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F-term uplifting and consistent D-terms

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- Moduli stabilisation via the racetrack

- KKLT and the cosmological constant

- Indirect uplifting via D-terms

The Setup: SUGRA

Our theoretical framework is low energy effective $\mathcal{N}=1$, $D=4$ supergravity. The object of interest to us is the scalar potential:

$$V = e^K \left(K_{i\bar{j}} F^i F^{\bar{j}} + \frac{1}{2}\frac{8\pi^2}{Re(T)}|D|^2 - 3|W|^2 \right)$$
$$= V_F + V_D + V_W$$

where $F_i = \frac{\partial W}{\partial \phi_i} + \frac{\partial K}{\partial \phi_i} W$ and $D = iK_i X^i$. X^i are Killing vectors, given by $\delta\phi_i = X^i \epsilon$.

For a $U(1)$ symmetry under which the fields $\phi_i \to e^{iq_i\epsilon}\phi_i$ we recover, for a canonical Kähler potential, the usual D-term potential:

$$V_D = g^2 \left(\sum_i q_i |\phi_i|^2 \right)$$

Stringy Moduli Stabilisation

The low energy effective SUGRA is an approximation to the KKLT class of models in which stringy effects stabilise the shape of the extra dimensions, but not their size. The final modulus is stabilised using

$$W_{KKLT} = Ae^{T/N} + W_0$$

at a supersymmetric point with non-zero $\langle W \rangle$.

Unfortunately this implies $\langle V \rangle < 0$.

Uplifting via D-terms

Clearly more work is required to break SUSY and lift the potential to positive values.

We consider the effects of gauging the shift of $T \to T + i\delta\Lambda$. This has two immediate effects:

- e^{-T} is no longer gauge invariant

- $U(1)$ D-terms appear in the potential

To restore gauge invariance we introduce additional fields M_1 and M_2 which transform like $M \to e^{-i\delta\Lambda} M$.

Racetrack

To allow tuning of the cosmological constant we introduce a second exponential:

$$W_{Race.} = A_1 N_1 \left(\frac{e^{-T}}{M_1}\right)^{\frac{1}{N_1}} - A_2 N_2 \left(\frac{e^{-T}}{M_2}\right)^{\frac{1}{N_2}} + W_0,$$

where W_0, $A_1 N_1$ and $A_2 N_1$ are constants determined by the high energy theory. M_1 and M_2 are the determinants of the quark bi-linears, which are treated as independent fields. Finally N_1 and N_2 are the number of colours in the two groups.

Solving the Racetrack

In addition to the superpotential we need a Kähler potential given by

$$K = -3 \ln (T + \bar{T}) + |M_1|^2 + |M_2|^2$$

Then the racetrack superpotential, in the absence of D-terms, stabilises T. It is sufficient to consider only

$$V = e^K \left(K_{i\bar{j}} F^i F^{\bar{j}} \right)$$

when stabilising t. The solution to $F_T = 0$ gives, in the limit of large t,

$$t \sim \ln \left(\frac{|A_1| x_2^{1/N_2}}{|A_2| x_1^{1/N_1}} \right) \frac{N_1 N_2}{N_2 - N_1}$$

where $T = t + ia$, $M_1 = x_1 e^{iN_1 \phi_1}$ and $M_2 = x_2 e^{iN_2 \phi_2}$.

This solution is not exact because it ignores F_{M_1}, F_{M_2}, D and W, however it serves as a good first approximation when $t \gg N_1, N_2$.

Indirect Lifting

V_D has two natural values, M_P^4 and zero. We now show a D-term that minimises to give $V_D = 0$ can lift $V_F + V_W$, albeit indirectly.

We extend the previous theory by the addition of one chiral field C that appears with the opposite sign charge to M_i, $C \to e^{iq\delta\Lambda}C$, hence generates the following D-term:

$$V_D = \frac{\pi^2\delta^2}{t}\left(\frac{3}{2t} + x_1^2 + x_2^2 - qc^2\right)^2$$

If we allow a natural $\delta \sim 1 - 10^{-2}$ then V_D dominates the shape of the potential, $qc^2 = \frac{3}{2t} + x_1^2 + x_2^2$ is enforced and a large mass term is introduced for $\frac{3}{2t} + x_1^2 + x_2^2 - qc^2$, splitting the moduli masses.

Constraint equation

It is clear that, if c has no potential, the constraint is satisfied trivially: c aligns to set $D = 0$. However, even if $\frac{\partial W}{\partial C} = 0$, a potential is generated for c via the Kähler derivatives:

$$V = e^K c^2 |W|^2$$

So it is clear that this introduces an energetic preference for small c. However, to good approximation the first derivative

$$\frac{\partial V}{\partial c} = -q \frac{4c\pi^2\delta^2}{t}\left(\frac{3}{2t} + x_1^2 + x_2^2 - qc^2\right)$$
$$+ 2c|W|^2 + \frac{\partial e^K}{\partial c} c^2 |W|^2$$

is dominated by the derivative of V_D. Enforcing $V_D = 0$ as a constraint, i.e. making the replacement $c^2 = \frac{1}{q}\left(\frac{3}{2t} + x_1^2 + x_2^2\right)$ is a good approximation, true in the limit $\delta \to \infty$.

Minimisation

The procedure for finding a minimum is as follows:

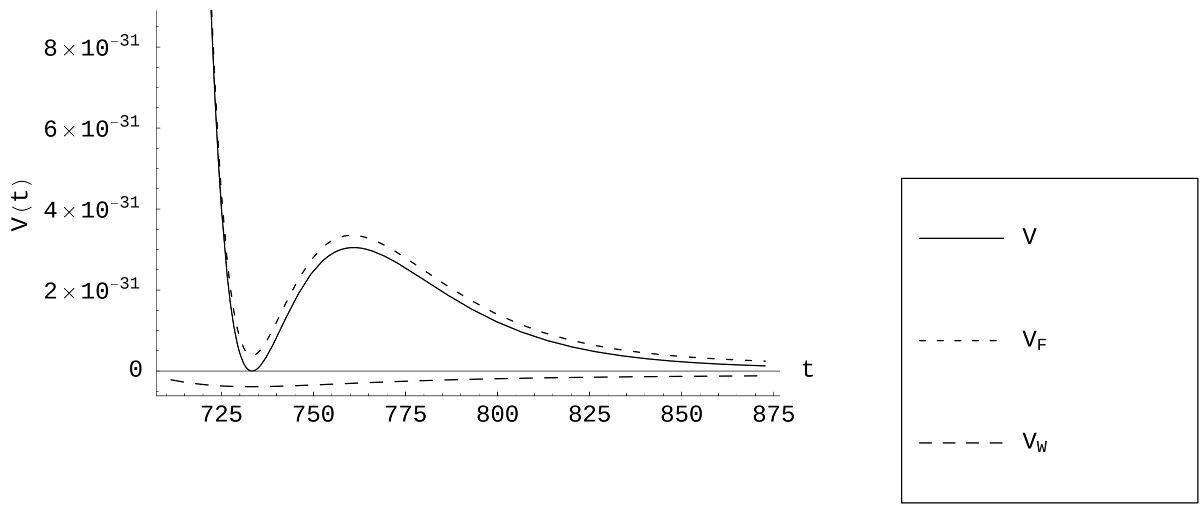
- D-term is set to zero
- c^2 is replaced by the constraint
- Constrained potential is minimised
- Solution is input into unconstrained potential

The unconstrained minimum is, by definition, the lowest value of the potential locally. Because the constraint equation has its own potential, with $|W|^2$ acting as a mass term, a balance must be achieved by moving to a higher minimum.

As a result the potential is lifted and the cosmological constant can be tuned with arbitrary precision by varying the input parameters.

Minimum of the potential

This plot is of the constrained potential, in this region the D-term is parabolic.



Input parameter values

$\|A_1\|$	2.62
$\|A_2\|$	0.3
N_1	25
N_2	27
δ	1
W_0	2.13×10^{-12}
q	1/16

Minimisation results

$m_{3/2}$	478.142
$V_0^{1/4}$	0
$V_F^{1/4}$	3.41239×10^{10}
$V_D^{1/4}$	2292.40
t	733.325
x_1	0.173619
x_2	0.237810
ϕ_1	π
ϕ_2	π
m_t	1.630×10^5
m_a	1.646×10^5
m_{x_1}	3.141×10^2
m_{x_2}	2.427×10^{17}
m_c	7.853×10^2
m_{ϕ_1}	6.000×10^2
m_{ϕ_2}	0
m_V	8.56×10^{17}

Conclusions

- All fields are stabilised with a realistic gravitino mass and vanishing or slightly positive cosmological constant

- Cancellable D-terms introduce a constraint indirectly lifting the potential and allowing for a natural δ