Spectral flow as a map between (2,0) models

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Introductory concepts

The two viewpoints in string theory

One can think about string theory in two different ways:

First viewpoint

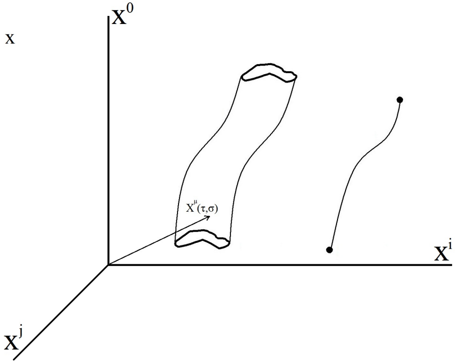
The spacetime viewpoint.

Second viewpoint

The worldsheet viewpoint.

The spacetime viewpoint

1. The string lives in D dimensions.
2. It spans a worldsheet (ws).
3. A point in the ws is described as $X^{\mu}(\tau, \sigma)$.
4. No consistent actions unless $D > 4$.
5. We have to explain why we don't observe these extra dimensions.



The worldsheet viewpoint

1. Ask what is the most general CFT we can write for a 2d surface...
2. ...assuming it satisfies the Virasoro algebra.
3. Consistency requires $c = 26$ for the bosonic string and $c = 15$ for the superstring.
4. We can split these as:

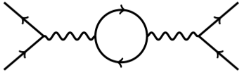
$$\begin{aligned} \text{Bosonic:} \quad & \text{CFT}_{c=26} &=& \ \text{CFT}_{c=4} \oplus \text{CFT}_{\text{internal},c=22} \\ \text{Superstring:} \quad & \text{CFT}_{c=15} &=& \ \text{CFT}_{c=6} \oplus \text{CFT}_{\text{internal},c=9} \end{aligned}$$

5. The internal CFT can be anything (**geometric** vs **non-geometric** theories).

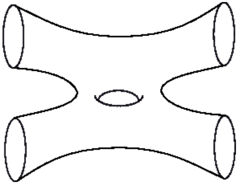
Modular invariance

One of the most important concepts in string theory.

1. In QFT loop calculations involve integrations over a circle.

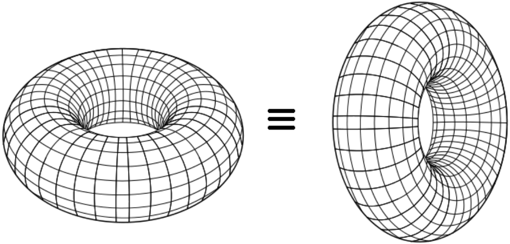


2. In string theory loop calculations involve integrations over a 2d torus.



Modular invariance

The symmetry group of the torus is $SL(2,\mathbb{Z})$ which is much bigger than the $U(1)$ symmetry of the circle. For example the following tori should be identified:



This is **VERY** restrictive!

Modular invariance

A string model is consistent whenever all physical quantities are invariant under these symmetries. (This is the definition of **modular invariance**.)

We routinely check that this is the case by looking at the simplest quantity: The integrand of the 1-loop vacuum-to-vacuum amplitude which we call the **partition function**.

Spectral flow as a map between (2,0) models

Motivation

- In heterotic string theory $(2,0)$ models play a prominent role.
- They are supersymmetric and they are in a sense "minimal" since supersymmetry requires (at least) $(2,0)$ superconformal symmetry.
- They naturally allow for $SO(10)$ unification, which is well motivated by the standard model data.
- Some of the most realistic heterotic string models including orbifolds and free fermionic models are of this type.

Spectral flow as a map between (2,0) models

By definition a CFT is said to have $N = 2$ world-sheet supersymmetry if it includes four fields:

$$T(z)\ ,\ G^{\pm}(z)\ ,\ J(z),$$

that satisfy the algebra:

$$\begin{aligned}
[L_m, L_n] &= (m-n)L_{m+n} + \frac{c}{12}(m^3 - m)\delta_{m+n,0}\ , \\
\left[L_m, G^{\pm}_{n\pm a}\right] &= (\frac{m}{2} - n \mp a)G^{\pm}_{m+n\pm a}\ , \\
[L_m, J_n] &= -nJ_{m+n}\ , \\
&\vdots
\end{aligned}$$

Motivation from spinor-vector duality

- $(2,0)$ models appear after the right $N = 2$ superconformal symmetry of (2,2) models breaks.
- $N = 2$ SCS on the bosonic side is equivalent to E_6 gauge symnmetry.
- When E_6 is broken down to $SO(10)$ the representations decompose as:

$$
\begin{aligned}
\mathbf{27} &= \mathbf{16} + \mathbf{10} + \mathbf{1} \\
\overline{\mathbf{27}} &= \overline{\mathbf{16}} + \mathbf{10} + \mathbf{1}
\end{aligned}
$$

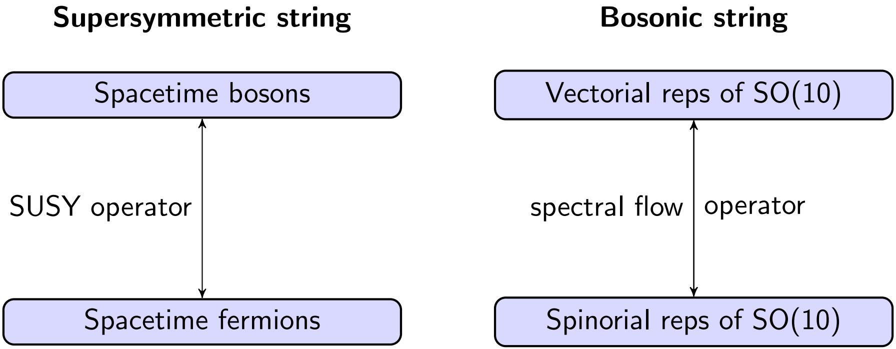
- If the breaking is implemented through a $\mathbb{Z}_2$ or $\mathbb{Z}_2 \times \mathbb{Z}_2$ orbifold we naturally end up with two models: One with massless spinors and one with massless vectors and singlets. (Spinor-vector duality)
- This agrees with our intuition from anomaly cancellation, but can we generalize this result? (Ans: No.)

Spectral flow as a **map** between (2,0) models

Spectral flow as a **map** between (2,0) models

Spectral flow as a map between (2,0) models

Every supersymmetric theory has an operator that relates bosons to fermions. On the bosonic sector of the heterotic string this operator relates vectorial and spinorial representations of $SO(10)$:



The tool: Simple currents

The simple current method of Schellekens and Yankielowicz provides a way to construct new modular invariants from a given one.

This generic CFT construction generalizes both the orbifold and the free fermionic construction.

Simple current modular invariants

For any model with a given partition function:

$$Z[\tau, \bar{\tau}] = \sum_{i,j} \chi_i(\tau) M_{ij} \chi_j(\bar{\tau}),$$

we can construct a new one with

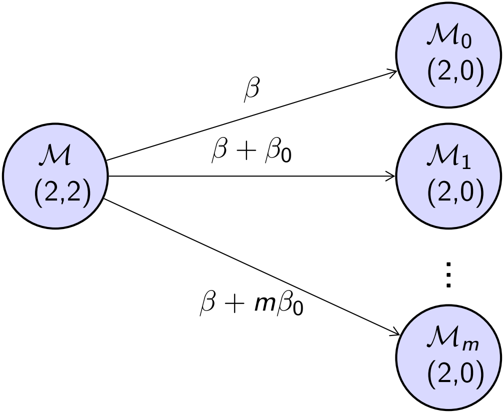
$$Z[\tau, \bar{\tau}] = \sum \chi_i(\tau) M_{ik} M_{kj}(\beta) \chi_j(\bar{\tau}),$$

where

$$M_{kj}(\beta) = \frac{1}{N} \sum_{n=1}^{N_J} \delta(\Phi_k, \Phi_j + n\beta) \delta_{\mathbb{Z}}(Q_\beta(\Phi_k) + \frac{n}{2} Q_\beta(\beta))$$

is called a *simple current modular invariant*. In the new model, some states have been projected out and some new states have appeared as well.

The basic idea



So the (2,2) model $\mathcal{M}$ is naturally associated with an entire family of (2,0) models. We studied how the models within each family are related.

Back where it all started from

Whenever the spectral flow operator is of order 2, we only get two (2,0) models in a family. This is particularly common in many free-fermionic models. This suggests a connection with the spinor-vector duality...

A direct check shows that indeed the previous construction reproduces the spinor-vector duality whenever the spectral flow operator is of order 2.

For these families the models have exactly the property that the massless spectrum of one of the models contains the spinorials of $SO(10)$ and the other one contains the vectorials.

Summary and outlook

1. The space of $(2,0)$ models is of great interest because of the requirement of spacetime SUSY and the accommodation of $SO(10)$ unification.
2. The spinor-vector duality is a very interesting feature of some classes of free fermionic models.
3. Generalizing the idea to arbitrary CFTs we discovered that it is the spectral flow operator that in reality induces a whole family of models.
4. We hope that the viewpoint we propose will prove useful in the long term goal of classifying completely all the $(2,0)$ models.

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Thank you very much!

Heterotic string: notation and conventions

We take the left-moving sector to be supersymmetric and the right-moving sector to be bosonic. A general state in a heterotic model is then of the form:

$$\Phi_L \otimes \Phi_R$$

where

$$\Phi_L = (w_L)(h_L, Q_L), \quad \Phi_R = (w)(h, Q)(p).$$

- w_L is an $SO(2)$ weight (o, v, s, c)
- w is an $SO(10)$ weight (o, v, s, c)
- p an E_8 weight.

The appearance of the $SO(10)$ and E_8 weights is because of the bosonic string map.

Spectral flow as a map between (2,0) models

The $N = 2$ algebra is invariant under the following transformation which is known as the *spectral flow*:

$$\begin{aligned} L_n^\eta &= L_n + \eta J_n + \frac{c}{6}\eta^2 \delta_{n,0} \ , \\ G_{n\pm a}^{\eta\pm} &= G_{n\pm(a+\eta)}^{\eta\pm} \ , \\ J_n^\eta &= J_n + \frac{c}{3}\eta\delta_{n,0}. \end{aligned}$$

This implies the existence of a *spectral flow operator* U_η that acts on states in the following way:

$$U_\eta|h, Q\rangle = |h_\eta, Q_\eta\rangle = \Big|h - \eta Q + \frac{\eta^2 c}{6}, Q - \frac{c\eta}{3}\Big\rangle.$$

Simple current modular invariants

Let us a have a closer look at the SCMI

$$M_{kj}(\beta) = \frac{1}{N}\sum_{n=1}^{N_J}\delta(\Phi_k, \Phi_j + n\beta)\delta_{\mathbb{Z}}(Q_\beta(\Phi_k) + \frac{n}{2}Q_\beta(\beta))$$

In practical terms, the above formula means that:

i) Only states whose left part is connected to the right through β will appear in the partition function, *i.e.* states with $\Phi_L = \Phi_R + n\beta$. This defines the n-th β-twisted sector.

ii) Only states invariant under the projection will appear in the partition function. This is expressed in the constraint $Q_\beta(\Phi) + \frac{n}{2}Q_\beta(\beta) \in \mathbb{Z}$. Q_β is called the monodromy charge and is defined as

$$Q_\beta(\Phi) = h(\Phi) + h(\beta) - h(\Phi + \beta) \qquad \text{mod } 1.$$

Some useful results

It can be proven that:

- The untwisted sector of all the models (w.r.t. to the simple current defining the model) is the same.
- The models will in general have a different number of twisted sectors and are therefore not identical.
- Massless states in the $n-$th twisted sector of the $m-$th model satisfy:

$$Q_\beta(\Phi_L) + \frac{n}{2} Q_\beta(\beta) + mnQ_{\beta_0}(\beta) \in \mathbb{Z},$$

$$n\Big(h(\beta) + \frac{1}{2} Q_\beta(\beta)\Big) \in \mathbb{Z}.$$