Phenomenology of top-quark pair production at the LHC: studies with DiffTop

Marco Guzzi

in collaboration with
Katerina Lipka and Sven-Olaf Moch

The University of Manchester

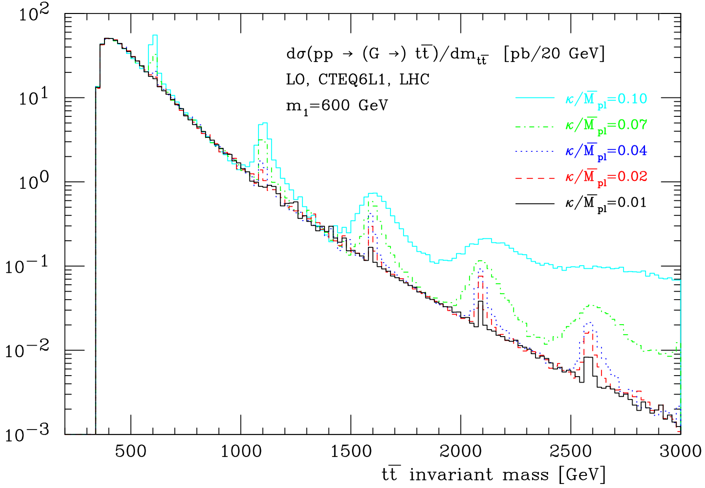
"High Energy Theory Divisional Seminars", Liverpool, March 2015

Outline and motivations

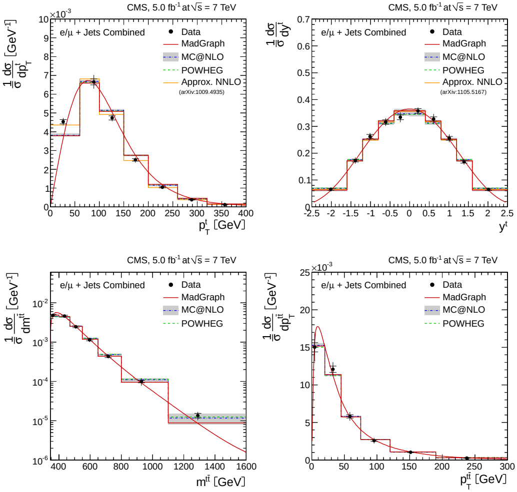
- Top-quark pair production at the LHC is crucial for many phenomenological applications/investigations:
  -Physics beyond the SM ($\Rightarrow$ distortions/bumps in distributions like $M_{t\bar{t}}$),
  -extent of QCD factorization,
  -PDFs determination in global QCD analyses:
  Clean constraints on the gluon at large x,
  Correlation between α_s, top-quark mass m_t, and the gluon.

- New data available: the CMS and ATLAS collaborations published measurements of differential cross sections for $t\bar{t}$ pair production ($\sqrt{S} = 7$ and 8 TeV) as a function of different observables of interest with unprecedented accuracy:

Physics Beyond the SM: is there any?

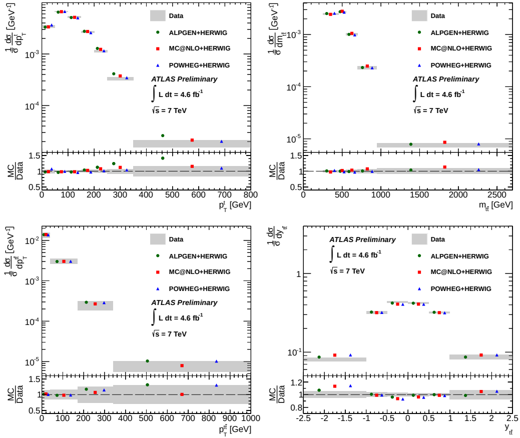


from F. Maltoni and R. Frederix, JHEP 0901 (2009) 047



The CMS Collaboration EPJC 2013, $\int Ldt = 5.0[\text{fb}]^{-1}$, $\sqrt{S} = 7$ TeV,

TOP-12-028 $\rightarrow \int Ldt \approx 12[\text{fb}]^{-1}$, $\sqrt{S} = 8$ TeV



The ATLAS Collaboration ATLAS-CONF-2013-099, lepton+jets, $\int Ldt = 4.6[\text{fb}]^{-1}$, $\sqrt{S} = 7$ TeV

Main focus: exploit the full potential of these new (and forthcoming) data in QCD analyses of PDFs.

- We need tools incorporating the current state-of-the-art of QCD calculations:
  some of them are already on the market,
  for some others work is still in progress.
- Global QCD analyses of the current measurements set specific requirements to the representation of the experimental data and availability of fast computing tools.
- We tried to address these requirements in the context of differential $t\bar{t}$ production cross sections by using approximate calculations.
- **DiffTop** calculates $t\bar{t}$ differential cross sections in 1PI kinematics at approximate NLO ($\mathcal{O}(\alpha_s^3)$), and NNLO $\mathcal{O}(\alpha_s^4)$.
- Exploratory work for future PDF fits using the exact NNLO theory when available and usable.

Available calculations

NLO exact computations available since many years:

- Nason, Dawson, Ellis (1988); Beenakker, Kuijif, Van Neerven, Smith (1989); Meng, Schuler, Smith, Van Neerven (1990); Beenakker, Van Neerven, Schuler, Smith (1991); Mangano, Nason, Rodolfi (1992).

The NNLO $O(\alpha_s^4)$ full QCD calculation for the $t\bar{t}$ total cross section has been accomplished:

- Czakon, Fiedler, Mitov (2013); Czakon, Mitov (2012), (2013); Baernreuther, Czakon, Mitov (2012)
- TOP++ Czakon, Mitov (2011); HATHOR Aliev, Lacker, Langenfeld, Moch, Uwer, Wiedermann (2011)

Exact NLO tools available

- MNR,HVQMNR Mangano, Nason, Ridolfi; MCFM Campbell, Ellis, Williams; MADGRAPH5 Alwall, Maltoni, et al.; MC@NLO Frixione, Stoeckli, Torrielli, Webber, White; POWHEG Alioli, Hamilton, Nason, Oleari, Re.

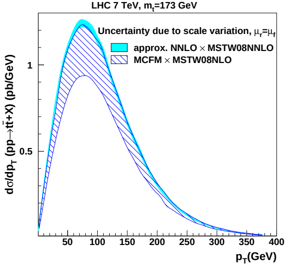
Exact NLO calculations for $t\bar{t}$ total and differential cross sections have been implemented into publicly available Monte Carlo numerical codes.

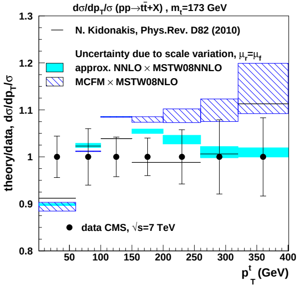
Full NNLO calculation for the $t\bar{t}$ production cross section at differential level is on the way.

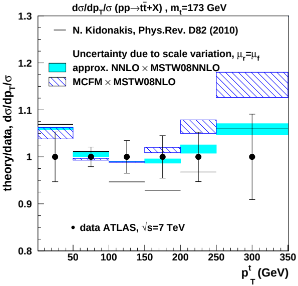
NLO predictions are not accurate enough to describe the data:

- perturbative corrections are large,
- systematic uncertainties associated to various scales entering the calculation are important.

In the meanwhile, one can use approximate calculations based on threshold expansions in QCD to make esploratory studies at phenomenological level


LHC 7 TeV, m_t=173 GeV
Uncertainty due to scale variation, $\mu_r=\mu_f$
approx. NNLO × MSTW08NNLO
MCFM × MSTW08NLO
$d\sigma/dp_T$ $(pp\rightarrow t\bar{t}+X)$ (pb/GeV)
p_T(GeV)


$d\sigma/dp_T/\sigma$ $(pp\rightarrow t\bar{t}+X)$, m_t=173 GeV
N. Kidonakis, Phys.Rev. D82 (2010)
Uncertainty due to scale variation, $\mu_r=\mu_f$
approx. NNLO × MSTW08NNLO
MCFM × MSTW08NLO
data CMS, √s=7 TeV
theory/data, $d\sigma/dp_T/\sigma$
p^t_T (GeV)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MCFM × MSTW08NLO
data ATLAS, √s=7 TeV
theory/data, $d\sigma/dp_T/\sigma$
p^t_T (GeV)

Development of tools for phenomenology

DiffTop: Fortran based computer code for calculating differential and total cross section for heavy-flavor production at hadron colliders at approximate NLO and NNLO by using threshold expansions in QCD.

Implementation based on the calculation by N.Kidonakis, S.-O.Moch, E.Laenen, R.Vogt (2001) - (Mellin-space resummation).

DiffTop stand alone (1PI kinematics branch) is now available at:
http://difftop.hepforge.org/
arXiv:1406.0386[hep-ph] published on JHEP (2014)

The FastNLO-DiffTop code to produce grids will be available soon. (few grids are already available)

What’s in the Box ?

Remnants of long-distance dynamics in a hard scattering function can be large in regions of phase space near partonic threshold and dominate higher order corrections: $\rightarrow$ logarithmic corrections

Threshold resummation organizes double-logarithmic corrections to all orders, thereby extending the predictive power of QCD to these phase space regions. G. Sterman (1987); S. Catani and L. Trentadue (1989); H. Contopanagos, E. Laenen, and G. Sterman (1997)

The kinematics of inclusive heavy quark hadroproduction depend on which final state momenta are reconstructed.
In threshold resummation, a kinematic choice manifests itself at next-to-leading-logarithmic level.

Single-particle inclusive (1PI) kinematics

In our calculation, heavy-quark hadroproduction near the threshold is approximated by considering the partonic subprocesses

$$q(k_1) + \bar{q}(k_2) \to t(p_1) + X[\bar{t}](p_2'),$$
$$g(k_1) + g(k_2) \to t(p_1) + X[\bar{t}](p_2') \quad p_2' = \bar{p}_2 + k, \quad (1)$$

where is k any additional radiation, and $s_4 = p_2' - m^2 \to 0$ momentum at the threshold.

This kinematic is used to determine the p_T^t and rapidity y^t distribution of the final-state top-quark.

Hard scattering functions are expanded in terms of

$$\left[\frac{\ln^l(s_4/m_t^2)}{s_4}\right]_+ = \lim_{\Delta\to 0}\left\{\frac{\ln^l(s_4/m_t^2)}{s_4}\theta(s_4-\Delta) + \frac{1}{l+1}\ln^{l+1}\left(\frac{\Delta}{m_t^2}\right)\delta(s_4)\right\},$$

where corrections are denoted as leading-logarithmic (LL) if $l = 2i+1$ at $\mathcal{O}(\alpha_s^{i+3})$ with $i = 0, 1, \ldots$, as next-to-leading logarithm (NLL) if $l = 2i$, as next-to-next-to-leading logarithm (NNLL) if $l = 2i-1$, and so on.

The hard scattering expansion

The factorized differential cross section is written as

$$S^2 \frac{d^2\sigma(S, T_1, U_1)}{dT_1 \; dU_1} = \sum_{i,j=q,\bar{q},g} \int_{x_1^-}^1 \frac{dx_1}{x_1} \int_{x_2^-}^1 \frac{dx_2}{x_2} f_{i/H_1}(x_1, \mu_F^2) f_{j/H_2}(x_2, \mu_F^2)$$
$$\times \omega_{ij}(s, t_1, u_1, m_t^2, \mu_F^2, \alpha_s(\mu_R^2)) + \mathcal{O}(\Lambda^2/m_t^2)\,,$$

$$\omega_{ij}(s_4, s, t_1, u_1) = \omega_{ij}^{(0)} + \tfrac{\alpha_s}{\pi}\omega_{ij}^{(1)} + \left(\tfrac{\alpha_s}{\pi}\right)^2 \omega_{ij}^{(2)} + \cdots$$

where $\omega_{ij}^{(2)}$ at parton level in 1PI kinematics is given by

$$\omega_{ij}^{(2)} = s^2 \frac{\hat{\sigma}_{ij}^{(2)}}{du_1 dt_1}\bigg|_{1PI} = F_{ij}^{Born} \frac{\alpha_s^2(\mu_R^2)}{\pi^2} \left\{ D_{ij}^{(3)} \left[\frac{ln^3(s_4/m_t^2)}{s_4}\right]_+ \right.$$
$$\left. + D_{ij}^{(2)} \left[\frac{ln^2(s_4/m_t^2)}{s_4}\right]_+ + D_{ij}^{(1)} \left[\frac{ln(s_4/m_t^2)}{s_4}\right]_+ + D_{ij}^{(0)} \left[\frac{1}{s_4}\right]_+ + R_{ij}^{(2)}\delta(s_4) \right\}.$$

Few more details...

- Hard and soft functions: $H_{ij} = H_{ij}^{(0)} + (\alpha_s/\pi) H_{ij}^{(1)} + \cdots$ and $S_{ij} = S_{ij}^{(0)} + (\alpha_s/\pi) S_{ij}^{(1)} + \cdots$,
  $H_{ij}^{(2)}$ and $S_{ij}^{(2)}$ are set to zero.
- Soft anomalous dimension matrices:
  $\Gamma_S = (\alpha_s/\pi)\Gamma_S^{(1)} + (\alpha_s/\pi)^2 \Gamma_S^{(2)} + \cdots$
  In our calculation, $\Gamma_S^{(2)}$ at two-loop for the massive case is included. Becher (2009), Kidonakis (2009).
- Anomalous dimensions of the quantum fields $i = q, g$:
  $\gamma_i = (\alpha_s/\pi)\gamma_i^{(1)} + (\alpha_s/\pi)^2 \gamma_i^{(2)} + \cdots$
- The Coulomb interactions, due to gluon exchange between the final-state heavy quarks, are included at 1-loop level.
- we work with the pole mass definition of the heavy quark.

Matching

The matching conditions are determined by comparing the expansion in the Mellin moment space to the exact results for the partonic cross section.

Matching terms at NLO

$$Tr\{H^{(1)}S^{(0)} + H^{(0)}S^{(1)}\} \tag{2}$$

are included. Beenakker, Kuijf, Van Neerven, Smith (1989), Beenakker, Van Neerven, Meng, Schuler, Smith (1991), Mangano, Nason, Ridolfi (1992).

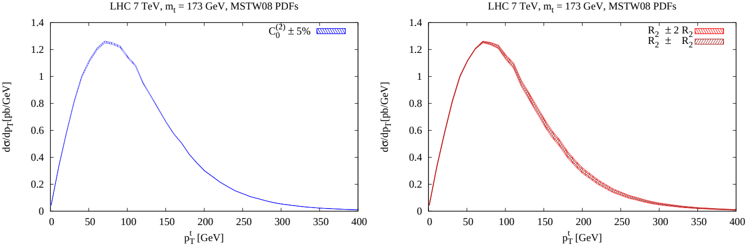
Matching terms at NNLO

$$Tr\{H^{(1)}S^{(1)}\}, Tr\{H^{(0)}S^{(2)}\}, Tr\{H^{(2)}S^{(0)}\} \tag{3}$$

are set to zero.

Systematic uncertainties due to missing terms

The uncertainties due to missing contributions in $D_{ij}^{(0)}$ and R_2 are part of the systematic uncertainty associated to approximate calculations of this kind which are based on threshold expansions.



Left: The coefficient $C_0^{(2)}$ (scale ind. contribution in $D_{ij}^{(0)}$) is varied within its 5% while R_2 is kept fixed. Right: here the coefficient R_2 is varied by adding and subtracting $2R_2$ while $C_0^{(2)}$ is kept fixed.

QCD Threshold expansions: "pros and cons"

Approximate calculations based on threshold expansions are not perfect, but can be (easily) highly improved once the full NNLO calculation will be available.

- ☺ provide a local effective description of the p_T and y distributions that captures the main features of the full calculation.
- ☺ relatively easy interface to FastNLO or ApplGrid.
- ☺ provide a fast tool for taking into account correlations (α_s, m_t, gluon); easy to implement different heavy-quark mass definitions. Dowling, Moch (2014)
- ☹ Very sensitive to the missing contribution in $D^{(0)}$ and R_2.
- ☹ Scale uncertainty is also affected (at approx NNLO is underestimated at the moment. We'll improve on this)
- ☹ At the moment the description is valid near the threshold.

Phenomenology: Exploratory studies at the LHC 7 TeV

What is it good for?

Top-quark pair production at LHC probes high-x gluon and the differential cross section is strongly correlated at $x \approx 0.1$:

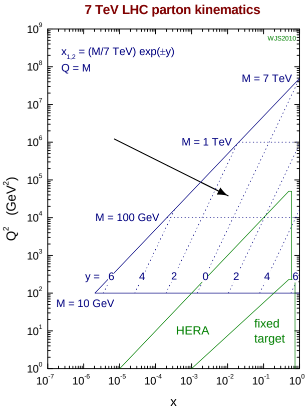
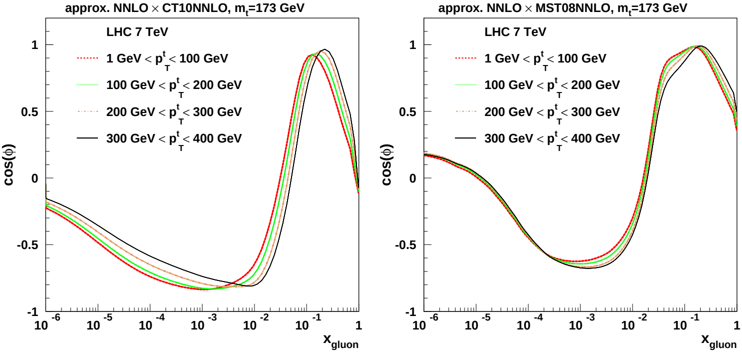


Figure by J. Stirling



Here we choose MSTW08 and CT10 as representative. ABM11, HERA1.5 and NNPDF2.3 show a similar behavior.

What is it good for?

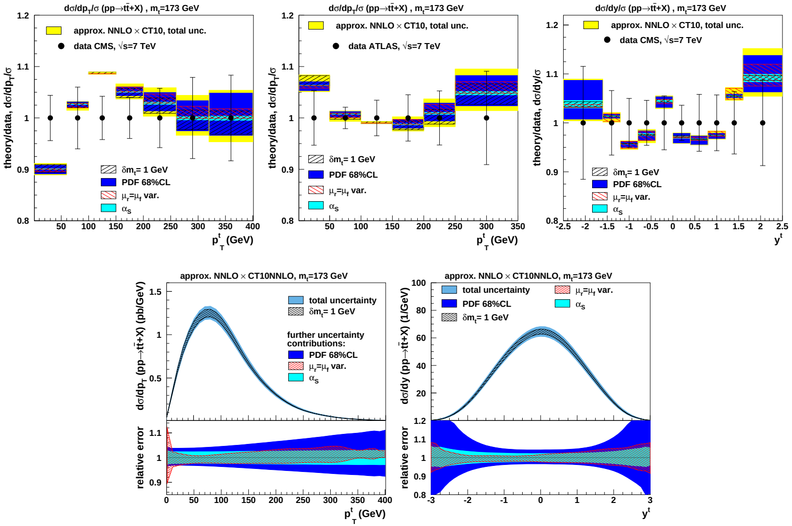
Top-quark pair production at LHC probes high-x gluon ($x \approx 0.1$): but there is a strong correlation between $g(x)$, α_s and the top-quark mass m_t that we want to pin down

- Precise measurements of the total and differential cross section of $t\bar{t}$ pair production provide us with a double handle on these quantities
- Precise measurements of the absolute differential cross section also provides us with important information to constrain PDFs (gluon)
- The shape of the differential cross section is modified by m_t and α_s (very sensitive)
- extraction of m_t will benefit from the interplay between these two measurements. (recent CMS paper PLB (2014))

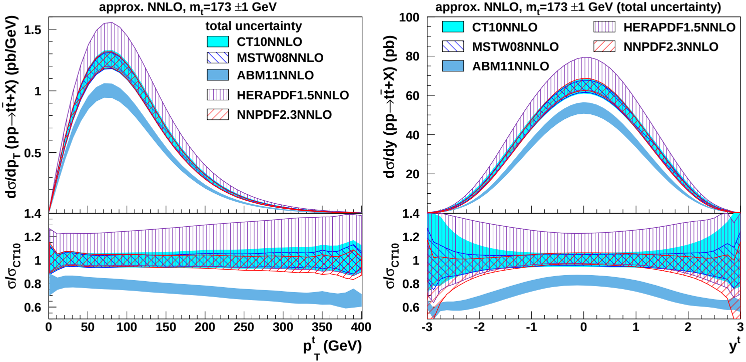
DiffTop Results

In what follows:

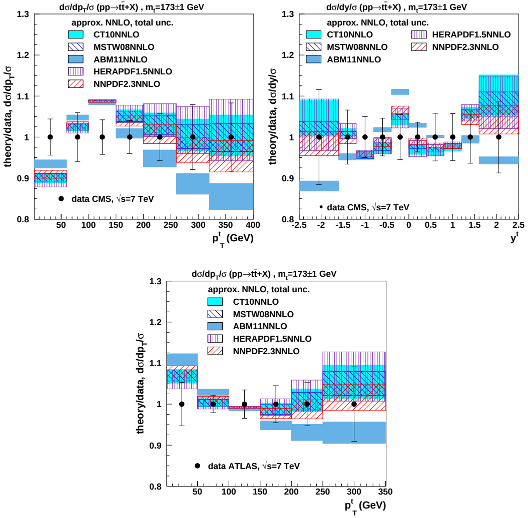
- PDF unc. are computed by following the prescription given by each PDF group at 68% CL ;
- The uncertainty associated to $\alpha_s(M_Z)$ is given by the central value as given by each PDF group $\pm\Delta\alpha_s(M_Z) = 0.001$;
- Scale unc. is obtained by variations $m_t/2 \leq \mu_R = \mu_F \leq 2m_t$;
- Uncertanty associated to the top-quark mass is estimated by using $m_t = 173$ GeV (Pole mass) $\pm\Delta m_t = 1$ GeV.



Uncertainties for the top p_T^t and y^t distribution obtained by using DiffTop with CT10 NNLO PDFs. PDF and $\alpha_s(M_Z)$ errors are evaluated at the 68% CL.



PDF uncertainties $\sqrt{S} = 7$ TeV p_T^t and y^t distributions: comparison between all PDF sets (bands are total unc.).



PDF uncertainties $\sqrt{S} = 7$ TeV p_T^t and y^t distributions: ratio to the LHC measurements (bands are total unc.).

Interface to *fast*NLO (In collaboration with D. Britzger)

DiffTop has been succesfully interfaced to fastNLO.

This is important for applications in PDF fits, because NNLO computations are generally CPU time consuming.

$$c_{i,n}(\mu_R,\mu_F) = c^0_{i,n} + \log(\mu_R)c^R_{i,n} + \log(\mu_F)c^F_{i,n} + \ldots$$

beyond the NLO one has double log contributions

$$.. + \log^2(\mu_F)c^{(2,F)}_{i,n} + \log^2(\mu_R)c^{(2,R)}_{i,n} + \log(\mu_F)\log(\mu_R)c^{(2,R\ F)}_{i,n}$$

♠**DiffTop is now included into HERAFitter for PDF analyses**

Work is in progress on fastNLO grids generation to make all publicly available soon.

HERAFitter

www.herafitter.org

QCD analysis using $t\bar{t}$ production measurements

Impact of the current measurements on PDF determination: Inclusion of differential $t\bar{t}$ production cross sections into NNLO QCD fits of PDFs.

- we interfaced DiffTop to the HERAFitter platform,
- HERAFitter uses QCDNUM for NNLO DGLAP evol.,
- W asymmetry at NNLO: MCFM ApplGrid + K-factors.

Data sets included in the analysis

- HERA I inclusive DIS,
- CMS electron and muon charge asymmetry in W-boson production at $\sqrt{S} = 7$ TeV,
- ATLAS and CMS total inclusive Xsec at $\sqrt{S}$ =7 and 8 TeV,
- CDF total inclusive Xsec, Tevatron Run-II,
- ATLAS and CMS normalized differential cross-sections at $\sqrt{S} = 7$ TeV as a function of p_T^t.

Particulars of the fit

The PDF determination follows the approach used in the QCD fits of the HERA and CMS coll.

- GM VFNS used is TR' at NNLO with $m_c = 1.4$ GeV and $m_b = 4.75$ GeV as input,
- $\alpha_s(m_Z) = 0.1176$; the Q^2 range of the HERA data restricted to $Q^2 \geq Q^2_{\min} = 3.5$ GeV^2.

At the scale $Q_0^2 = 1.9$ GeV^2, the parton distributions are represented by

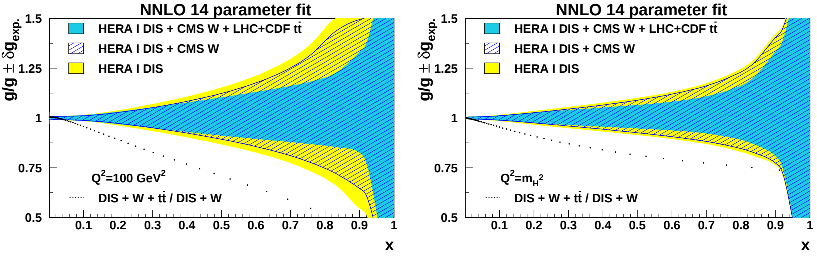
$$\begin{aligned}
xu_{\rm v}(x) &= A_{u_{\rm v}}\, x^{B_{u_{\rm v}}}\, (1-x)^{C_{u_{\rm v}}}\, (1 + D_{u_{\rm v}} x + E_{u_{\rm v}} x^2), \\
xd_{\rm v}(x) &= A_{d_{\rm v}}\, x^{B_{d_{\rm v}}}\, (1-x)^{C_{d_{\rm v}}}, \\
x\overline{\rm U}(x) &= A_{\overline{\rm U}}\, x^{B_{\overline{\rm U}}}\, (1-x)^{C_{\overline{\rm U}}}, \\
x\overline{\rm D}(x) &= A_{\overline{\rm D}}\, x^{B_{\overline{\rm D}}}\, (1-x)^{C_{\overline{\rm D}}}, \\
xg(x) &= A_g\, x^{B_g}\, (1-x)^{C_g} + A'_g\, x^{B'_g}\, (1-x)^{C'_g}.
\end{aligned} \tag{4}$$

where $x\overline{\rm U}(x) = x\bar{u}(x)$ and $x\overline{\rm D}(x) = x\bar{d}(x) + x\bar{s}(x)$.

Results of the fit: experimental uncertainty

The analysis is performed by fitting 14 free parameters.

A moderate impact on the large-x gluon exp. unc. is observed.



Uncertainties of the gluon distribution as a function of x, as obtained in our NNLO fit by using: inclusive DIS measurements only (light shaded band), DIS and W lepton charge asymmetry data (hatched band), and DIS, lepton charge asymmetry and the $t\bar{t}$ production measurements (dark shaded band), shown at the scales of $Q^2 = 100$ GeV^2 (left) and $Q^2 = m_H^2$ (right). The ratio of $g(x)$ obtained in the fit including $t\bar{t}$ data to that

Results of the fit: experimental uncertainty

Data set χ^2 / dof	
NC cross section HERA-I H1-ZEUS combined e^- p	109 / 145
NC cross section HERA-I H1-ZEUS combined e^+ p	461 / 379
CC cross section HERA-I H1-ZEUS combined e^- p	19 / 34
CC cross section HERA-I H1-ZEUS combined e^+ p	30 / 34
CMS W charge ele Asymmetry	8.1 / 11
CMS W charge muon Asymmetry	18 / 11
CDF inclusive ttbar cross section	0.64 / 1
CMS Norm. differential $t\bar{t}$ vs p_T 7 TeV	11 / 7
CMS total $t\bar{t}$ 8TeV m_t=173.3 GeV	2.0 / 1
CMS total $t\bar{t}$ 7TeV m_t=173.3 GeV	1.5 / 1
ATLAS Norm. diff. $t\bar{t}$ vs p_T 7 TeV	3.6 / 6
ATLAS total $t\bar{t}$ 7TeV m_t=173.3 GeV	0.11 / 1
ATLAS total $t\bar{t}$ 8TeV m_t=173.3 GeV	0.080 / 1
Total χ^2 / dof	664 / 618

A few considerations on the results

- HERA + CMS W lepton charge asy. vs HERA only ⇒ impact on light quarks central val. and reduction of the uncertainties
- Slight reduction of the gluon unc. in HERA + CMS W lepton asy. with respect to HERA only ⇒
  ascribed to the improved constraints on the light-quark distributions through the sum rules.
- Inclusion of $t\bar{t}$ measurements in the NNLO PDF fit ⇒ change in the shape of the gluon distribution (softens), moderate improvement of its uncertainty at large x.
- A similar effect is observed, although less pronounced, when only the total or only the differential $t\bar{t}$ cross section measurements are included in the fit.
- Correlations between $t\bar{t}$ measurements are not included here.
- **Correlations with m_t and α_s must be included in the fit.**

Conclusions

- We have shown phenomenological studies in which differential $t\bar{t}$ measurements are used in exploratory determination of the impact of such measurements on the PDFs of the proton.
- Theoretical predictions at approximate NNLO are provided by fastNLO-DiffTop.
- given the current accuracy of the data, the improvements on the gluon are still moderate (Correlations with m_t and α_s not included).
- More data is needed: absolute differential cross section data will bring more information.
- It will be interesting to see how this scenario will evolve once the full NNLO calculation will be available.
- Looking forward to see all this machinery at work in more extensive global PDF fits.

BACKUP

Behavior around the threshold

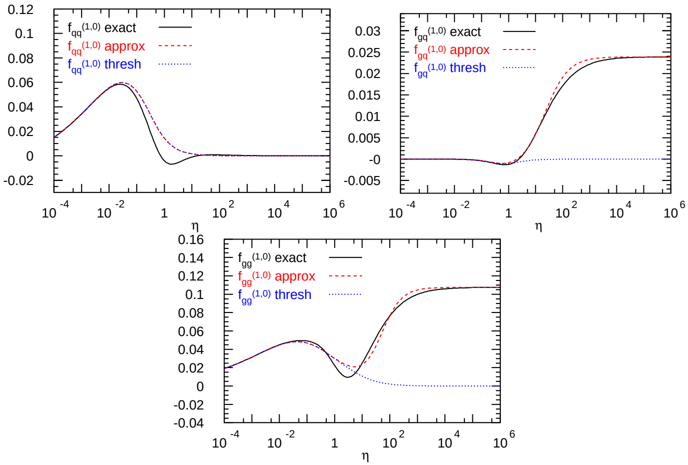
By setting $\mu_R = \mu_F = \mu$ one can write the inclusive total partonic cross section in terms of scaling functions $f_{ij}^{(k,l)}$ that are dimensionless and depend only on the variable $\eta = s/(4m_t^2) - 1$

$$\sigma_{ij}(s, m_t^2, \mu^2) = \frac{\alpha_s^2(\mu)}{m_t^2} \sum_{k=0}^{\infty} (4\pi\alpha_s(\mu))^k \sum_{l=0}^{k} f_{ij}^{(k,l)}(\eta) \ln^l \left(\frac{\mu^2}{m_t^2}\right) \quad (5)$$

$\eta = s/(4m_t^2) - 1$ **distance from the threshold**.

Recent analysis by Moch, Vogt, and Uwer PLB (2012): known threshold corrections and improved approximate NNLO results are given over the full kinematic range.

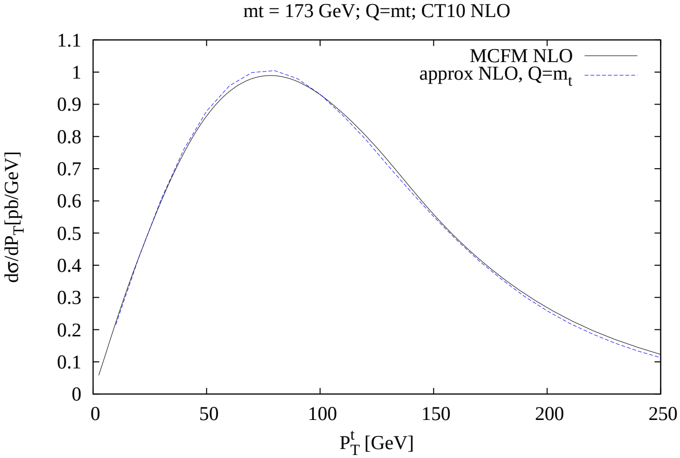
Behavior around the threshold: NLO



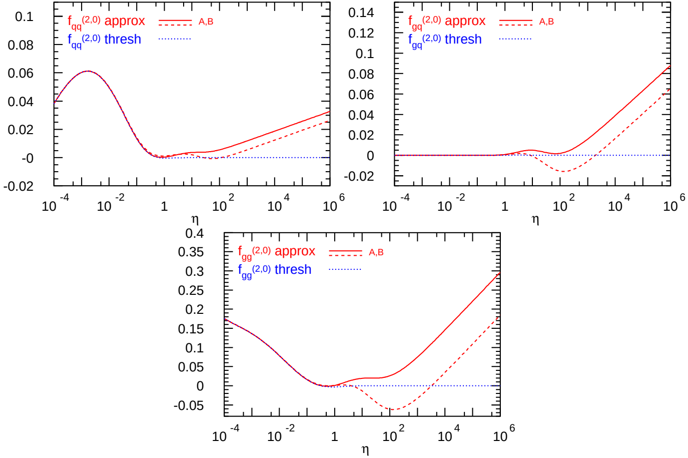
From Moch, Vogt, Uwer PLB (2012)

Quality check:

NLO Exact Calculation vs DiffTop approx NLO

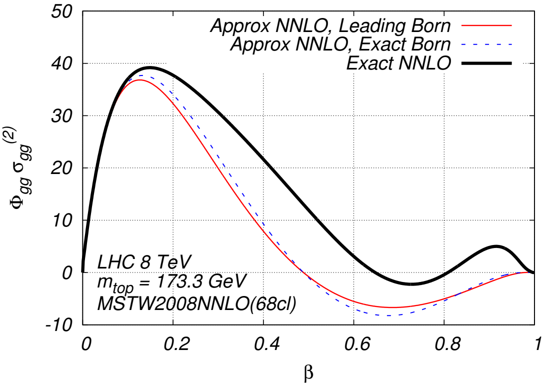


Behavior around the threshold: NNLO



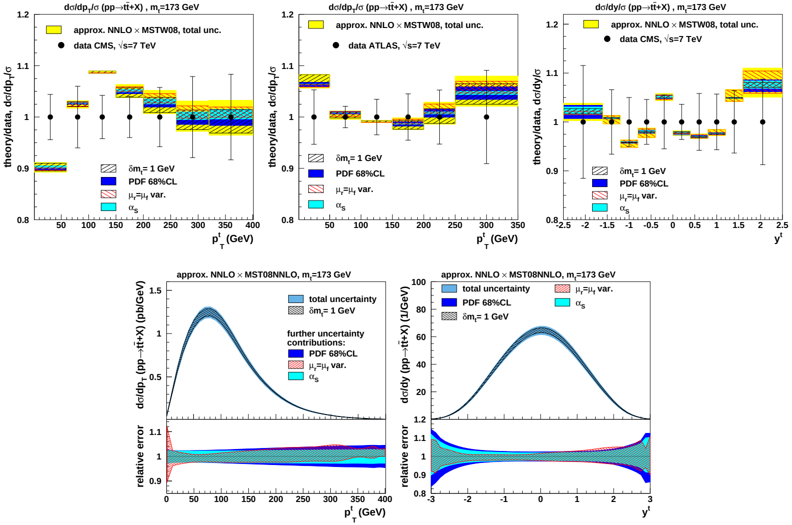
From Moch, Vogt, Uwer PLB (2012)

Exact calculation for the gg channel at NNLO

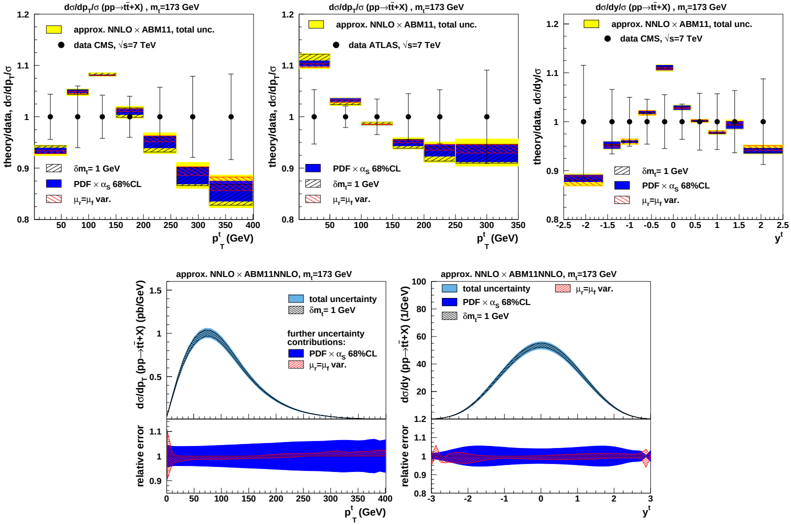


$\beta = \sqrt{1 - 4m^2/s}$

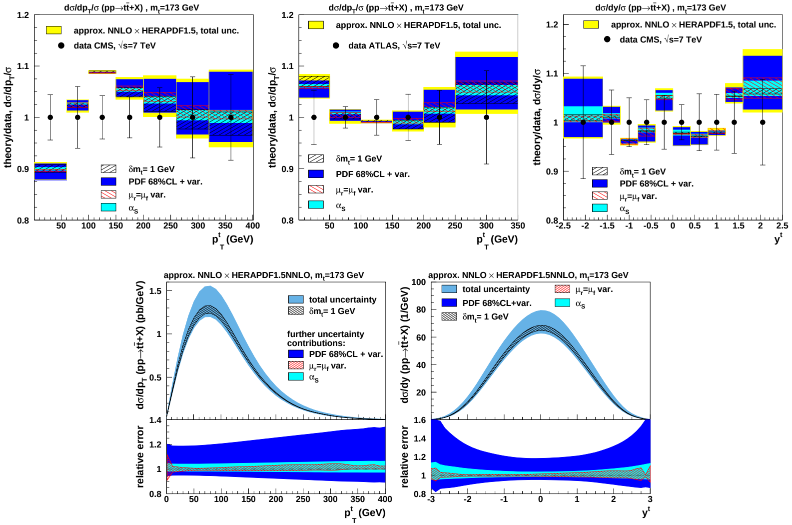
From Czakon, Fiedler, Mitov PRL (2013)



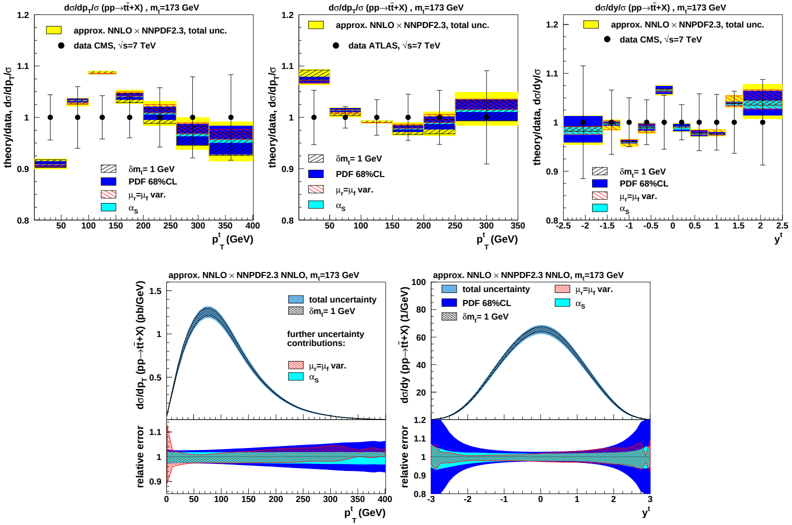
As in the previous slide but with MSTW08 NNLO PDFs. PDF and $\alpha_s(M_Z)$ errors are evaluated at the 68% CL.



As in the previous slide but with ABM11 NNLO PDFs. Here the uncertainty on $\alpha_s(M_Z)$ is already part of the total PDF uncertainty.



As in the previous slide but with HERA1.5 NNLO PDFs.



As in the previous slide but with NNPDF2.3 NNLO PDFs.