Mass Unification

G. Ross, Liverpool, 23 March 2007

Unification

- Gauge Unification

$$g_i(\mu) = g_i(M_{GUT}, M_{SUSY})$$

- Yukawa Unification

$$m_b(M_{GUT}) = m_\tau(M_{GUT})$$

$$\Rightarrow \quad m_{b,\tau}(\mu) = m_{b,\mu}(M_{GUT}, M_{SUSY})$$

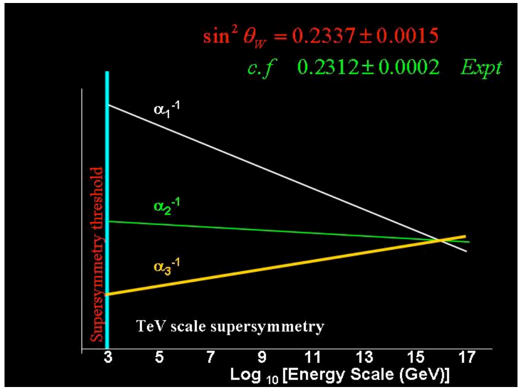
- Soft mass unification

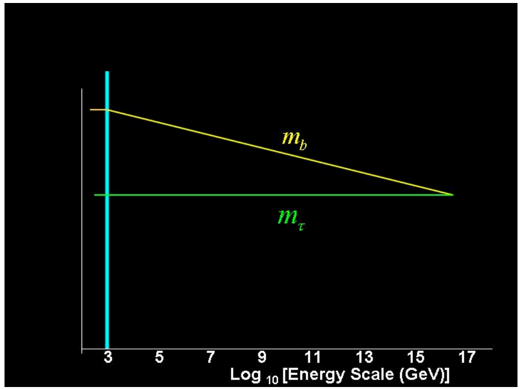
$$\tilde{m}_i(M_{GUT}) = m_0$$

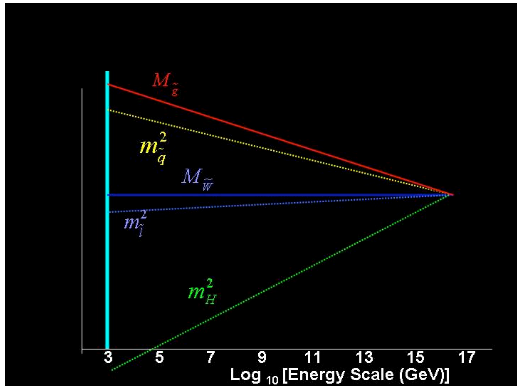
$$\tilde{M}_i(M_{GUT}) = m_{1/2}$$

+3 other "soft" parameters

$$\Rightarrow \quad m_{\phi_i}(\mu) = m_{\phi_i}(M_{GUT}, m_0, m_{1/2}, \ldots)$$







$$SU(3) \otimes SU(2) \otimes U(1) \rightarrow SU(3) \otimes U(1)_{EM}$$

Yukawa Unification

$$\psi_\alpha = \begin{pmatrix} d \\ d \\ d \\ l \end{pmatrix}$$

$$\frac{m_s}{m_\mu}(M_X) = \frac{1}{3}$$

$$\frac{m_d}{m_e}(M_X) = 3$$

Georgi Jarlskog

$$|V_{us}| = \left| \sqrt{\frac{m_d}{m_s}} - e^{i\delta}\sqrt{\frac{m_u}{m_c}} \right|$$

$$\frac{M^{d,l}}{m_3} = \begin{pmatrix} < \varepsilon^4 & \varepsilon^3 & \\ \varepsilon^3 & a^{d,l}\varepsilon^2 & \\ & & 1 \end{pmatrix}$$

$\varepsilon \simeq 0.15$

$a^d = 1 \quad a^l = -3$

$\overline{\psi}^\alpha \; \psi_\alpha$

$$\frac{m_b}{m_\tau}(M_X) = 1$$

$$\overline{\psi}^\alpha \begin{pmatrix} 1 & & & \\ & 1 & & \\ & & 1 & \\ & & & -3 \end{pmatrix} \psi_\alpha$$

$$Det(M^l) = Det(M^d)\big|_{M_X}$$

Low-Energy Parameter	Value(Uncertainty in last digit(s))	Notes and Reference
$m_u(\mu_L)/m_d(\mu_L)$	0.45(15)	PDB Estimation [1]
$m_s(\mu_L)/m_d(\mu_L)$	19.5(1.5)	PDB Estimation [1]
$m_u(\mu_L) + m_d(\mu_L)$	[8.8(3.0), 7.6(1.6)] MeV	PDB, Quark Masses, pg 15 [1]. (Non-lattice, Lattice)
$Q = \sqrt{\frac{m_s^2 - (m_d+m_u)^2/4}{m_d^2 - m_u^2}}$	22.8(4)	Martemyanov and Sopov [2]
$m_s(\mu_L)$	[103(20) , 95(20)] MeV	PDB, Quark Masses, pg 15 [1]. [Non-lattice, lattice]
$m_u(\mu_L)$	3(1) MeV	PDB, Quark Masses, pg 15 [1]. Non-lattice.
$m_d(\mu_L)$	6.0(1.5) MeV	PDB, Quark Masses, pg 15 [1]. Non-lattice.
$m_c(m_c)$	1.24(09) GeV	PDB, Quark Masses, pg 16 [1]. Non-lattice.
$m_b(m_b)$	4.20(07) GeV	PDB, Quark Masses, pg 16,19 [1]. Non-lattice.
M_t	170.9 (1.9) GeV	CDF & D0 [3] Pole Mass
(M_e, M_μ, M_τ)	(0.511(15), 105.6(3.1), 1777(53)) MeV	3% uncertainty from neglecting Y^e thresholds.
A Wolfenstein parameter	0.818(17)	PDB Ch 11 Eq. 11.25 [1]
$\overline{\rho}$ Wolfenstein parameter	0.221(64)	PDB Ch 11 Eq. 11.25 [1]
λ Wolfenstein parameter	0.2272(10)	PDB Ch 11 Eq. 11.25 [1]
$\overline{\eta}$ Wolfenstein parameter	0.340(45)	PDB Ch 11 Eq. 11.25 [1]
$\lvert V_{CKM}\rvert$	$\begin{pmatrix} 0.97383(24) & 0.2272(10) & 0.00396(09) \\ 0.2271(10) & 0.97296(24) & 0.04221(80) \\ 0.00814(64) & 0.04161(78) & 0.999100(34) \end{pmatrix}$	PDB Ch 11 Eq. 11.26 [1]
$\sin 2\beta$ from CKM	0.687(32)	PDB Ch 11 Eq. 11.19 [1]
Jarlskog Invariant	$3.08(18) \times 10^{-5}$	PDB Ch 11 Eq. 11.26 [1]
$v_{Higgs}(M_Z)$	246.221(20) GeV	Uncertainty expanded. [1]
($\alpha_{EM}^{-1}(M_Z)$, $\alpha_s(M_Z)$, $\sin^2\theta_W(M_Z)$)	(127.904(19), 0.1216(17), 0.23122(15))	PDB Sec 10.6 [1]

Table 1: Low-energy observables. Masses in lower-case m are $\overline{MS}$ running masses. Capital M indicates pole mass. The light quark's (u,d,s) mass are specified at a scale $\mu_L = 2$ GeV. V_{CKM} are the standard model's best fit values.

$$m_u(\mu_L) = 2.7 \pm 0.53 \text{ MeV} \quad m_d(\mu_L) = 5.25 \pm 0.44 \text{ MeV} \quad m_s(\mu_L) = 103 \pm 12 \text{ MeV}.$$

Parameters	Input SUSY Parameters					
$\tan\beta$	1.3	10	38	50	38	38
γ_b	0	0	0	0	-0.22	$+0.22$
γ_d	0	0	0	0	-0.21	$+0.21$
γ_t	0	0	0	0	0	-0.44
Parameters	Corresponding GUT-Scale Parameters with Propagated Uncertainty					
$y^t(M_X)$	6^{+1}_{-5}	0.48(2)	0.49(2)	0.51(3)	0.51(2)	0.51(2)
$y^b(M_X)$	$0.0113^{+0.0002}_{-0.01}$	0.051(2)	0.23(1)	0.37(2)	0.34(3)	0.34(3)
$y^\tau(M_X)$	0.0114(3)	0.070(3)	0.32(2)	0.51(4)	0.34(2)	0.34(2)
$(m_u/m_c)(M_X)$	0.0027(6)	0.0027(6)	0.0027(6)	0.0027(6)	0.0026(6)	0.0026(6)
$(m_d/m_s)(M_X)$	0.051(7)	0.051(7)	0.051(7)	0.051(7)	0.051(7)	0.051(7)
$(m_e/m_\mu)(M_X)$	0.0048(2)	0.0048(2)	0.0048(2)	0.0048(2)	0.0048(2)	0.0048(2)
$(m_c/m_t)(M_X)$	$0.0009^{+0.001}_{-0.00006}$	0.0025(2)	0.0024(2)	0.0023(2)	0.0023(2)	0.0023(2)
$(m_s/m_b)(M_X)$	0.014(4)	0.019(2)	0.017(2)	0.016(2)	0.018(2)	0.010(2)
$(m_\mu/m_\tau)(M_X)$	0.059(2)	0.059(2)	0.054(2)	0.050(2)	0.054(2)	0.054(2)
$A(M_X)$	$0.56^{+0.34}_{-0.01}$	0.77(2)	0.75(2)	0.72(2)	0.73(3)	0.46(3)
$\lambda(M_X)$	0.227(1)	0.227(1)	0.227(1)	0.227(1)	0.227(1)	0.227(1)
$\bar\rho(M_X)$	0.22(6)	0.22(6)	0.22(6)	0.22(6)	0.22(6)	0.22(6)
$\bar\eta(M_X)$	0.33(4)	0.33(4)	0.33(4)	0.33(4)	0.33(4)	0.33(4)
$J(M_X)\times 10^{-5}$	$1.4^{+2.2}_{-0.2}$	2.6(4)	2.5(4)	2.3(4)	2.3(4)	1.0(2)
Parameters	Comparison with GUT Mass Ratios					
$(m_b/m_\tau)(M_X)$	$1.00^{+0.04}_{-0.4}$	0.73(3)	0.73(3)	0.73(4)	1.00(4)	1.00(4)
$(3m_s/m_\mu)(M_X)$	$0.70^{+0.8}_{-0.05}$	0.69(8)	0.69(8)	0.69(8)	0.9(1)	0.6(1)
$(m_d/3\,m_e)(M_X)$	0.82(7)	0.83(7)	0.83(7)	0.83(7)	1.05(8)	0.68(6)
$(\frac{\det Y^d}{\det Y^e})(M_X)$	$0.57^{+0.08}_{-0.26}$	0.42(7)	0.42(7)	0.42(7)	0.92(14)	0.39(7)

Table 2: The mass parameters continued to the GUT-scale for various values of $\tan\beta$ and threshold corrections $\gamma_{t,b,d}$. These are calculated with the 2-loop gauge coupling and 2-loop Yukawa coupling RG equations assuming $M_S = 500$ GeV.

SUSY threshold effects:

$$m_d m_d^0 (1+\gamma_d+\gamma_u)^{-1}$$

$$m_s m_s^0 (1+\gamma_d+\gamma_u)^{-1}$$

$$m_b m_b^0 (1+\gamma_b+\gamma_t)^{-1}$$

$$\frac{V_{ub}^{SM}-V_{ub}^{MSSM}}{V_{ub}^{MSSM}}\,\frac{V_{cb}^{SM}-V_{cb}^{MSSM}}{V_{cb}^{MSSM}}-\left(\gamma_t-\gamma_u\right).$$

$$\gamma_t \approx -y_t^2\,\mu\,A^t\,\frac{\tan\beta}{16\pi^2}I_3(m_{\tilde{t}_1}^2,m_{\tilde{t}_2}^2,\mu^2)\sim -y_t^2\frac{\tan\beta}{32\pi^2}\frac{\mu\,A^t}{m_{\tilde{t}}^2}$$

$$\gamma_u \approx -g_2^2\,M_2\,\mu\frac{\tan\beta}{16\pi^2}I_3(m_{\chi_1}^2,m_{\chi_2}^2,m_{\tilde{u}}^2)\sim 0$$

$$\gamma_b \approx \frac{8}{3}g_3^2\frac{\tan\beta}{16\pi^2}M_3\,\mu\,I_3(m_{\tilde{b}_1}^2,m_{\tilde{b}_2}^2,M_3{}^2)\sim\frac{4}{3}g_3^2\frac{\tan\beta}{16\pi^2}\frac{\mu\,M_3}{m_{\tilde{b}}^2}$$

$$\gamma_d \approx \frac{8}{3}g_3^2\frac{\tan\beta}{16\pi^2}M_3\,\mu\,I_3(m_{\tilde{s}_1}^2,m_{\tilde{s}_2}^2,M_3{}^2)\sim\frac{4}{3}g_3^2\frac{\tan\beta}{16\pi^2}\frac{\mu\,M_3}{m_{\tilde{d}}^2}$$

$$\gamma_b+\gamma_t \approx -0.22\pm0.02$$

$$\gamma_d+\gamma_u \approx -0.21\pm0.02$$

$$\frac{\mu\,M_3}{m_{\tilde{b}}^2}\sim -0.5,\qquad \frac{m_{\tilde{b}}^2}{m_{\tilde{d}}^2}\sim 1.0$$

$$g-2 \;\Rightarrow\; \mu>0 \qquad \Rightarrow \qquad M_3<0 \quad c.f.\ \text{anomaly mediation}$$

Parameter	2001 RRRV	Fit A0	Fit B0	Fit A1	Fit B1	Fit A2	Fit B2
$\tan\beta$	Small	1.3	1.3	38	38	38	38
a'	$\mathcal{O}(1)$	0	0	0	0	-2.0	-2.0
ϵ_u	0.05	0.030(1)	0.030(1)	0.0491(16)	0.0491(15)	0.0493(16)	0.0493(14)
ϵ_d	0.15(1)	0.117(4)	0.117(4)	0.134(7)	0.134(7)	0.132(7)	0.132(7)
$\|b'\|$	1.0	1.75(20)	1.75(21)	1.05(12)	1.05(13)	1.04(12)	1.04(13)
$\arg(b')$	90^o	$+93(16)^o$	$-93(13)^o$	$+91(16)^o$	$-91(13)^o$	$+93(16)^o$	$-93(13)^o$
a	1.31(14)	2.05(14)	2.05(14)	2.16(23)	2.16(24)	1.92(21)	1.92(22)
b	1.50(10)	1.92(14)	1.92(15)	1.66(13)	1.66(13)	1.70(13)	1.70(13)
$\|c\|$	0.40(2)	0.85(13)	2.30(2)	0.78(15)	2.12(36)	0.83(17)	2.19(38)
$\arg(c)$	$-24(3)^o$	$-39(18)^o$	$-61(14)^o$	$-43(14)^o$	$-59(13)^o$	$-37(25)^o$	$-60(13)^o$

Table 3: A χ^2 fit of eqs(15,16) to Table 2 in the absence of threshold corrections. We set a' as indicated and set $c' = d' = d = 0$ and $f = f' = 1$ at fixed values. Uncertainty in final digit(s) reflected in parenthesis gives widest 1σ axis on the error ellipse.

$$Y^u(M_X) = y_{33}^u \begin{pmatrix} 0 & b'\varepsilon_u^3 & c'\varepsilon_u^3 \\ b'\varepsilon_u^3 & \varepsilon_u^2 & a'\varepsilon_u^2 \\ c'\varepsilon_u^3 & a'\varepsilon_u^2 & 1 \end{pmatrix}, \qquad Y^d(M_X) = y_{33}^d \begin{pmatrix} 0 & b\varepsilon_d^3 & c\varepsilon_d^3 \\ b\varepsilon_d^3 & \varepsilon_d^2 & a\varepsilon_d^2 \\ c\varepsilon_d^3 & a\varepsilon_d^2 & 1 \end{pmatrix}$$

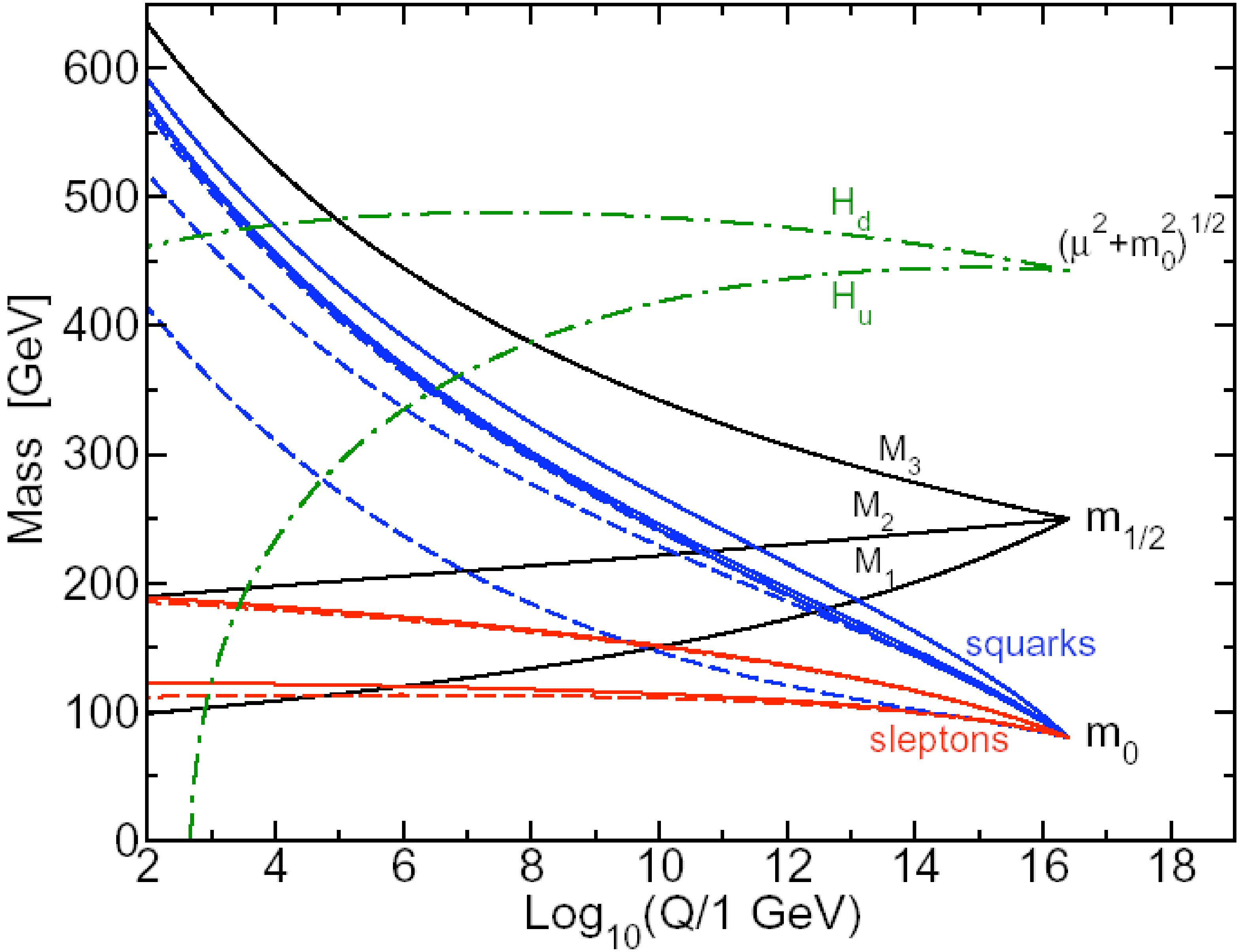
Parameter	A	B	C	B2	C2
$\tan\beta$	30	38	38	38	38
γ_b	0.20	-0.22	$+0.22$	-0.22	$+0.22$
γ_t	-0.03	0	-0.44	0	-0.44
γ_d	0.20	-0.21	$+0.21$	-0.21	$+0.21$
a'	0.0	0.0	0.0	-2	-2
ϵ_u	0.0495(17)	0.0483(16)	0.0483(18)	0.0485(17)	0.0485(18)
ϵ_d	0.131(7)	0.128(7)	0.102(9)	0.127(7)	0.101(9)
$\|b'\|$	1.04(12)	1.07(12)	1.07(11)	1.05(12)	1.06(10)
$\arg(b')$	$90(12)^o$	$91(12)^o$	$93(12)^o$	$95(12)^o$	$95(12)^o$
a	2.17(24)	2.27(26)	2.30(42)	2.03(24)	1.89(35)
b	1.69(13)	1.73(13)	2.21(18)	1.74(10)	2.26(20)
$\|c\|$	0.80(16)	0.86(17)	1.09(33)	0.81(17)	1.10(35)
$\arg(c)$	$-41(18)^o$	$-42(19)^o$	$-41(14)^o$	$-53(10)^o$	$-41(12)^o$
Y^u_{33}	0.48(2)	0.51(2)	0.51(2)	0.51(2)	0.51(2)
Y^d_{33}	0.15(1)	0.34(3)	0.34(3)	0.34(3)	0.34(3)
Y^e_{33}	0.23(1)	0.34(2)	0.34(2)	0.34(2)	0.34(2)
$(m_b/m_\tau)(M_X)$	0.67(4)	1.00(4)	1.00(4)	1.00(4)	1.00(4)
$(3m_s/m_\mu)(M_X)$	0.60(3)	0.9(1)	0.6(1)	0.9(1)	0.6(1)
$(m_d/3\,m_e)(M_X)$	0.71(7)	1.04(8)	0.68(6)	1.04(8)	0.68(6)
$\left\|\frac{\det Y^d(M_X)}{\det Y^e(M_X)}\right\|$	0.3(1)	0.92(14)	0.4(1)	0.92(14)	0.4(1)

Table 4: The top-down best-fit values at $\tan\beta = 38$. We perform the fits using different threshold corrections $\gamma_{t,d,b,d}$. Uncertainty in final digit(s) reflected in parenthesis. Below the second horizontal line, we show interesting ratios extracted from the GUT-scale fit

$$Y^u(M_X) = y^u_{33}\begin{pmatrix} 0 & e^{i\pi/2}\varepsilon_u^3 & 0 \\ e^{i\pi/2}\varepsilon_u^3 & \varepsilon_u^2 & 0 \\ 0 & 0 & 1 \end{pmatrix},$$

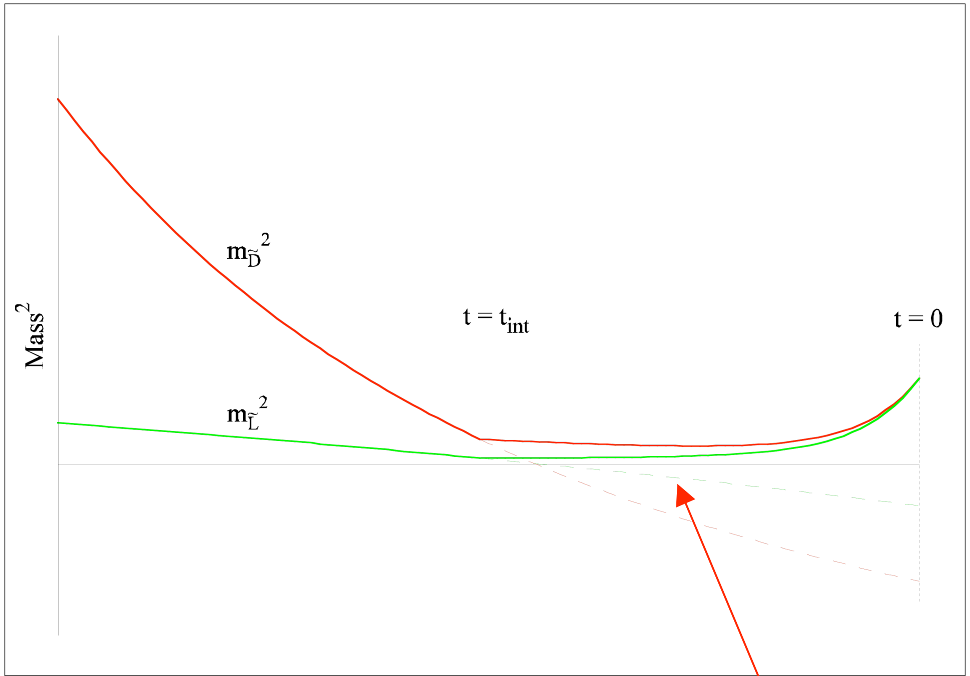
$$Y^d(M_X) = y^d_{33}\begin{pmatrix} d\,\varepsilon_d^4 & 1.7\,\varepsilon_d^3 & e^{-i\pi/4}\varepsilon_d^3 \\ 1.7\,\varepsilon_d^3 & \varepsilon_d^2 & 2\,\varepsilon_d^2 \\ e^{-i\pi/4}\varepsilon_d^3 & 2\,\varepsilon_d^2 & 1 \end{pmatrix},$$

Scalar mass unification:



In general WRONG!

Cohen, Roy, Schmaltz



Hidden sector effects

Soft masses

$$m_\phi^2 = \frac{|F_X|^2}{M^2} \qquad m_\lambda = \frac{F_X}{M}$$

$$\int d^4\theta\, k_i \frac{X^\dagger X}{M^2} \Phi_i^\dagger \Phi_i + \int d^2\theta\, w \frac{X}{M} W_n W_n$$

$$e.g. \quad W_h = \frac{\lambda}{3!} X^3.$$

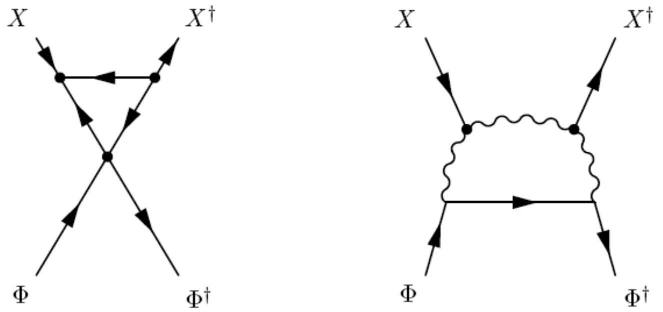


FIG. 1: Renormalization of the operators responsible for scalar masses.

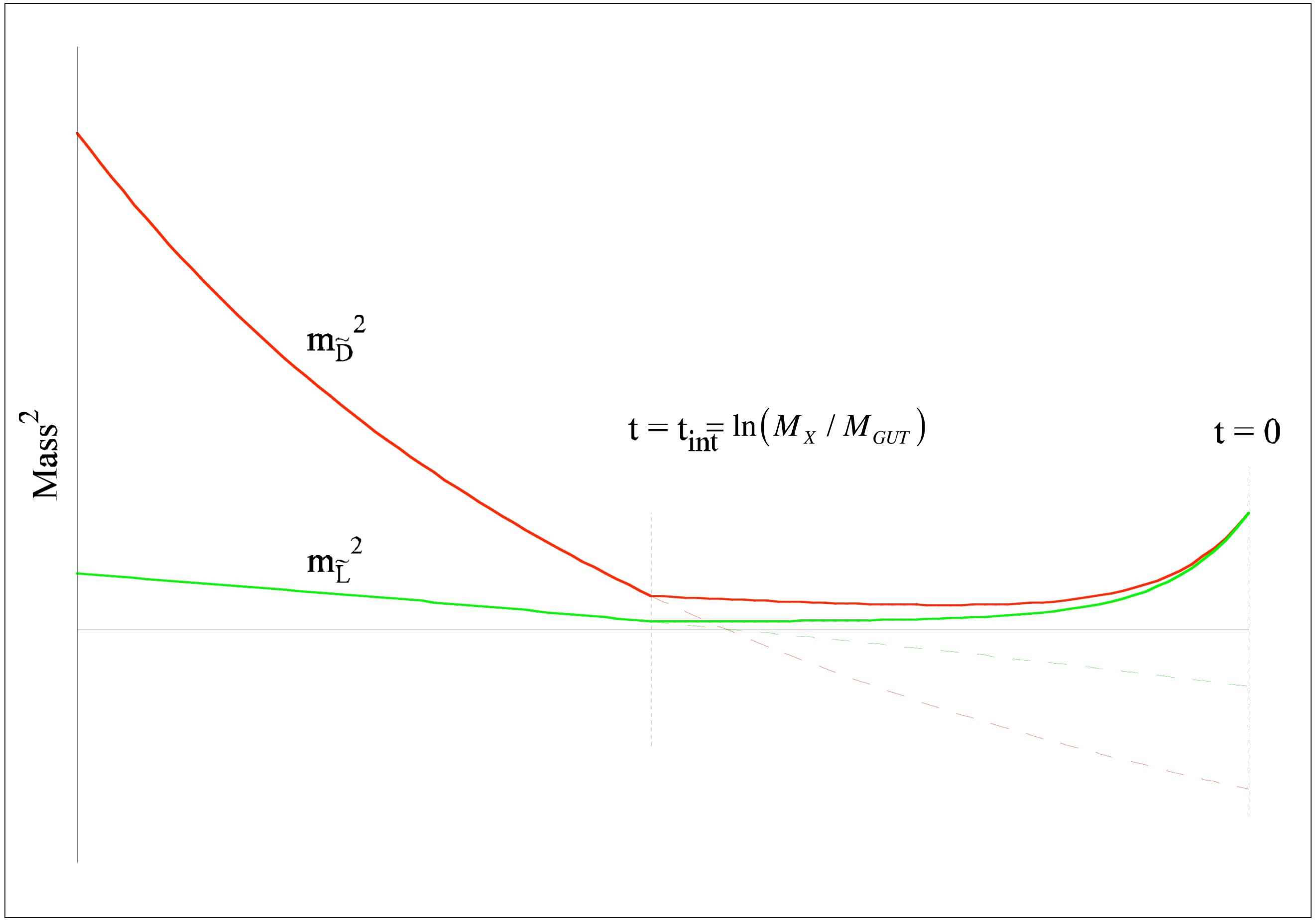
$$\frac{d}{dt} k_i = \frac{2\lambda^*\lambda}{16\pi^2} k_i - \frac{1}{16\pi^2} \sum_n 8C_2^n(R_i) g_n^6 w^* w$$

$$\equiv \gamma k_i - \frac{1}{16\pi^2} \sum_n 8C_2^n(R_i) g_n^6 G.$$

Missing term

Cohen, Roy, Schmaltz

$$k_i(t) = \exp\left(-\int_t^0 dt' \gamma(t')\right) k_i(0) + \frac{1}{16\pi^2} \sum_n 8C_2^n(R_i) \int_t^0 ds\, g_n^6(s) \exp\left(-\int_t^s dt' \gamma(t')\right) G.$$



In general

$$m_i^2(t_{int}) = k_i(t_{int})\frac{F^2}{M^2} = N_0 + \sum_{n=1}^{3} 8C_2^n(R_i)N_n.$$

4 unknowns

$$N_0 = \frac{F^2}{M^2}\exp\left(-\int_{t_{int}}^{0} dt'\,\gamma(t')\right)k(0) \qquad N_n = \frac{F^2}{M^2}\frac{1}{16\pi^2}\int_{t_{int}}^{0} ds\exp\left(-\int_{t_{int}}^{s} dt'\,\gamma(t')\right)g_n^6(s)G$$

Possibilities:

$$k_i(t) = \exp\left(-\int_t^0 dt'\gamma(t')\right)k_i(0) + \frac{1}{16\pi^2}\sum_n 8C_2^n(R_i)\int_t^0 ds\, g_n^6(s)\exp\left(-\int_t^s dt'\gamma(t')\right)G.$$

- Hidden sector non-interacting e.g. $\langle F_X \rangle \neq 0$, X moduli; e.g. gaugino condensate

$$k_i(t) = k_i(0) + \frac{1}{16\pi^2}\sum_n 8C_2^n(R_i)\frac{G}{b_n^3}\left(-\frac{1}{2a_n^2} + \frac{1}{2(t-a_n)^2}\right)$$

$$g_n^2(t) = \frac{1}{b_n(t-a_n)} \equiv \frac{1}{b_n \ln(\mu / M_{np})}$$

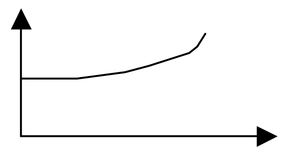
"walking".

- Hidden sector IR free

$$\gamma(t) = \frac{1}{b_\gamma(t-a_\gamma)} \qquad b_\gamma = -1,\ a_\gamma = 1.$$

$$k_i(t) = (t-a_\gamma)^{-1}\left[-a_\gamma k_i(0) + \frac{1}{16\pi^2}\sum_n 8C_2^n(R_i)\frac{G}{b_n^3}\left(\frac{(a_n+a_\gamma)}{2a_n^2} + \frac{1}{(t-a_n)} - \frac{(a_\gamma - a_n)}{2(t-a_n)^2}\right)\right]$$

$c.f.\ b_3 = 2(11 - 2n_f/3)$

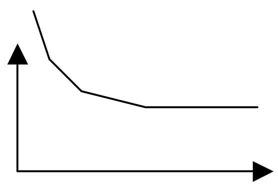


$k_i(t_i)$ suppressed by factor $(t_i)^{-1} \simeq 1/7$ – "Mirage unification"

- Hidden sector UV free

$$b_\gamma = -1, \quad a_\gamma = \ln(M_i / M).$$

$$k_i(t) = (t-a_\gamma)^{-1}\left[-a_\gamma k_i(0) + \frac{1}{16\pi^2}\sum_n 8C_2^n(R_i)\frac{G}{b_n^3}\left(\frac{(a_n+a_\gamma)}{2a_n^2} + \frac{1}{(t-a_n)} - \frac{(a_\gamma - a_n)}{2(t-a_n)^2}\right)\right]$$



$k_i(t_i)$ enhanced by factor $(2 + a_\gamma - a_n)(t_{np} - a_\gamma)^{-1} \simeq 10$ (i dependent)

Conclusions

- Yukawa Unification (d,l,(u)) ✓

  Finite SUSY threshold corrns, M_3 negative

  (d,l,u,ν) ✓

  Sequential domination, non-Abelian discrete family symmetry

- Soft mass unification

  Gaugino mass ✓

  Scalar masses ? Sensitive to hidden sector