Magnetic moment $(g-2)_\mu$ and SUSY

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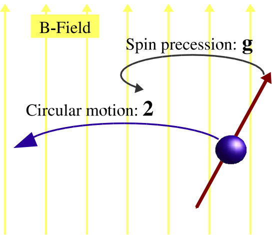
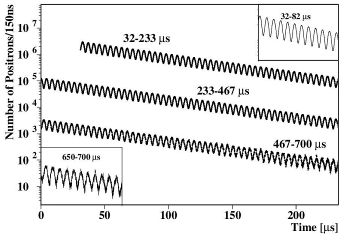
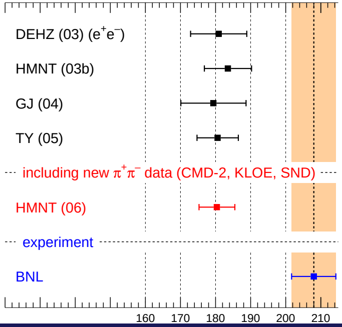
[DS '07]

$(g-2)$: Magnetic Moment of the Muon







$$a_\mu(\text{exp}) = 11\,659\,208(6) \times 10^{-10}$$

$$a_\mu(\text{exp} - \text{SM}) = 28(8) \times 10^{-10}$$

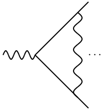
3.4σ deviation from SM-prediction!

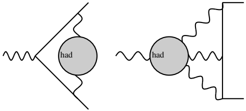
Outline

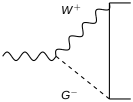
1. Status: SM and experiment
2. a_μ and SUSY
3. Numerical results in SUSY
4. Campaign for new, better measurement
5. Conclusions

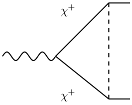
Outline

Classification of SM contributions

QED:  $\sim 10^{-3}$ 7000σ

had:  $\sim 10^{-7}$ 100σ

weak:  $\sim 10^{-9}$ 2.5σ

SUSY:  $\sim 10^{-8\ldots-10}$ $O(10\sigma)$

Era of the Brookhaven experiment

'78 – 2001: Theory precision increased to $\pm 10 \times 10^{-10}$

2001: BNL experiment:	$a_\mu^{\rm exp} - a_\mu^{\rm th} =$	43 ± 16	2.7σ
2002: correction of lbl sign error:		28 ± 16	1.7σ
2002: BNL experiment:		26 ± 11	2.4σ
2003: 4-Loop error, new LBL result:		20 ± 11	1.8σ

'03–'06: Consolidation of SM result by effort of many groups

spring 2006: BNL experiment (final):	24 ± 10	2.4σ

>summer 2006: new SM evaluations: spectacular improvement

Current status

- Exp: finalized
- Th:
  - new SM evaluations, based on new exp data for $a_\mu^{\rm had}$:

$$a_\mu(\text{Exp-SM}) = \left\{ \begin{array}{ll} [HMNT06] & 28(8) \\ [DEHZ06] & 28(8) \\ [FJ07] & 29(9) \end{array} \right\} \times 10^{-10}$$

- better agreement between evaluations, more precise, larger deviation from exp than ever before

Outline

a_μ and SUSY

Two questions:

Could SUSY be the origin of the $(28 \pm 8) \times 10^{-10}$ deviation?

Which restrictions on SUSY follow from (e.g. 3σ band)

$$3 \times 10^{-10} \; < \; a_\mu^{\rm SUSY} \; < \; 51 \times 10^{-10}?$$

$g-2$ in the MSSM

Key to understand $g-2$:

chiral symmetry

$g-2$ = chirality-flipping interaction

$$\bar{u}_R(p')\frac{\sigma_{\mu\nu}q^\nu}{2m_\mu}u_L(p) + (L \leftrightarrow R)$$

in each Feynman diagram we need to pick up one transition

$$\mu_L \rightarrow \mu_R \text{ or } \tilde{\mu}_L \rightarrow \tilde{\mu}_R$$

Chiral symmetry would forbid $g-2$

$g-2$ in the MSSM

chiral symmetry broken by λ_μ, $m_\mu = \lambda_\mu \langle H_1 \rangle$

- second Higgs doublet H_2 important

$$\tan\beta = \frac{\langle H_2 \rangle}{\langle H_1 \rangle}, \qquad \mu = H_2 - H_1 \text{ transition}$$

some terms

$$\propto \lambda_\mu \langle H_1 \rangle = m_\mu \qquad \to a_\mu^{\mathrm{SUSY}} \propto \frac{m_\mu^2}{M_{\mathrm{SUSY}}^2}$$

some terms

$$\propto \lambda_\mu\, \mu \langle H_2 \rangle = m_\mu\ \mu\ \tan\beta \qquad \to a_\mu^{\mathrm{SUSY}} \propto \tan\beta\, \mathrm{sign}(\mu)\, \frac{m_\mu^2}{M_{\mathrm{SUSY}}^2}$$

potential enhancement $\propto \tan\beta = 1 \ldots 50$

a_μ in the MSSM

1-Loop result if μ, $m_{\tilde{\mu}}$, $m_{\tilde{\chi}} \approx M_{\rm SUSY}$

$$a_\mu^{\rm SUSY} \approx \frac{\alpha}{\pi\, 8 s_W^2} \tan\beta\ {\rm sign}(\mu)\ \frac{m_\mu^2}{M_{\rm SUSY}^2}$$

numerically

$$a_\mu^{\rm SUSY} \approx 12 \times 10^{-10} \tan\beta\ {\rm sign}(\mu) \left(\frac{100{\rm GeV}}{M_{\rm SUSY}}\right)^2$$

- $\propto \tan\beta\ {\rm sign}(\mu)$
- $\propto 1/M_{\rm SUSY}^2$, but complicated dependence on individual masses

a_μ in the MSSM

1-Loop result if μ, $m_{\tilde{\mu}}$, $m_{\tilde{\chi}} \approx M_{\rm SUSY}$

$$a_\mu^{\rm SUSY} \approx \frac{\alpha}{\pi\, 8 s_W^2} \tan\beta \,{\rm sign}(\mu)\, \frac{m_\mu^2}{M_{\rm SUSY}^2}$$

numerically

$$a_\mu^{\rm SUSY} \approx 12 \times 10^{-10} \tan\beta \,{\rm sign}(\mu) \left(\frac{100{\rm GeV}}{M_{\rm SUSY}}\right)^2$$

e.g. $a_\mu^{\rm SUSY} = 24 \times 10^{-10}$ for

$$\begin{array}{ll} \tan\beta = 2, & M_{\rm SUSY} = 100 \text{ GeV} \\ \tan\beta = 50, & M_{\rm SUSY} = 500 \text{ GeV} \end{array} \quad (\mu > 0)$$

$\Rightarrow$ SUSY could easily be the origin of the observed deviation!

a_μ in the MSSM

1-Loop result if μ, $m_{\tilde{\mu}}$, $m_{\tilde{\chi}} \approx M_{\rm SUSY}$

$$a_\mu^{\rm SUSY} \approx \frac{\alpha}{\pi\, 8 s_W^2} \tan\beta\, {\rm sign}(\mu)\, \frac{m_\mu^2}{M_{\rm SUSY}^2}$$

numerically

$$a_\mu^{\rm SUSY} \approx 12 \times 10^{-10} \tan\beta\, {\rm sign}(\mu) \left(\frac{100{\rm GeV}}{M_{\rm SUSY}}\right)^2$$

e.g. $a_\mu^{\rm SUSY} = -96 \times 10^{-10}$ for

$$\tan\beta = 50, \quad M_{\rm SUSY} = 250 \text{ GeV} \quad (\mu < 0)$$

$\Rightarrow$ such parameter points are ruled out by a_μ!

a_μ in the MSSM

Answers:

SUSY could be the origin of the observed $(28 \pm 8) \times 10^{-10}$ deviation!

a_μ significantly restricts the SUSY parameters

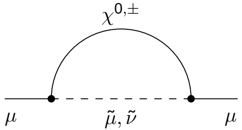
$\rightarrow$ generically, positive μ, large tan β/small $M_{\rm SUSY}$ preferred

Precise analysis justified!

Status of SUSY prediction

1-Loop

$\propto \tan\beta$



[Fayet '80],. . .

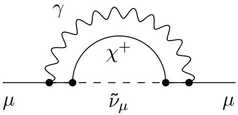
[Kosower et al '83],[Yuan et al '84],. . .

[Lopez et al '94],[Moroi '96]

complete

2-Loop (SUSY 1L)

$\propto \log \frac{M_{\text{SUSY}}}{m_\mu}$

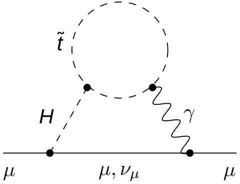


[Degrassi,Giudice '98]

leading log

2-Loop (SM 1L)

$\propto \tan\beta\, \mu\, m_t$



[Chen,Geng'01][Arhib,Baek '02]

[Heinemeyer,DS,Weiglein '03]

[Heinemeyer,DS,Weiglein '04]

complete

SUSY prediction

Implementation of 1-Loop and leading 2-Loop straightforward:

1-Loop:

$$a_\mu^{\chi^0} = \frac{m_\mu}{16\pi^2} \sum_{i,m} \left\{ -\frac{m_\mu}{12 m^2_{\tilde\mu m}} (|n^L_{im}|^2 + |n^R_{im}|^2) F^N_1(x_{im}) + \frac{m_{\chi^0_i}}{3 m^2_{\tilde\mu m}} \mathrm{Re}[n^L_{im} n^R_{im}] F^N_2(x_{im}) \right\},$$

$$a_\mu^{\chi^\pm} = \frac{m_\mu}{16\pi^2} \sum_k \left\{ \frac{m_\mu}{12 m^2_{\tilde\nu_\mu}} (|c^L_k|^2 + |c^R_k|^2) F^C_1(x_k) + \frac{2 m_{\chi^\pm_k}}{3 m^2_{\tilde\nu_\mu}} \mathrm{Re}[c^L_k c^R_k] F^C_2(x_k) \right\},$$

$$n^L_{im} = \frac{1}{\sqrt{2}}(g_1 N_{i1} + g_2 N_{i2}) U^{\tilde\mu}_{m1}{}^* - y_\mu N_{i3} U^{\tilde\mu}_{m2}{}^*,$$
$$n^R_{im} = \sqrt{2} g_1 N_{i1} U^{\tilde\mu}_{m2} + y_\mu N_{i3} U^{\tilde\mu}_{m1},$$
$$c^L_k = -g_2 V_{k1},$$
$$c^R_k = y_\mu U_{k2}. \; F^N_1(x) = \frac{2}{(1-x)^4}\left[1 - 6x + 3x^2 + 2x^3 - 6x^2 \log x\right],$$
$$F^N_2(x) = \frac{3}{(1-x)^3}\left[1 - x^2 + 2x \log x\right],$$
$$F^C_1(x) = \frac{2}{(1-x)^4}\left[2 + 3x - 6x^2 + x^3 + 6x \log x\right],$$
$$F^C_2(x) = \frac{3}{(1-x)^3}\left[-3 + 4x - x^2 - 2 \log x\right],$$

SUSY prediction

Implementation of 1-Loop and leading 2-Loop straightforward:

2-Loop: $a_\mu^{\rm logs} = -\frac{4\alpha}{\pi} \log \frac{M_{\rm SUSY}}{m_\mu} a_\mu^{1-{\rm Loop}}$

$$a_\mu^{(\chi\gamma H)} = \frac{\alpha^2 m_\mu^2}{8\pi^2 M_W^2 s_W^2} \sum_{k=1,2} \left[{\rm Re}[\lambda_\mu^{A^0} \lambda_{\chi_k^+}^{A^0}]\, f_{PS}(m_{\chi_k^+}^2/M_{A^0}^2) + \sum_{S=h^0,H^0} {\rm Re}[\lambda_\mu^S \lambda_{\chi_k^+}^S]\, f_S(m_{\chi_k^+}^2/M_S^2) \right],$$
$$a_\mu^{(\tilde f\gamma H)} = \frac{\alpha^2 m_\mu^2}{8\pi^2 M_W^2 s_W^2} \sum_{\tilde f=\tilde t,\tilde b,\tilde\tau} \sum_{i=1,2} \left[\sum_{S=h^0,H^0} (N_c Q^2)_{\tilde f}\, {\rm Re}[\lambda_\mu^S \lambda_{\tilde f_i}^S]\, f_{\tilde f}(m_{\tilde f_i}^2/M_S^2) \right].$$

$$\lambda_\mu^{\{h^0,H^0,A^0\}} = \left\{ -\frac{s_\alpha}{c_\beta}, \frac{c_\alpha}{c_\beta}, t_\beta \right\},$$
$$\lambda_{\chi_k^+}^{\{h^0,H^0,A^0\}} = \frac{\sqrt{2} M_W}{m_{\chi_k^+}} \left(U_{k1} V_{k2} \{ c_\alpha, s_\alpha, -c_\beta \} + U_{k2} V_{k1} \{ -s_\alpha, c_\alpha, -s_\beta \} \right).$$
$$\lambda_{\tilde t_i}^{\{h^0,H^0\}} = \frac{2 m_t}{m_{\tilde t_i}^2 s_\beta} \left(+\mu^* \{ s_\alpha, -c_\alpha \} + A_t \{ c_\alpha, s_\alpha \} \right) (U_{i1}^{\tilde t})^*\, U_{i2}^{\tilde t},$$
$$\lambda_{\tilde b_i}^{\{h^0,H^0\}} = \frac{2 m_b}{m_{\tilde b_i}^2 c_\beta} \left(-\mu^* \{ c_\alpha, s_\alpha \} + A_b \{ -s_\alpha, c_\alpha \} \right) (U_{i1}^{\tilde b})^*\, U_{i2}^{\tilde b},$$
$$\lambda_{\tilde\tau_i}^{\{h^0,H^0\}} = \frac{2 m_\tau}{m_{\tilde\tau_i}^2 c_\beta} \left(-\mu^* \{ c_\alpha, s_\alpha \} + A_\tau \{ -s_\alpha, c_\alpha \} \right) (U_{i1}^{\tilde\tau})^*\, U_{i2}^{\tilde\tau}.$$

$$f_{PS}(z) = \frac{2z}{y} \left[{\rm Li}_2\left(1 - \frac{1-y}{2z}\right) - {\rm Li}_2\left(1 - \frac{1+y}{2z}\right) \right]$$
$$f_S(z) = (2z-1) f_{PS}(z) - 2z(2 + \log z),$$
$$f_{\tilde f}(z) = \frac{z}{2} \left[2 + \log z - f_{PS}(z) \right].$$

SUSY prediction

- 1-loop and most 2-loop contributions known
- remaining theory uncertainty of SUSY prediction: [DS '06]

$$\delta a_\mu^{\mathrm{SUSY}} \approx 3 \times 10^{-10}$$

while

$$\delta a_\mu^{\mathrm{exp-SM}} \approx 8 \times 10^{-10}$$

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Numerical results

Generic behaviour understood.
Now study details → interesting aspects:

- “typical” / “most general” results
- “aggressive” / “conservative” bounds on SUSY parameters

Example for “typical” behaviour

benchmark point SPS1a

$$a_\mu(\text{SUSY, SPS1a}) = 29.8(3.1) \times 10^{-10}$$ [DS ’06]

$$a_\mu(\text{Exp.} - \text{SM}) =$$

$$M_W(\text{SPS1a}) =$$

$$M_W(\text{Exp.}) =$$

Agreement with experiment?

Example for “typical” behaviour

benchmark point SPS1a

$$a_\mu(\text{SUSY, SPS1a}) = 29.8(3.1) \times 10^{-10}$$ [DS ’06]

$$a_\mu(\text{Exp.} - \text{SM}) = 28.7(9.1) \times 10^{-10}$$ [Jegerlehner ’07]

$$M_W(\text{SPS1a}) =$$

$$M_W(\text{Exp.}) =$$

Agreement with experiment?

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$$M_W(\text{SPS1a}) = 80.381(18)\text{ GeV}$$ [Heinemeyer, Hollik, DS, Weber, Weiglein '06]

$$M_W(\text{Exp.}) =$$

Agreement with experiment?

Example for "typical" behaviour

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$$a_\mu(\text{Exp.} - \text{SM}) = 28.7(9.1) \times 10^{-10}$$ [Jegerlehner '07]

$$M_W(\text{SPS1a}) = 80.381(18)\ \text{GeV}$$ [Heinemeyer, Hollik, DS, Weber, Weiglein '06]

$$M_W(\text{Exp.}) = 80.398(25)\ \text{GeV}$$ [CDF, LEPEWWG '07]

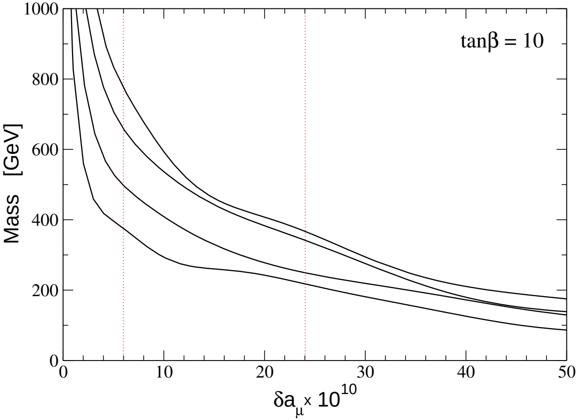
Agreement with experiment? very good!

Numerical results

SPS Point	$a_\mu^{\mathrm{SUSY,1L}}$(improved)	$a_\mu^{(\chi\gamma H)}$	$a_\mu^{(\tilde{f}\gamma H)}$	$a_\mu^{\mathrm{SUSY},\chi+\not{f},\mathrm{rest}}$	$a_\mu^{\mathrm{SUSY,ferm+bos,2L}}$
SPS 1a	29.29	0.168	0.029	0.056	0.267
SPS 1b	31.84	0.273	0.044	0.106	0.222
SPS 2	01.65	0.032	-0.002	0.027	0.068
SPS 3	13.55	0.078	0.009	0.029	0.187
SPS 4	49.04	0.786	0.085	0.288	0.349
SPS 5	08.59	0.029	0.135	-0.046	0.153
SPS 6	16.87	0.125	0.015	0.044	0.230
SPS 7	23.71	0.236	0	0.089	0.282
SPS 8	17.33	0.163	-0.001	0.062	0.211
SPS 9	-08.98	-0.046	-0.002	-0.018	0.115

[DS ’06]

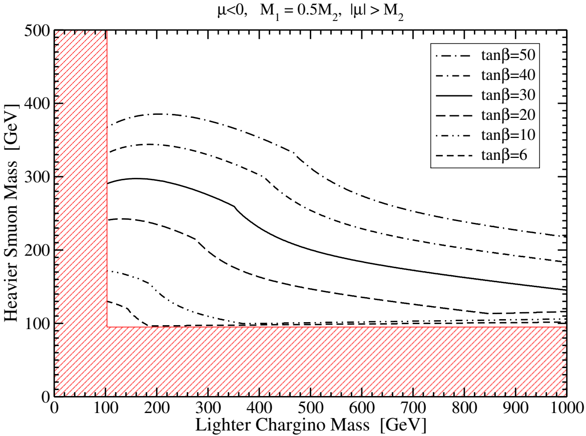
Numerical results



“aggressive”: require a_μ^{SUSY} within 2σ band [Byrne,Kolda,Lennon ’02]

$\Rightarrow$ upper mass bounds on four lightest sparticles

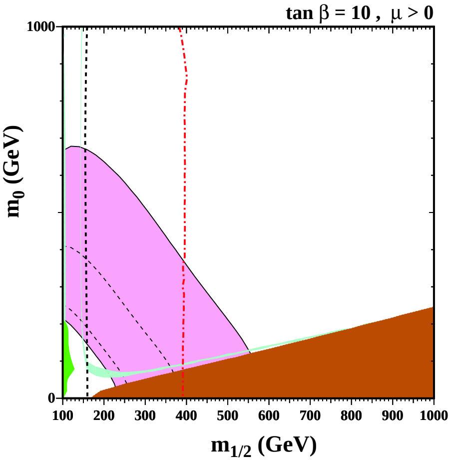
Numerical results



“conservative”: require $a_\mu^{\rm SUSY}$ within 5σ band [Martin Wells '02]

$\Rightarrow$ lower mass limits

Numerical results

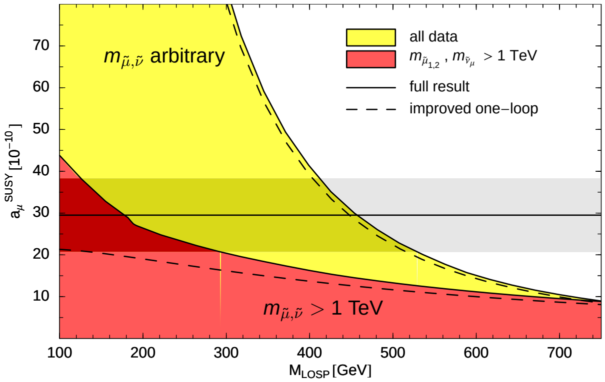


[Ellis, Olive, Sandick ’06]

- MSugra scenario ($\rightarrow$ only two parameters)
- even in very restricted scenarios, SUSY can accomodate the observed value of a_μ consistently with many other constraints from dark matter, b-decays, direct SUSY+Higgs searches

Numerical results

Summary: scan for $\tan\beta = 50$, all parameters < 3 TeV [DS '06]



- typically: $12 \times 10^{-10} \tan\beta \times \left(\frac{100\,\text{GeV}}{M_{\text{SUSY}}}\right)^2 \text{sign}(\mu)$

SUSY contributions in the observed range for low M_{SUSY}!

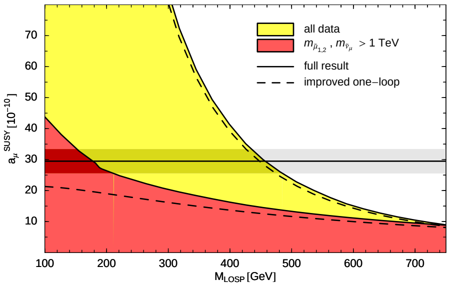
Outline

Potential of improved measurement

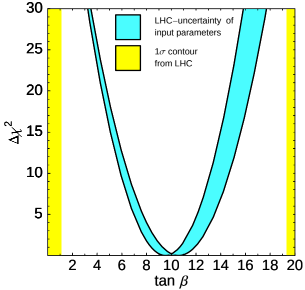
- new Brookhaven experiment proposed and feasible
- improved SM evaluation possible
- projected accuracy: $a_\mu(\text{Exp-SM}) = 29.5(3.9) \times 10^{-10}$ [Roberts et al 07]

Would be of tremendous importance as a complement of LHC

Constrain SUSY



Measure tan β (case SPS1a)



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Conclusions I

- history: fantastic experiments & calculations, many errors...
- experiment finalized, SM prediction has recently improved (and will further improve!)

$$a_\mu^{\text{exp}} - a_\mu^{\text{SM,HMNT,DEHZ}} = (28 \pm 8) \times 10^{-10} \quad 3.4\sigma$$

- significance of deviation gets stronger!
- further improvement of hadronic contributions can be expected!

Conclusions II

$$a_\mu^{\text{exp}} - a_\mu^{\text{SM,HMNT,DEHZ}} = (28 \pm 8) \times 10^{-10} \quad 3.4\sigma$$

- Case for new physics gets stronger!
- current measurements sensitive to 2-Loop SM and SUSY effects
- 2-Loop SUSY contributions:
  - reliable theory prediction

  [Degrassi, Giudice '98]
  [Heinemeyer, DS, Weiglein '03,'04]
  [DS '06]
- SUSY with low mass scale $\sim 200 \ldots 600$ GeV fits very well and large parameter regions already excluded

Future, more precise measurements very important and promising!