University of Liverpool

Automatic generation of RGEs at two-loop: PyR@TE

arXiv:1309.7030

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Motivations

Description

- Generate the **Renormalization Group Equations** for non-supersymmetic theories @ 2-loop

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  - collider experiments
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  - collider experiments
  - direct DM detection experiments
- Systematic studies of non-SUSY models require the RGEs
- One possible application: constraining non-SUSY BSM models via the stability bound

- RGEs for general gauge theories known for a long time:
  - *M. Machacek and M. T. Vaughn, 1983 Nuc.Phys.B222*
  - *M. Luo et al. Phys.Rev. D67 (2003) 065019*
- Calculation of beta functions "by hand" is time consuming and prone to error $\Rightarrow$ Difficult to use in practice.
- Full set of 2-loop RGEs known only for few specific cases:
  - SM + Neutrinos

    from *A. Wingerter Phys.Rev. D84 (2011) 095012*
  - SM + chiral fourth generation

    from *C. Cheung et al. JHEP 1207 (2012) 105*
  - SM + real singlet scalar
  - SM + real triplet scalar
  - SM + complex doublet scalar
  - ...

SUSY

- SARAH *Comp. Phys. Com. 182 (2011) pp. 808-833* (spectrum generator generator)
- SUSYNO *Comput.Phys.Commun. 183 (2012) 2298-2306*

NON-SUSY

- Two implementations in parallel in Python and Mathematica
- Python $\Rightarrow$ PyR@TE
- Mathematica $\Rightarrow$ merged with SARAH 4.0.
- Numerous cross checks between the two versions

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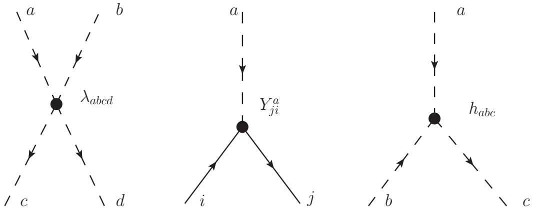
Renormalization Group Equations

- Renormalization scale μ
  $\Rightarrow g_{10}, \alpha_{S0}, \lambda_0 \cdots \Rightarrow \tilde{g}_1(\mu), \tilde{\alpha}_S(\mu), \tilde{\lambda}(\mu)$.
- RGEs : ensure the invariance of the observables.
  - e.g. : $\mu \dfrac{d}{d\mu}\tilde{\alpha}_S(\mu) = \beta_{\alpha_S}$



- β functions depend on the theory i.e. **particles and gauge groups**.
- Can be approximated in perturbation theory.

Renormalization Group Equations

- The RG gives the dependence of the system on the energy probing it.
- Beta functions can be calculated from the renormalization constants.
- The RGEs depend on the renormalization scheme.
- $\overline{\mathrm{MS}}$ scheme and regularization in d dimensions.

Renormalization Group Equations

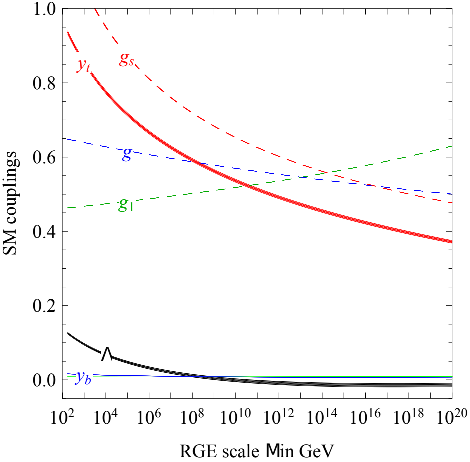


Fig: from G. Degrassi et al. arXiv:1205.6497

Definition

- Take a general gauge field theory

$$G_1 \times G_2 \times \cdots \times G_n \text{ direct product of simple groups}$$

$$\begin{aligned}\mathcal{L} \supset \quad & - \quad N_a Y^a_{jk} \psi_j \xi \psi_k \phi_a + h.c. \Rightarrow \beta^a_{jk} \\ & - \quad N_\lambda \lambda_{abcd} \phi_a \phi_b \phi_c \phi_d \Rightarrow \beta_{abcd} \\ & - \quad N_{mf} (mf)_{jk} \psi_j \xi \psi_k + h.c. \Rightarrow (\beta_{mf})_{jk} \\ & - \quad N_{mab} m^2_{ab} \phi_a \phi_b \Rightarrow \beta_{ab} \\ & - \quad N_h \phi_a \phi_b \phi_c \Rightarrow \beta_{abc},\end{aligned}$$

$\Rightarrow$ 6 types of beta functions to calculate:

- $\beta(g) \Rightarrow$ gauge couplings
- $\beta^a_{jk} \Rightarrow$ yukawas
- $\beta_{abcd} \Rightarrow$ quartic couplings
- $\beta_{ab} \Rightarrow$ scalar mass
- $(\beta_{mf})_{jk} \Rightarrow$ fermion mass
- $\beta_{abc} \Rightarrow$ trilinear couplings

Results

- Known @two-loop:
  - Machacek and M. T. Vaughn, 1983 Nuc.Phys.B222
  - Corrected/enhenced M. Luo et al. Phys.Rev. D67 (2003)
  - Multiple $U(1)$ factors, M. Luo et al Phys.Lett. B555 (2003)
    - Also see, R. Fonseca, M. Malinsky, F. Staub, arXiv:1308.1674
- e.g. gauge coupling constant for unique gauge group factor :

$$
\begin{aligned}
\beta(g) = &-\frac{g^3}{(4\pi)^2}\left\{\frac{11}{3}C_2(G) - \frac{4}{3}\kappa S_2(F) - \frac{1}{6}S_2(S) + 2\frac{\kappa}{(4\pi)^2}Y_4(F)\right\} \\
&+\frac{g^5}{(4\pi)^4}\left\{\frac{34}{3}[C_2(G)]^2 - \kappa[4C_2(F) + \frac{20}{3}C_2(G)]S_2(F)\right. \\
&\left. -[2C_2(S) + \frac{1}{3}C_2(G)]S_2(S)\right\},
\end{aligned}
$$

$$
Y_4(F) = \frac{1}{d(G)}Tr\left(C_2(F)Y^aY^{\dagger a}\right)
$$

Results

- Notation extremely compact, difficult to find the correct multiplicity!

E.g.(1) : two-loop gauge couplings beta function

- $g_k^4(S(R)C(R))_k \rightarrow$
$$\sum_r \sum_l g_k^2 g_l^2 \mathcal{N}_r \mathcal{S}_k(\Lambda(r)) \mathcal{C}_l(\Lambda(r)) \prod_m \tilde{N}(\Lambda(r))_{mk}$$
- r is running over the scalars $(R = S)$ or fermions $(R = F)$ of the model.
- $\mathcal{C}_l$ is the quadratic casimir of the irrep $\Lambda(r)$.
- $\mathcal{S}_k$ is the dynkin index of the irrep $\Lambda(r)$.
- $N_l(\Lambda)$ is the dimension of the irrep Λ in

$$\tilde{N}(\Lambda)_{lk} = \begin{cases} N_l(\Lambda) & \text{if } l \neq k, \\ 1 & \text{else if } l = k\,. \end{cases}$$

Results

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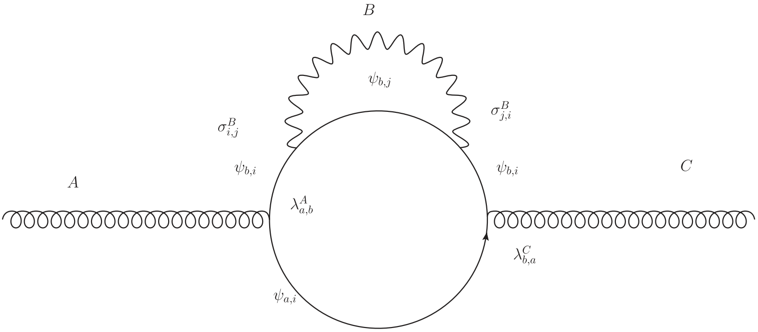
- E.g. in the SM the quark doublet $Q \sim (3,2)$ contribution to this term for the g_3 couplings is :

$$(S(R)C(R))_{\mathrm{SU}(3)}(Q) : g_3^2 g_3^2 \cdot S(\mathbf{3})_{\mathrm{SU}(3)} \cdot C_{\mathrm{SU}(3)}(\mathbf{3}) \cdot n_g(1 \cdot 2)$$
$$+\ \mathrm{g}_3^2 g_2^2 \cdot S(\mathbf{3})_{\mathrm{SU}(3)} \cdot C_{\mathrm{SU}(2)}(\mathbf{2}) \cdot n_g(2 \cdot 1)$$



E.g. (2): $g_2^2 g_3^3$ contribution to g_3 in the SM

$$diag \sim g_2^2 g_3^2 \sum_{a,b,i,j,B} \lambda^A_{a,b}\sigma^B_{i,j}\sigma^B_{j,i}\lambda^C_{b,a}$$

SUSY vs Non-SUSY RGEs

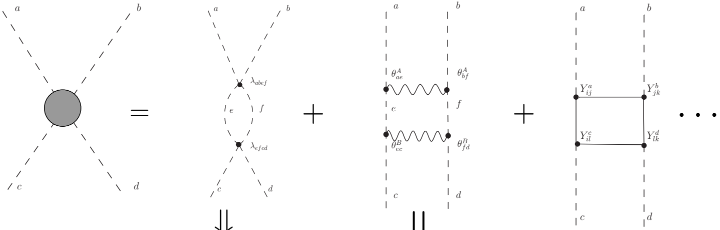
- Non SUSY case ⇒Quartic Terms
- Expressions more involved ⇒ more time consuming
- One needs the explicit matrices of the representation for the scalars and fermions:
  - $D_\mu\phi_a = \partial_\mu\phi_a - ig\theta^A_{ab}V^A_\mu\phi_b$
- θ^A_{ab} assumed purely imaginary and antisymmetric in the calculation. ⇒ Hermitian Basis
  - complex hermitian field with n components ⇒ $2n$ components real vector transforming as

$$L_i = \frac{1}{2}\begin{pmatrix} \tilde{L}_i - \tilde{L}_i^* & i(\tilde{L}_i + \tilde{L}_i^*) \\ -i(\tilde{L}_i + \tilde{L}_i^*) & \tilde{L}_i - \tilde{L}_i^* \end{pmatrix}$$

$$L^1_{\phi_h} = \frac{i}{2}\begin{pmatrix} 0 & \tau^1 \\ -\tau^1 & 0 \end{pmatrix}, L^2_{\phi_h} = \frac{1}{2}\begin{pmatrix} \tau^2 & 0 \\ 0 & \tau^2 \end{pmatrix}, L^3_{\phi_h} = \frac{i}{2}\begin{pmatrix} 0 & \tau^3 \\ -\tau^3 & 0 \end{pmatrix}$$

$$\phi_h = (\phi_1, \phi_2, \phi_3, \phi_4)^T, \phi^+ = (\phi_1 + i\phi_2)/\sqrt{2}, \phi^0 = (\phi_3 + i\phi^4)/\sqrt{2}$$

The Quartic Terms

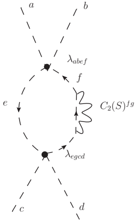


$$\sim \sum_{perm} \lambda_{abef}\lambda_{efcd}$$

$$\sim \sum_{perms,k,l} g^{2k} g^{2l} \{\theta^A, \theta^B\}_{ab} \{\theta^A, \theta^B\}_{cd}$$

$$\sim \sum_{perms} \sum_{i,j,k,l} Y^a_{ij} Y^{b\dagger}_{jk} Y^c_{kl} Y^{d\dagger}_{li}$$

$+$



$\cdots$

$$\sim \sum_{perm} g^2 C_2^{fg}(S) \lambda_{abef}\lambda_{cdeg}$$

Summary

What are the different ingredients needed ?

- C_2, S_2 for all the representations involved
- θ^A, t^A matrix representation for the scalars and fermions
- Contract the different terms in the Lagrangian into singlets :
  - CGCs, database built from Susyno arxiv: 1106.5016
- Replacement rules to go from single gauge group factor to product :
  - $G \rightarrow G_1 \times G_2 \times \cdots \times G_n$
  - e.g. $g^4 C_2(R) C_2(R') \rightarrow \sum_{k,l} g_k^2 g_l^2 C_2^k(R) C_2^l(R')$

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Main features

- Public code for any non-SUSY theories, RGEs at 2-loop .
- Version 1.0.2 is out : http://pyrate.hepforge.org
- Gauge Groups : $U(1); SU(n), n = 2, ..., 6$ (no kinetic mixing).
- shell and interactive mode (IPython notebook)

Validation

- Collaborator F. Staub implemented same RGEs in SARAH 4, arXiv: 1309.7223 $\Rightarrow$ independent cross check.
- All the models from C. Cheung et al. JHEP 1207 (2012) 105
- Cross checking the beta functions that are not in the SM :
  - SM + one real scalar field $\Rightarrow$ Trilinear term
  - SM + t' vector like quark $\Rightarrow$ Fermion mass term

Future developments :

- Extend the group part i.e. more groups, more irreps
- Generation indices for scalars
- Multiple $U(1) \Rightarrow$ Kinetic mixing
- Running of the vevs, arXiv: 1305.1548
- Include available three loops results

Structure of PyR@TE

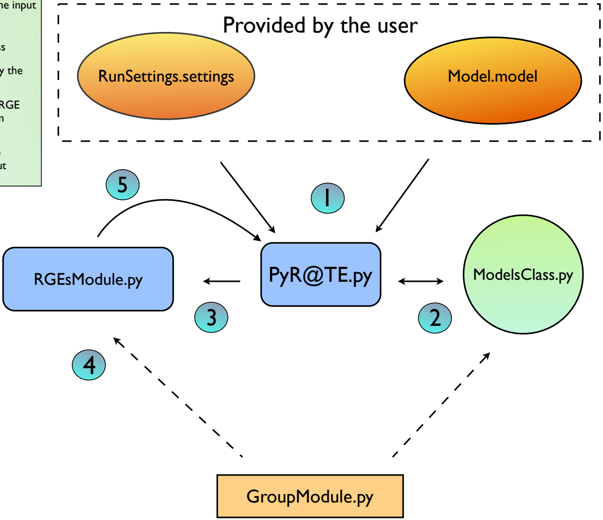
1 : read the settings from the input files

2 : construct the Model class

3 : each RGE is calculated by the RGEsModule

4 : the calculation of each RGE requires information from GroupModule.py

5 : the result is sent back to PyR@TE.py for final output



Input files

- *.model* required to run and *.settings*.

.model

- we are using text files for the input (YAML)
- keys :
  - Author Date Name

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  - *Fermions*: Label: Gen: ng, Qnb:{GroupName:value}

    e.g. Q: Gen: ng, Qnb:{ U1: 1/6, SU2L: 2, SU3c: 3}

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  - Potential ⇒ is given in a similar way:

    e.g. Yukawas: Y_u : $Fields$: $[Qbar, H*, uR]$, $Norm : 1$

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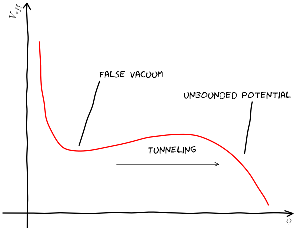
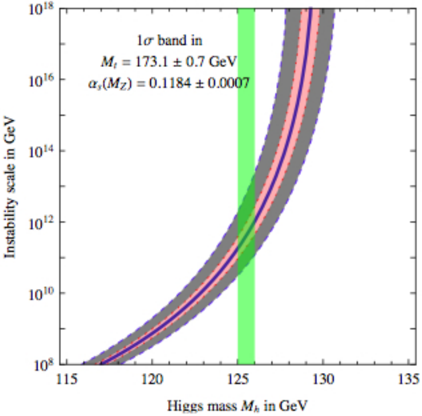
Stability bound

- Our existence demands that the minimum of the EW potential be stable !
  - Stable : Only one minimum
  - meta-Stable : Two minima but $\tau_{minimum} >$ age of the universe $\Rightarrow$ avoid tunneling !
  - Potential Stable up to scale $\Lambda \Leftrightarrow \lambda(\Lambda) > 0$
- $\lambda(\mu)$ calculated from β_λ, depends on $m_H, m_t, \ldots$
- Stability bound :
  - $m_H(m_Z), m_t(m_Z), \ldots$
  - Calculate the RGEs (PyR@TE !) and solve them $\Rightarrow \Lambda_{max}!\lambda(\Lambda_{max}) = 0; (m_H(m_Z), \Lambda_{max})$



Stability bound SM



- State of the art : NNLO, G. Degrassi et al JHEP 1208 (2012) 098
  - Two-loop potential improved
  - Three-loop gauge couplings beta function
  - Leading three-loop contribution to λ and top yukawa
- Absolute stability of the Higgs Potential excluded at 98% C.L. for $M_h < 126\text{GeV}$
- Inflation tends to disfavor the meta-stability, A. Kobakhidze et al. arXiv:1301.2846v2 [hep-ph]

Vector like t' model

Vector like quarks

Simple extension of the SM

- One vector like $t' \sim (3,1)_{2/3} \Rightarrow$ vector like mass.

Lagrangian

$$\mathcal{L} \supset -\underbrace{m_t {t'_L}^\dagger t'_R}_{\beta_{m_t}} - \underbrace{Y_t^i Q^{i\dagger} H^c t'_R}_{\beta_{Y_t}}$$

- t' modifies the RGEs $\Rightarrow Y_t$ enters β_λ at 1-loop.

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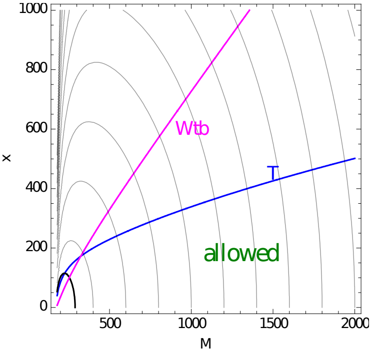
$\Rightarrow$ Time to have a look at PyR@TE!

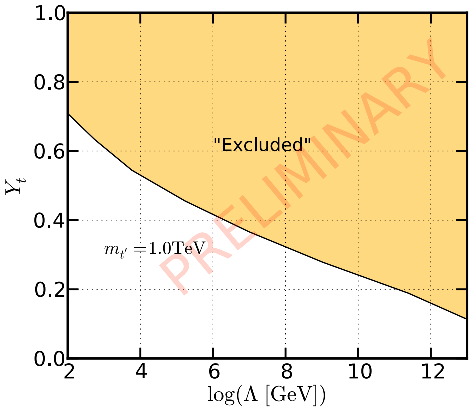
Constrains

- Constrains from Wtb and T parameter by G. Cacciapaglia et al. *JHEP11(2010)159*
- $x \sim \frac{y_t v}{\sqrt{2}}, M = 1\text{TeV} \Rightarrow y_t \sim 1.06$



Stability Bound

- Estimated the stability bound for this model.
- Impose the higgs mass : $m_H \sim 125 GeV$
- No matching corrections for now.
- Possibility of extracting constrains in the plane (Y_t, Λ).



Conclusion and outlook

- For a more systematic study of non SUSY models RGEs are needed.
- We developed a tool that generates the RGEs @2-loop $\Rightarrow$ PyR@TE
- Have fun !