s
  - top-quark mass least precise parameter
- Vacuum stability bound at M_P in terms of m_t

$$m_t \; \leq \; (171.53 \pm 0.15 \pm 0.23_{\alpha_3} \pm 0.15_{m_h}) \text{ GeV } \; = \; (171.53 \pm 0.42) \text{ GeV}$$

Summary

Top-quark mass

- Running mass ($\overline{\mathrm{MS}}$ scheme) at NNLO in QCD

$$m_t(m_t) = 162.3 \pm 2.3 \pm 0.7\,\mathrm{GeV}$$

Higgs mass

- Known to very high precision (pole mass)

$$m_H = 125.15 \pm 0.24\,\mathrm{GeV}$$

Fate of the universe

- Still undecided ...

Summary

Physics at the Terascale

- Discovery of Higgs boson opens new avenue for studies of Standard Model physics and beyond
- QCD and electroweak corrections at higher orders are crucial
- Precision tests of SM at LHC depend on non-perturbative parameters
  - masses m_t, M_W, m_H, ...
  - coupling constant $\alpha_s(M_Z)$
  - parton content of proton (PDFs)

Top-quark mass

- Top-quark mass is parameter of Standard Model Lagrangian
- Measurements of m_t require careful definition of observable
- Quality of perturbative expansion depends on scheme for top-quark mass
- Relation of Monte Carlo mass m^{MC} to pole mass with additional theory uncertainty Δm_t(th)

Future tasks

- Joint effort theory and experiment