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Heterotic $Z_2 \times Z_2$ orbifolds of non factorisable lattices

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Summary

1. Heterotic strings.

2. Compactifications on orbifolds.

3. $Z_2 \times Z_2$ orbifolds.

4. Connection between free fermionic models and orbifolds.

5. Non factorisable lattices.

6. Other ingredients in model building.

7. Conclusions.

• Conformal anomaly $\longrightarrow$ extra degrees of freedom:

- compactified extra dimensions or

- internal free fermions on the worldsheet

Heterotic strings

• Closed strings: left movers and right movers are independent up to level matching

Left sector (Susy, D=10)	Right sector (Bosonic, D=26)
$X_L^\mu,\ \psi_L^\mu,\ \mu = 1,2$	$X_R^\mu, \mu = 1,2$
$X_L^i,\ \psi_L^i,\ \ i = 1,...,6$	$X_R^i,\ i = 1,...,6$
	$X_R^I,\ I = 1,...,16$

X_R^I : internal, compactified on a lattice $E_8 \times E_8$ or $SO(32)$

4D models : compactify $X^i = X_L^i + X_R^i \longrightarrow X^i = X^i + 2\pi$ (T^6) $\rightarrow$ non-chiral, N=4 SUSY

$\downarrow$

orbifolds

Orbifolds

$$T^6/P \times T_L^{16}/G \times M^4$$

P - discrete symmetry (point group)

G - embedding of the point group in the gauge degrees of freedom

Geometrically: $g \in P, \quad gX \sim X$; fixed points: $gX = X$ (singular)

Example: $S^1 = [0,1), \; X \sim X + 1$

$Z_2 : \; X \sim -X$; fixed points $X = 0, 1/2$

$S^1/Z_2 = [0, 1/2]$

String theory orbifold: • keep states that are invariant under P

• add twisted sectors (also projected into P)

Twisted sectors (at the fixed points): $X(\sigma + 2\pi) = g \; X(\sigma), \forall g \in P$

Wilson lines: embedding of the torus lattice in the gauge degrees of freedom. They break the gauge group and modify the spectrum.

$Z_2 \times Z_2$ orbifolds

$T^6 = T^2 \times T^2 \times T^2$

$\theta_1 = (+\ ,\ \ -\ \ ,\ \ -)$

$\theta_2 = (-\ ,\ \ -\ \ ,\ \ +)$

Three twisted sectors : θ_1 , θ_2 , $\theta_1\theta_2$

θ_1 fixed points : $X^{(2)}_{1,2} = 0, 1/2$, $X^{(3)}_{1,2} = 0, 1/2$ $\rightsquigarrow$ 2^4 fixed tori

Similar for θ_2 and $\theta_1\theta_2$ $\rightsquigarrow$ 3 x 16 = 48 fixed tori

1 fixed point $\longrightarrow$ 1 Standard Model generation

Wilson lines lift the degeneracy of the fixed points

Constructing element:

$$(x, y, 1/2, 0, 0, 0) \xrightarrow{\theta_1} (x, y, -1/2, 0, 0, 0) = (x, y, 1/2, 0, 0, 0) + (0, 0, 1, 0, 0, 0)$$

$$(x, y, 1/2, 0, 0, 0) \sim (\theta_1, e_3)$$

Free fermionic models

$\psi_L^\mu,\ \mu = 1, 2$

$\psi_L^i,\ i = 1, ..., 6 \ \longrightarrow\ \chi^{1...6}\ (r)$

$X_L^i,\ i = 1, ..., 6 \ \longrightarrow\ y^{1...6}, \omega^{1...6}\ (r)$

$X_R^i,\ i = 1, ..., 6 \ \longrightarrow\ \bar{y}^{1...6}, \bar{\omega}^{1...6}\ (r)$

$X_R^I, I = 1, ..., 16 \ \longrightarrow\ \bar{\psi}^{1...5}, \bar{\eta}^{1,2,3}\ (E_8),\ \bar{\phi}^{1...8}\ (E_8')\ (c)$

$$\{\ \psi^\mu_{1,2}\ ,\ \chi^{1...6}\ ,\ y^{1...6}\ ,\ \omega^{1...6}\ |\ \bar{y}^{1...6}\ ,\ \bar{\omega}^{1...6}\ ,\ \bar{\psi}^{1...5}\ ,\ \bar{\eta}^{1,2,3}\ ,\ \bar{\phi}^{1...8}\ \}$$

$$\boxed{f \longrightarrow e^{-i\alpha(f)\pi}\ f} \qquad \alpha = 0\ (NS)\ ;\ \alpha = 1\ (R)$$

Basis vectors: $\vec{b} = \{\ \alpha(\psi^\mu_{1,2})\ ,\ \alpha(\chi^1)\ ,\ \alpha(\chi^2)\ ,\ \ ...\ |\ ...\ \}$

Semi-realistic models

- gauge group includes Standard Model with $SO(10)$ embedding
- 3 chiral generations
- N=1 SUSY
- minimal Higgs spectrum, proton stability

Correspondence between free fermionic models and orbifolds

[Faraggi,1993]

Model: $\{\ 1,\ S,\ \xi_1,\ \xi_2\ \}$

	ψ^μ	χ^{12}	χ^{34}	χ^{56}	y^{12}	$y^{3\ldots6}$	$\omega^{1\ldots4}$	ω^{56}	$\bar\psi^{1,\ldots,5}$	$\bar\eta^1$	$\bar\eta^2$	$\bar\eta^3$	$\bar\phi^{1,\ldots,8}$
S	1	1	1	1	0	0	0	0	0	0	0	0	0
ξ_1	0	0	0	0	0	0	0	0	1	1	1	1	0
ξ_2	0	0	0	0	0	0	0	0	0	0	0	0	1

$\mathcal{N}=4$ SUSY. Gauge group : $\underbrace{SO(12)}_{\{\bar y^{1\ldots6},\,\bar\omega^{1\ldots6}\}} \times \underbrace{E_8}_{\{\bar\psi^{1\ldots5},\,\bar\eta^{1,2,3}\}} \times \underbrace{E_8}_{\{\bar\phi^{1\ldots8}\}}$

Add $\{b_1, b_2\}$

	ψ^μ	χ^{12}	χ^{34}	χ^{56}	y^{12}	$y^{3\ldots6}$	$\omega^{1\ldots4}$	ω^{56}	$\bar\psi^{1,\ldots,5}$	$\bar\eta^1$	$\bar\eta^2$	$\bar\eta^3$	$\bar\phi^{1,\ldots,8}$
b_1	1	1	0	0	0	1	0	0	1	1	0	0	0
b_2	1	0	1	0	1	0	0	1	1	0	1	0	0

$\boxed{\mathcal{N}=4 \rightarrow \mathcal{N}=1}$ $[SO(4)]^3 \times E_6 \times U(1)^2 \times E_8$; 24 generations

$\boxed{b_1\ \&\ b_2}$ - translate to twists by $\frac{1}{\sqrt{2}}\,(y^i + i\omega^i) = -ie^{iX_i}$

y, ω	X_L	X_R	$X = X_L + X_R$
0 0	$X_L + \pi$	$X_R + \pi$	$X + 2\pi$
1 0	$-X_L$	$-X_R$	$-X$
0 1	$-X_L + \pi$	$-X_R + \pi$	$-X + 2\pi$
1 1	X_L	X_R	X

- $b_1 \longrightarrow \theta_1 = (+, -, -)$
- $b_2 \longrightarrow \theta_1\theta_2 = (-, +, -)$
- $b_3 = 1 + \xi_2 + b_1 \longrightarrow \theta_2 = (-, -, +)$

The orbifold formulation: compactification on the $SO(12)$ root lattice at a specific point, $R_i = \sqrt{2}$, in the moduli space and with the background fields:

$$g_{ij} = \text{Cartan matrix of } SO(12), \qquad b_{ij} = \begin{cases} g_{ij} & , i > j \\ 0 & , i = j \\ -g_{ij} & , i < j \end{cases}$$

$SO(12)/Z_2 \times Z_2$

$Z_2 \times Z_2$ orbifolds of non factorisable tori

$SO(12)$ root lattice:

$$e_1 = (1, -1, 0, 0, 0, 0)$$
$$e_2 = (0, 1, -1, 0, 0, 0)$$
$$e_3 = (0, 0, 1, -1, 0, 0)$$
$$e_4 = (0, 0, 0, 1, -1, 0)$$
$$e_5 = (0, 0, 0, 0, 1, -1)$$
$$e_6 = (0, 0, 0, 0, 1, 1)$$

$$\begin{pmatrix} x^1 \\ \vdots \\ x^6 \end{pmatrix} \rightarrow \theta_1 \begin{pmatrix} x^1 \\ \vdots \\ x^6 \end{pmatrix}, \quad \theta_1 = \begin{pmatrix} 1 & 0 & 0 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 & 0 & 0 \\ 0 & 0 & -1 & 0 & 0 & 0 \\ 0 & 0 & 0 & -1 & 0 & 0 \\ 0 & 0 & 0 & 0 & -1 & 0 \\ 0 & 0 & 0 & 0 & 0 & -1 \end{pmatrix}$$

θ_1 fixed tori:

$$\{(x, y, 0, 0, 0, 0) \,|\, x, y \in \mathbb{R}^2/\Lambda^2\}, \qquad \Lambda^2 : \{(1, 1),\ (1, -1)\}$$

$$\{(x, y, 1, 0, 0, 0) \,|\, x, y \in \mathbb{R}^2/\Lambda^2\}$$

$$\left\{\left(x, y, \underline{\tfrac{1}{2}, \tfrac{1}{2}, 0, 0}\right) \,\middle|\, x, y \in \mathbb{R}^2/\Lambda^2\right\} \qquad \begin{pmatrix} 4 \\ 2 \end{pmatrix} = 6$$

$$\left\{\left(x, y, \underline{\tfrac{1}{2}, -\tfrac{1}{2}, 0, 0}\right) \,\middle|\, x, y \in \mathbb{R}^2/\Lambda^2\right\}$$

$$\left\{\left(x, y, \tfrac{1}{2}, \tfrac{1}{2}, \tfrac{1}{2}, \pm\tfrac{1}{2}\right) \,\middle|\, x, y \in \mathbb{R}^2/\Lambda^2\right\}$$

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16 fixed tori

But:

$$(x, y, 1, 0, 0, 0) + e_1 + e_2 = (x + 1, y, 0, 0, 0, 0) \sim (x, y, 0, 0, 0, 0)$$

Similar for the other fixed tori → only 8 tori are inequivalent.

3 twisted sectors → 24 fixed tori (as in the free fermionic model)

Less fixed points

Lattice	fixed tori	(generations, anti-generations)	net # of generations
(SO(12))	24	(27,3)	24
$(SO(6)^2$-A)	20	(19,7)	12
$(SO(6)^2$-B)	12	(15,3)	12
$(SU(3)^3)$	12	(12,6)	6

Minimal number of fixed points per twisted sector is four.

[S. Forste, T. Kobayashi, H. Ohki, K. Takahashi; 2006]

$[SO(6)]^2$ root lattice (skew)

$$
\begin{aligned}
e_1 &= (1, \; 0, -1, \; 0, \; 0, \; 0) \\
e_2 &= (0, \; 0, \; 1, \; 0, -1, \; 0) \\
e_3 &= (0, \; 0, \; 1, \; 0, \; 1, \; 0) \\
e_4 &= (0, \; 1, \; 0, -1, \; 0, \; 0) \\
e_5 &= (0, \; 0, \; 0, \; 1, \; 0, -1) \\
e_6 &= (0, \; 0, \; 0, \; 1, \; 0, \; 1)
\end{aligned}
$$

12 fixed tori

$A_2 = A_3, \; A_5 = A_6$

θ_2	θ_1	$\theta_1\theta_2$
$(\theta_2, 0)$	$(\theta_1, 0)$	$(\theta_1\theta_2, 0)$
$(\theta_2, A_2), \; A_1$	$(\theta_1, A_1), \; A_2$	$(\theta_1\theta_2, A_1 + A_2), A_1, \; A_2$
$(\theta_2, A_5), \; A_4$	$(\theta_1, A_4), \; A_5$	$(\theta_1\theta_2, A_4 + A_5), \; A_4, \; A_5$
$(\theta_2, A_2 + A_5), \; A_1 + A_4$	$(\theta_1, A_1 + A_4), \; A_2 + A_5$	$(\theta_1\theta_2, \sum_{1,2,4,5} A_i), \; A_1 + A_4, \; A_2 + A_5$

A 3 generation model

$[SO(6)]^2/Z_2 \times Z_2$

$$V_1 = (\tfrac{1}{2}, -\tfrac{1}{2}, 0^6)\,(0^8) \quad , \quad V_2 = (0, \tfrac{1}{2}, -\tfrac{1}{2}, 0^5)\,(0^8)$$

$$\begin{aligned}
A_1 &= (0^8)\left(0^3, \frac{1}{2}, \frac{1}{2}, -\frac{1}{2}, -\frac{1}{2}, 0\right)\\
A_2 = A_3 &= (0^7, 1)\,(1, 0^7)\\
A_4 &= (0^8)\left(0, -\frac{1}{2}, -\frac{1}{2}, 0, 0, \frac{1}{2}, \frac{1}{2}, 0\right)\\
A_5 = A_6 &= (0^8)\left(0, \frac{1}{2}, \frac{1}{2}, -\frac{1}{2}, -\frac{1}{2}, 0^3\right)
\end{aligned}$$

$$E_8 \times E_8 \xrightarrow{V_1, V_2} E_6 \times U(1)^2 \times E_8 \xrightarrow{A_{1\ldots6}} SO(10) \times U(1)^3 \times [SU(2)]^8$$

One generation per twisted sector

Other ingredients in orbifold constructions

Asymmetric boundary conditions: in the free fermionic models they allow to obtain a minimal Higgs spectrum

$$\downarrow$$

asymmetric twists

$$\downarrow$$

moduli stabilisation, doublet-triplet splitting

Shifts : identify some of the fixed points $\rightsquigarrow$ less generations to start with $\rightsquigarrow$ less Wilson lines needed

Conclusions

Free fermionic models produce in a simple and elegant manner semi-realistic models.

They seem to be related to $Z_2 \times Z_2$ orbifolds of the $SO(12)$ lattice. One can use the free fermionic models as inspiration for model building in orbifold compactifications.

Non factorisable lattices offer an interesting, yet simple set-up for model building.

Asymmetric boundary conditions play an important role in the free fermionic models. One should investigate asymmetric orbifolds.