Cosmology of the string tachyon

Progress in cubic string field theory

Gianluca Calcagni

University of Sussex

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Outline

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For a review see Sen (hep-th/0410103).

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where $\lambda = 3^{9/2}/2^6 \approx 2.19$.

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$\Rightarrow$ A comparison would open up interesting possibilities!

Friedmann–Robertson–Walker background

Flat FRW metric:

$$ds^2 = -dt^2 + a^2(t)\, dx_i dx^i .$$

The Hubble parameter is defined as $H \equiv \dot{a}/a = d_t a/a$.

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Equations of motion (EH action):

$$H^2 = \rho_T + \rho_m + \rho_r$$

$$\frac{\ddot{T}}{1-\dot{T}^2} + 3H\dot{T} + \frac{V_{,T}}{V} = 0$$

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- This is a toy model valid on cosmological scales.
- More problematic to justify at late times.

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- No reheating with runaway string effective potentials.
- Large density perturbations with such potentials.
- Reheating can be achieved with a negative KKLT-like Λ. Anisotropies adjusted with small warp factor [Garousi, Sami, Tsujikawa 2004]
- With phenomenological potentials, good inflation and non-Gaussianity but non-characteristic predictions.

DBI tachyon as dark energy

with Andrew R. Liddle – PRD **74**, 043528 (2006), astro-ph/0606003

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- The tachyon is poorly effective as dark energy (the cosmological constant problem is NOT solved).
- The tachyon cannot decay faster than dust matter, $\rho_T \sim a^{-3(1+w_T)} > \rho_m \sim a^{-3} \Rightarrow$ cannot be used as quintessential inflaton.

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$$e^{r_*\Box} = \sum_{\ell=0}^{+\infty}\frac{r_*^\ell}{\ell!}\Box^\ell \equiv \sum_{\ell=0}^{+\infty} c_\ell\Box^\ell$$

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Susy CSFT when $m^2 = -1/2, \ \sigma = \sigma(\lambda), \ n = 4$.

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In terms of $\tilde{\phi}$:

$$-(\Box - m^2)e^{-2r_*\Box}\tilde{\phi} + \tilde{U}' = 0$$

Energy density and pressure

$$\begin{aligned} \rho &= -T_0{}^0 = \frac{\dot{\phi}^2}{2}(1-\mathcal{O}_2) + \tilde{V} - \mathcal{O}_1 \\ p &= T_i{}^i = \frac{\dot{\phi}^2}{2}(1-\mathcal{O}_2) - \tilde{V} + \mathcal{O}_1 \end{aligned}$$

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\mathcal{O}_2 &= \frac{2}{\dot\phi^2}\int_0^{r_*} ds\,(e^{s\Box}\tilde U')^{\cdot}(e^{-s\Box}\tilde\phi)^{\cdot}.
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- Non-standard Cauchy problem: infinite initial conditions, to know them means to find the solution! [Moeller and Zwiebach 2002]
- Nonlocal theories are at odds with the inflationary paradigm: while the latter tends to erase any memory of the initial conditions, the formers do preserve this memory.

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Possibility: p is a positive even number, so that the series ends at $\ell_* \equiv p/2$ and $\tilde{\phi} \sim t^p$.

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$$\Box^\ell \phi = (-1)^\ell t^{p-2\ell} \prod_{n=0}^{\ell-1} (p-2n)(p-2n-1+3H_0) .$$

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$$\lim_{\ell\to\infty} \left| \frac{c_{\ell+1}\Box^{\ell+1}\phi}{c_\ell \Box^\ell \phi} \right| = \lim_{\ell\to\infty} |c_1(p-2\ell)(p-2\ell-1+3H_0)| \frac{t^{-2}}{\ell+1} = +\infty ,$$

Possibility: p is a positive even number, so that the series ends at $\ell_* \equiv p/2$ and $\tilde{\phi} \sim t^p$. Another case is $p - 1 + 3H_0 = 2n$.

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NO!

What is not defined is the nonlocal solution expressed as an infinite series of powers of the d'Alembertian.

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- Define

$$\boxed{\phi(r,t)\equiv e^{r(\beta+\Box/\alpha)}\phi_{\rm loc}(t)}$$

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$$(\Box - m^2)\phi(r,t) = e^{-r\beta}\tilde{U}'[e^{r\beta}\phi((1+2\alpha)r,t)]$$

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4. Check that $\phi(r,t)$ is an approximated solution of the nonlocal e.o.m.s for some α, β, C_i.

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3. Quantization is well-defined, as in the closely related $1+1$ Hamiltonian formalism [Llosa and Vives 1994; Gomis et al. 2001,2004; Cheng et al. 2002]

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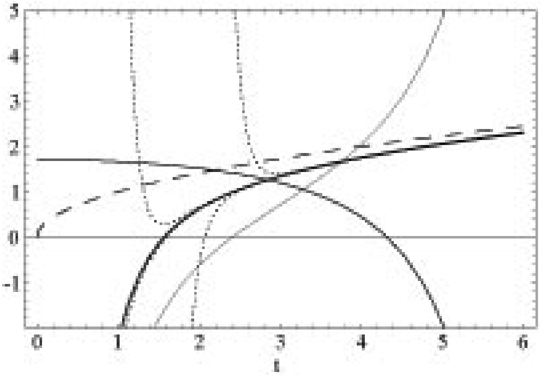

Figure: $p = 1/2$, $\nu = -3/2$, $r = 1$

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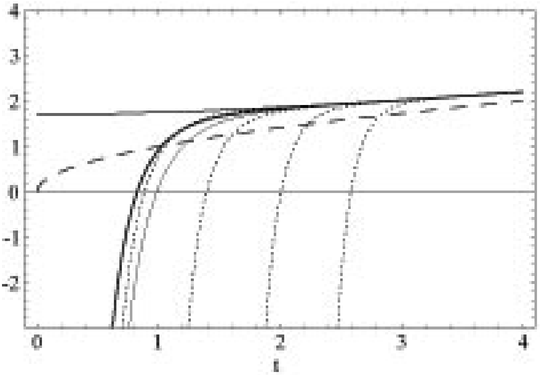

Figure: $p = 1/2$, $\nu = -3/2$, $r = -1$

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- Evidence for a nontrivial relation between SBFT and CSFT! (to appear)