Flavour blindness in QCD: Sigma - Lambda mixing

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arXiv:1411:7665 [hep-lat]

Introduction

The QCD interaction is flavour-blind. Neglecting electromagnetic and weak interactions, the only difference between flavours comes from the mass matrix. We investigate how flavour-blindness constrains hadron masses after flavour SU(3) is broken by the mass difference between the strange and light quarks, to help us extrapolate 2+1 flavour lattice data to the physical point.

We have our best theoretical understanding when all 3 quark flavours have the same masses (because we can use the full power of flavour $SU(3)$); nature presents us with just one instance of the theory, with $m_s/m_l \approx 25.$ We are interested in interpolating between these two cases.

Introduction

Standard Theorist's Approach:

$$\text{Action} = \text{Large Piece} + \text{Small Piece}$$

Treat the Small Piece as a perturbation.
Apply this to QCD.

Introduction

This Talk

$$
\begin{aligned}
\text{Large Piece} &= \text{Kinetic Terms} \\
&+ \text{Gluon-Gluon Vertices} \\
&+ \text{Quark-Gluon Vertices} \\
&+ \text{Singlet Quark Mass Term}
\end{aligned}
$$

$$
\text{Small Piece} = \text{Non-Singlet Quark Mass Terms}
$$

Perturb about $SU(3)$ symmetric QCD.

Introduction

This Talk

$$
\begin{aligned}
\text{Large Piece} &= \text{Kinetic Terms} \\
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&\quad + \text{Singlet Quark Mass Term}
\end{aligned}
$$

$$
\text{Small Piece} = \text{Non-Singlet Quark Mass Terms}
$$

Long history: M. Gell Man, Phys Rev 125 (1962) 1067.
S. Okubo, Prog Theor Phys 27 (1962) 949.
S. R. Beane, K. Orginos and M. J. Savage, *Phys. Lett.* **B654** (2007) 20 [`arXiv:hep-lat/0604013`].

Introduction

This Talk

$$\text{Large Piece} = \text{Kinetic Terms} + \text{Gluon-Gluon Vertices} + \text{Quark-Gluon Vertices} + \text{Singlet Quark Mass Term}$$

$$\text{Small Piece} = \text{Non-Singlet Quark Mass Terms}$$

Not as familiar as chiral perturbation theory,
but useful for organising and analysing the data.

Quark Masses

Notation

$$
\begin{aligned}
\overline{m} &\equiv \frac{1}{3}(m_u + m_d + m_s) \\
\delta m_u &\equiv m_u - \overline{m} \\
\delta m_d &\equiv m_d - \overline{m} \\
\delta m_s &\equiv m_s - \overline{m} \\
\\
\delta m_u + \delta m_d + \delta m_s &= 0 \\
\\
m_l &\equiv \frac{1}{2}(m_u + m_d) \\
\delta m_l &\equiv m_l - \overline{m}
\end{aligned}
$$

Quark Masses

The quark mass matrix is

$$\mathcal{M} = \begin{pmatrix} m_u & 0 & 0 \\ 0 & m_d & 0 \\ 0 & 0 & m_s \end{pmatrix}$$
$$= \overline{m} \begin{pmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{pmatrix} + \frac{1}{2}(\delta m_u - \delta m_d) \begin{pmatrix} 1 & 0 & 0 \\ 0 & -1 & 0 \\ 0 & 0 & 0 \end{pmatrix} + \frac{1}{2}\delta m_s \begin{pmatrix} -1 & 0 & 0 \\ 0 & -1 & 0 \\ 0 & 0 & 2 \end{pmatrix}$$

$\mathcal{M}$ has a flavour singlet part (proportional to I) and a flavour octet part, proportional to λ_3, λ_8.
In clover case, the singlet and non-singlet parts of the mass matrix renormalise differently.

Flavour Hierarchy

$$\begin{aligned}\text{Large Piece} &= \text{Kinetic Terms}\\ &+ \text{Gluon-Gluon Vertices}\\ &+ \text{Quark-Gluon Vertices}\\ &+ \text{Singlet Quark Mass Term}\end{aligned}$$

$$\text{Small Piece} = \text{Non-Singlet Quark Mass Terms}$$

All terms in Large Piece are flavour singlets, leave $SU(3)$ unbroken.
Small Piece is pure flavour octet.
Higher $SU(3)$ representations completely absent from QCD action.

Flavour Hierarchy

Higher representations of $SU(3)$ are absent from the QCD action, but they appear at higher orders in the perturbation. Square an octet — generates 27-plet.

δm_q^0	1						1
δm_q^1		8					8
δm_q^2	1	8			27		$\frac{1}{2!} 8 \times 9 = 36$
δm_q^3	1	8	10	$\overline{10}$	27	64	$\frac{1}{3!} 8 \times 9 \times 10 = 120$

Flavour Hierarchy

Decuplet mass matrix

$$10 \otimes \overline{10} = 1 \oplus 8 \oplus 27 \oplus 64$$

Δ^-	Δ^0	Δ^+	Δ^{++}	Σ^{*-}	Σ^{*0}	Σ^{*+}	Ξ^{*-}	Ξ^{*0}	Ω^-	$SU(3)$
1	1	1	1	1	1	1	1	1	1	1
-1	-1	-1	-1	0	0	0	1	1	2	8
3	3	3	3	-5	-5	-5	-3	-3	9	27
-1	-1	-1	-1	4	4	4	-6	-6	4	64

Flavour Hierarchy

$$
\begin{array}{rcll}
4M_{\Delta} + 3M_{\Sigma^*} + 2M_{\Xi^*} + M_{\Omega} & = & 13.82 \text{ GeV} & \text{singlet} \\
-2M_{\Delta} \qquad\qquad + M_{\Xi^*} + M_{\Omega} & = & 0.742 \text{ GeV} & \text{octet} \\
4M_{\Delta} - 5M_{\Sigma^*} - 2M_{\Xi^*} + 3M_{\Omega} & = & -0.044 \text{ GeV} & 27-\text{plet} \\
-M_{\Delta} + 3M_{\Sigma^*} - 3M_{\Xi^*} + M_{\Omega} & = & -0.006 \text{ GeV} & 64-\text{plet} \quad ,
\end{array}
$$

[PDG masses]

Strong Hierarchy:

1	8	27	64
$(m_s - m_l)^0$	$(m_s - m_l)^1$	$(m_s - m_l)^2$	$(m_s - m_l)^3$

Strategy

Keep Large Piece constant,
Vary Small Piece until we reach the physical point.

Strategy

Start from a point with all 3 sea quark masses equal,

$$m_u = m_d = m_s \equiv m_0$$

and extrapolate towards the physical point, keeping the average sea quark mass

$$\overline{m} \equiv \frac{1}{3}(m_u + m_d + m_s)$$

constant.
Starting point has

$$m_0 \approx \frac{1}{3} m_s^{phys}$$

As we approach the physical point, the u and d become lighter, but the s becomes heavier. Pions are decreasing in mass, but K and η increase in mass as we approach the physical point.

Singlet Quantities

Consider a flavour singlet quantity (eg plaquette P) at the symmetric point (m_0, m_0, m_0).

$$\frac{\partial P}{\partial m_u} = \frac{\partial P}{\partial m_d} = \frac{\partial P}{\partial m_s} \; .$$

If we keep $m_u + m_d + m_s$ constant, $dm_s = -dm_u - dm_d$ so

$$dP = dm_u \frac{\partial P}{\partial m_u} + dm_d \frac{\partial P}{\partial m_d} + dm_s \frac{\partial P}{\partial m_s} = 0$$

The effect of making the strange quark heavier exactly cancels the effect of making the light quarks lighter, so we know that P must have a stationary point at the symmetrical point.

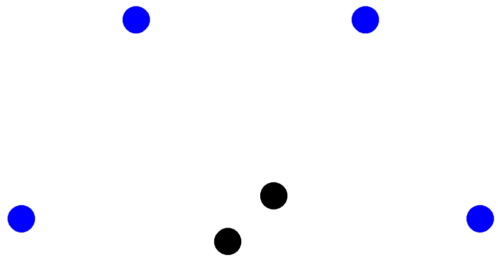
Singlet Quantities

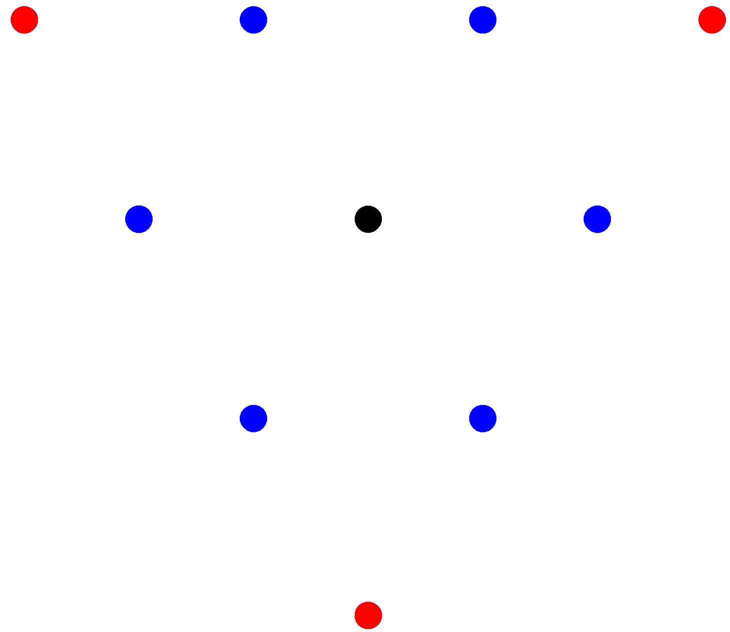
Any permutation of the quarks, eg

$$u \leftrightarrow s, \qquad u \to d \to s \to u$$

doesn't really change physics, it just renames the quarks.
Group S_3 , permutations of three objects, symmetry group of the equilateral triangle.
Any quantity unchanged by all permutations will also be flat at the symmetric point.

Singlet Quantities





$$2M_\Delta + M_\Omega$$

$$2(M_N + M_\Sigma + M_\Xi) \qquad 2(M_\Delta + M_{\Sigma^*} + M_{\Xi^*})$$

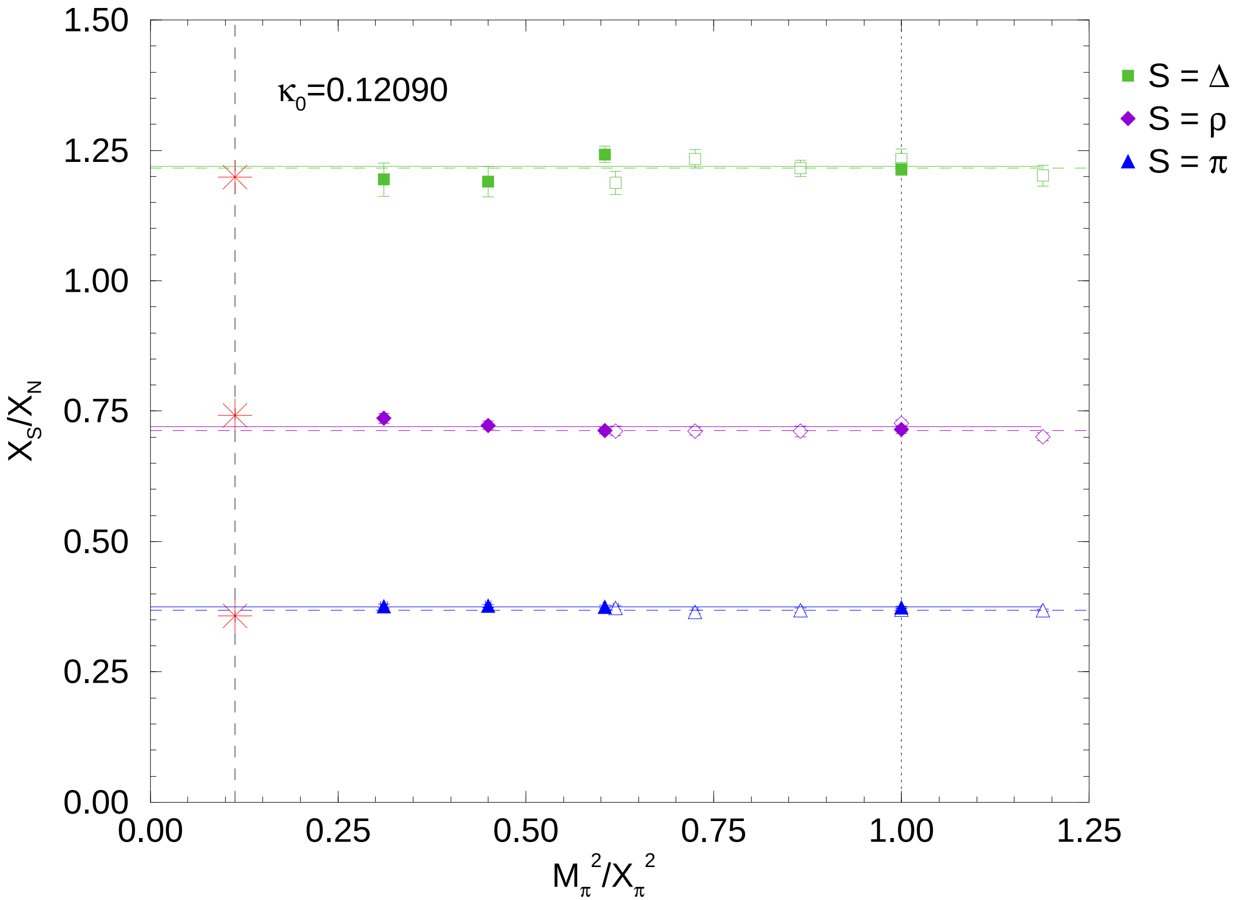
$$M_\Sigma + M_\Lambda \qquad M_{\Sigma^*}$$

Singlet Quantities

$$
\begin{aligned}
X_\pi^2 &= (M_\pi^2 + 2M_K^2)/3 \\
X_\rho &= (M_\rho + 2M_{K^*})/3 \\
X_N &= (M_N + M_\Sigma + M_\Xi)/3 \\
X_\Delta &= (2M_\Delta + M_\Omega)
\end{aligned}
$$

Multiplet Centre-of-Mass
Use octet baryons (X_N) to set scale for the other three multiplets.

Singlet Quantities



X_S so flat because we keep $m_u + m_d + m_s$ constant.
Choose initial m_0 to make X_S/X_N equal to physical value.

SU(3) classification

- Classify physical quantities by $SU(3)$ and permutation group S_3 (which is a subgroup of SU(3)).
- Classify quark mass polynomials in same way.
- Quantity of Known Symmetry $=$ Polynomials of Matching Symmetry
- Taylor expansion about (m_0, m_0, m_0) strongly constrained by symmetry.

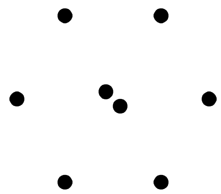
SU(3) classification

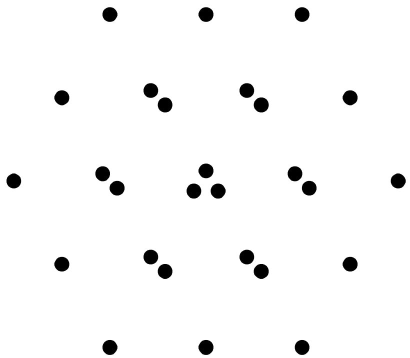
Polynomial		S_3	$SU(3)$		
1	✓	A_1	1		
$(\overline{m} - m_0)$		A_1	1		
δm_s	✓	E^+		8	
$(\delta m_u - \delta m_d)$	✓	E^-		8	
$(\overline{m} - m_0)^2$		A_1	1		
$(\overline{m} - m_0)\delta m_s$		E^+		8	
$(\overline{m} - m_0)(\delta m_u - \delta m_d)$		E^-		8	
$\delta m_u^2 + \delta m_d^2 + \delta m_s^2$	✓	A_1	1		27
$3\delta m_s^2 - (\delta m_u - \delta m_d)^2$	✓	E^+		8	27
$\delta m_s(\delta m_d - \delta m_u)$	✓	E^-		8	27

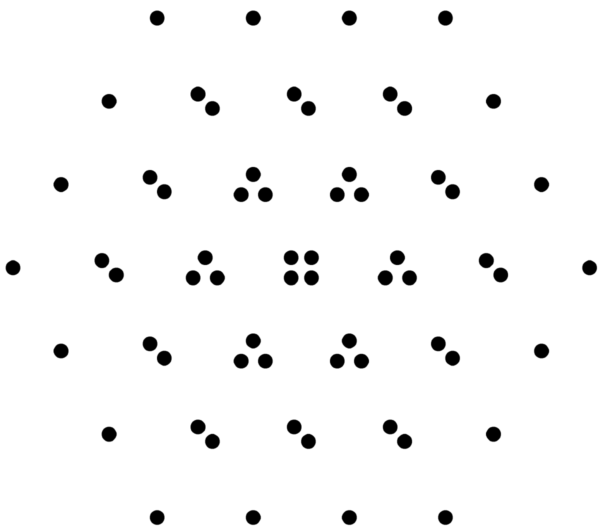
SU(3) classification

Polynomial		S_3	$SU(3)$					
$(\overline{m} - m_0)^3$		A_1	1					
$(\overline{m} - m_0)^2 \delta m_s$		E^+		8				
$(\overline{m} - m_0)^2 (\delta m_u - \delta m_d)$		E^-		8				
$(\overline{m} - m_0)(\delta m_u^2 + \delta m_d^2 + \delta m_s^2)$		A_1	1				27	
$(\overline{m} - m_0) \left[3\delta m_s^2 - (\delta m_u - \delta m_d)^2\right]$		E^+		8			27	
$(\overline{m} - m_0)\delta m_s (\delta m_d - \delta m_u)$		E^-		8			27	
$\delta m_u \delta m_d \delta m_s$	✓	A_1	1				27	64
$\delta m_s (\delta m_u^2 + \delta m_d^2 + \delta m_s^2)$	✓	E^+		8			27	64
$(\delta m_u - \delta m_d)(\delta m_u^2 + \delta m_d^2 + \delta m_s^2)$	✓	E^-		8			27	64
$(\delta m_s - \delta m_u)(\delta m_s - \delta m_d)(\delta m_u - \delta m_d)$	✓	A_2			10	$\overline{10}$		64

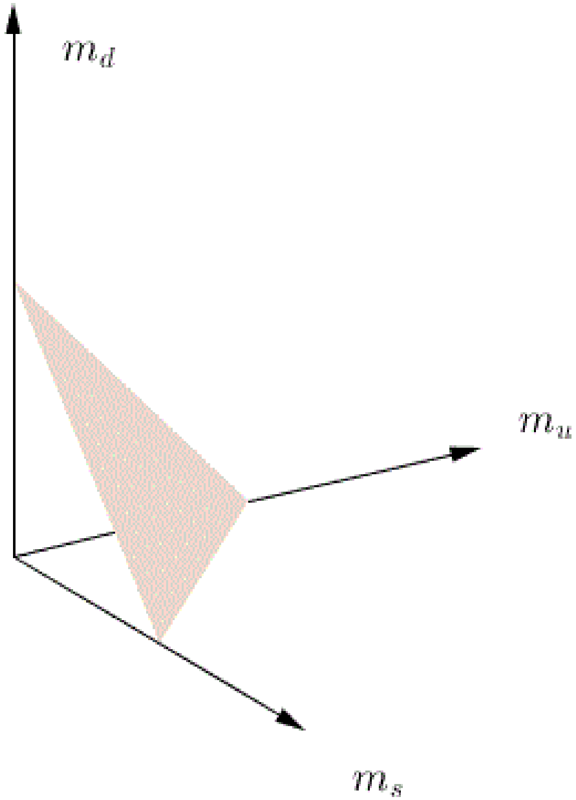
SU(3) classification





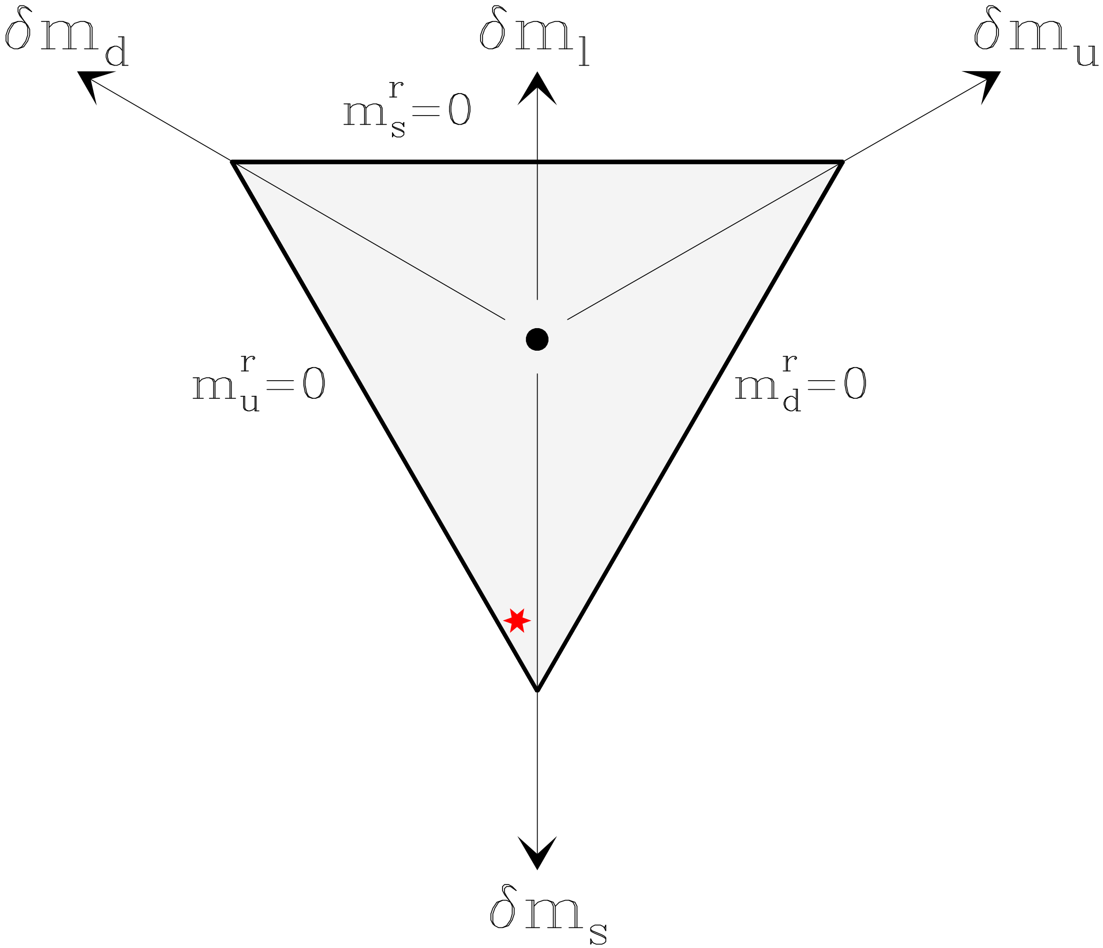


SU(3) classification

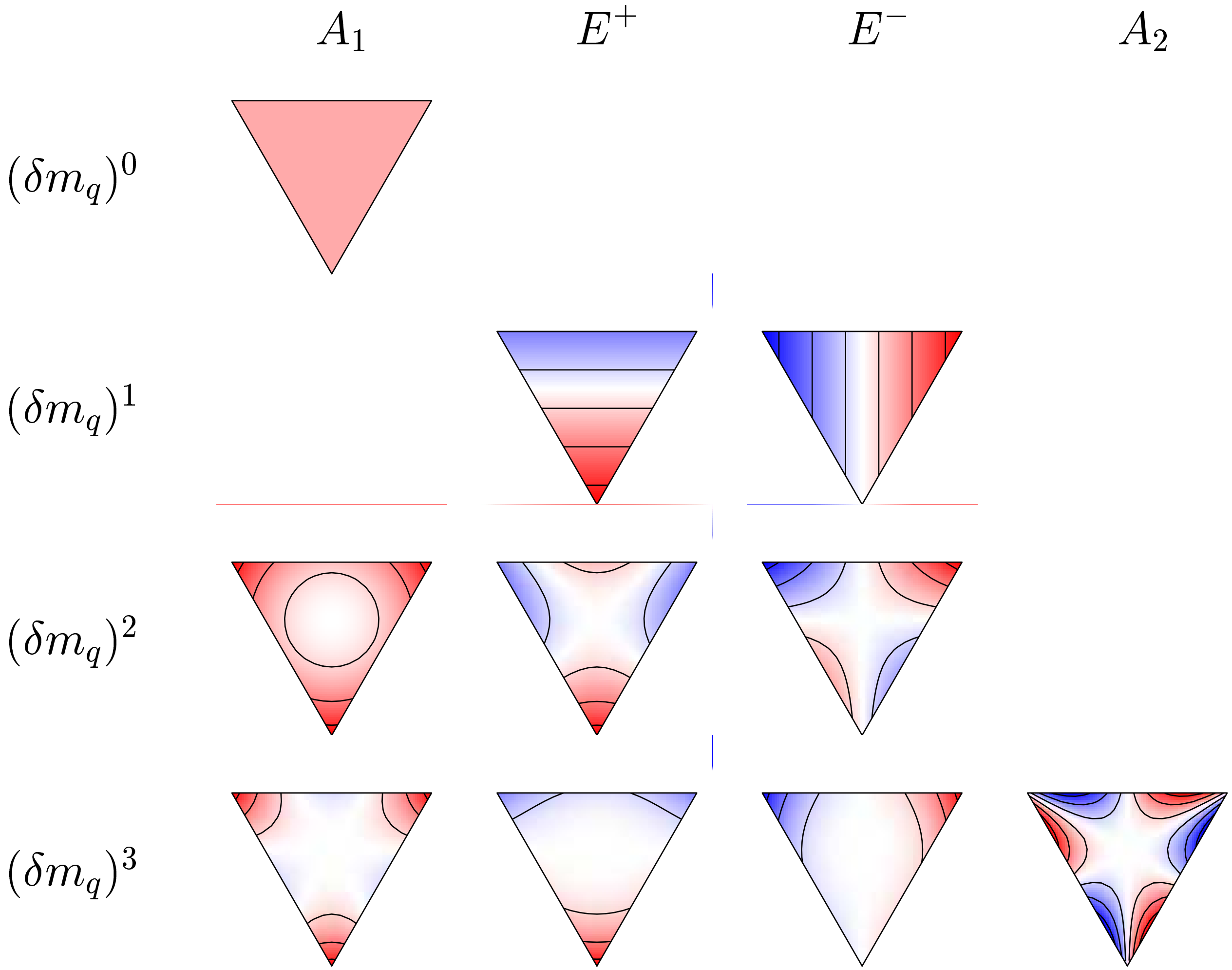


$$(m_u + m_d + m_s) = const, \qquad m_q \geq 0$$

SU(3) classification



SU(3) classification



$2 + 1$ **Simulation**

Tree-level Symanzik glue, $\beta = 5.50$

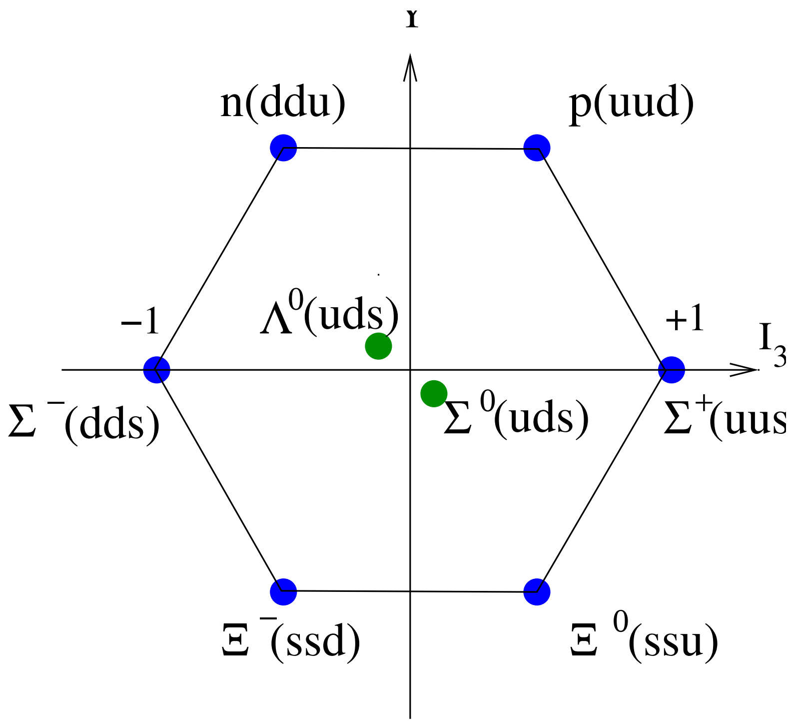
Clover Fermions, non-pert c_{SW}.

To to keep the action highly local, the hopping terms use a stout smeared link (‘fat link’) with $\alpha = 0.1$ ‘mild smearing’ for the Dirac kinetic term and Wilson mass term.

Symmetric point $\kappa_0 = 0.12090$

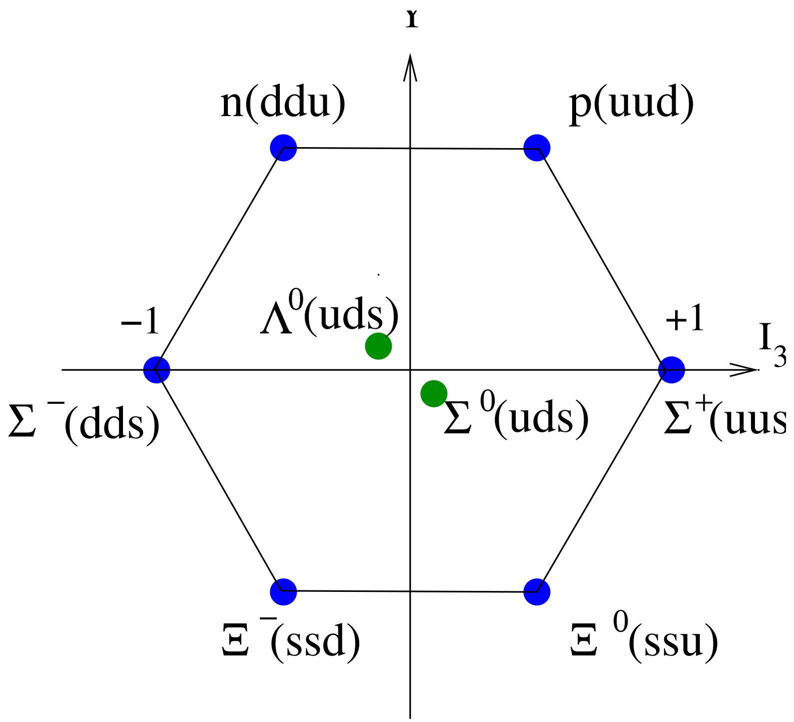
$24^3 \times 48$ lattices and $32^3 \times 64$ lattices

Octet Baryons



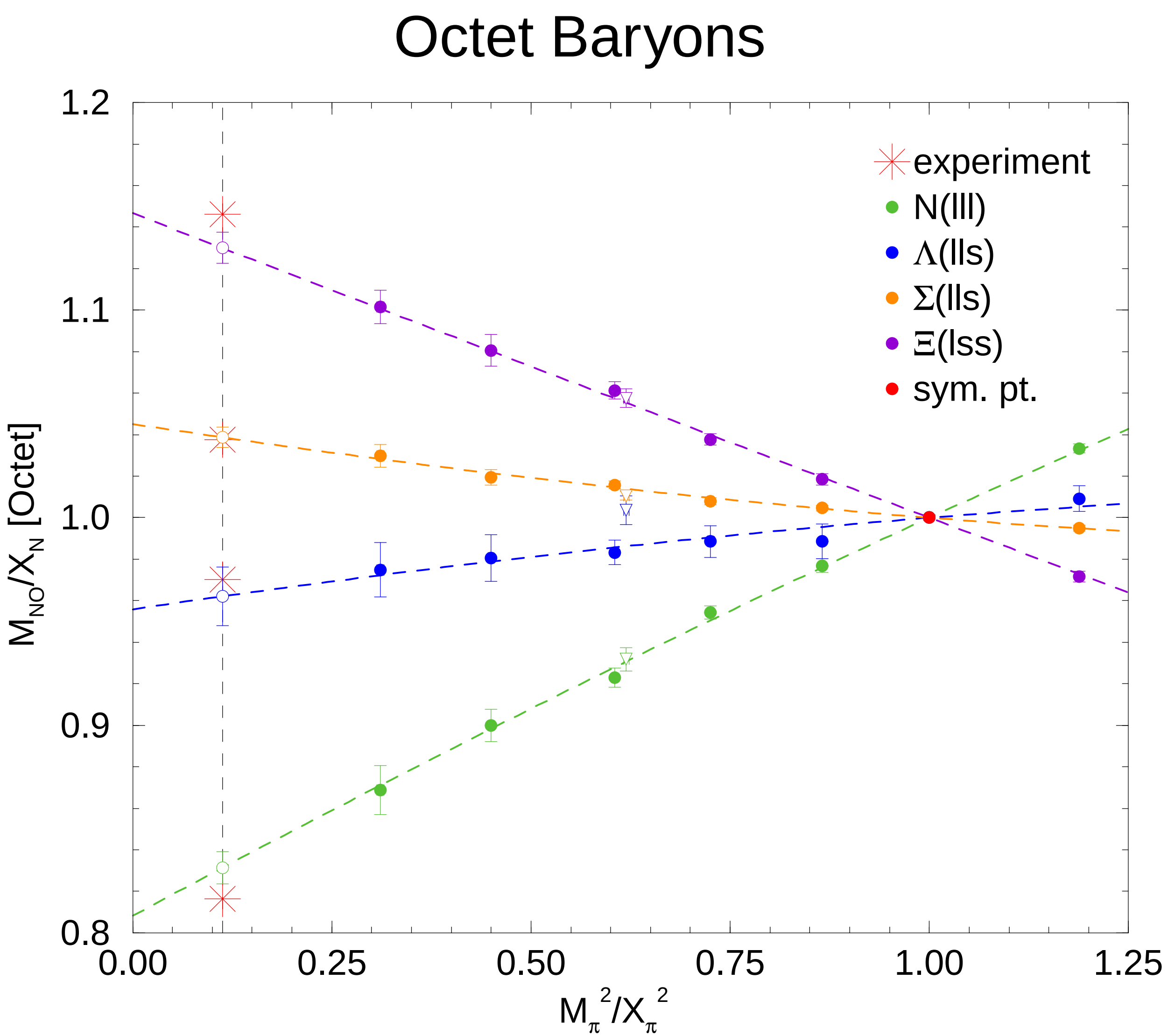
The spin $\frac{1}{2}$ baryons (partners of the proton and neutron) form an octet under SU(3).

Octet Baryons



The central baryons (Λ^0, Σ^0) have the same quark content, uds, but different wave functions, (in particular, different arrangements of quark spin).

Fan Plot


Octet Baryons
1.2
1.1
1.0
0.9
0.8
0.00
0.25
0.50
0.75
1.00
1.25
M_{NO}/X_N [Octet]
M_π^2/X_π^2
experiment
N(lll)
Λ(lls)
Σ(lls)
Ξ(lss)
sym. pt.

$m_u \neq m_d$

The fan plot shows that we predict the masses of the octet baryons well when $m_u = m_d$. What changes if $m_u \neq m_d$?

In the outer ring, degeneracies are split,

$$\begin{aligned} M_p &\neq M_n \\ M_{\Sigma^-} &\neq M_{\Sigma^+} \\ M_{\Xi^-} &\neq M_{\Xi^0} \end{aligned}$$

Investigated in our framework in Phys Rev D86 (2012) 114511

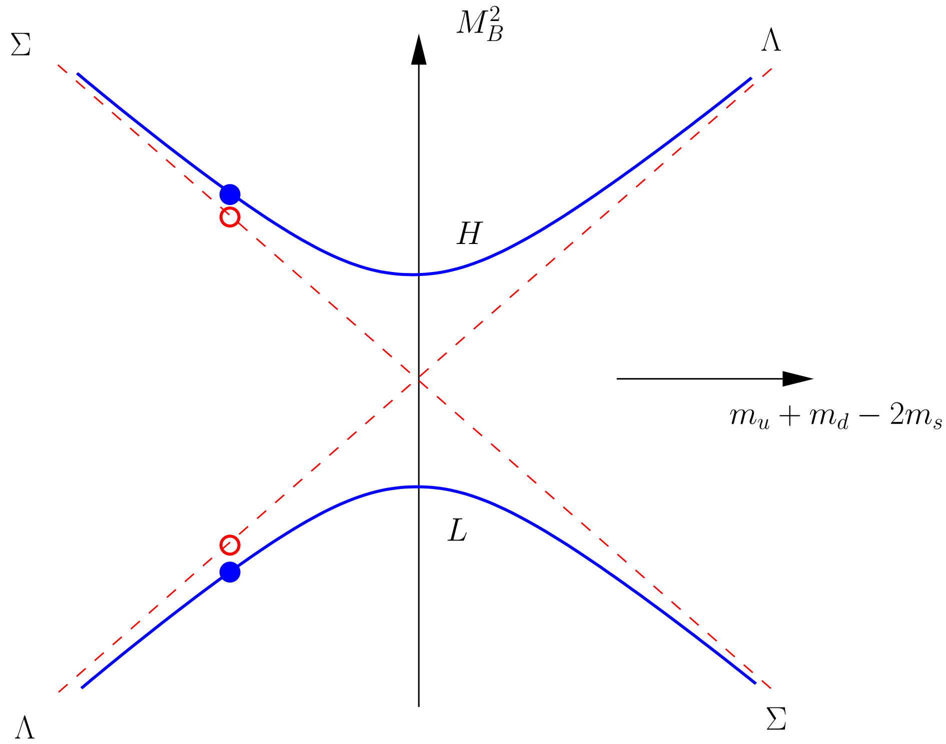
$m_u \neq m_d$

The fan plot shows that we predict the masses of the octet baryons well when $m_u = m_d$. What changes if $m_u \neq m_d$?

Inner states
Total Isospin is no longer a good quantum number, the Σ^0 and Λ^0 will mix.

Topic of the rest of this this talk.

$m_u \neq m_d$



Σ-Λ mixing

Two methods of calculating the masses and mixings in the Σ-Λ system.

Calculate masses in the $m_u = m_d$ case, and use group theory to predict the $m_u \neq m_d$ case. ("Rotate" the sensitivity to $m_s - m_l$ to find the sensitivity to $m_u - m_d$.)

Directly calculate masses with $m_u \neq m_d \neq m_s$. Lattice splitting $m_u - m_d$ is much larger than the physical, so the mixing and mass shifts are much larger than in the real world. We then interpolate down to find what the real-world result is.

We have used both types of data.

Σ-Λ mixing

Directly calculate masses with $m_u \neq m_d \neq m_s$. Lattice splitting $m_u - m_d$ is much larger than the physical, so the mixing and mass shifts are much larger than in the real world. We then interpolate down to find what the real-world result is.

In this mixed data the lattice correlators form a 2×2 matrix - eigenvectors correspond to M_H and M_L in the level-crossing sketch.

Σ-Λ mixing

Baryon mass matrix

$$M^2(\mathcal{M}) = \begin{pmatrix} M_n^2 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & M_p^2 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & M_{\Sigma^-}^2 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & M_{\Sigma\Sigma}^2 & M_{\Sigma\Lambda}^2 & 0 & 0 & 0 \\ 0 & 0 & 0 & M_{\Lambda\Sigma}^2 & M_{\Lambda\Lambda}^2 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & M_{\Sigma^+}^2 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & M_{\Xi^-}^2 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & M_{\Xi^0}^2 \end{pmatrix}$$

Mostly diagonal: One mixing block

Σ-Λ mixing

8×8 mass matrix, made of a basis of 10 matrices:

$$M^2 = \sum_{i=1}^{10} K_i(m_q) N_i$$

Σ-Λ mixing

n	p	Σ^-	Σ^0	Λ^0	Σ^+	Ξ^-	Ξ^0	S_3	SU(3)
1	1	1	1	1	1	1	1	A_1	1
-1	-1	0	0	0	0	1	1	E^+	8_a
-1	1	-2	0	0	2	-1	1	E^-	8_a
1	1	-2	-2	2	-2	1	1	E^+	8_b
-1	1	0	mix		0	1	-1	E^-	8_b
1	1	1	-3	-3	1	1	1	A_1	27
1	1	-2	3	-3	-2	1	1	E^+	27
-1	1	0	mix		0	1	-1	E^-	27
1	-1	-1	0	0	1	1	-1	A_2	10,$\overline{10}$
0	0	0	mix		0	0	0	A_2	10,$\overline{10}$

$$N_1 = \begin{pmatrix} 1&0&0&0&0&0&0&0\\ 0&1&0&0&0&0&0&0\\ 0&0&1&0&0&0&0&0\\ 0&0&0&1&0&0&0&0\\ 0&0&0&0&1&0&0&0\\ 0&0&0&0&0&1&0&0\\ 0&0&0&0&0&0&1&0\\ 0&0&0&0&0&0&0&1 \end{pmatrix} \quad N_2 = \begin{pmatrix} -1&0&0&0&0&0&0&0\\ 0&-1&0&0&0&0&0&0\\ 0&0&0&0&0&0&0&0\\ 0&0&0&0&0&0&0&0\\ 0&0&0&0&0&0&0&0\\ 0&0&0&0&0&0&0&0\\ 0&0&0&0&0&0&1&0\\ 0&0&0&0&0&0&0&1 \end{pmatrix}$$

$$N_3 = \begin{pmatrix} -1&0&0&0&0&0&0&0\\ 0&1&0&0&0&0&0&0\\ 0&0&-2&0&0&0&0&0\\ 0&0&0&0&0&0&0&0\\ 0&0&0&0&0&0&0&0\\ 0&0&0&0&0&2&0&0\\ 0&0&0&0&0&0&-1&0\\ 0&0&0&0&0&0&0&1 \end{pmatrix} \quad N_4 = \begin{pmatrix} 1&0&0&0&0&0&0&0\\ 0&1&0&0&0&0&0&0\\ 0&0&-2&0&0&0&0&0\\ 0&0&0&-2&0&0&0&0\\ 0&0&0&0&2&0&0&0\\ 0&0&0&0&0&-2&0&0\\ 0&0&0&0&0&0&1&0\\ 0&0&0&0&0&0&0&1 \end{pmatrix}$$

$$N_5 = \begin{pmatrix} -1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 2/\sqrt{3} & 0 & 0 & 0 \\ 0 & 0 & 0 & 2/\sqrt{3} & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & -1 \end{pmatrix}$$

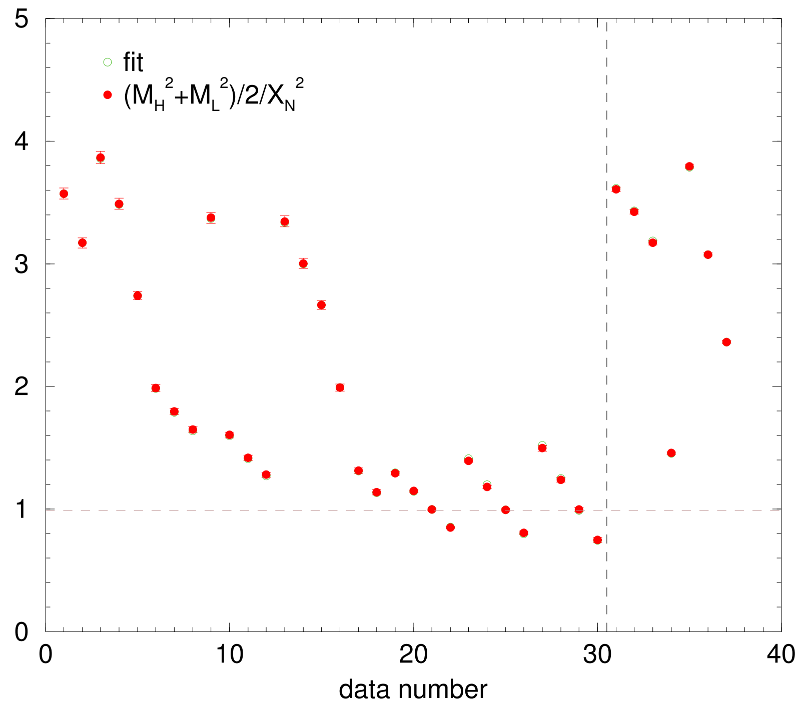
First mixing term:
Coefficient $\propto (m_u - m_d)$
Contributes both to n-p splitting and to Σ-Λ mixing

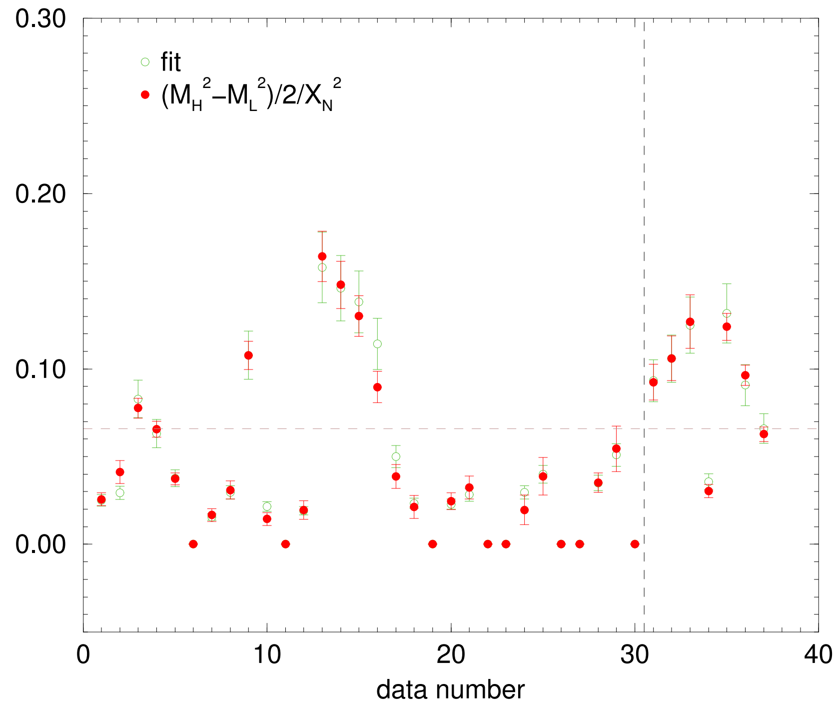
Results

We have a 2×2 mixing matrix.
From symmetry, we know the allowed polynomials in each entry.
(Diagonal terms even under $m_u \leftrightarrow m_d$, mixing terms odd under $m_u \leftrightarrow m_d$).
From lattice data, know the coefficient of each allowed term.
Can calculate splitting and mixing for any m_u, m_d, m_s.
Put in the physical mass values (fixed from π, K).

Data — Fit

Data: $M_H(aab)$, $M_L(aa'b)$, $M_H(abc)$, $M_L(abc) \leq 2.0$ GeV [wf $\Sigma(abc), \Lambda(abc)$]





Results

- Mixing angle [as anticipated very small $\theta \sim 1^\circ$]

$$\tan 2\theta = 0.0123(45)(25)$$

- Mass difference [mixing contribution to mass difference ~ 1 MeV]

$$M_{\Sigma^0} - M_{\Lambda^0} = 79.4(7.4)(3.4)\,\text{MeV}$$

$$[(M_{\Sigma^0} - M_{\Lambda^0})^{\text{exp}} = 76.959(23)\,\text{MeV}]$$

Measuring the Mixing Angle

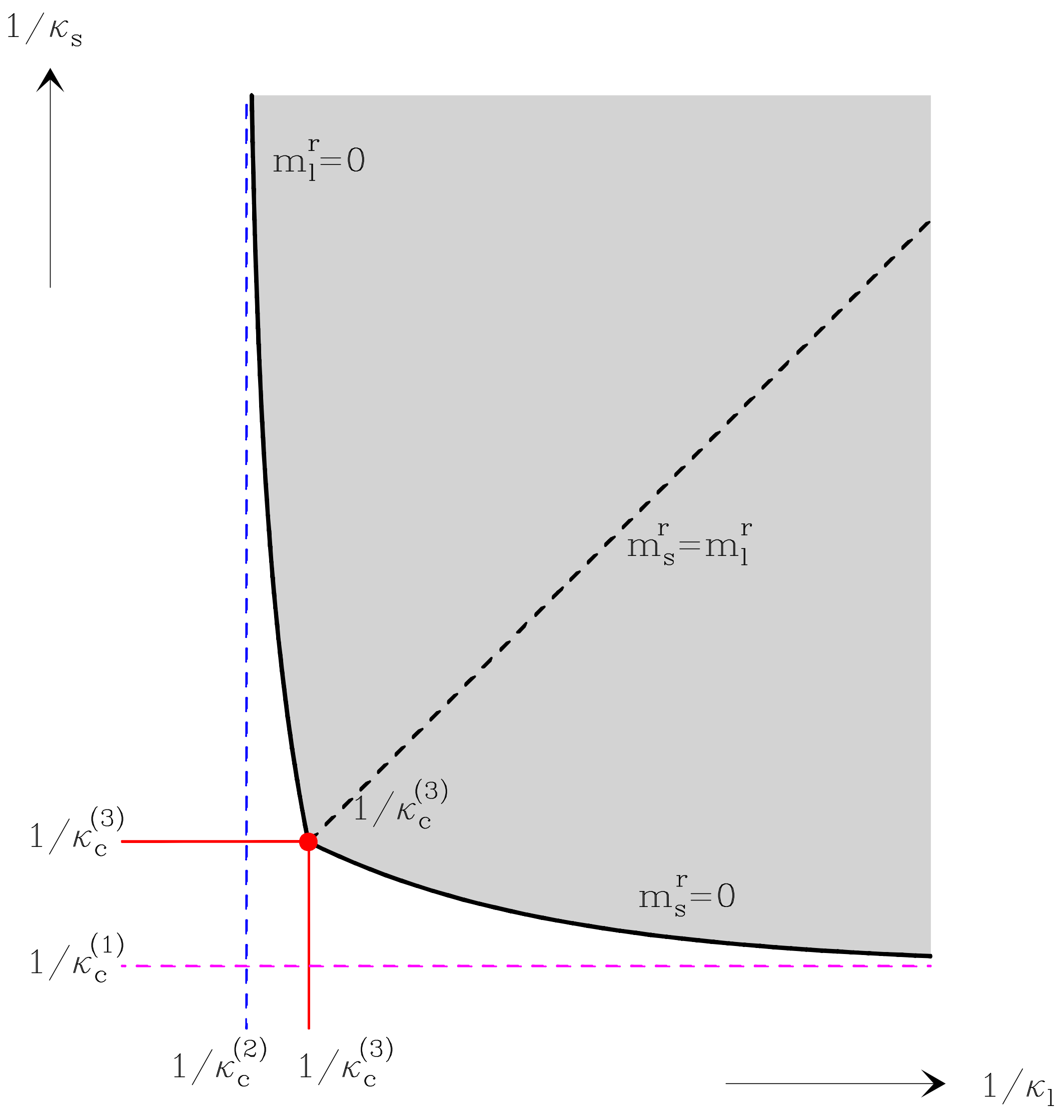
We know of no results, but there is an old proposal
G. Karl, Phys. Lett. B328 (1994) 149 [Erratum-ibid. B341 (1995) 449].
Need a quantity linear in θ (θ^2 too small).
Compare $\Sigma^+ \to \Lambda e^+ \nu_e$ and $\Sigma^- \to \Lambda e^- \bar{\nu}_e$.

Conclusions

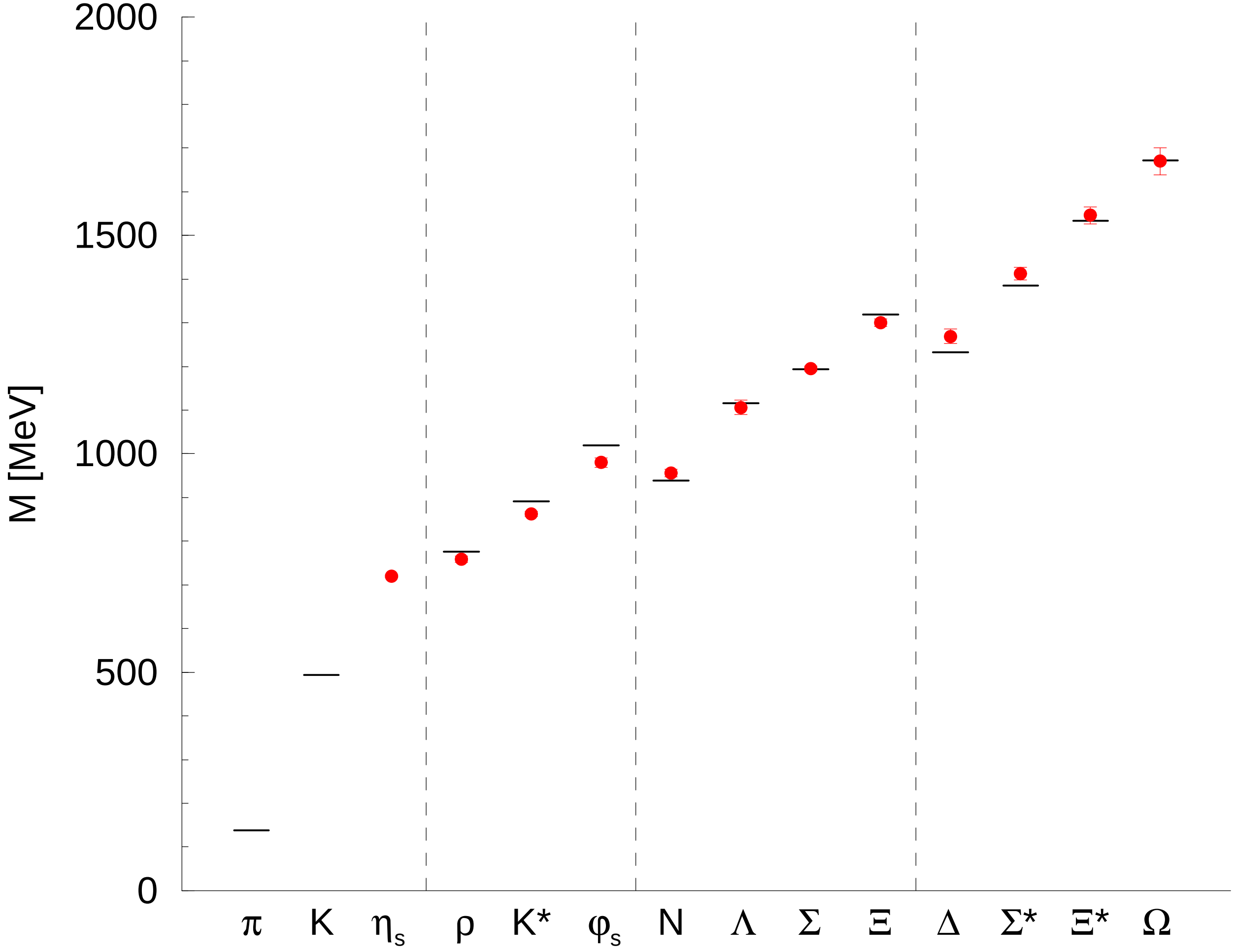
- Extrapolating from lattice simulations to the physical quark masses is made much easier by keeping $m_u + m_d + m_s$ constant.
- Flavour SU(3) analysis strongly constrains Taylor expansions in quark masses.
- Spectrum, splitting, well reproduced.
- So, presumeably mixing angle is reliable too.
- Caveat: QED corrections

Extra

Allowed Region



Hadron Spectrum



Extra

Op	A_1	E: E^+	E: E^-	A_2
Identity	$+$	$+$	$+$	$+$
$u \leftrightarrow d$	$+$	$+$	$-$	$-$
$u \leftrightarrow s$	$+$	mix		$-$
$d \leftrightarrow s$	$+$	mix		$-$
$u \rightarrow d \rightarrow s \rightarrow u$	$+$	mix		$+$
$u \rightarrow s \rightarrow d \rightarrow u$	$+$	mix		$+$