Heterotic Orbifold and Resolution Model Building

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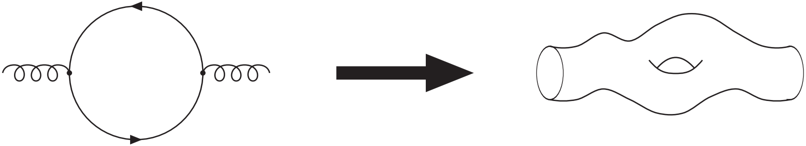
Based on work with: N. Cabo Bizet, S. Groot Nibbelink, H.P. Nilles, M. Ratz, F. Rühle, M.Trapletti, P.K.S. Vaudrevange

Liverpool, 11.12.13

Motivation

- SUSY: hirachy problem, gauge coupling unification,...
- GUTs: multiplet structure, gauge coupling unification,...
- discrete symmetries: Proton stability, mu-problem,...
- extra dimensions: origin of symmetires, local GUTs,...
- SUGRA: incorporation of gravity
- ...

Heterotic String



provides:

- $E_8 \times E_8$ gauge group
- UV complete theory of gravity
- $N = 1$ SUSY in $d = 10$
- Properties of low energy effective theory from compactification

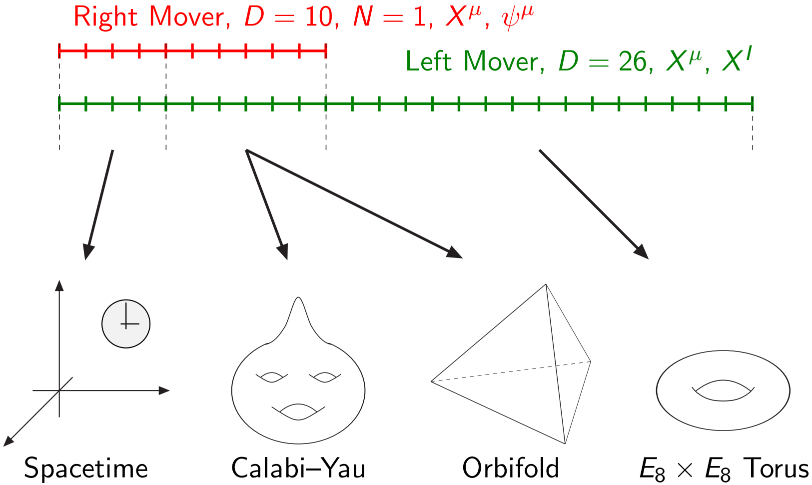
Outline

- Heterotic Strings on Orbifolds
- Resolutions
- Phenomenology

Heterotic String

- String Theories: Super Conformal 2d Quantum Field Theories
- Heterotic String: Gauged $N = (1,0)$ SUSY, closed worldsheet $\Rightarrow$ left-/right movers decouple:
  - Right Movers $\cong$ $10d$ target space with $N = 1$ SUSY
  - Left Movers $\cong$ $26d$ bosonic target space
  - $\Rightarrow$ compactify 16 leftover dimensions of $T^{16}_{E_8 \times E_8}$
- In perturbative heterotic String: $M_{\text{string}} \sim 10^{17}\, GeV$ $\Rightarrow$ look at low energy theory
- $10d$ Minkowski spectrum: $N = 1$ SUGRA:
  - SUGRA multiplet: graviton $g_{\mu\nu}$, Kalb–Ramond $B_{\mu\nu}$, dilaton ϕ
  - $E_8 \times E_8$ gauge multiplet A^a_μ, λ^a

Heterotic String



Heterotic String Compactification

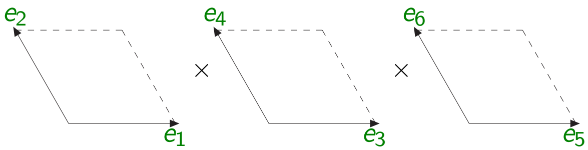
- Need to compactify
  $(10d, N = 1, E_8 \times E_8) \longmapsto (4d, N = 1, G_{\mathrm{SM}} \times G_{\mathrm{hidden}})$
- problem: for generic metric: quantization very difficult
- ways out:
  - flat metric: soluable free CFT $\Rightarrow$ **Toroidal Orbifold** $T^6/\mathbb{Z}_N$
  - take SUGRA approximation: smooth CY compactification, (later)
  - other exactly soluable models: free fermionic, Gepner,...

$\mathbb{Z}_3$ Orbifold

Dixon, Harvey, Vafa, Witten; Ibanez, Nilles, Quevedo

Torus

$$T^6 = \mathbb{C}^3/\Lambda\,, \qquad \Lambda = \left\{ n_i e_i \middle| n_i \in \mathbb{Z} \right\}$$



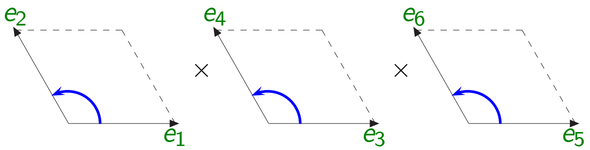
Dixon, Harvey, Vafa, Witten; Ibanez, Nilles, Quevedo

$\mathbb{Z}_3$ Orbifold = Torus + mod out Point Group P

$$T^6 = \mathbb{C}^3/\Lambda\,, \qquad \Lambda = \left\{ n_i e_i \middle| n_i \in \mathbb{Z} \right\}$$

$$\mathcal{O} = T^6/P\,, \qquad P = \left\{1, \theta, \theta^2\right\} \cong \mathbb{Z}_3$$

$$\theta : (z_1, z_2, z_3) \to \left(e^{2\pi \mathrm{i}/3} z_1, e^{2\pi \mathrm{i}/3} z_2, e^{2\pi \mathrm{i}/3} z_3\right)$$



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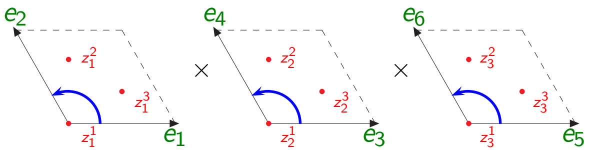


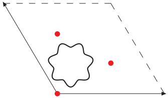
27 Fixed Points = Singularities :

$$F_{\alpha\beta\gamma} = \{z_1 = z_1^{\alpha},\ z_2 = z_2^{\beta},\ z_3 = z_3^{\gamma}\}$$

Embed Space Group $S = P \ltimes \Lambda$ into gauge group
$\rightarrow$ discrete **Shift Vectors** V_i and discrete **Wilson Lines** W_i

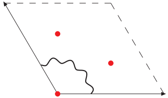
- break Gauge Group and determine N=1 chiral Spectrum
- must satisfy Modular Invariance

Untwisted String



- SUGRA Multiplet
- Gauge Multiplets
- untwisted Moduli
- few chiral Matter

Twisted String



- lots of Matter
- can act as **Blow-up modes**

Orbifold MSSM models

- $\mathbb{Z}_3$ models

  Ibañez, Kim, Nilles, Quevedo; Casas, Munoz

- $\mathbb{Z}_7$ models

  Casas, de la Maccora, Mondragon, Munoz

- $\mathbb{Z}_{6-II}$ models ("Mini-Landscape")

  Buchmüller, Hamaguchi, Lebedev, Ratz; Lebedev, Nilles, Raby, Ramos-Sánchez, Ratz, Vaudrevange, Wingerter

- $\mathbb{Z}_{12-II}$ model

  Kim, Kim, Kyae

- $\mathbb{Z}_2 \times \mathbb{Z}_2 \times \mathbb{Z}_{2,\text{free}}$ model

  M.B., Groot Nibbelink, Ratz, Rühle, Trapletti, Vaudrevange

- $\mathbb{Z}_2 \times \mathbb{Z}_4$ models

  Mayorga–Peña, Nilles, Oehlmann

Properties of Orbifold Models

- local GUTs: complete vs. split multiplets (Matter vs. Higgs)
- discrete symmetries (R and non-R) with geometric origin
- hidden sector dynamics (2^{nd} E_8 factor) for potential SUSY breaking at low scale via gaugino condensate
- big parameter space by choice of space group, shift vector, Wilson lines
- full computability of 4d effective action in string perturbation theory

Outline

- Heterotic Strings on Orbifolds
- Resolutions
- Phenomenology

Motivation for Resolutions

- Orbifold anomalous $U(1)$ induces FI term $\rightarrow$ requires vevs to restore SUSY
- Many Exotic states at Orbifold point need to be decoupled
- Huge Symmetry needs to be reduced
- Vevs of twisted fields $\Rightarrow$ geometric backreaction

Attempts:

- K3 orbifold resolutions

  Honecker, Trapletti; Correia, Groot Nibbelink, Trapletti

- Local $\mathbb{C}^3/\mathbb{Z}_N$ resolutions

  Groot Nibbelink, Ha, Trapletti, Walter

- Global $T^6/\mathbb{Z}_N$ resolutions by gluing

  Lust, Reffert, Scheidegger, Stieberger;

  M.B., Cabo Bizet, Groot Nibbelink, Held, Klevers, Nilles, Plöger, Rühle, Vaudrevange, Trapletti

- Global $T^6/\mathbb{Z}_N$ resolutions as Complete Intersections

  M.B., Groot Nibbelink, Rühle

Local Singularity Resolution

Resolution of local singularity $\mathbb{C}^3/P$ in toric geometry:

- add coordinates x_r and $\mathbb{C}^*$ scalings: $(z_i, x_r) \sim (\lambda^{q_i} z_i, \lambda^{q_r} x_r)$ with charges q
- $x_r \neq 0$ breakes $\mathbb{C}^{*N}$ to point group P $\Rightarrow U = \{x, z \neq 0\}$ looks like vicinity of singularity
- for blow-up: singularity $\{z_i = 0\}$ replaced by smooth divisors $E_r = \{x_r = 0\}$
- size controlled by Kähler moduli $J = \sum_r b_r E_r$

$$\Rightarrow \mathrm{Res}(\mathbb{C}^3/P) = \frac{\mathbb{C}^{3+N} - Z}{(\mathbb{C}^*)^N}$$

$Z =$ exclusion set to guarantee smooth space, not unique!

Gauged Linear Sigma Models

- $N = (2,0)$ or $N = (2,2)$ SUSY models in $d = 2$ with gauge group $U(1)^N$
- D–terms + $U(1)^N$ gaugings $\rightarrow$ toric ambient spaces $\mathcal{A} \approx \mathbb{C}^n/(\mathbb{C}^*)^N$
- F–terms $\rightarrow$ holomorphic hypersurface
  $\rightarrow$ target space = Calabi–Yau with monad vector bundle Witten
- gauge couplings and superpotential parameters dimensionful
  $\rightarrow$ take conformal limit to NLSM
- gauge anomalies can appear, have to be cancelled

M.B., S. Groot Nibbelink, F. Rühle; Quigley, Sethi

Local Orbifold Resolutions as GLSM: $\mathbb{C}^3/\mathbb{Z}_3$

S. Groot Nibbelink

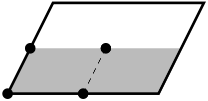
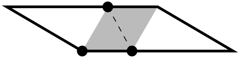
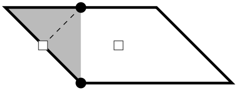
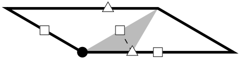
	$z_i,\ i=1,2,3$	x	Λ^a
charge	1	-3	Q^a

- D-term: $|z_1|^2+|z_2|^2+|z_3|^3-3|x|^2-b=0$
  - $b<0 \ \rightarrow\ x\neq 0\ \rightarrow\ \mathbb{Z}_3$ action on $z_i\ \rightarrow\ \mathbb{C}^3/\mathbb{Z}_3$
  - $b>0 \rightarrow \{z_1=z_2=z_3=0\}$ removed, exceptional divisor $\{x=0\}$ resolves singularity

blowup mode = twisted oscillatorless state with charge $Q^a \leftrightarrow$ oscillator blowup mode, multiple blowup modes

Orbifold CFT	GLSM	SUGRA
modular invariance	anomaly freedom	Bianchi Identity
$V_{\rm loc}=kV+n_\alpha W_\alpha$	$\mathcal{V}=\bigoplus\mathcal{O}(q_i)\,,q_i\in\mathbb{Z}$	flux quantization

GLSMs for Elliptic Curves

Point Group	Geometry	*n*-volution	Elliptic Curve
$\mathbb{Z}_2$		$\mathbb{Z}_2 \times \mathbb{Z}_2$	$\mathbb{P}^3_{(1,1,1,1)}[2,2]$
$\mathbb{Z}_3$		$\mathbb{Z}_3$	$\mathbb{P}^2_{(1,1,1)}[3]$
$\mathbb{Z}_4$		$\mathbb{Z}_2$	$\mathbb{P}^2_{(1,1,2)}[4]$
$\mathbb{Z}_6$		–	$\mathbb{P}^2_{(1,2,3)}[6]$

Global Orbifold Resolutions

- write $T^6 = T_1^2 \times T_2^2 \times T_3^2$, T_i^2 suitably chosen elliptic curves
  $\rightarrow$ homogeneous coordinates $z_{i,\rho}$
- add exceptional coordinates $x_{k,\alpha\beta\gamma}$ and $U(1)_{k,\alpha\beta\gamma}$ gaugings
  - discrete group action gets induced:
    point group $\times$ discrete n-volution
  - FI-parameters $b_{k,\alpha\beta\gamma}$ control blowup process
    $\rightarrow$ various phases accessible
- choice of x_r coordinates can
  - switch Wilson lines on/off ($\rightarrow$ discrete family symmetries)
  - blowup more or fewer fixed points
  - create non-factorizable Orbifolds

Example: $T^6/\mathbb{Z}_3$

	$z_{1,\alpha}$	$z_{2,\beta}$	$z_{3,\gamma}$
$U(1)_1$	1	0	0
$U(1)_2$	0	1	0
$U(1)_3$	0	0	1

$\alpha, \beta, \gamma = 1, 2, 3$

- Torus $T^6 = \mathbb{P}^2[3] \times \mathbb{P}^2[3] \times \mathbb{P}^2[3]$

Example: $T^6/\mathbb{Z}_3$

	$z_{1,\alpha}$	$z_{2,\beta}$	$z_{3,\gamma}$	x_{111}
$U(1)_1$	1	0	0	0
$U(1)_2$	0	1	0	0
$U(1)_3$	0	0	1	0
$U(1)_{111}$	$\delta_{\alpha 1}$	$\delta_{\beta 1}$	$\delta_{\gamma 1}$	-3

- Orbifold $\mathbb{P}^2[3] \times \mathbb{P}^2[3] \times \mathbb{P}^2[3] \ / \ \mathbb{Z}_{3,\text{Orbi}}$

Aspinwall, Plesser

- one blowup mode for all 27 fixed points simultaneously
- multiplicity 27 via factorization of F-terms

Example: $T^6/\mathbb{Z}_3$

	$z_{1,\alpha}$	$z_{2,\beta}$	$z_{3,\gamma}$	x_{111}	x_{211}
$U(1)_1$	1	0	0	0	0
$U(1)_2$	0	1	0	0	0
$U(1)_3$	0	0	1	0	0
$U(1)_{111}$	$\delta_{\alpha 1}$	$\delta_{\beta 1}$	$\delta_{\gamma 1}$	-3	0
$U(1)_{211}$	$\delta_{\alpha 2}$	$\delta_{\beta 1}$	$\delta_{\gamma 1}$	0	-3

- Orbifold $\mathbb{P}^2[3] \times \mathbb{P}^2[3] \times \mathbb{P}^2[3]$ / $\mathbb{Z}_{3,\text{Orbi}} \times \mathbb{Z}_{3,\text{vol}}$
- Wilson line in first torus switched on
- fixed points/exceptional divisors appear in 3 groups of 9 each
- fixed points at $z_{1,3} = z_{2,1} = z_{3,1} = 0$ cannot be resolved $\rightarrow$ need coordinate x_{311}

Example: $T^6/\mathbb{Z}_3$

	$z_{1,\alpha}$	$z_{2,\beta}$	$z_{3,\gamma}$	x_{111}	x_{222}
$U(1)_1$	1	0	0	0	0
$U(1)_2$	0	1	0	0	0
$U(1)_3$	0	0	1	0	0
$U(1)_{111}$	$\delta_{\alpha 1}$	$\delta_{\beta 1}$	$\delta_{\gamma 1}$	-3	0
$U(1)_{222}$	$\delta_{\alpha 2}$	$\delta_{\beta 2}$	$\delta_{\gamma 2}$	0	-3

- Orbifold $\mathbb{P}^2[3] \times \mathbb{P}^2[3] \times \mathbb{P}^2[3] \;/\; \mathbb{Z}_{3,\mathrm{Orbi}} \times \mathbb{Z}_{3-\mathrm{vol}-\mathrm{non}-\mathrm{fac.}}$
- $\mathbb{Z}_{3-\mathrm{vol}-\mathrm{non}-\mathrm{fac.}}$ makes torus non-factorizable, (E_6 lattice)

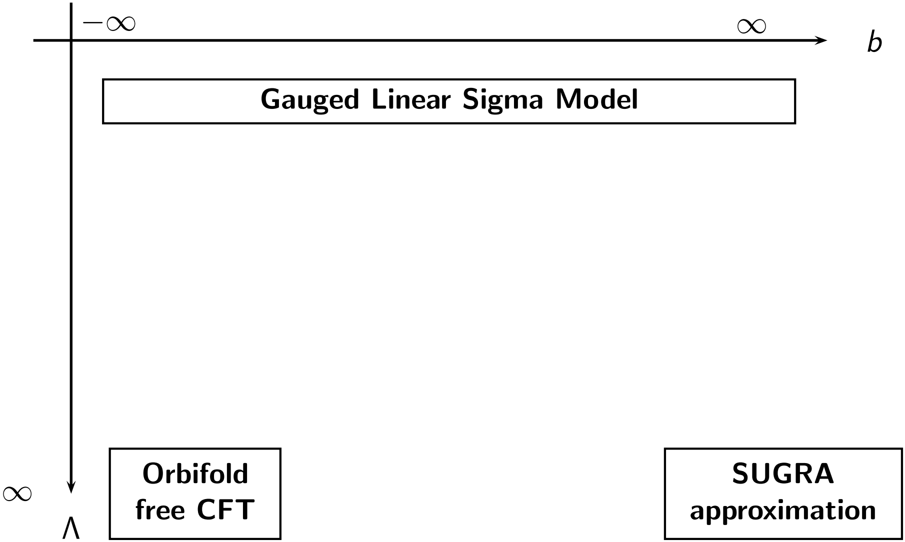
Example: $T^6/\mathbb{Z}_3$

	$z_{1,\alpha}$	$z_{2,\beta}$	$z_{3,\gamma}$	$x_{\alpha\beta\gamma}$
$U(1)_1$	1	0	0	0
$U(1)_2$	0	1	0	0
$U(1)_3$	0	0	1	0
$U(1)_{\alpha'\beta'\gamma'}$	$\delta_{\alpha\alpha'}$	$\delta_{\beta\beta'}$	$\delta_{\gamma\gamma'}$	$-3\delta_{\alpha\alpha'}\delta_{\beta\beta'}\delta_{\gamma\gamma'}$

$\alpha', \beta', \gamma' = 1, 2, 3$

- Orbifold $\mathbb{P}^2[3] \times \mathbb{P}^2[3] \times \mathbb{P}^2[3] \ / \ \mathbb{Z}_{3,\mathrm{Orbi}} \times \mathbb{Z}^3_{3-\mathrm{vol}}$
- all Wilson lines available
- every fixed point has own blowup mode $b_{\alpha\beta\gamma}$

"Moduli Space" of Theories



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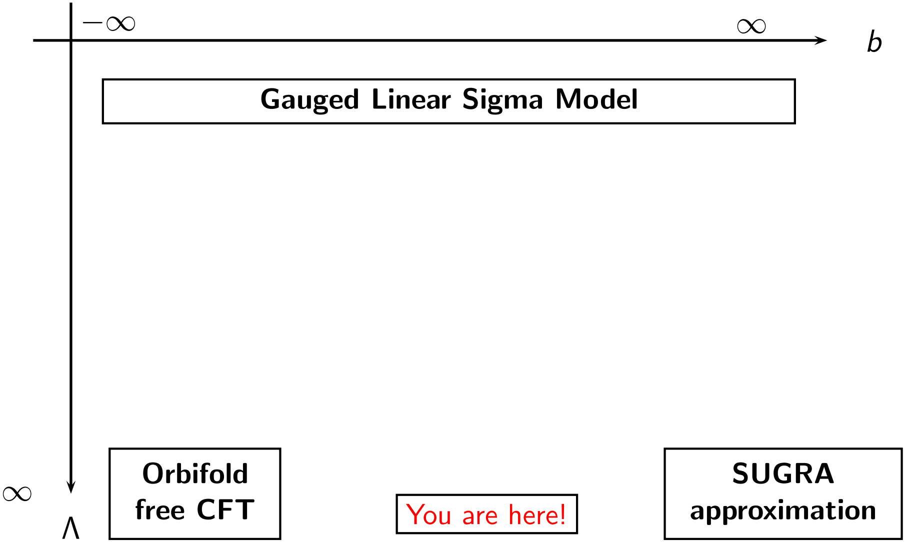
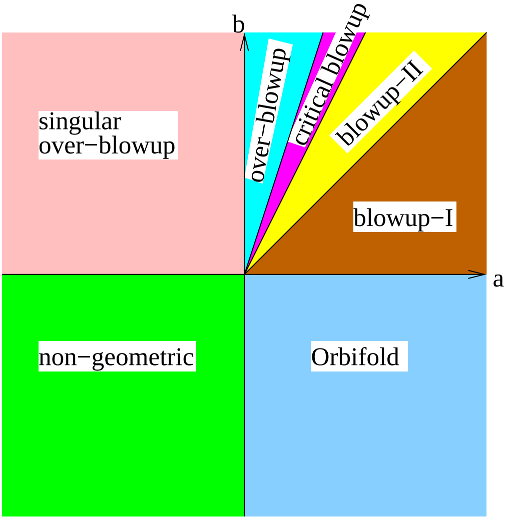


Moduli Space of (2, 2) Minimal $T^6/\mathbb{Z}_3$ Model



- $a \cong$ size of the two-tori
- $b \cong$ blow-up mode $\cong$ size of exceptional divisor

GLSMs for $T^6/\mathbb{Z}_N$ and $T^6/\mathbb{Z}_N \times \mathbb{Z}_M$ resolutions

Point group	twist vector(s)	T^6 torus lattice	exc. gaugings	Invisible moduli $h^{1,1}_{\text{off-diag}}$	Invisible moduli $h^{1,2}_{\text{twisted}}$	Indistinguishable fixed points/tori
$\mathbb{Z}_3$	$\frac{1}{3}(1,1,-2)$	A_2^3	27	6	0	0
$\mathbb{Z}_4$	$\frac{1}{4}(1,1,-2)$	$D_2^2 \times A_1^2$ $D_2 \times A_1 \times A_3$ A_3^2	23 6 8	2 2 2	6 2 0	1×2 FT 2×8 FP, 2×2 FT 4×4 FP
$\mathbb{Z}_{6-\mathrm{I}}$	$\frac{1}{6}(1,1,-2)$	$G_2^2 \times A_2$	17	2	5	1×3 FT , 1×2 FP
$\mathbb{Z}_{6-\mathrm{II}}$	$\frac{1}{6}(1,2,-3)$	$G_2 \times A_2 \times A_1^2$	32	0	10	0
$\mathbb{Z}_2 \times \mathbb{Z}_2$	$\frac{1}{2}(1,-1,0)$, $\frac{1}{2}(0,1,-1)$	A_1^6	48	0	0	0
$\mathbb{Z}_2 \times \mathbb{Z}_4$	$\frac{1}{2}(0,1,-1)$, $\frac{1}{4}(1,-1,0)$	$D_2^2 \times A_1^2$	57	0	0	1×2 FT
$\mathbb{Z}_2 \times \mathbb{Z}_{6-\mathrm{I}}$	$\frac{1}{2}(0,1,-1)$, $\frac{1}{6}(1,1,-2)$	G_2^3	26	0	0	3×3 FT , 1×2FP
$\mathbb{Z}_2 \times \mathbb{Z}_{6-\mathrm{II}}$	$\frac{1}{6}(1,-1,0)$, $\frac{1}{2}(0,1,-1)$	$G_2^2 \times A_2$	46	0	2	1×3 FT
$\mathbb{Z}_3 \times \mathbb{Z}_3$	$\frac{1}{3}(1,-1,0)$, $\frac{1}{3}(0,1,-1)$	A_2^3	81	0	0	0
$\mathbb{Z}_3 \times \mathbb{Z}_6$	$\frac{1}{3}(0,1,-1)$, $\frac{1}{6}(1,-1,0)$	$G_2^2 \times A_2$	65	0	1	2×2 FT , 3×2 FP
$\mathbb{Z}_4 \times \mathbb{Z}_4$	$\frac{1}{4}(1,-1,0)$, $\frac{1}{4}(0,1,-1)$	D_2^3	87	0	0	0
$\mathbb{Z}_6 \times \mathbb{Z}_6$	$\frac{1}{6}(1,-1,0)$, $\frac{1}{6}(0,1,-1)$	G_2^3	80	0	0	1×2 FP

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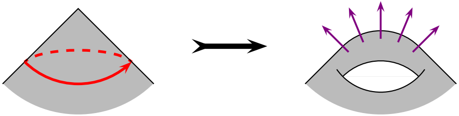
Calabi–Yau model building

- valid in SUGRA limit, α' corrections
- dimensionally reduce $N = 1$ $d = 10$ SUGRA on CY three-fold $\mathcal{M}$ with stable vector bundle $\mathcal{V}$

$$\mathrm{Adj}_{E_8 \times E_8} \mapsto \underbrace{\mathrm{Adj}_G \otimes 1}_{\text{gauge multiplets}} \oplus \underbrace{1 \otimes \mathrm{Adj}_H}_{\text{bundle moduli}} \oplus \bigoplus_i \underbrace{R_i \otimes \tilde{R}_i}_{\text{chirals}}$$

- Properties mainly determined by topology of $\mathcal{M}$, $\mathcal{V}$ like cohomology groups $H^{p,q}$, intersection numbers
- Consistency:
  - Bianchi Identity: $\mathrm{d}H_3 = \mathrm{tr}\,\mathcal{R}^2 - \mathrm{tr}\,\mathcal{F}^2$
  - HYM equations: $\mathcal{F}_{ab} = \mathcal{F}_{\bar{a}\bar{b}} = 0$, $g^{a\bar{b}}\mathcal{F}_{a\bar{b}} = 0$

$\mathbb{Z}_{6-II}$ Resolution Phenomenology



- local shifts $V_{\text{loc}} = kV + n_\alpha W_\alpha$ are gauge potential $\mathcal{A}$ around singularity
- in blow-up: line bundle corresponding to gauge flux $\mathcal{F} = V_r E_r$
- Mini-Landscape: GUT $\to$ SM breaking via Wilson Line
  $\Rightarrow \mathcal{F}_Y \neq 0$ in blow up
  $\Rightarrow$ corresponding axions $\beta_r = B_2\big|_{E_r}$ break Hypercharge via Stückelberg mechanism

Groot Nibbelink, Held, Rühle, Trapletti, Vaudrevange

- no full resolution possible

$\mathbb{Z}_2 \times \mathbb{Z}_2 \times \mathbb{Z}_{2,\mathrm{free}}$ Resolution Phenomenology

Way out: non-local GUT breaking

M.B., Groot Nibbelink, Rühle, Trapletti, Vaudrevange

- 6 family $SU(5)$ model on $\mathbb{Z}_2 \times \mathbb{Z}_2$ Orbifold
- mod out freely acting $\mathbb{Z}_{2,\mathrm{free}}$ to break $SU(5) \to G_{\mathrm{SM}}$ via associated Wilson Line
  $\Rightarrow \mathcal{F} \perp U(1)_Y$ in full blowup
- full resolution line bundle model still difficult:
  - Bianchi Identities very restrictive
  - HYM equations ($\cong$ D-flatness) at 1-loop: Divisor Volumes finite, SUGRA not trustable,
    true model between Orbifold and SUGRA point

Matching of Models

Idea: match Orbifold and Resolution models
$\rightarrow$ what happens in between?

M.B., Cabo Bizet, Groot Nibbelink, Nilles, Rühle, Trapletti, Vaudrevange

- Blow-up mode becomes Kähler modulus + axion
  $\Psi_{\mathrm{BU-mode}} \propto e^{b_r + \mathrm{i}\beta_r}$
- chiral states get redefined $\Psi_{\mathrm{Orb}} = e^{b_r + \mathrm{i}\beta_r} \Psi_{\mathrm{Res}}$
- On Orbifold: discrete torsion alters spectrum; Which is correct model?
- On Resolution: topology not completely fixed, flop transitions: states can appear/disappear
  $\Rightarrow$ instantonic mass terms $\mathcal{W} \supset e^{-\mathrm{Vol}(\mathcal{C})} \Psi_1 \Psi_2$
- in SUGRA: symmetries (like R-symmetries) that are broken by string effects
  $\Rightarrow$ SUGRA sees too many massless states

Matching of Anomalies

- On Orbifold: one anomalous $U(1)$, cancelled by Green–Schwarz mechanism with universal axion a_{Orb}
- On Resolution: many anomalous $U(1)$'s, many axions to cancel anomaly
- 't Hooft anomaly matching: Anomalies must coincide!
- $\Rightarrow$ Extra contribution from redefinition
  Axion Matching:
  - β_r = phase of BU-mode
  - $a_{\mathrm{uni}} = B_2\big|_{4d} \cong a_{\mathrm{Orb}} + d_r \beta_r$

  $\Rightarrow$ agreement of Spectrum and Anomalies!

Outlook

- precise map between orbifold CFT and $(2,0)$ GLSM
- GLSM techniques to understand 4d theory in any phase
- Construction of more general vector bundles for general vev configuration
- Incorporate non-perturbative effects, torsion,..

Thank You for Your Attention!