ORIENTIFOLDS of NON FACTORISABLE TORI

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talk at BSM L'pool
30/3/07

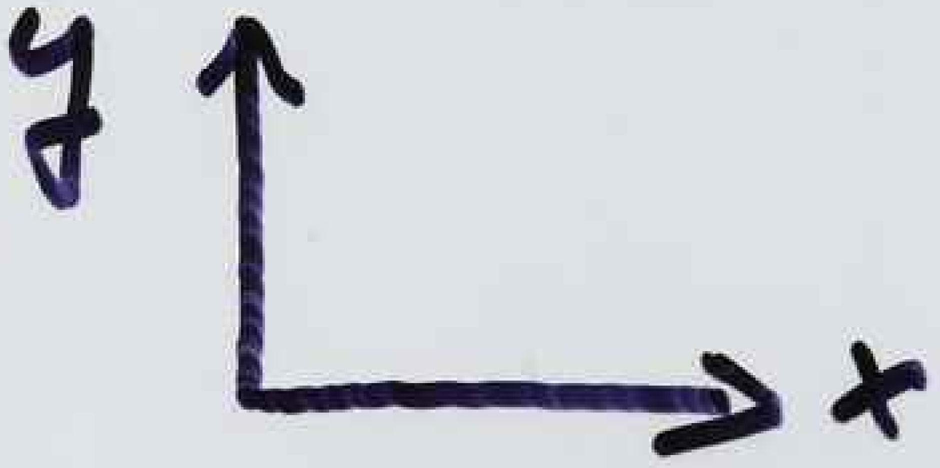
work in progress with
Cristina Timirgazin
& Ivonne Zavala

(related:
- A. Faraggi, S.F., C. Timirgazin
JHEP '06
- S.F., T. Kobayashi, H. Ohki, K.J. Takahashi, JHEP '07)

Preliminaries

toy: 2 extra dimensions

coordinates: $z = x + iy$



coordinate system

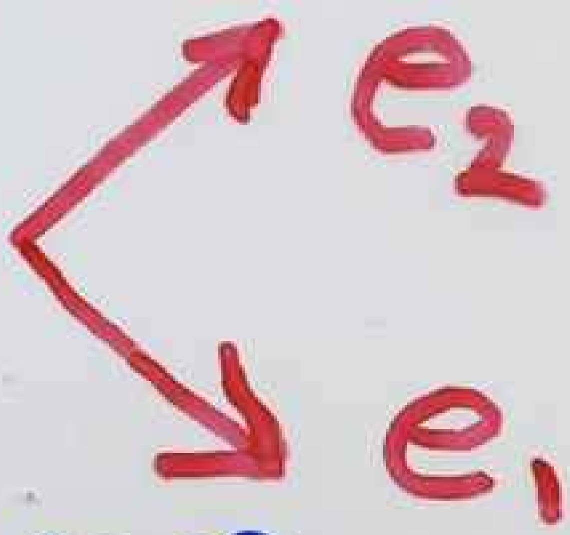
· orbifold: $\mathcal{R}: y \to -y$

$\Rightarrow$ two compactifications

A lattice

factorisable

B lattice



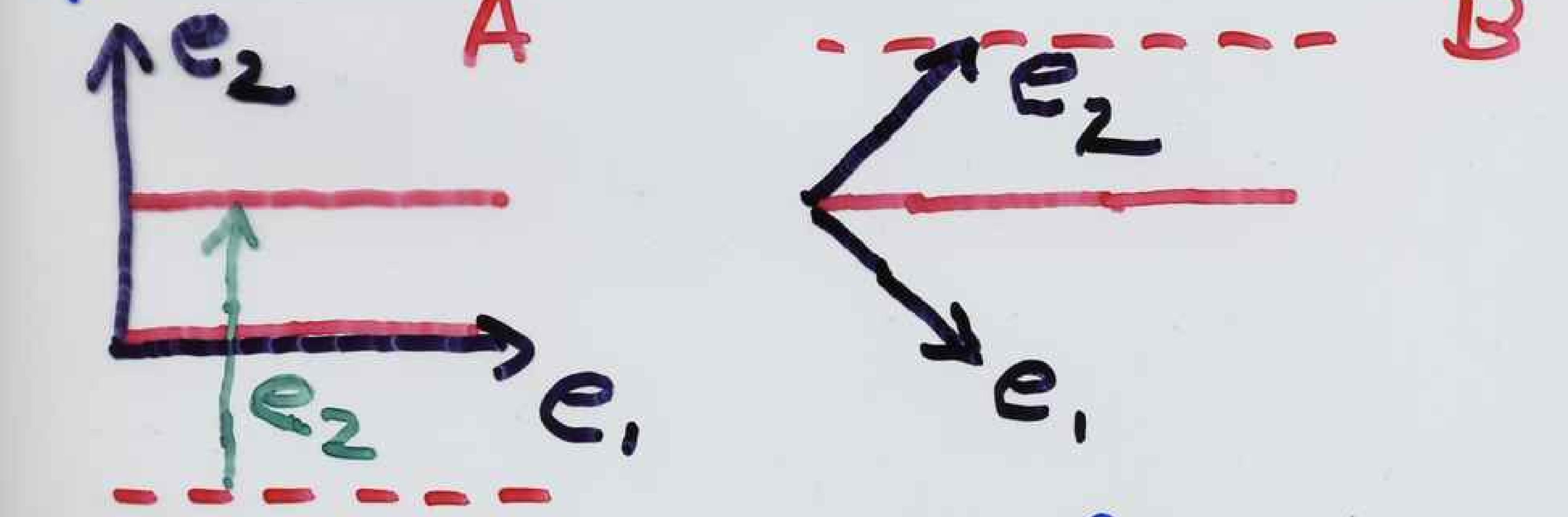
non factorisable

w.r.t. $\mathcal{R}$!

$e_2 \to -e_2$

$e_1 \leftrightarrow e_2$

fixed lines



2 fixed lines   1 fixed l.

Lefshetz Theor.

$$\# FL = \left| \frac{N}{(1-\theta)\Lambda} \right|$$

Narain, Samardi, Vafa '87

N = lattice $\perp$ to invariant

(A) $N = e_2\,,\ (1-\theta)\Lambda :\ 2e_2$

$$\# FL = vol((1-\theta)\Lambda)/vol(N) = 2$$

(B) $N = e_2 - e_1$

$$(1-\theta)\Lambda = e_2 - e_1$$

$$\Longrightarrow \# FL = 1$$

strings: 6 extra dim.

$z^i = x^i + i y^i$

$i = 1, 2, 3$

$\mathbb{Z}_2 \times \mathbb{Z}_2$ orbifold:

$$\Theta: \begin{array}{l} z^1 \to e^{i\pi} z^1 \\ z^2 \to e^{-i\pi} z^2 \\ z^3 \to z^3 \end{array}, \qquad \omega: \begin{array}{l} z^1 \to z^1 \\ z^2 \to e^{i\pi} z^2 \\ z^3 \to e^{-i\pi} z^3 \end{array}$$

$$\Theta\omega: \begin{array}{l} z^1 \to e^{i\pi} z^1 \\ z^2 \to z^2 \\ z^3 \to e^{-i\pi} z^3 \end{array}$$

- compactification on
  $T^6 = \Lambda$
- factorisable / non factoris. ~~refers~~ to
  $\Theta, \omega, \Theta\omega$ action!

- heterotic string: see Cristina's talk
- this talk:

ORIENTIFOLDS

outline:

- O - planes
- D-branes parallel to O-planes
- D-branes at angles to O-planes (N=1 SUSY & chirality)
- outlook

O-planes

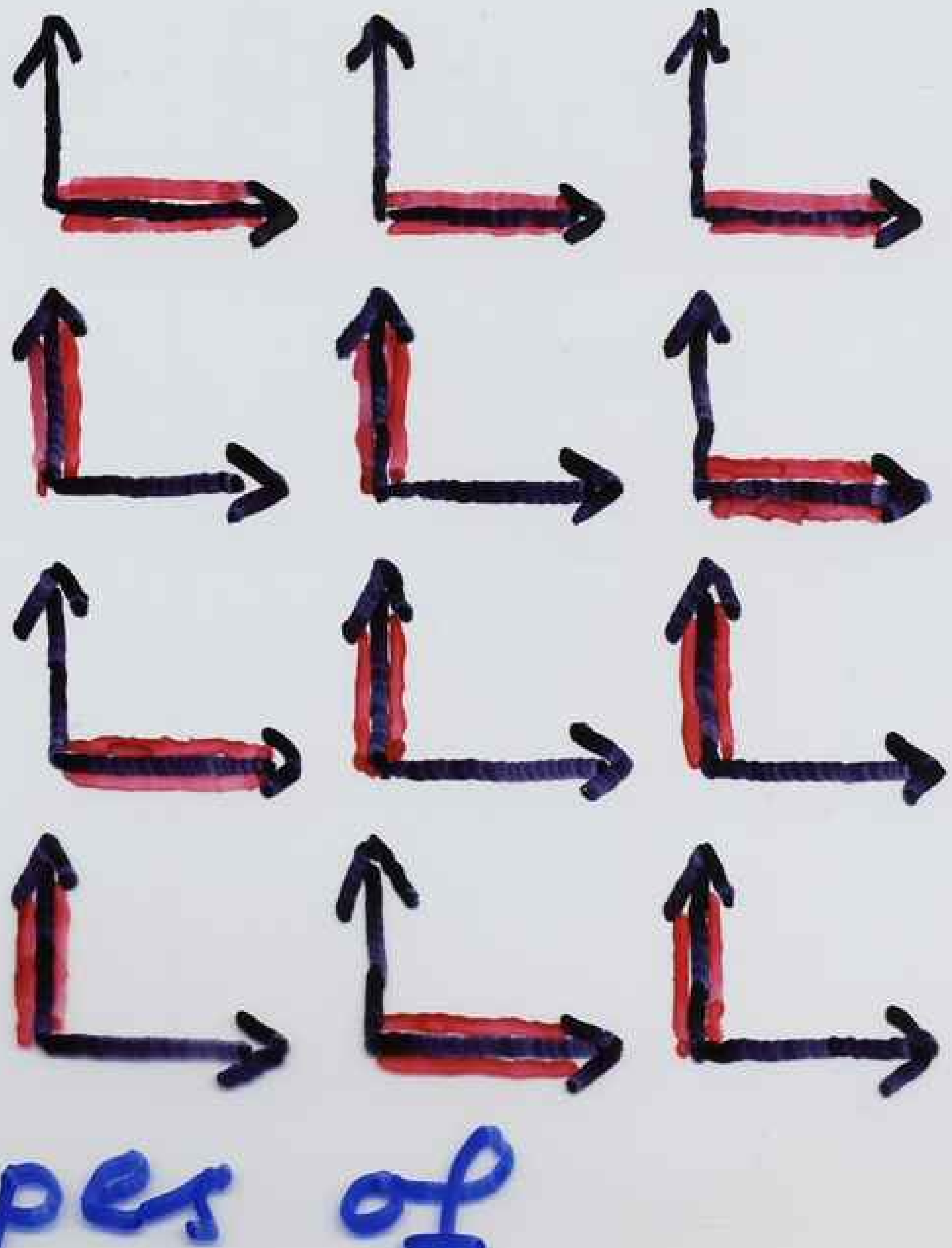
type IIA on $T^6/\mathbb{Z}_2 \times \mathbb{Z}_2$

$\rightarrow \mathcal{N}=2$ SUSY

$\rightarrow$ mod out $\Omega\mathcal{R}$,

$\mathcal{R}: z^i \rightarrow \bar{z}^i \rightarrow$ $\mathcal{N}=1$ SUSY

- $\Omega\mathcal{R}$-fixed plane:
- $\Omega\mathcal{R}\Theta$-fixed plane:
- $\Omega\mathcal{R}\omega$-fixed plane:
- $\Omega\mathcal{R}\Theta\omega$-fixed plane:



$\Rightarrow$ 4 types of O6-planes

factorisable T^6

Berkooz, Leigh '96
S.F., Honecker, Schreyer '00

AAA 

$2^3 = 8$ O6-planes
each charge -4

$\Rightarrow M = 32$ parallel D6 br.

replace $A \to B$ 

$\to$ reduces # O6 by $\frac{1}{2}$

· tadpole cancellation:

AAA: $(32-M)^2=0$

AAB: $(16-M)^2=0$

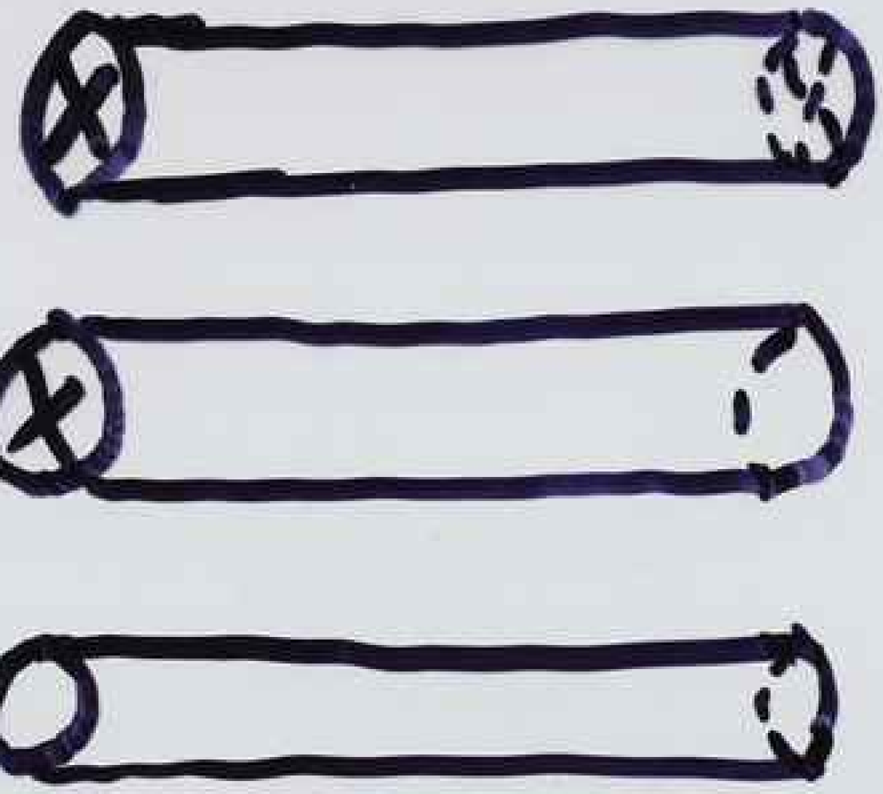
ABB: $(8-M)^2=0$

BBB: $(4-M)^2=0$

gauge group $\left[Sp\left(\frac{M}{4}\right)\right]^4$

computation

- relevant for RR charge



- suitable for computation

KB: 

MS: 

A: 

modular trans-formation

||

Poisson resummation of zero modes

KB: $\mathrm{Tr}(\Omega\mathcal{R} + \Omega\mathcal{R}\Theta + \dots)$

→ momenta & windings invariant under $\Omega\mathcal{R}$

- windings on $\Lambda_{-\mathcal{R}\,\mathrm{inv}}$
- momenta on $(\Lambda^*)_{\mathcal{R}\,\mathrm{inv}}$

$$= (\Lambda_{\mathcal{R},\perp})^* \equiv \left[\frac{1+\mathcal{R}}{2}\Lambda\right]^*$$

Poisson resum.: $\dfrac{\mathrm{vol}(\Lambda_{\mathcal{R},\perp})}{\mathrm{vol}(\Lambda_{-\mathcal{R}\,\mathrm{inv}})}$

A $\Lambda_{\mathcal{R},\perp}: (1,0)$

$\Lambda_{-\mathcal{R}\,\mathrm{inv}}: (0,1)$ ⇒ factor is <u>one</u>

B $\Lambda_{\mathcal{R},\perp}: (1,0)$

$\Lambda_{-\mathcal{R}\,\mathrm{inv}}: (0,2)$ factor is <u>1/2</u>

$\frac{1+\mathcal{R}}{2}(1,1) = (1,0)$

$(1,1) - (1,-1)$

A: · contribution from D6 // $\Omega\mathcal{R}$ fixed O6

· momenta on lattice dual to the one wrapped by D6 $(\Lambda_{\mathcal{R}\,\mathrm{inv}})^*$

· windings $\Lambda_{-\mathcal{R},\perp}$

$\Rightarrow$ factor

$$\frac{\mathrm{vol}\,(\Lambda_{\mathcal{R}\,\mathrm{inv}})}{\mathrm{vol}\,(\Lambda_{-\mathcal{R}\perp})} = \begin{cases} 1 & \text{(A)} \\ 2 & \text{(B)} \end{cases}$$

MS: $\mathrm{Tr}\,(\Omega\mathcal{R} + \dots)$

$$\frac{\mathrm{vol}\,(\Lambda_{\mathcal{R}\,\mathrm{inv}})}{\mathrm{vol}\,(\Lambda_{-\mathcal{R}\,\mathrm{inv}})} = 1 \quad \text{(A) \& (B)}$$

→ tadpole cancellation condition

$$0 = 2^{-\sum_{i=1}^{3}\delta_{B_i}}(32)^2 - 2\cdot M\cdot 32 + 2^{+\sum \delta_{B_i}} M^2 =$$

$$= 2^{\sum_i \delta_{B_i}}\left(\frac{32}{2^{\sum \delta_{B_i}}} - M\right)^2$$

- easy to generalise to non-factorisable T^6.

non factorisable T^6
SO(12) root lattice

$e_1 = (1,-1,0,0,0,0)$ $e_2 = (0,1,-1,0,0,0)$

$e_3 = (0,0,1,-1,0,0)$ $e_4 = (0,0,0,1,-1,0)$

$e_5 = (0,0,0,0,1,-1)$ $e_6 = (0,0,0,0,1,1)$

$\Rightarrow \ldots \Rightarrow$ tadpole cancellation

$$(16-M)^2 = 0$$

• 2d sublattices $(1,1,0,0,0,0)$, $(1,-1,0,0,0,0)$, …

a bit like BBB

• Def.: $B \leftrightarrow A$ $\longleftrightarrow$ $\mathcal{R} \longleftrightarrow \mathcal{R}'$

$y \to -y$ $\quad x \leftrightarrow y$

$\Rightarrow$

BBB	$(16-M)^2 = 0$
ABB	$(8-M)^2 = 0$
AAB	$(4-M)^2 = 0$
AAA	$(8-M)^2 = 0$

- so far D6 || O6

$\rightarrow$ any two D6 branes are related by an $SU(2) \subset SO(6)$ rotation

$\Rightarrow$ $\mathcal{N}=2$ SUSY spectrum from $D6_i D6_j$ open strings

$\rightarrow$ non-chiral

- add non-parallel D6 branes such that $\mathcal{N}=1$ SUSY preserved

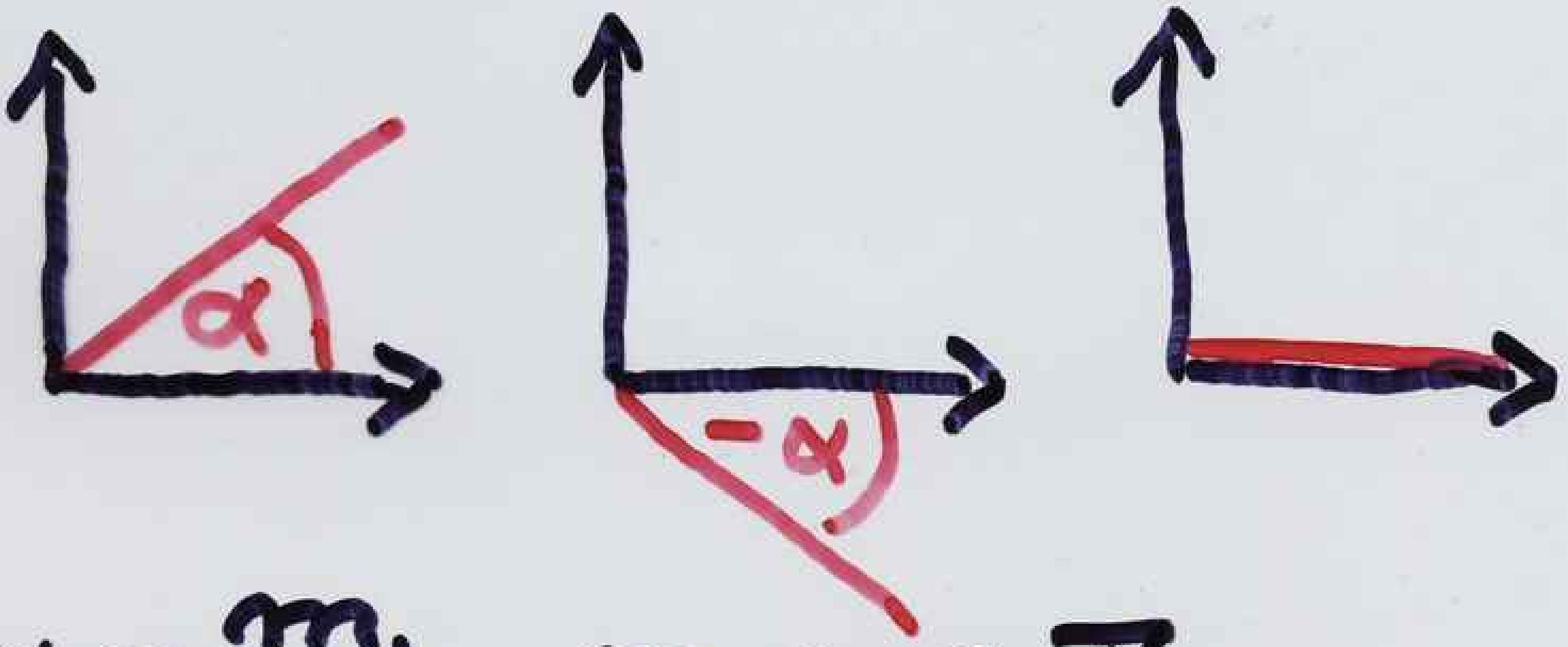
(Berkooz, Douglas, Leigh '96)

(Cvetič, Shiu, Uranga '01)

- must be related by $SU(3)$ to O6

- here, pick different $SU(2)$'s $\subset SU(3)$

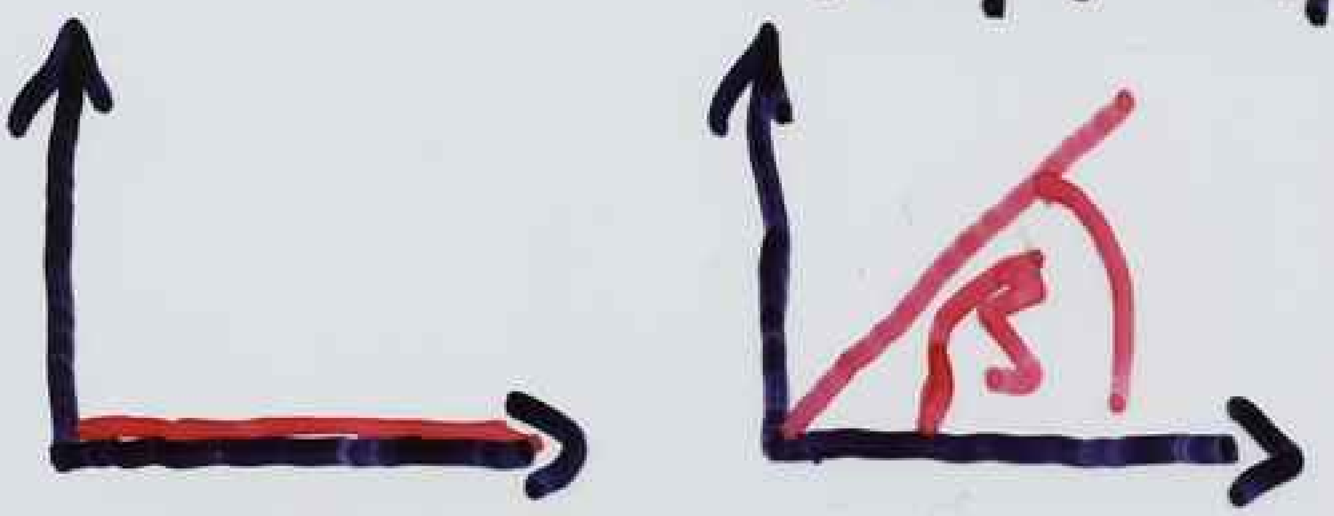
D6$_1$

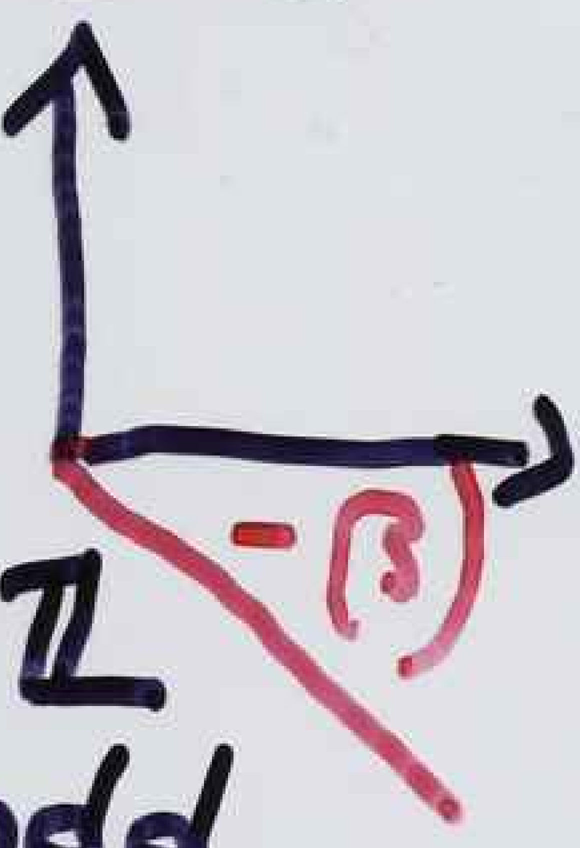


$$\cot\alpha = \frac{m_1}{n_1}, \quad m_1, n_1 \in \mathbb{Z}$$

$m_1 + n_1$ odd

D6$_2$

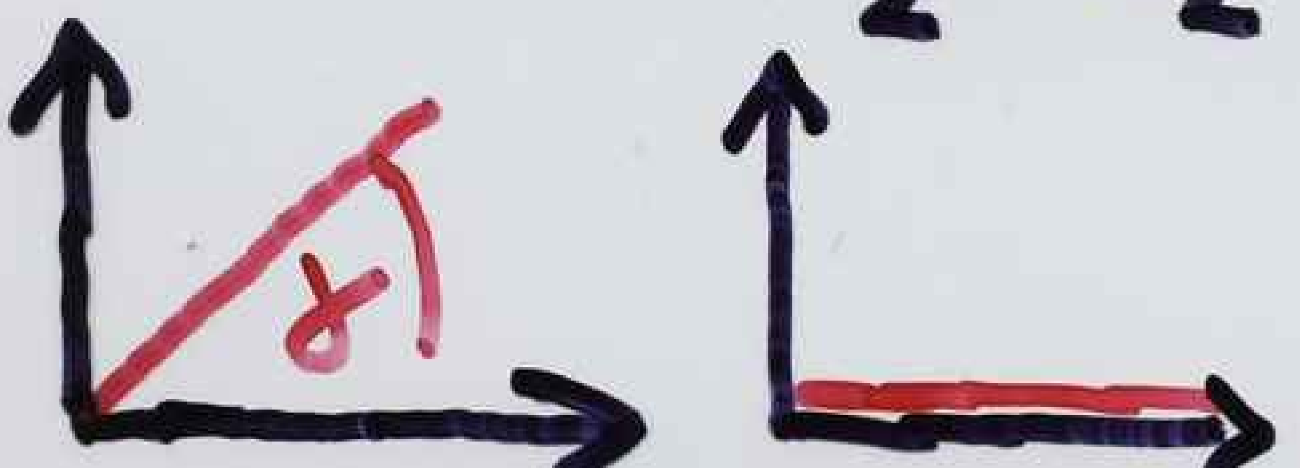


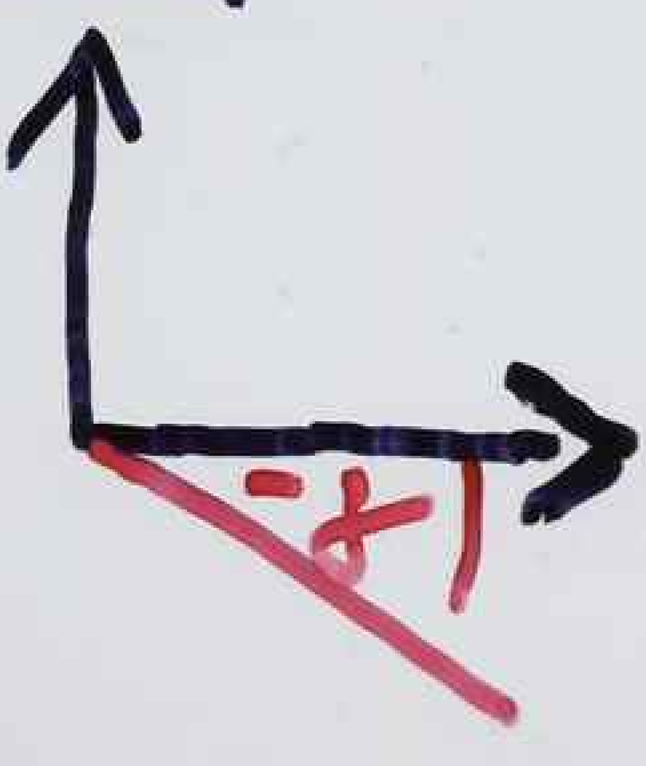


$$\cot\beta = \frac{m_2}{n_2}, \quad m_2, n_2 \in \mathbb{Z}$$

$m_2 + n_2$ odd

D6$_3$





$$\cot\gamma = \frac{m_3}{n_3}$$

$m_3, n_3 \in \mathbb{Z}$; $m_3 + n_3$ odd

· RR tadpole cancellation

write flux through 3-cycles

e.g. horizontal O6 plane

$$\underbrace{-4}_{\text{charge}} \cdot \underbrace{4}_{\# O6} \cdot (1,0,-1,0,0,0) \wedge (0,0,1,0,-1,0) \wedge (0,0,1,0,1,0)$$

... ⇒ set of equations

⇒ $$\sum_a (n_i^a)^2 - 8 = 0 \,, \quad i = 1,2,3$$

$$\sum_a \sum_{i=1}^{3} (m_i^a)^2 - 8 = 0$$

1 solution

	#	m_1	n_1	m_2	n_2	m_3	n_3
$D6_1$	2	1	2	0	0	0	0
$D6_2$	2	0	0	1	2	0	0
$D6_3$	2	0	0	0	0	1	2
$D6_h$	2	1	0	0	0	0	0

→ $SU(2)_h \times U(1)_1 \times U(1)_2 \times U(1)_3$

intersection

= # chiral multiplets

$D6_a$ wraps $E_1^a \wedge E_2^a \wedge E_3^a$ once

$D6_b$ —"— $E_1^b \wedge E_2^b \wedge E_3^b$ —"—

$$\cdot\ E_1^a \wedge E_1^b \wedge E_2^a \wedge E_2^b \wedge E_3^a \wedge E_3^b$$

$$= n(a,b)\, e_1 \wedge e_2 \wedge e_3 \wedge e_4 \wedge e_5 \wedge e_6$$

- $n(a,b)$ = intersection #
  = # chiral multiplets from $D6_a$ $D6_b$ open string

'chiral spectrum'

	$n(a,b)$	$U(1)_1$	$U(1)_2$	$U(1)_3$
$D6_1 D6_2$	16	1	-1	0
$D6_1 D6_2'$	-16	-1	1	0
$D6_1 D6_3$	0	1	0	-1
$D6_1 D6_3'$	-8	-1	0	1
$D6_2 D6_3$	0	0	1	-1
$D6_2 D6_3'$	20	0	1	-1

looks chiral

but only

$U(1)_1 + U(1)_2 + U(1)_3$

anomaly free

outlook

- may be better model with 'hybrid' lattice

$$\left.\begin{aligned} e_1 &= (1,0,-1,0,0,0) \\ e_3 &= (0,0,1,0,-1,0) \\ e_5 &= (0,0,1,0,1,0) \end{aligned}\right\} \begin{array}{l} SO(6) \\ \text{root} \\ \text{lattice} \end{array}$$

$$\left.\begin{aligned} e_2 &= (0,x_1,0,0,0) \\ e_3 &= (0,0,0,x_2,0,0) \\ e_4 &= (0,0,0,0,0,x_3) \end{aligned}\right\} \begin{array}{l} SU(2)^3 \\ \text{lattice} \end{array}$$

closer to Cvetič, Shiu, Uranga type semi-realistic models

→ let's see & hope!