How Matrix Models thermalize and what they tell us about black holes.

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Liverpool, 22.10.2014

Motivation

Quantum gauge theory can be Quantum gravity

Usual List of ingredients

- Large N — $G_N \simeq 1/N^2$
- Gauge theory — Singlet constraint: few operators of low dimension, enforces planarity
- Thermal states — Black holes
- Strong coupling — Weakly curved gravity: strings decoupled

Strong coupling is not absolutely required:
we generally expect to get a stringy geometry,
even here we can talk about stringy black holes.

What can we say in this case?

$$g^2 N \simeq \hbar$$

So, we can imagine that

$$g^2 N \to 0$$

$$\hbar \to 0$$

What do we want to compute?

- Real time dynamics of a quench: pick initial condition (can easily do in gravity). Late time is described by hydrodynamics.
- Real time dynamics is almost impossible in quantum systems.
- Can we cheat with classical physics? UV catastrophe is cured by quantum mechanics or by having finite number of degrees of freedom in the first place.
- Want to study the second option
- Putting ingredients together we want to study real time dynamics of gauged matrix classical systems and compare qualitatively to gravity.

Issues in gauge/gravity duality

- Gravity can live in more than d+1 dimensions: more than one dimension can be emergent.

- Black hole dynamics is sensitive to dimensionality (instabilities can appear based on shape: Gregory-Laflamme). This can lead to a richer phase diagram.

- GR has Locality (and causality) in all of these dimensions.

- Diffusive physics of horizons can be in these extra dimensions, but at a point from the point of view of the boundary.

- Do we need to generalize the notions of hydrodynamics?

Rest of the talk

- Holographic matrix models
- Choosing initial conditions: “colliding D-branes”.
- Thermalization and pre-thermalization.
- Finite time correlation functions and “generalized hydro”.
- Adding angular momentum: Rotating black hole instabilities

Multi-Matrix models

Is there a multi-matrix model that is holographic?

YES

Banks, Fischler, Shenker ,Susskind, [hep-th/9610043](hep-th/9610043)

$$S_{BFSS} = \frac{1}{2g^2} \int dt \left((D_t X^I)^2 + \frac{1}{2}[X^I, X^J]^2 \right) + \text{fermions}$$

$$\ddot{X}^i \propto \sum_j [X^j, [X^j, X^i]]$$

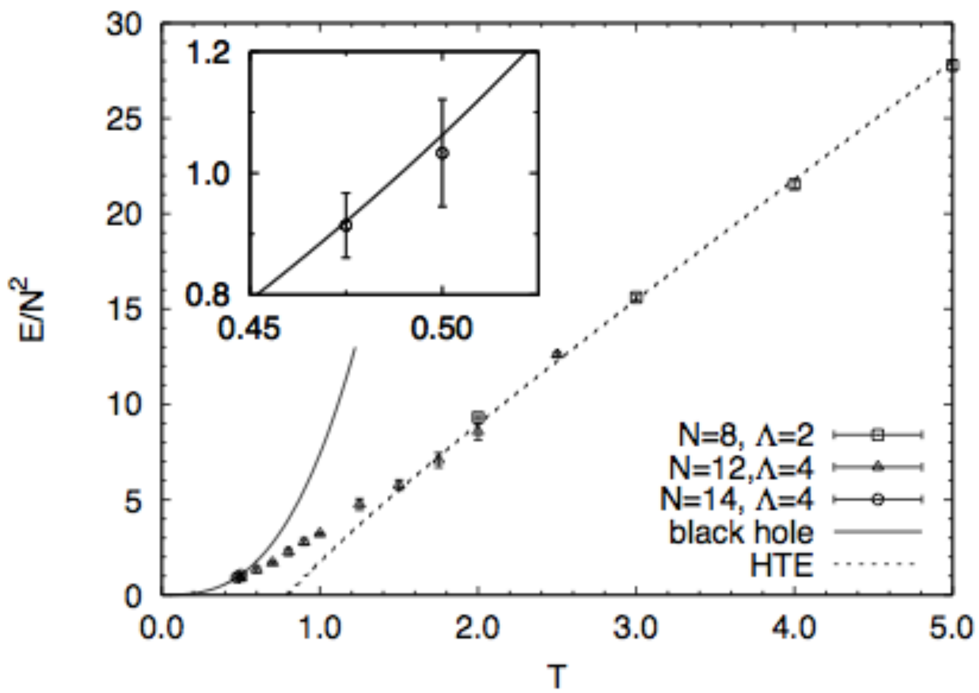
g has units

“weakly coupled” at high temperature:
non-linear (chaotic) classical physics.

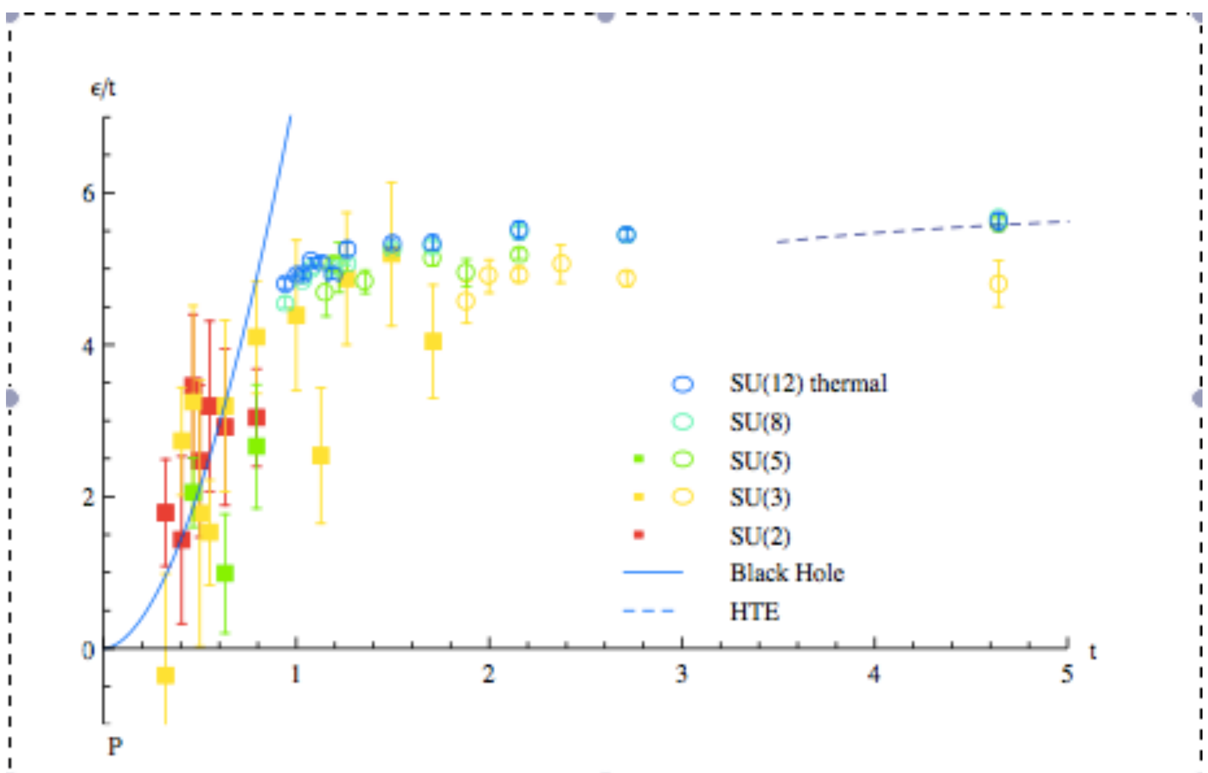
Can describe 10D black holes at strong coupling (low T).

What about large T?

Mote Carlo Lattice



Agnastopoulos, Hanada,Nishimura, Takeuchi, 2007



Caterall, Wiseman 2008

High temperature and low temperature in same phase: but no real time dynamics in MCL.

Hope:

High temperature dynamics qualitatively similar to black hole.

`Fast thermalization'
Hydrodynamic behavior
Phase diagram for rotations (chem. potential)

A massive deformation (BMN),

$$S_{BMN} = S_{BFSS} - \frac{1}{2g^2}\int dt\, \left(\mu^2 (X^i)^2 + \frac{\mu^2}{4}(Y^a)^2 + 2\mu i\, \epsilon_{\ell jk} X^\ell X^j X^k\right)$$

$$+\text{fermions}$$

D.B., Maldacena, Nastase, hep-th/0202021

BMN model is convenient for numerics and interesting initial conditions, then we can quench to BFSS

Initial conditions in BMN

$$\langle X^i \rangle = \begin{pmatrix} L^i_{(n_1)} + \Re e(b^i_1 * \mathbb{1}_{(n_1)} \exp(it)) & 0 & \cdots \\ 0 & L^i_{(n_2)} + \Re e(b^i_2 * \mathbb{1}_{(n_2)} \exp(it)) & \cdots \\ \vdots & \vdots & \ddots \end{pmatrix}$$

Exact classical solutions in BMN:
Rigid fuzzy spheres oscillating around origin.

Because trajectory is periodic, can do Floquet analysis for small perturbations.

Off-diagonal perturbations are subject to parametric resonance.

$$\ddot{q}_\ell(t) + (m_\ell^\pm(t))^2 q(t) = 0\,.$$

$$\begin{pmatrix} q_1(t+2\pi) \\ q_2(t+2\pi) \end{pmatrix} = \begin{pmatrix} A & B \\ C & D \end{pmatrix} \begin{pmatrix} q_1(t) \\ q_2(t) \end{pmatrix}$$

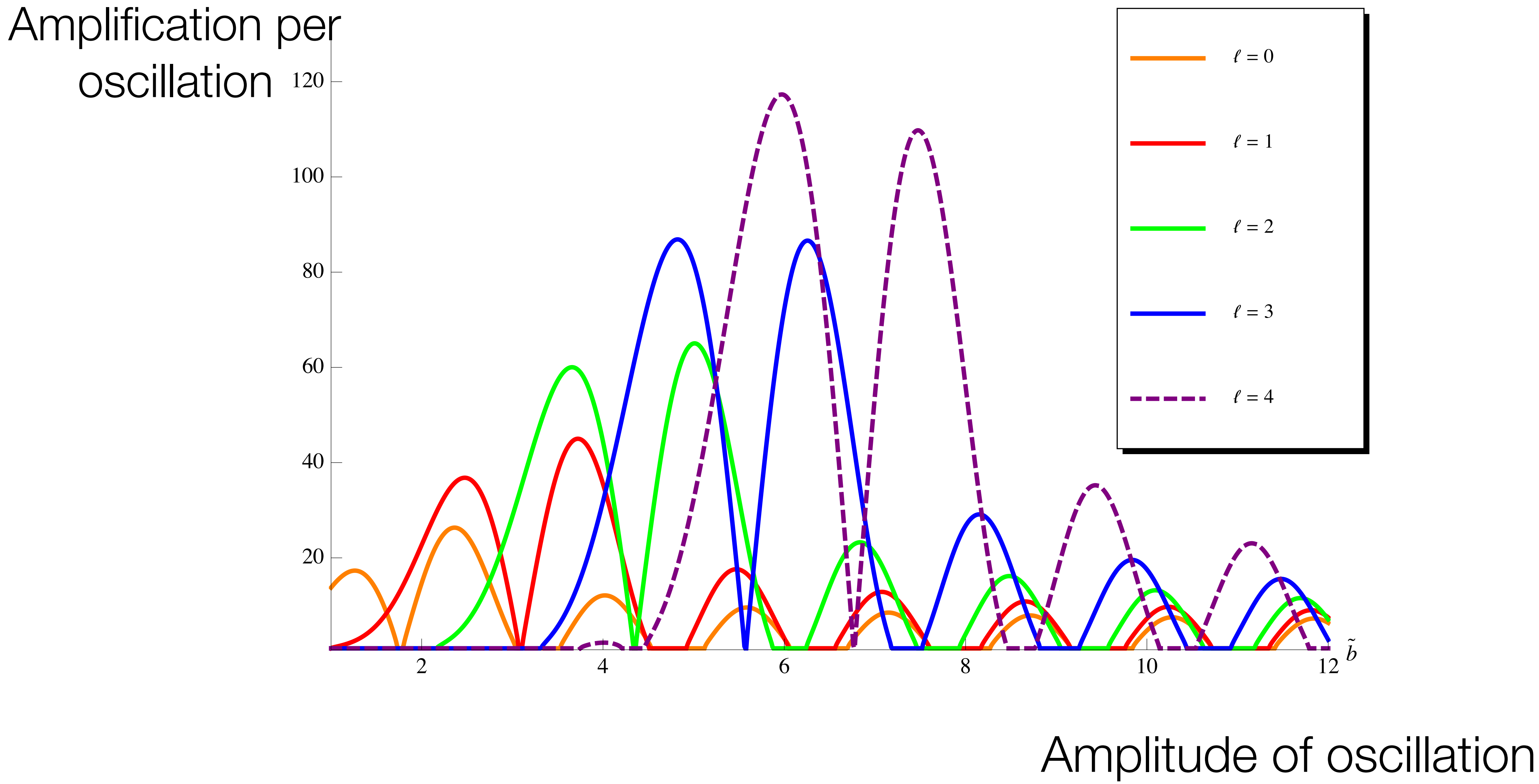
Eigenvalues of matrix determine if amplification occurs or not.

Simplifies for highest weight states (decompose using spherical symmetry)

$$(\omega^{-}_{\ell,\ell+1})^2 = -b + (b - \ell - 1)^2\,,$$
$$(\omega^{+}_{\ell,-\ell-1})^2 = b + (b + \ell + 1)^2\,.$$

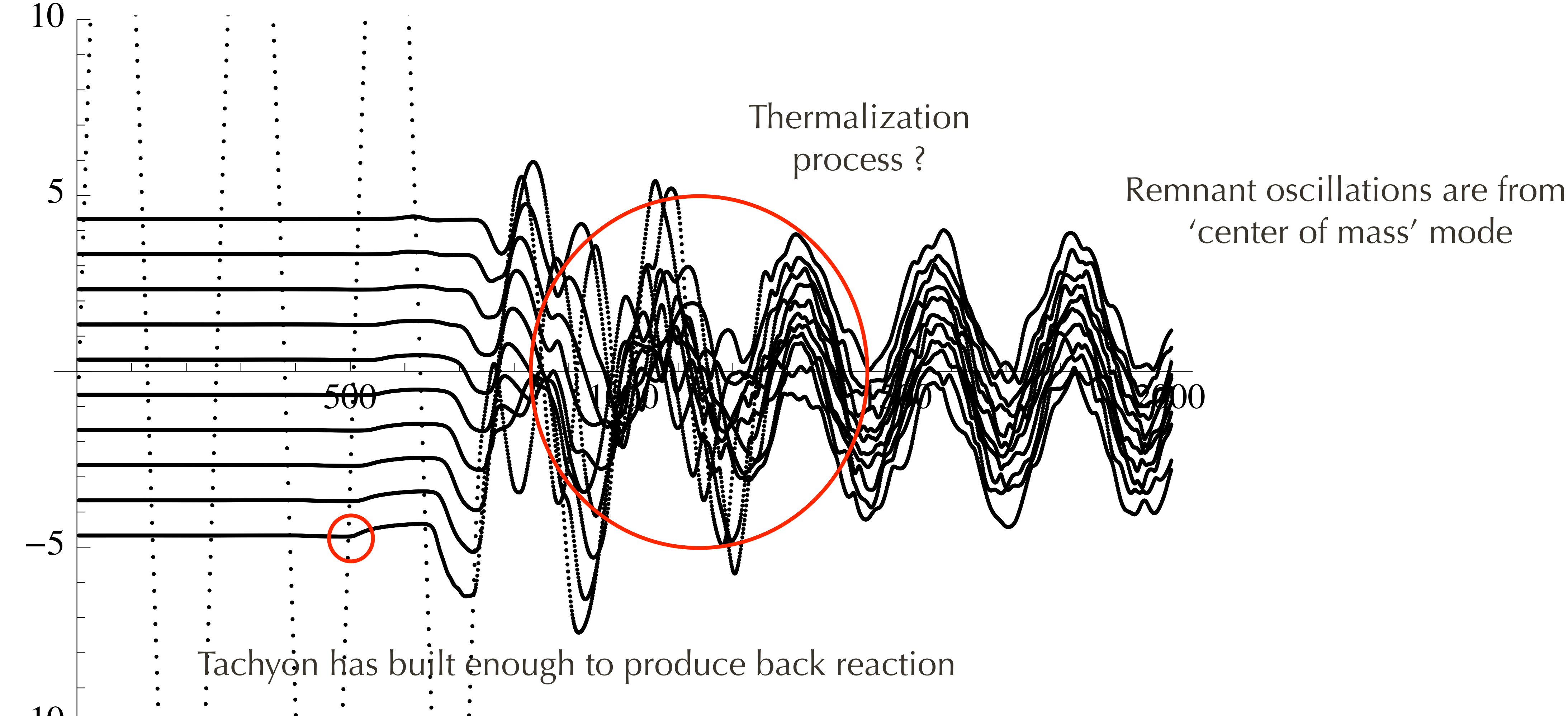
$$b(t) = \tilde{b}\sin(t)$$

Most unstable mode typically has highest l



D.B. + D. Trancanelli, arXiv:1011.2749

Full simulation

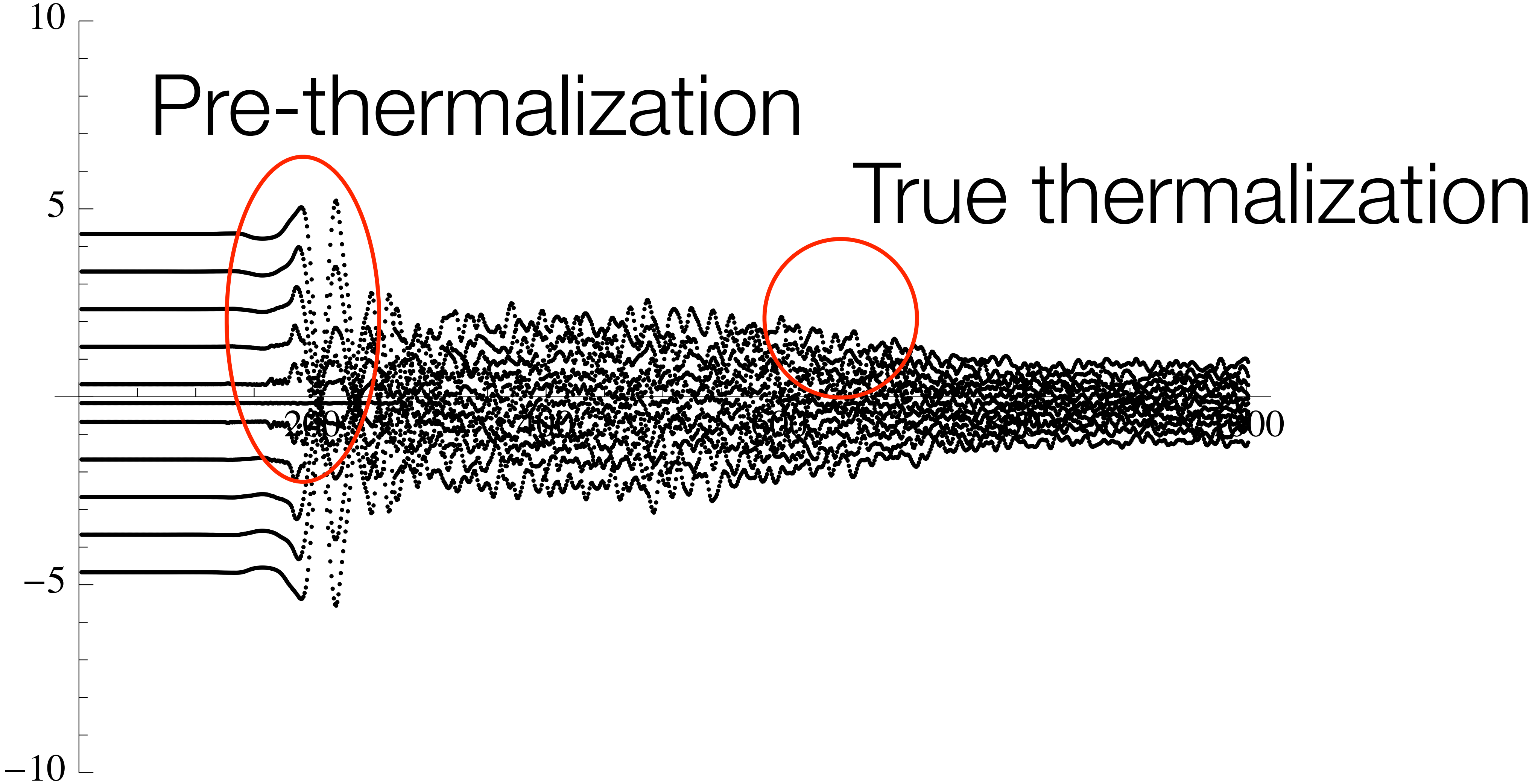


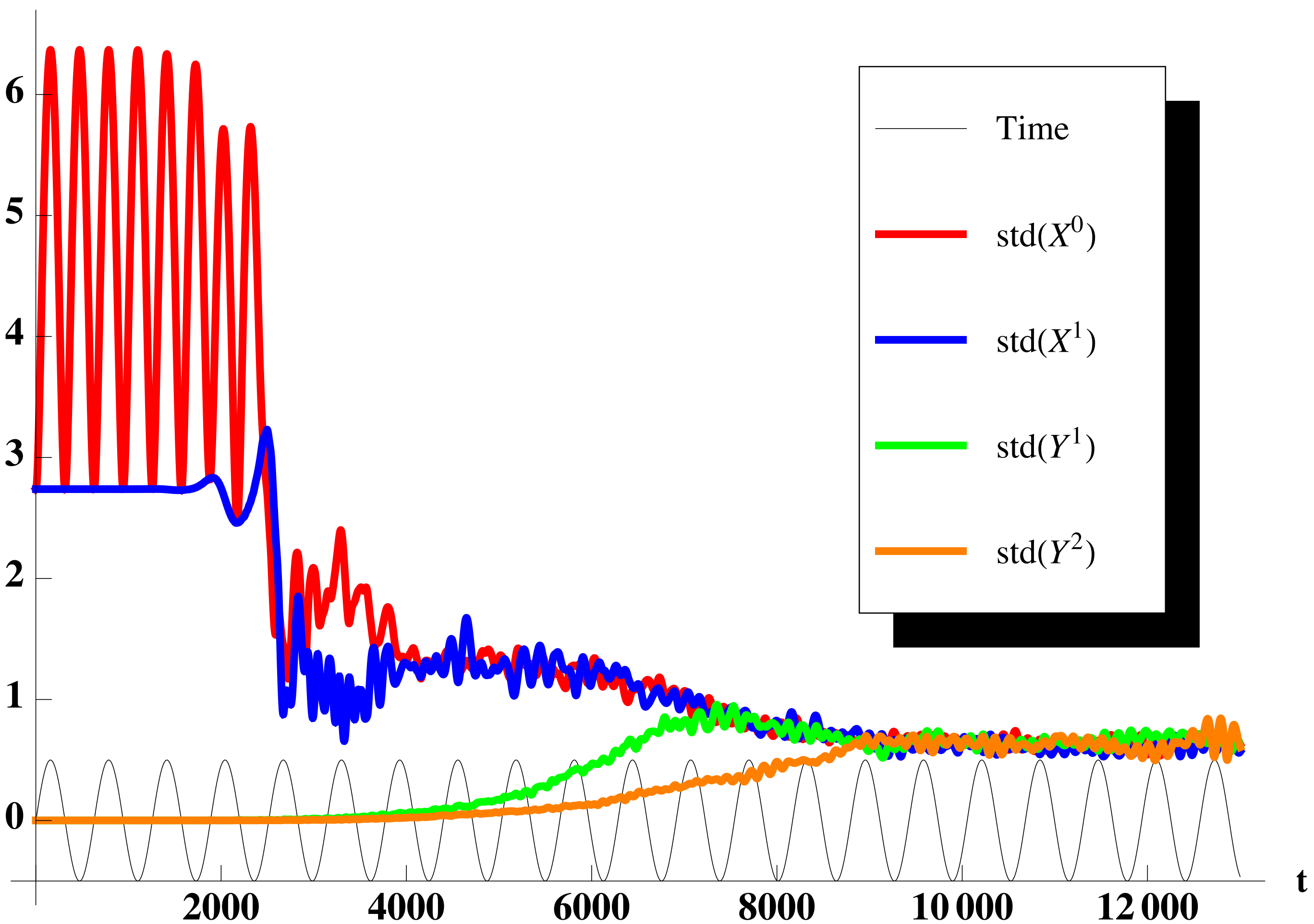
Eigenvalue spectrum for XO

C. Asplund, D.B., D. Trancanelli arXiv:1104.5469

Phys.Rev.Lett. 107 (2011) 171602

In another axis





Trace of X,Y decoupled: serves as physical clock.

Secondary shrinkage is from growth of Y matrices (parametric resonance due to random X interactions)

Similar results by

Riggins+ Sahakian, **arXiv:1205.3847**

Work in BFSS with collapsing (and bouncing) fuzzy sphere as set of initial conditions.

Tests of thermality

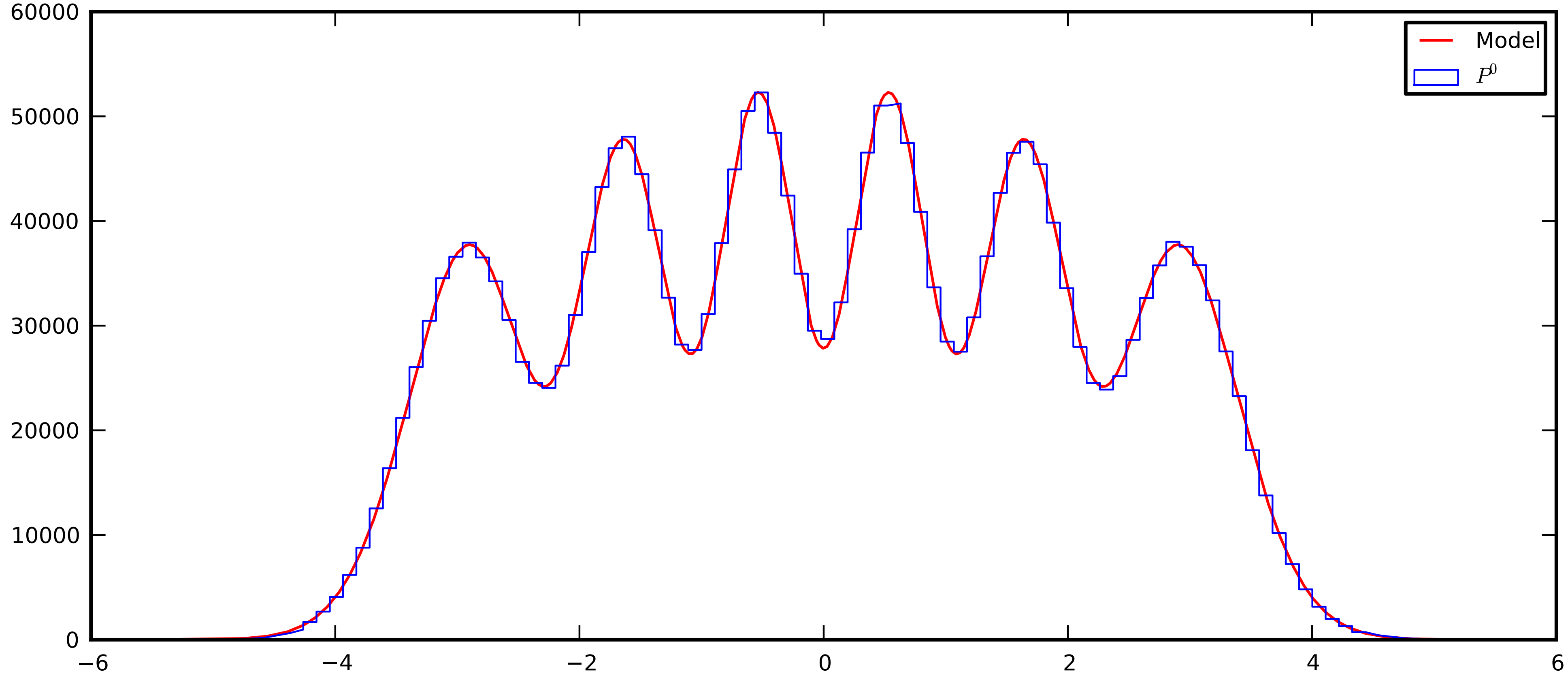
$$H \simeq \frac{P^2}{2} + V(X)$$

Thermal implies time averaged distribution of some quantities (momenta) should match the Gibbs ensemble.

$$\mathcal{P}(P) \simeq \exp(-\beta \frac{P^2}{2})$$

This is the standard gaussian matrix model ensemble.

C.Asplund, D. B., E. Dzienkowski arXiv:1211.3425



Need to study it with traceless matrices.

The ridges include the finite N exact soln.

Effective temperature is given by second moment.

N	$\langle \mathrm{Tr}(P_0^2) \rangle_0$	$\langle \mathrm{Tr}(P_1^2) \rangle_0$	$\langle \mathrm{Tr}(P_2^2) \rangle_0$	$\langle \mathrm{Tr}(Q_1^2) \rangle_0$	$\langle \mathrm{Tr}(Q_2^2) \rangle_0$	$\langle \mathrm{Tr}(Q_3^2) \rangle_0$	$\langle \mathrm{Tr}(Q_4^2) \rangle_0$	$\langle \mathrm{Tr}(Q_5^2) \rangle_0$	$\langle \mathrm{Tr}(Q_6^2) \rangle_0$
4	23.2 ± 0.6	23.3 ± 0.4	23.2 ± 0.5	21.3 ± 0.5	21.3 ± 0.5	21.2 ± 0.6	21.2 ± 0.4	21.3 ± 0.4	21.0 ± 0.4
11	26.9 ± 0.3	27.2 ± 0.2	27.0 ± 0.3	26.6 ± 0.2	26.5 ± 0.3	26.6 ± 0.2	26.6 ± 0.3	26.6 ± 0.2	26.5 ± 0.2
23	32.2 ± 0.3	32.2 ± 0.2	32.1 ± 0.2	31.9 ± 0.2	31.9 ± 0.2	31.9 ± 0.2	31.9 ± 0.2	31.9 ± 0.2	32.0 ± 0.2

$$T_{effX} \neq T_{effY}?$$

Need to be careful: there is a constraint.

$$\delta([X, P_X] + [Y, Q_Y])$$

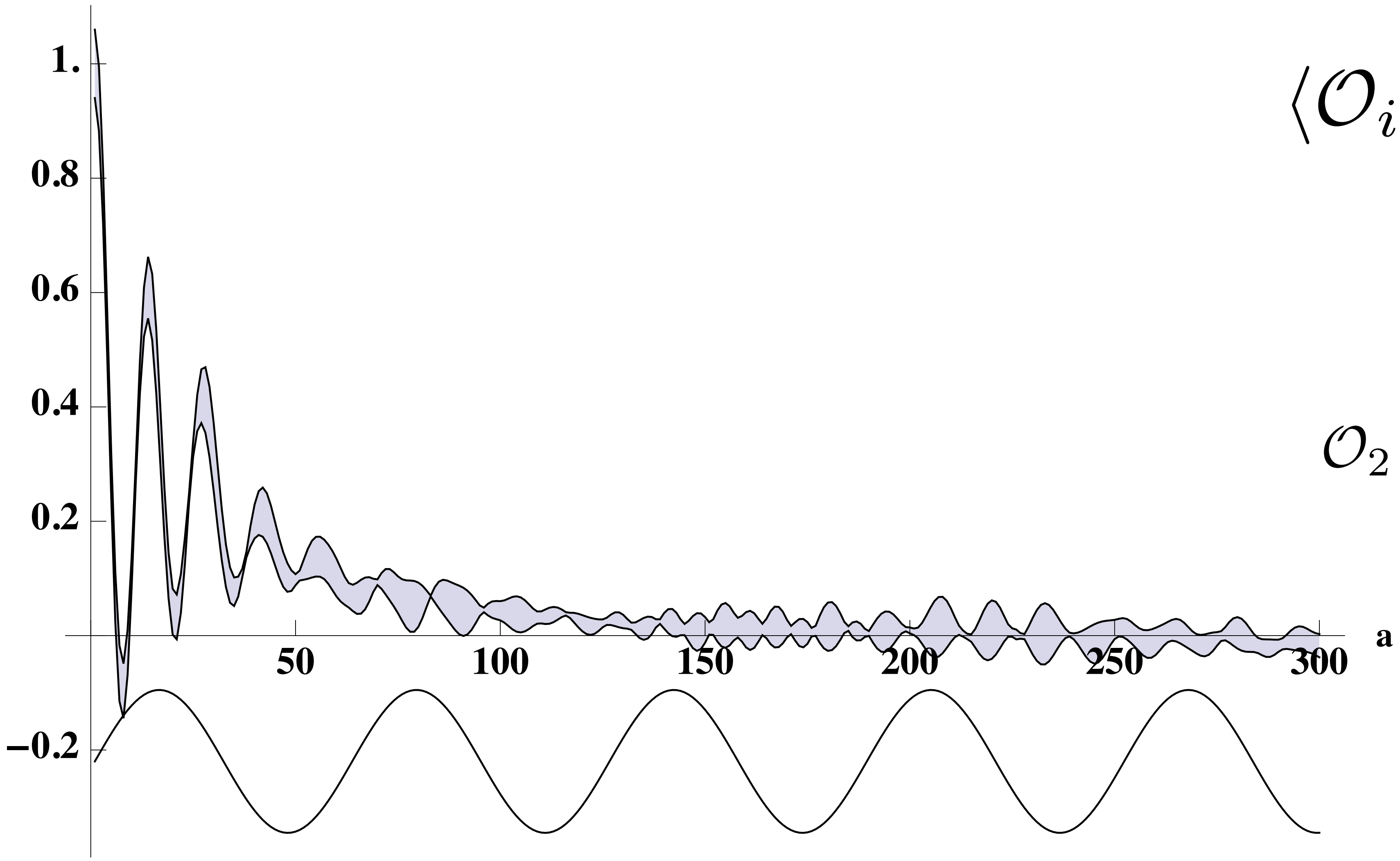
Symmetry breaking of X,Y eom's

⇒ symmetry breaking between X, Y,

⇒ symmetry breaking between P, Q

Fast thermalization?

Test via Normalized Autocorrelation functions



$$\langle \mathcal{O}_i(t)\mathcal{O}_i^\dagger(t+a)\rangle$$

$$\mathcal{O}_2 = \mathrm{tr}[(X^1 + iX^2)^2]$$

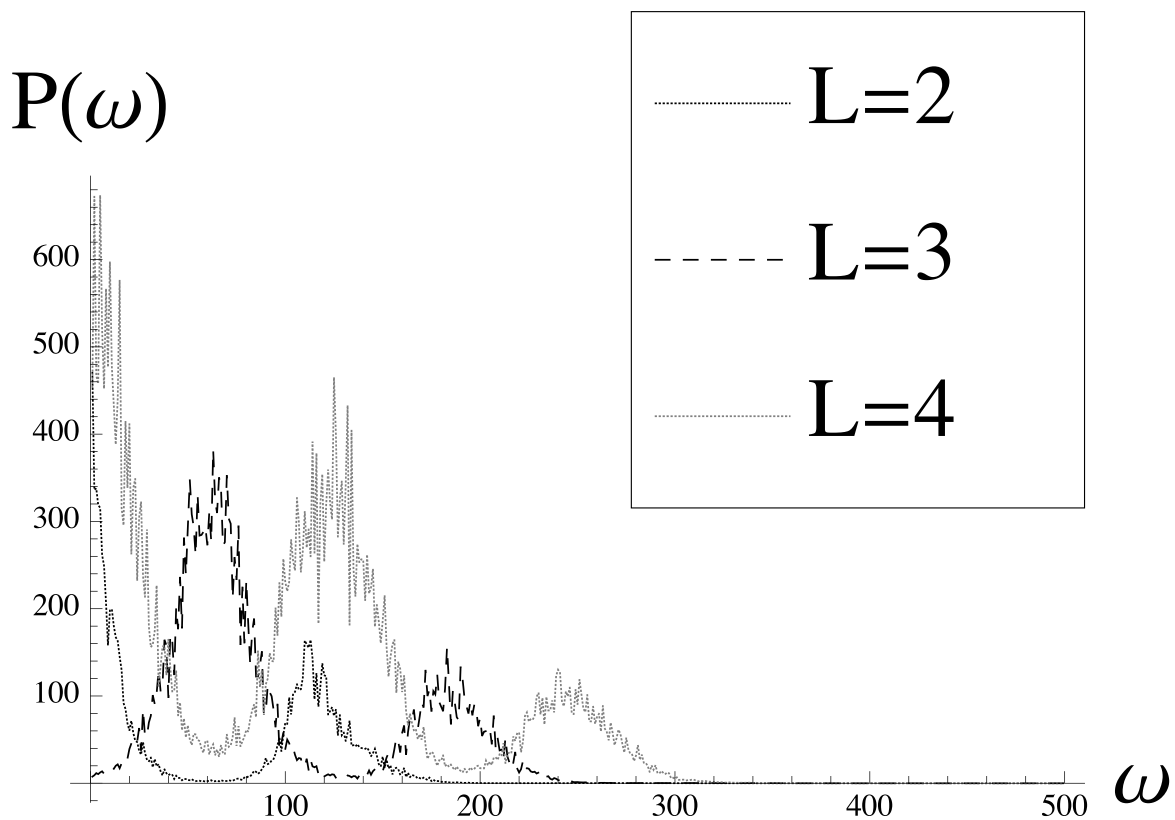
Autocorrelations

Better in Fourier space.

Autocorrelation function is
Fourier transform of power spectrum

$$\langle \mathcal{O}_i(t)\mathcal{O}_i^\dagger(t+a)\rangle = \int dw P(w)\exp(iwa)$$

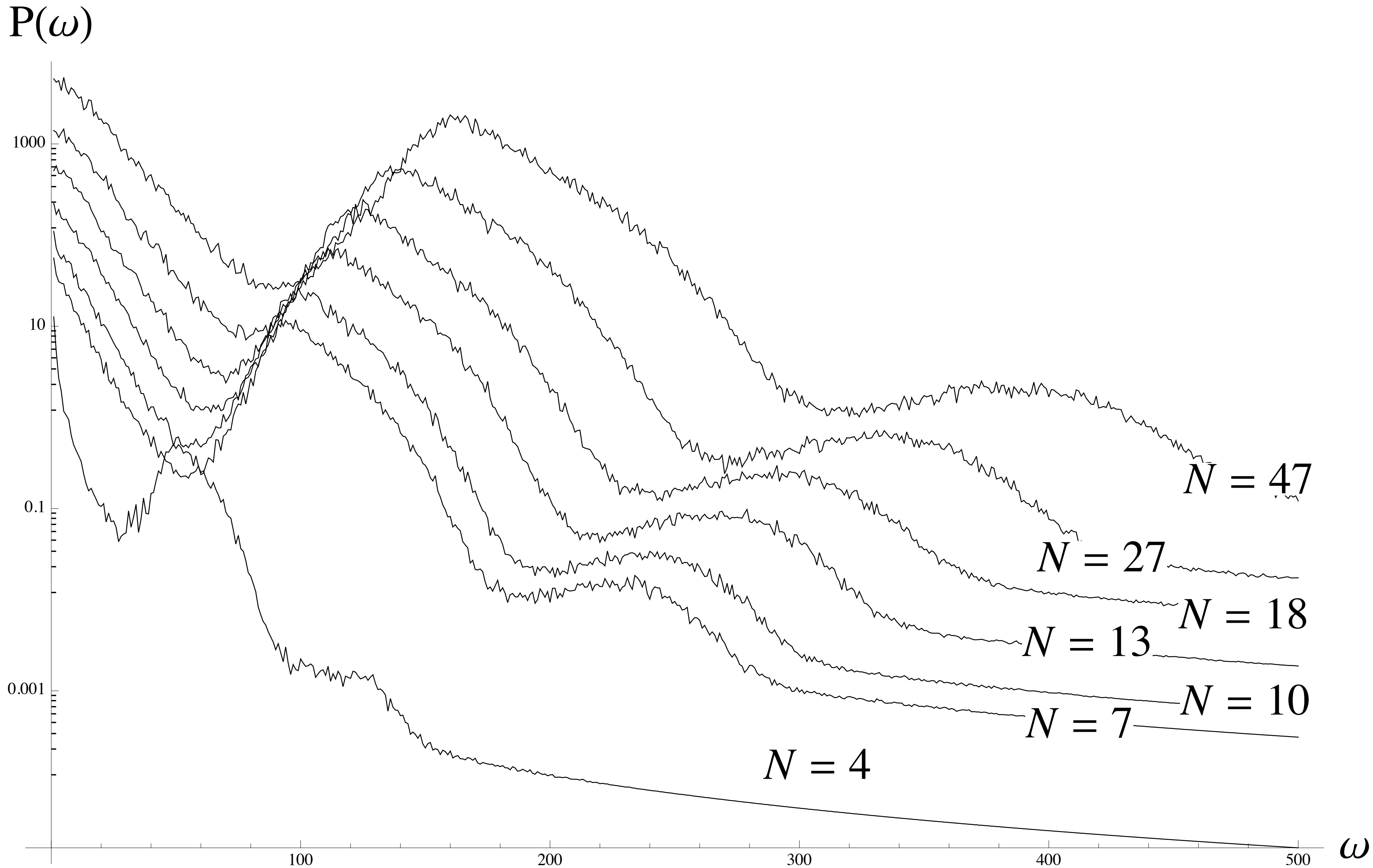
Look at BFSS (classical model with no scale, only depends on N up to rescalings)



Broadband spectrum indicates chaos.

Chaos in dim. reduction of YM is well known since 80's

(Chirikov et al. Mantiyan et al. - In russian)



Can compare different N: neither frequency nor amplitude normalized. Fit to max, and rescale power.

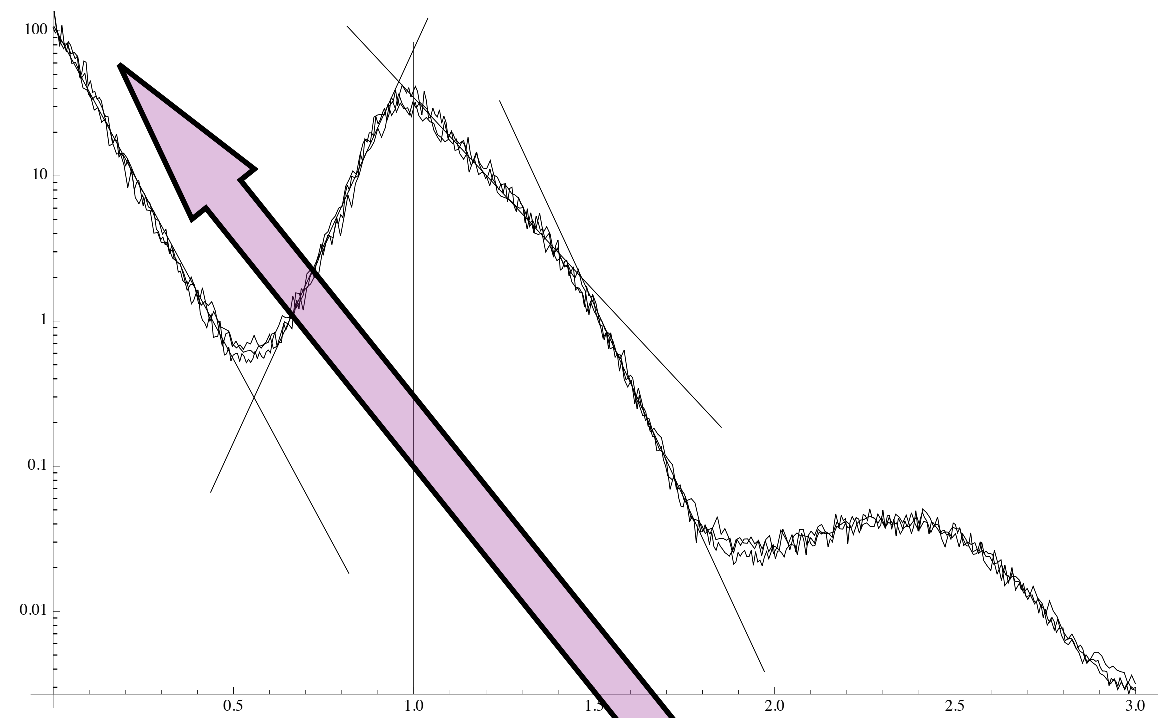


FIG. 8. The power spectrum of $\mathrm{tr}(X^1 + iX^2)^2(t)$ for various sizes of $N \times N$ matrices. The axis of frequency has been rescaled for each N, to the frequency ω_N, and we have also rescaled the power spectrum. The reference frequency for each N is located at 1 in the graph. Results shown for $N = 7, 10, 47$. We also have drawn additional suggestive straight lines superposed on the graph that serve as distinctive features of the power spectrum.

Notice Log spectrum seems to have an
absolute value singularity at 0:
this would imply power
law decays of correlation functions.

Interesting IR

- Power spectrum seems almost singular at zero.
- The log of power spectrum seems to have an absolute value singularity. Such singularity would imply polynomial decay of autocorrelation functions for asymptotically long times.
- Hydrodynamics: collective degrees of freedom whose time dependent autocorrelation functions are N independent.

Matching to black holes?

- Absolute value singularity can be approximated by square root

$$|\omega| \simeq \sqrt{\omega^2 + \epsilon}$$

Branch cuts in the complex plane?

Can also think as closely packed sequence of poles.

Similar to spectrum of quasi-normal modes for black holes.

Hydro?

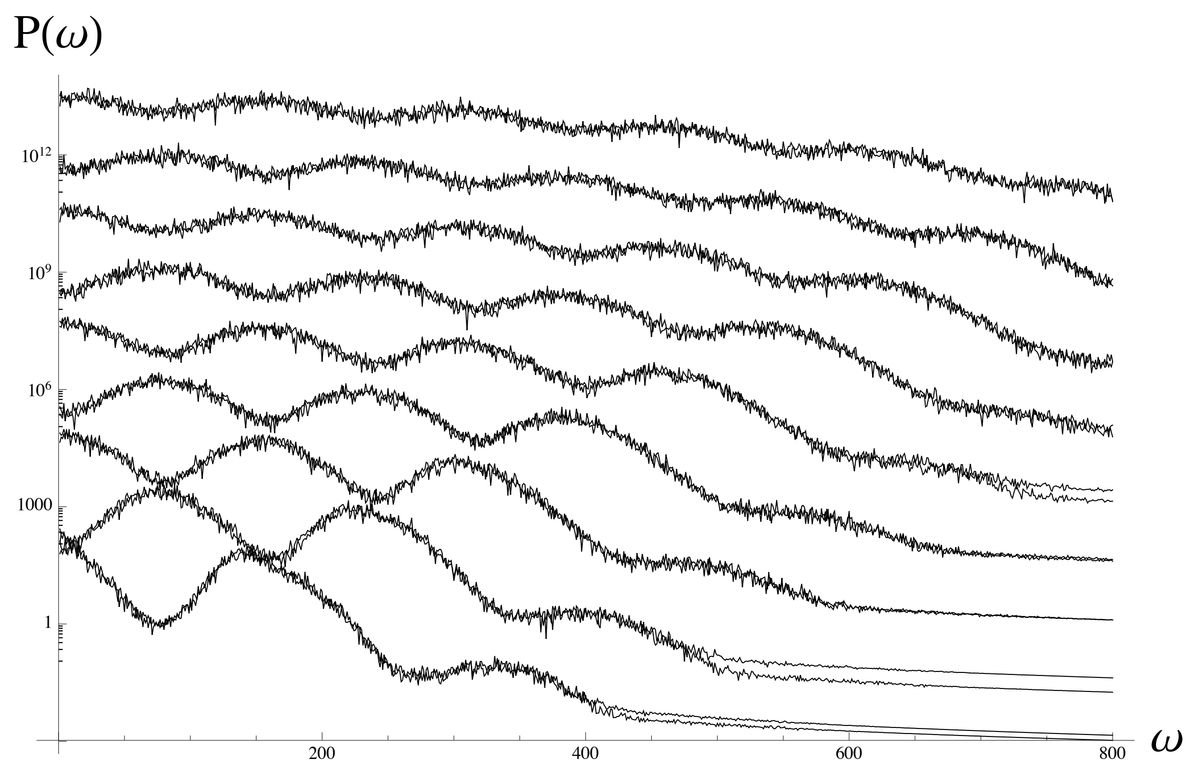
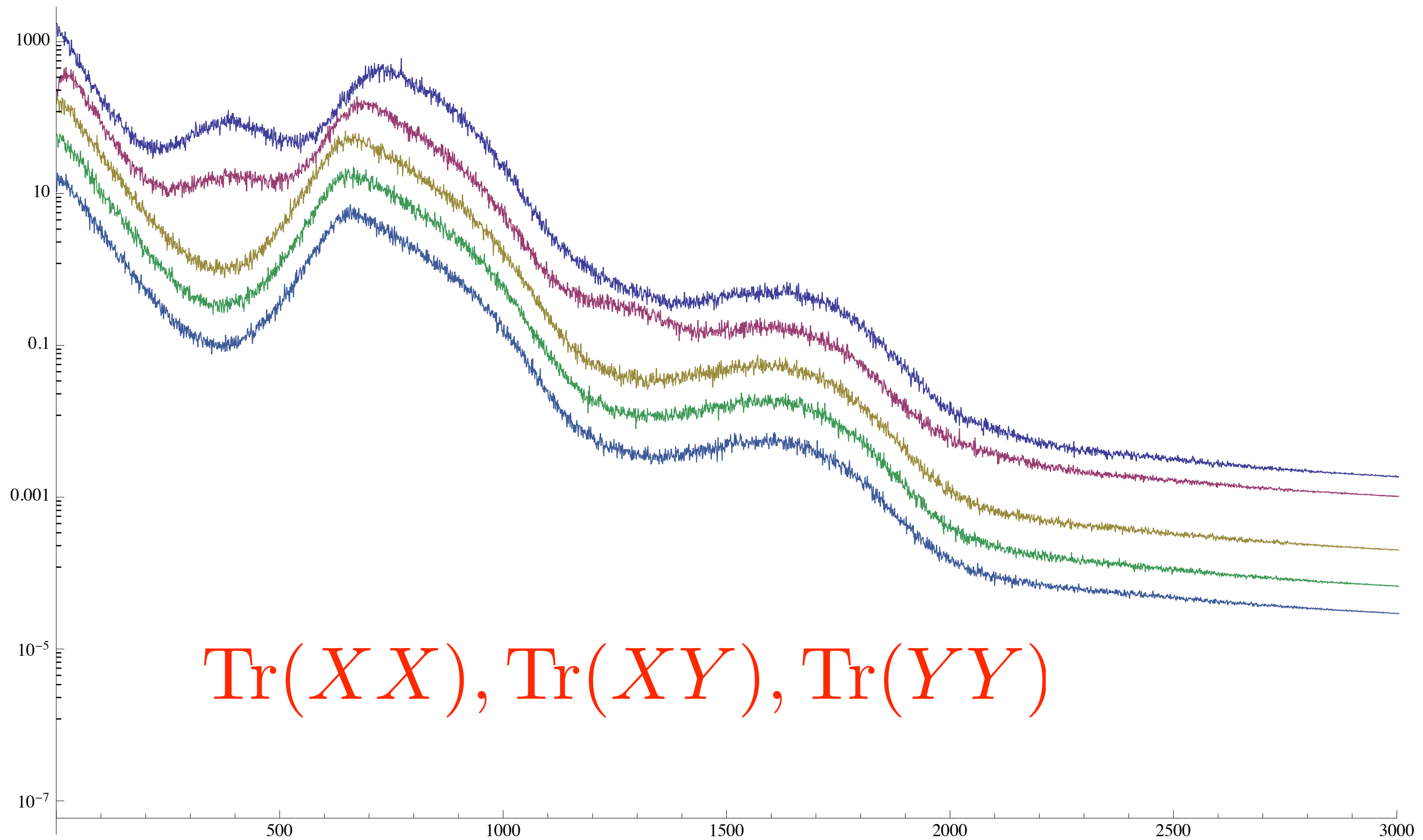


FIG. 10. Power spectrum in arbitrary units for $\mathcal{O}_L$, with $L = 2, \ldots 10$, with values of L increasing from bottom to top in the graph. For each L we show two such sets. This data is from $N = 27$.

Power spectrum again: BMN



Deformation adds peaks from mixing between modes with different symmetry.

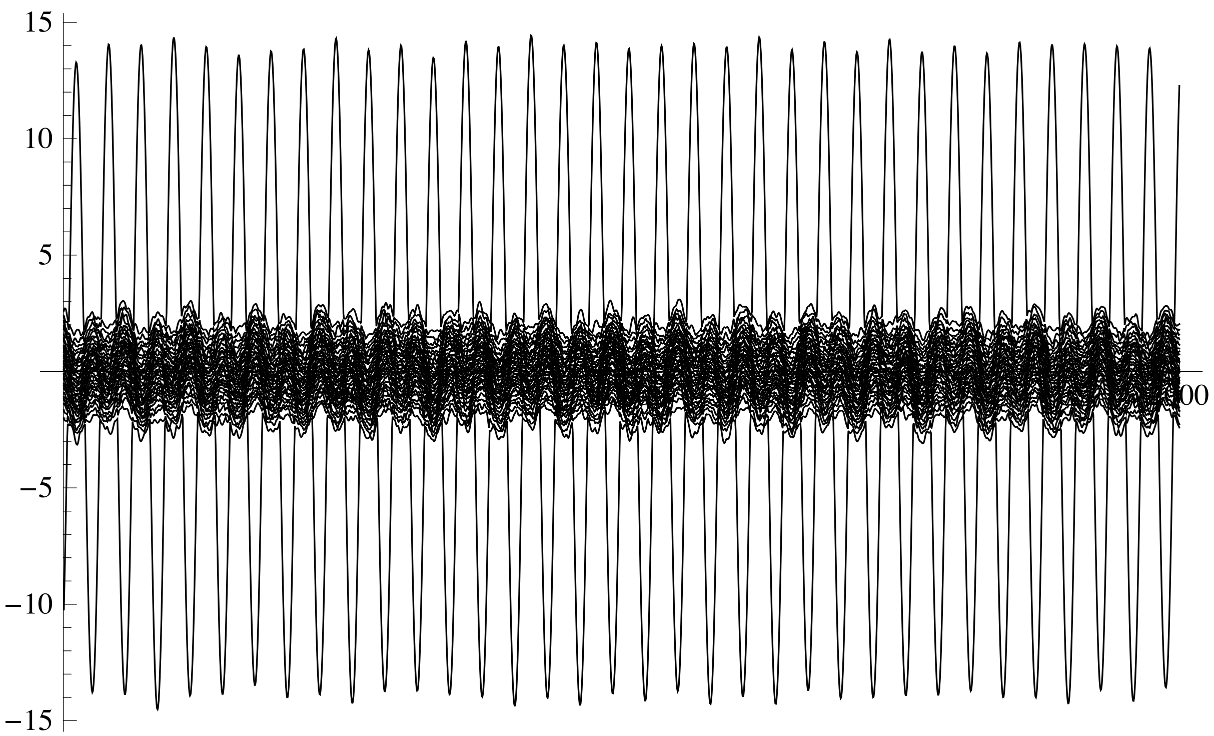
1/N corrections

$$\frac{\langle \mathcal{O}_L(t)\mathcal{O}_M(t)\bar{\mathcal{O}}_{L+M}(t+a)\rangle_t}{\sqrt{A_1^L A_1^M A_1^{L+M}}} \sim \frac{C_{L,M,L+M}(a)}{N} + O(1/N^3),$$

	$N=10$	$N=13$	$N=18$	$N=87$
$C_{2,2,4}$	4.97 ± 0.51	4.54 ± 0.15	4.94 ± 0.8	4.97 ± 1.2
$C_{3,3,6}$	6.97 ± 0.86	6.9 ± 0.4	7.58 ± 0.5	8.36 ± 1.4

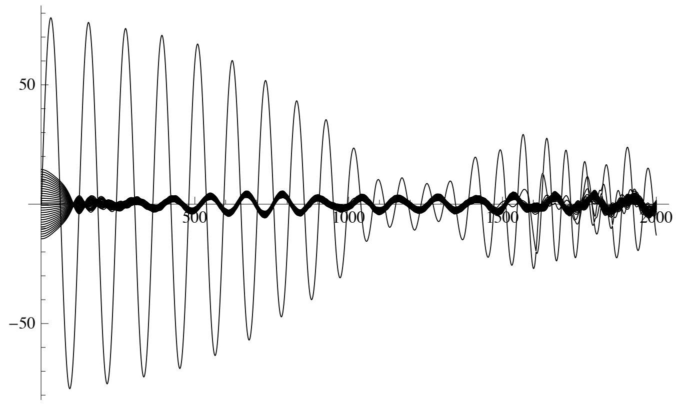
TABLE III. Values of $C_{L,M,L+M}$ at various values of N

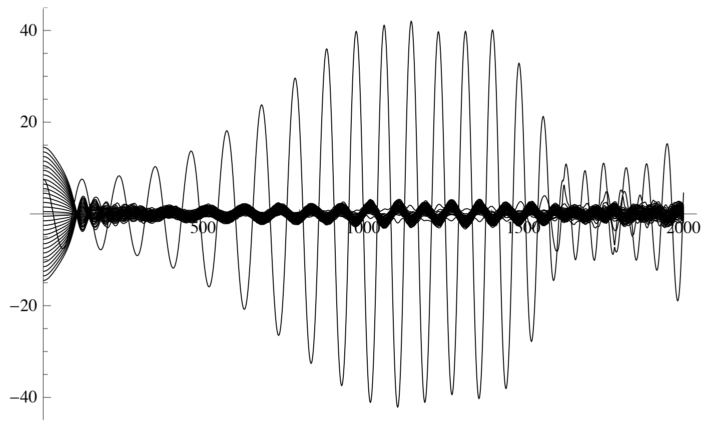
Add (large) angular momentum in initial condition

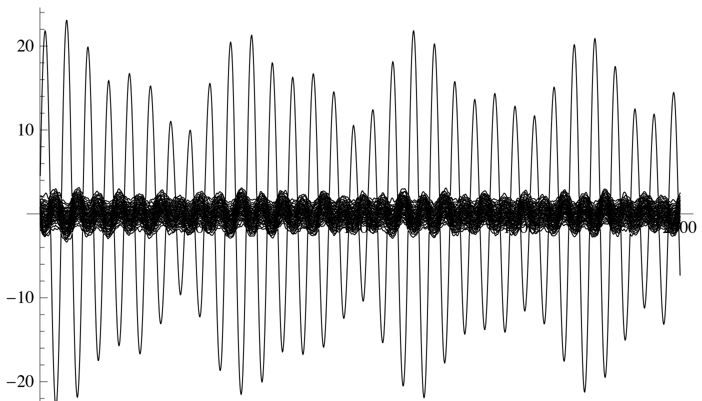


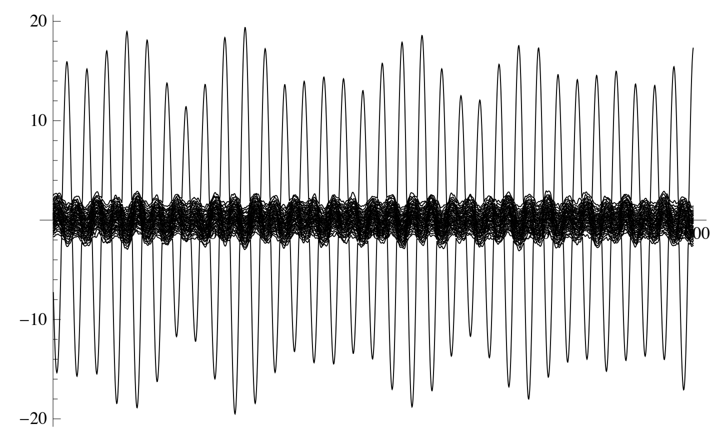
After thermalization, one eigenvalue is expelled

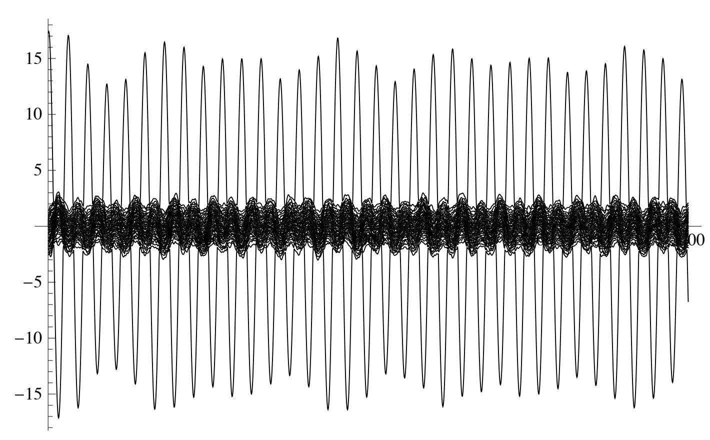
In progress.





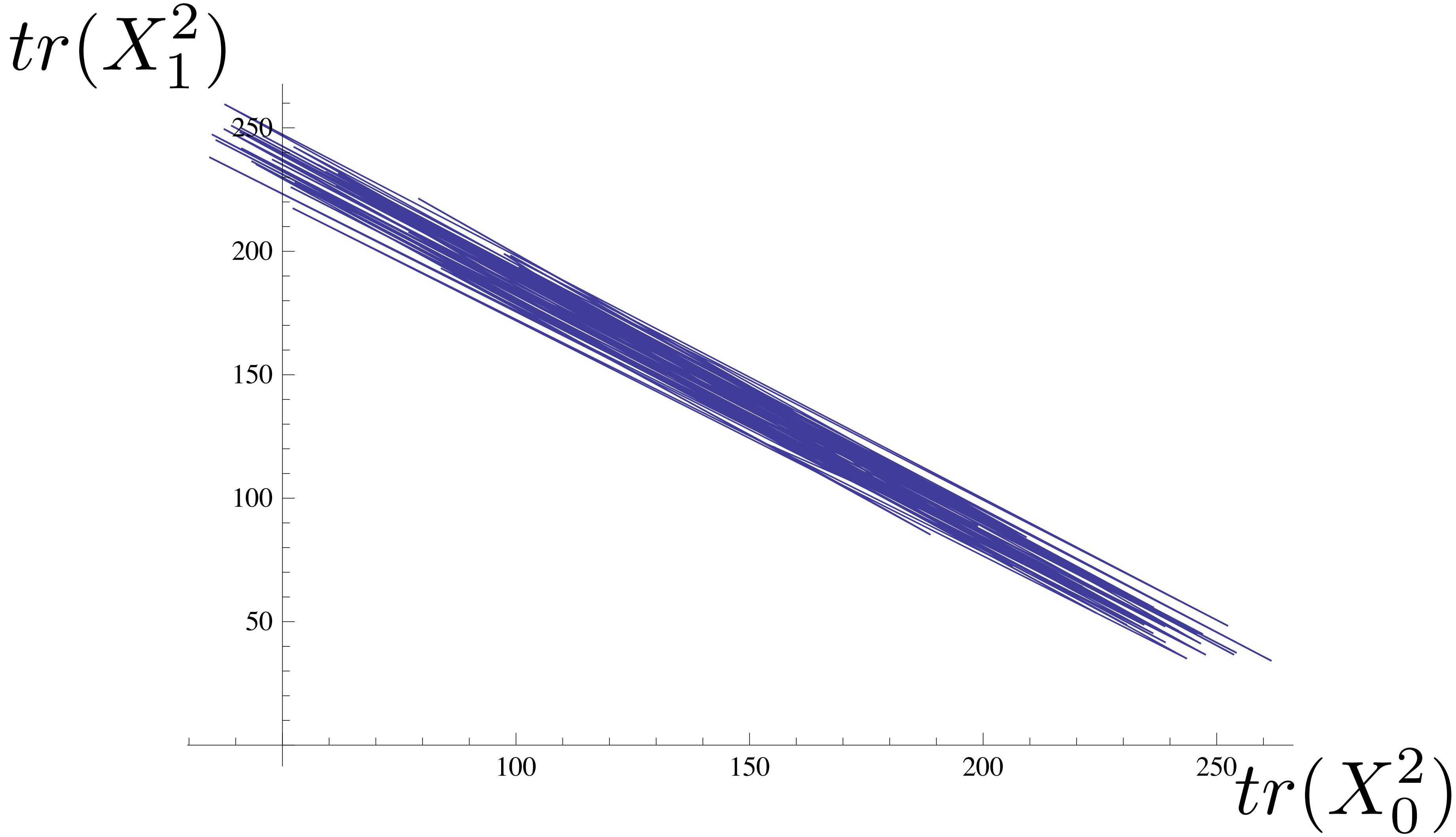


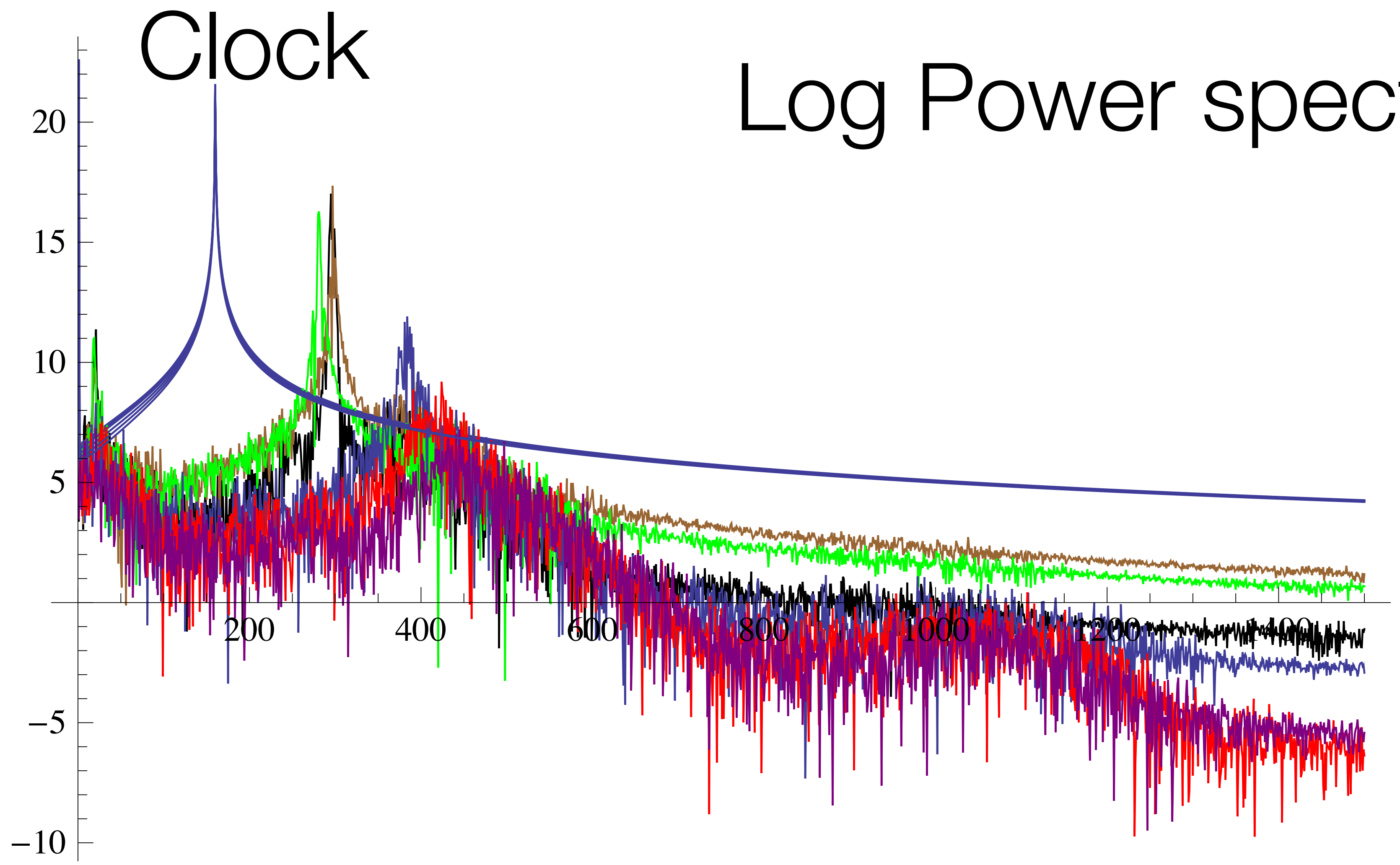




Orbit Circularizes

System rotates “rigidly” at constant speed





Log Power spectrum vs L

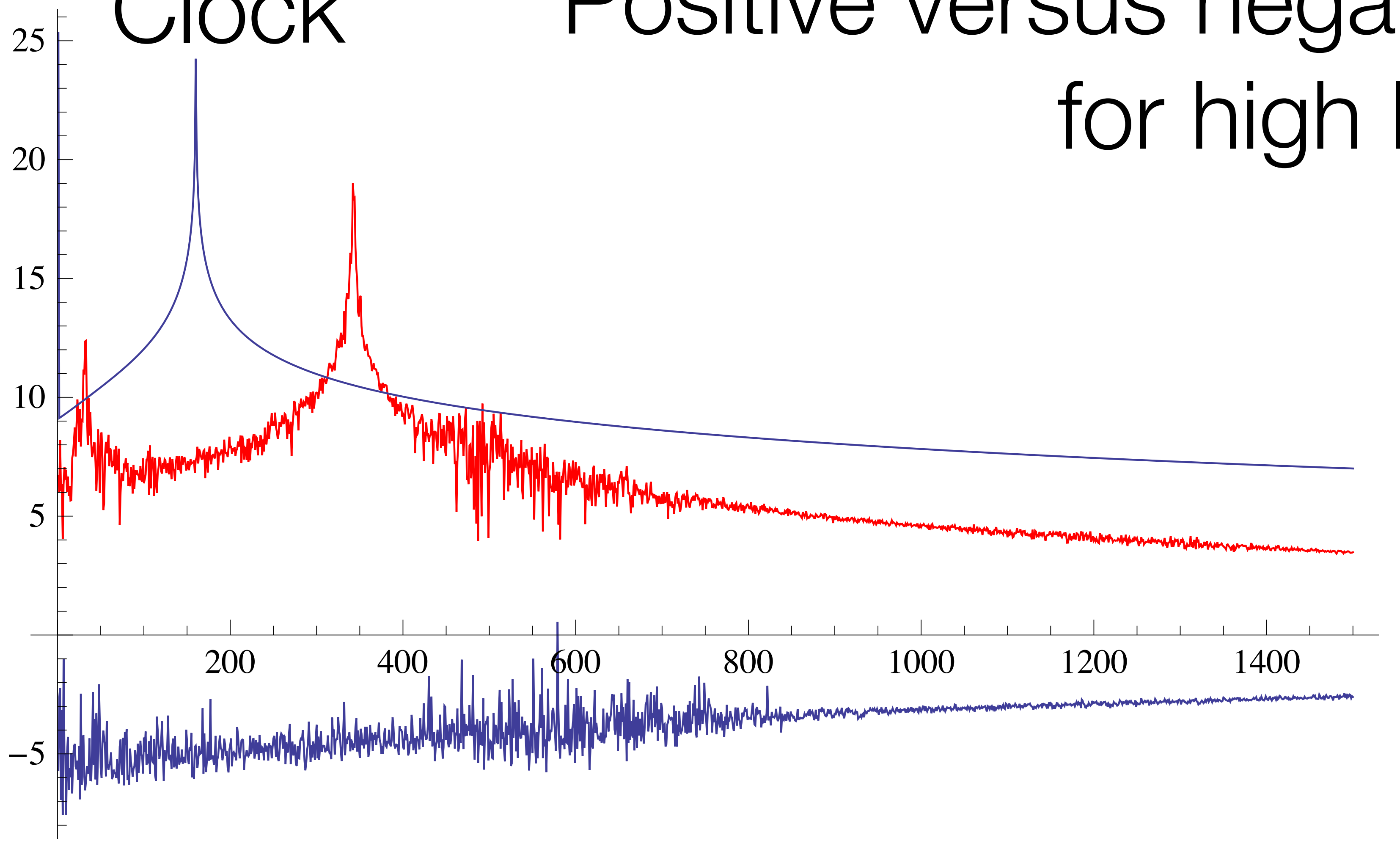
L=2,4,6,8,9,10 in random units

Critical angular momentum where eigenvalue peels off, and another one where the distribution deforms.

Extra force on expelled eigenvalue: should be entropic force

Clock

Positive versus negative frequency for high L.



Seems to be related to hyper-spinning instabilities in black holes.

We seem to match two known results (phases):

Emparan-Myers, ‘03

Black saturns: Elvang, Emparan, Figueras, ’07

Geometry of a probe

Typical idea of matrix models: add eigenvalue.

One can always make the matrices bigger.

By one.

By direct sum.

Ask about the degrees of freedom connecting the one to the rest.

$$\begin{pmatrix} X & * \\ *^\dagger & x \end{pmatrix}$$

Fermion mass matrix

$$\sum_i (X^i - x^i) \otimes \gamma^i$$

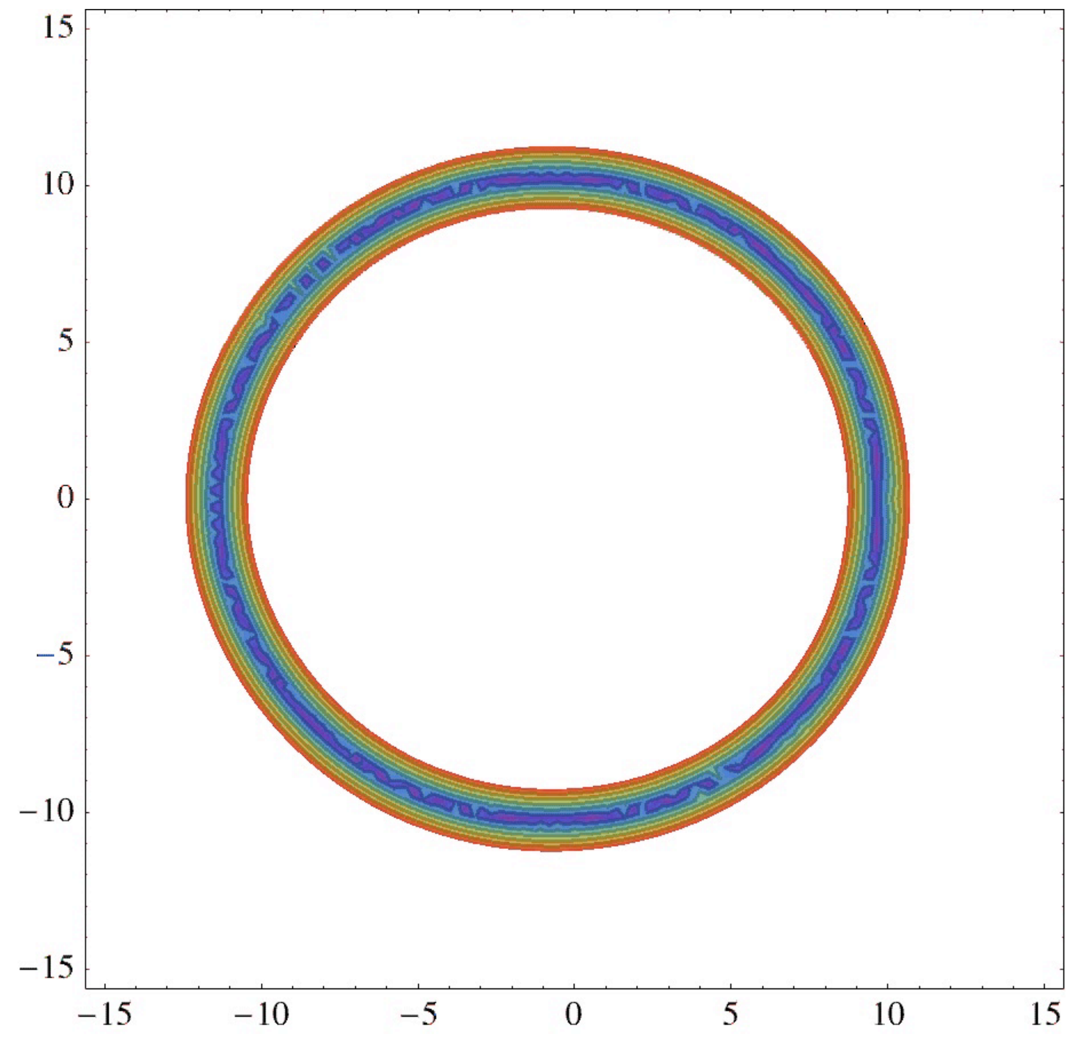
What matters is the spectrum of this one matrix
(provided by dynamics)

defines a spectral Distance:

$$d(X, x) \simeq (\min(\text{Abs}(\text{ Eigenvalues})))$$

D.B. + E. Dzienkowski arXiv:1204.2788

Movie of collapse by a 2D slice

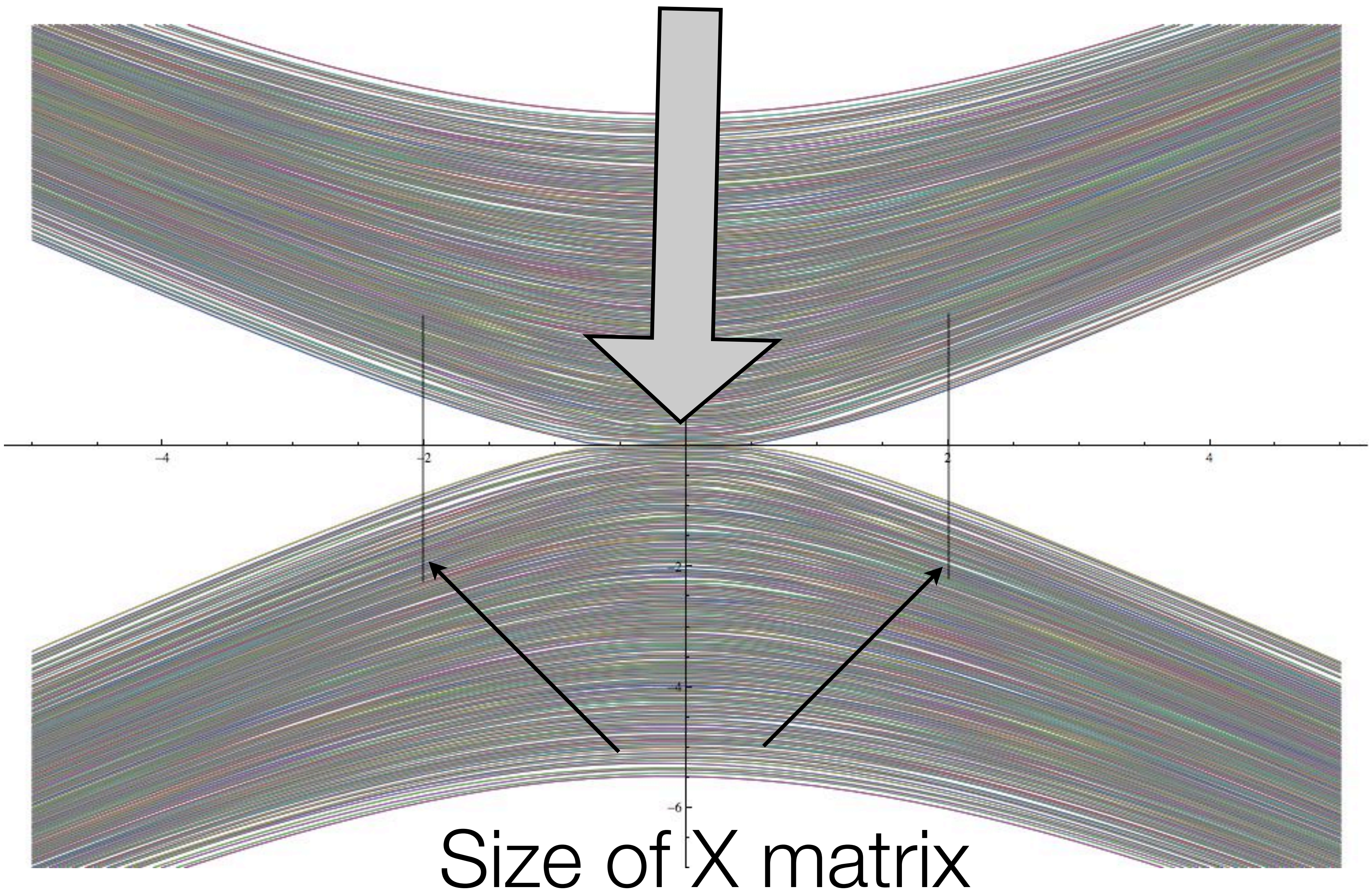


Spectral dimension

D.B and E. Dzienkowski **arXiv:1311.1168**

Scan over a 1 parameter set at fixed time

Fermions are gapless in a region



Effective field theory breaks down in gapless region: can’t integrate out off-diagonal fermion modes.

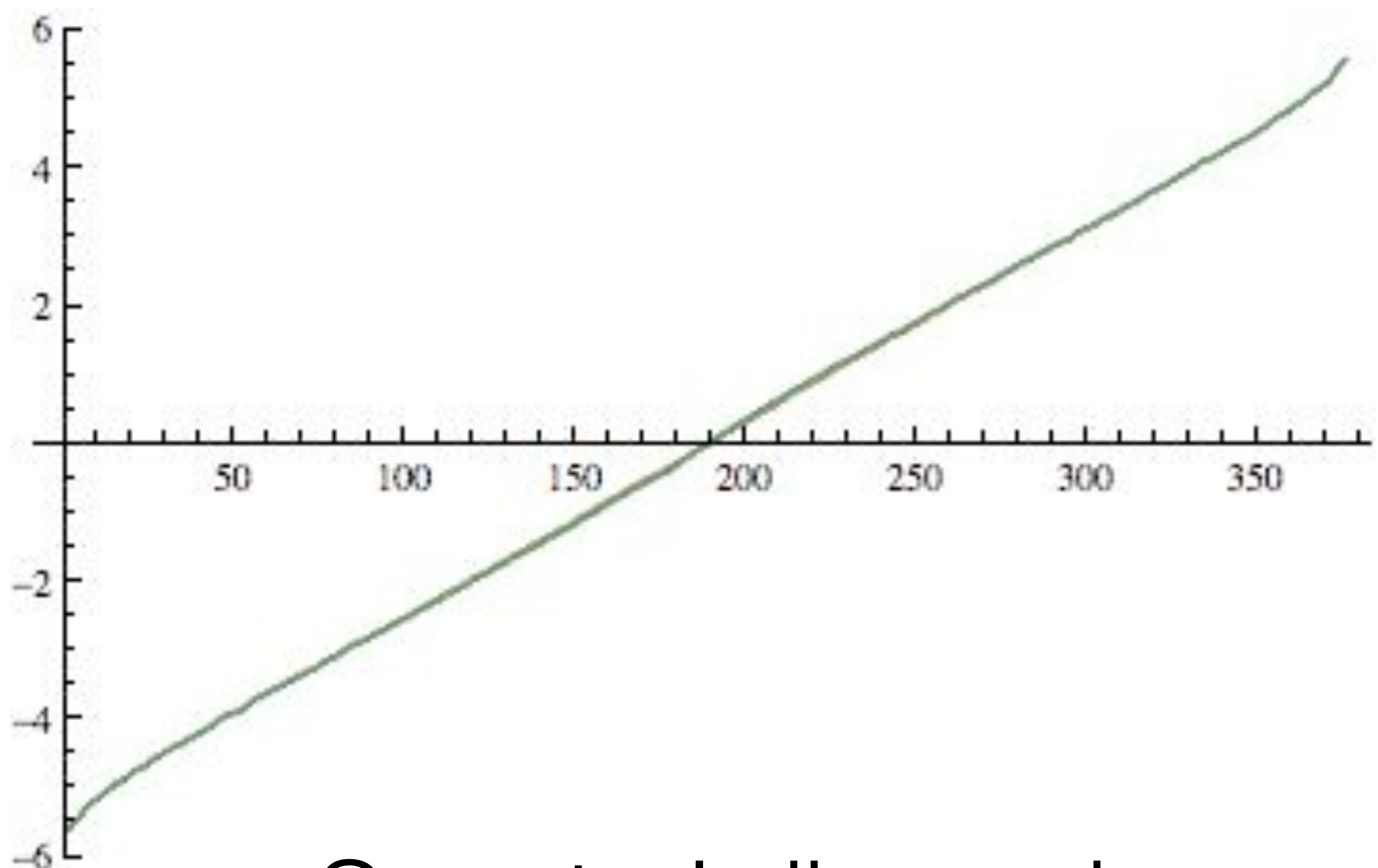
Fix position of probe inside gapless region

Define spectral dimension using density of states near zero

$$\frac{dn}{dE}|_{E\simeq 0} \simeq E^{\gamma-1}$$

$$\text{spectral dim} = \gamma$$

Same density of degrees of freedom as field theory in $\gamma + 1$ dimensions



Spectral dimension = 1

Effective 1+1 field theory:
seems to effectively change spacetime dimension inside black hole.

Some conclusions

- Some interesting classical Multi matrix models thermalize, initially fast, but with some polynomial-like intermediate regime, and a slow exponential tail.
- Large N limit is “hydrodynamic”: has N independent dynamics for many observables and 1/N correction are ‘non-linearities’.
- Adding angular momentum seems to give rise to interesting phase diagram that can be related to known black hole instabilities.
- Interesting effective dynamics for probes suggests a change in space-time dimension inside black holes.

Some geometric stuff