Heterotic String Theory and its Applications in Studying Gauge Enhancements in Standard-Like Models

Tom Radburn
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Abstract: The aim of this report is to analyse some of the sector enhancements involved in a heterotic string model in which the *SO(10)* gauge symmetry is broken to the $SU(3) \times SU(2) \times U(1)^2$ gauge group at the string level. We will first show how such models can be constructed using the free fermionic method and how will obtain an *SO(10)* gauge group. We will see that there are a number of sectors that are produced but specifically we are interested in the untwisted sector and will be analysing the sectors within this which produce massless spacetime vector bosons and hence have the capability to enhance the gauge symmetry. We will use a tree diagram based method to study these sectors and then, will conclude by presenting any resulting enhancements to the untwisted sectors gauge group.

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1 Introduction

The standard model is a perturbative quantum field theory and to date is the chosen method to understand the quantum universe. It provides us with the tools to understand particle interactions and allows for predictions of properties of said particles. So why would we need string theory? Well, in short, quantum field theory can not provide us with a complete understanding of our observable universe. There are four fundamental forces that we observe in nature; electromagnetism, the weak nuclear force, the strong nuclear force and gravity. Quantum field theory accounts for the first three, but does not include gravity. The supposed graviton (the particle which would mediate the gravitational force) itself induces a gravitational field, which then again induces another gravitational field ad infinitum. Because quantum field theory is explained through the use of Feynman diagrams, we get a huge number of loops being added to these diagrams which leads to divergences which are uncorrectable and hence the theory breaks down.
Therefore to understand the observable universe with one overarching theory we must look elsewhere. We now turn to string theory, these quantised strings on the Planck scale are the building blocks for this theory. This paper is organised as follows; in section 2 we will take a look at the standard model in brief detail in order to identify some features of it which we will work off comparitively, specifically how the standard model fits into a representation of *SO(10)*. In section 3 we will briefly cover an introduction to string theory in order to cover the foundations for which this paper will work off. Section 4 is where we will introduce the free fermionic formulation and show in some detail the ABK rules which we will be using throughout the rest of the paper. In section 5 we will look at the NAHE set of basis vectors which were used to produce realistic string models and take our first look at gauge enhancements. Next, in section 6, we will present the basis vectors that we will use in the paper. These we first formulated by Faraggi et.al in the 2018 paper which can be found in [1]. Next we will show an example of some projections of a sector and then show the 'tree' method for the enhancement sectors for this model. Doing this will allow us to find the conditions on the sectors for them to enhance the gauge symmetries. Finally in the last section we will present the results and then finish the paper by drawing some conclusions from the work that has been done.

2 The Standard Model

The standard model is an example of a perturbative quantum field theory, which gives us a very good understanding of the matter states that we observe in nature and their interactions under the strong and weak nuclear force along with the electromagnetic force. As eluded to before the standard model is very successful in its description of the universe (up to its limits), and so if we are to obtain a string model that describes the universe properly it makes sense that it should take on some of the properties of the standard model. In this paper we are analysing standard-like models, meaning they have a gauge group of the form: *SU(3)* x *SU(2)* x *U(1)*^{2}.

We will now go on to show how the standard model can be described in terms of group theory, and specifically show how it fits into a 16 (or $\bar{1}6$) representation of *SO(10)*. Firstly we see this group is of the form *SO(2n)*, and so is a rank 5 group, hence the five charges labelled at the top of figure.1 from 1-5. These will each be charged as $\frac{1}{2}$ under *SO(10)*. Another important property of the standard model is that the fermions are chiral, meaning for any fermion field we can define both left and right projections, defined by:

$$\psi = \begin{pmatrix} \psi_L \\ \psi_R \end{pmatrix}. \tag{1}$$

And $\psi_L and \psi_R$ are calculated by using:

$$\psi_L = \psi P_L = \frac{1-\gamma^5}{2}\psi \tag{2}$$

$$\psi_R = \psi P_L = \frac{1+\gamma^5}{2}\psi. \tag{3}$$

Where γ^5 is the Dirac gamma matrix and $P_L = \begin{pmatrix} 0 & 0 \\ 0 & I \end{pmatrix}$ and $P_R = \begin{pmatrix} I & 0 \\ 0 & 0 \end{pmatrix}$.

Our fermions are represented as 2-component Weyl Spinors. Now because the gauge bosons act only on left handed fields, then the *SU(2)* coupling is chiral, and so we get quark doublets and Lepton doublets for each of the three generations of fermions. We see that because of this any charges we find for a right handed fermion field will determine the charges for the corresponding anti-fermion chiral field. We will represent our particles as f_g where g=1,2,3 representing the three generations and the superscript c denotes its conjugate charge. This will be of importance later once we have extracted states from the representation. Now we will show an explicit form of the *SO(10)* spinorial weight lattice.

Q_1	Q_2	Q_3	Q_4	Q_5
+	+	+	+	+
-	-	+	+	+
-	+	-	+	+
-	+	+	-	+
-	+	+	+	-
+	-	-	+	+
+	-	+	-	+
+	-	+	+	-
+	+	-	-	+
+	+	-	+	-
+	+	+	-	-
-	-	-	-	+
-	-	-	+	-
-	-	+	-	-
-	+	-	-	-
+	-	-	-	-

Figure 1: Spinorial 16 weight lattice of *SO(10)*

As we can see in Fig.1 the 16 representation splits into three parts which we express in the following way. The initial state: $\binom{5}{0}$ which represents the 5 charges, of which, 0 are negative. Then the next 10 states: $\binom{5}{2}$ were in a similar fashion, of the 5 charges 2 are negative and finally the last 5 states: $\binom{5}{4}$ of the 5 charges, 4 are negative.
The weight lattice can be also be expressed as follows:

$$\left[16 \rightarrow 1_{\frac{5}{2}} \oplus 10_{\frac{1}{2}} \oplus \bar{5}_{-\frac{3}{2}}\right]. \tag{4}$$

Now we will look more in depth at the gauge group representation of the decomposition, we will use Fig.1 to do this. We are looking to see how *SO(10)* decomposes under the *SU(3)* x *SU(2)* x *U(1)*^{2} subgroup. First we check that the group is compatible for this decomposition, we check that the rank of the groups are equivalent before and after (which it is). Then we check the dimensionality of the group, *SO(10)* is a group with dimensionality of 45 and the decomposed group has dimension 13 + 16 + $\bar{1}$6 which is also 45. Now as we did earlier, we will use Fig.1 to obtain a list of states from the representation of the subgroup. We will take the first three columns, labelled Q_1, Q_2, Q_3 to represent the *SU(3)*x*U(1)* group and the last two columns labelled Q_4 and Q_5 to represent the *SU(2)*x*U(1)*. Still all with $\frac{1}{2}$ charge under the *U(1)* symmetry. By doing this we then see that we have the following:

$$\left[\binom{3}{0} + \binom{3}{2}\right]\left[\binom{2}{0} + \binom{2}{2}\right] + \left[\binom{3}{1} + \binom{3}{3}\right]\binom{2}{1}. \tag{5}$$

This is all of the possibilities of the even-even combinations of the *SU(3)* $\times$ *U(1)* $\times$ *SU(2)* $\times$ *U(1)* along with those for the odd-odd combinations. When they are expanded they give the following full set:

$$\left[\binom{3}{0}\binom{2}{0}+\binom{3}{0}\binom{2}{2}+\binom{3}{2}\binom{2}{0}+\binom{3}{2}\binom{2}{2}\right]+\left[\binom{3}{1}\binom{2}{1}+\binom{3}{3}\binom{2}{1}\right]. \quad (6)$$

Now again referring back to Fig.1 we can match up these states with rows in this table and we find that for the combinations listed above Fig.1 returns:

$$\binom{3}{0}\binom{2}{0}=1 \quad \tbinom{3}{0}\tbinom{2}{2}=1 \quad (7)$$

$$\binom{3}{2}\binom{2}{2}=\overline{3} \quad \tbinom{3}{2}\tbinom{2}{0}=3 \quad (8)$$

$$\binom{3}{1}\binom{2}{1}=6 \quad \tbinom{3}{3}\tbinom{2}{1}=\overline{2} \quad (9)$$

$$(10)$$

Hence now we can express equation (4) as the following:

$$16\rightarrow\binom{3}{0}\binom{2}{0}+\binom{3}{0}\binom{2}{2}+\binom{3}{2}\binom{2}{2}+\binom{3}{2}\binom{2}{0}+\binom{3}{1}\binom{2}{1}+\binom{3}{3}\binom{2}{1}. \quad (11)$$

Which can be written in terms of the charges as:

$$16\rightarrow(\frac{3}{2},+1)+(\frac{3}{2},-1)+(-\frac{1}{2},+1)+(-\frac{1}{2},-1)+(-\frac{3}{2},0)+(\frac{1}{2},0). \quad (12)$$

We will say here that from this 16 representation we can extract the standard model states, but this is an exercise not completed here.
To conclude we will state the things that we would want our string theory to have that are requirements for a realistic Standard-Like Model. First of all we require that there are no negative norm states (i.e. any tachyonic states should be projected out), and that any states in the current standard model that are observed in nature are included in our theory.

3 Introduction to String Theory

In this chapter we will briefly give an introduction to string theory, we will cover some basics that we will use and build upon in order to construct our model later in the free fermionic formulation of the heterotic string. String theory came about due to the inability of our best current theories which explain the observable universe, namely General Relativity and Quantum Field Theory. A main problem being the inability of a quantum field theory to include a consistent description of the gravitational interactions. String theory is the best current attempt at unity of the two pillars of modern physics. It can be shown that in the study of bosonic string theory a graviton can be modelled, and that this particle is an accurate description of what would be required for a boson to mediate gravitational interactions.

We will start by reviewing the origins of string theory, this is the bosonic string theory. So named as it models only bosonic particles. This theory is built upon the classical physics of an oscillating string. We incorporate Relativity early on in the procedure and use the theory to model first, a relativistic point particle and then on to a relativistic string. We will parameterize the space of the system and begin to work in the target space defined by: $X^{\mu}(\tau, \sigma)$ which we will use to define an area-functional for time-space surfaces. Then from here we will calculate this area functional and use it to go on and define the Nambu-Goto string action, which is given by:

$$S = -\frac{T_0}{c}\int_{\tau_i}^{\tau_f} d\tau \int_0^{\sigma_1} \sqrt{(\dot{X}\cdot X')^2 - (\dot{X})^2(X')^2} d\sigma. \tag{13}$$

We can use variational techniques, specifically making use of Hamilton's principle to vary this action and obtain equation of motion for the system. After this we introduce the light-cone gauge to our theory, we will use this gauge when we are quantizing our string. On making this choice of gauge it is important to note a couple of things; firstly that we will lose manifest Lorentz covariance (meaning we must do some extra work to convince ourselves that at all times the theory is Lorentz covariant) and secondly using this gauge will incorporate a constraint which will allow us to be certain that any states we obtain will be physical (i.e. there will be no negative norm states, ghosts or spurious states). We will state here the light-cone coordinates that we use:

$$X^{\mu} = (X^0, X^1, X^2, X^3) \to (X^+, X^-, X^2, X^3). \tag{14}$$

Where:

$$X^{\pm} = \frac{1}{\sqrt{2}}(X^0 \pm X^1). \tag{15}$$

During this process we find some interesting relations for the relativistic string. For example, the solution to the obtained wave equation which is given by:

$$X^{\mu}(\tau, \sigma) = x_0^{\mu} + \sqrt{2\alpha'}\alpha_0^{\mu}\tau + i\sqrt{2\alpha'}\sum_{n\neq 0}^{\infty}\Big(\frac{1}{n}\alpha_n^{\mu}e^{-in\tau}\Big)cos(n\sigma). \tag{16}$$

We can use all of the previous work to now look towards obtaining particle states with the end goal of analysing them in the hope they match states we observe in everyday life. We begin this by now looking to quantise our relativistic string, which we will do by the use of operators and canonical commutation relations. We know that whenever we have a physical system which satisfy the harmonic commutation relations : $[\hat{a}, \hat{a}^{\dagger}] = 1$ then we can build a Hilbert space by acting with creation operators. We can show that all our operators are Hermitian and that they satisfy a closed algebra. We then define a pair of operators as α_{-m}^{μ} , α_m^{μ} as creation and annihilation operators respectively. We can then begin to act on a well defined vacuum with our new operators and begin to produce states, the first massless excited state of the open string is given by: $\alpha_{-1}^{\mu}|k > \otimes|0 >$. This state can be interpreted as the photon, it is a massless vector boson. We can do similar for the closed string were the first massless excited state we obtain is given by: $\alpha_{-1}^{\mu}\tilde{\alpha}_{-1}^{\nu}|k > \otimes|0 >$. From this we obtain a constraint which gives us the following:

$$L_1(\rho_{\mu\nu}\alpha^{\mu}_{-1}\tilde{\alpha}^{\nu}_{-1})|k>=0. \tag{17}$$

Where, $\rho_{\mu\nu}$ is a second rank tensor of the SO(1,D-1) Lorentz group. This tensor can be split into its symmetric and asymmetric parts were we will find that the traceless part of the symmetric part of this second rank tensor describes the graviton. And for completeness, the trace part describes the dilaton.
So we have seen in this brief introduction that bosonic string theory can be used to describe particles from both the areas of physics that the theory is attempting to unify, this stands us in good stead to move on.

4 The Free-Fermionic Formulation

Earlier on when we discussed string theory we only talked about a theory which contained bosonic states, and especially it includes tachyonic states which are not realistic and so we will need to modify this. This was done using supersymmetry, and in our particular case, heterotic string theory with N=1 supersymmetry. We start this process by utilising a symmetry of the worldsheet, namely the worldsheet supersymmetry which will relate the fermions to the bosons. We will do this by introducing an anti-commutating dynamic variable $\psi^{\mu}_{\alpha}(\tau,\sigma)$. If we recall from our understanding of the standard model, we know that a quantum state of identical fermions is antisymmetric under the exchange of two fermions and hence the variable must be anti-commutating. We can think of this variable as being analogous to the bosonic dynamical variable $X^{\mu}(\tau,\sigma)$, this variable commutes, and so we can say now that $\psi^{\mu}_{\alpha}(\tau,\sigma)$ defines a worldsheet fermion fo a given value of μ. The theory has reduced dimensionality D=10 which we will expand upon later, and then under quantization we see that we obtain two sectors: The Ramond sector, where the fermions are periodic over one loop phases and the Neveu-Schwarz sector where the fermions are anti-periodic over one loop phases. The closed superstring is then constructed by combining left and right moving copies of the open superstring. These are called type II superstring theories. For us, instead of combining a left and a right superstring, we combine a left moving bosonic string with a right moving superstring, this is known as heterotic string theory. There are certain requirements that we put on our theory in order for us to call them "realistic", these include N=1 supersymmetry in 4 spacetime physical dimensions and the left and right moving modes in the theory are completely decoupled. Lets now consider the right moving superstring fields: X^{μ}_{+} and ψ^{μ}_{+} where $\mu=0,...,9$. We still have dimensionality D=10. Now the left moving sector, consisting of effectively 10 left moving bosonic fields X^{μ}_{-} where $\mu=0,...,9$. Noting here that bosons carry unit charge and the fermions carry half-unit charge. So for the two sides of the theory we need to make sure that we obtain the critical dimension D=10 on either side. This is an example of a conformal anomaly, which we can correct by manipulating the worldsheet charge via addition of worldsheet fermions. We add 32 Majorana-Weyl right-moving free fermions λ^{i} where $i=1,...,32$, note that these do not have a spacetime index as free fermions do not contribute to this property. At this point we can write down the string action:

$$S = \frac{1}{\pi}\int \Big(2\partial_- X_\mu \partial_+ X^\mu + i\psi^\mu \partial_- \psi_\mu + i\sum_{i=1}^{32} \lambda^i \partial_+ \lambda^i\Big) d^2\sigma. \tag{18}$$

4.1 The ABK Rules

Now that we have formulated the basics we can now proceed, recall that a requirement for our theory was to have properties of four flat space-time dimensions, and N=1 supersymmetry, this is problematic because at the moment we are in D=10. This problem was solved by Antoniadis, Bachas and Kounnas [6] whose work is directly built into four spacetime dimensions and along with a simple set of rules can be used to find the spectrum of realistic four dimensional models. We will go through the above in this part of the paper. Now we will address the compactification from D=10 to D=4, we will treat this in the same way as we did before and so arrive at the conclusion that we need 18 Majorana-Weyl left moving fields (λ^j) and 12 Majorana-Weyl right moving fields (λ^i). So finally we can write our complete set of right and left moving fermions as:

$$\textbf{Right movers:}\{\psi_1^\mu, \psi_2^\mu, \chi^{1\ldots6}, y^{1\ldots6}, \omega^{1\ldots6}\}. \tag{19}$$

$$\textbf{Left movers:}\{\bar{y}^{1\ldots6}, \bar{\omega}^{1\ldots6}, \bar{\lambda}^{1\ldots32}\}. \tag{20}$$

All of the extra quantum numbers associated with the string are carried by periodic and anti-periodic free fermions, which gives a representation of the worldsheet supersymmetry which is non-linear. The right and left movers listed above come directly from the fermionization of these internal degrees of freedom on an E_8 x E_8 heterotic string, which, for ease we can also complexify in the following way:

$$\Phi = \frac{1}{\sqrt{2}}(f^n + if^m). \tag{21}$$

Where Φ is a complex fermion and $f^{n,m}$ are real fermions.
Now we have this we will write out the set of basis vectors that we will be using throughout the remainder of this paper:

$$\textbf{Right movers:}\{\psi^\mu, \chi^{1,2}, \chi^{3,4}, \chi^{5,6}, y^{1\ldots6}, \omega^{1\ldots6}\}. \tag{22}$$

$$\textbf{Left movers:}\{\bar{y}^{1\ldots6}, \bar{\omega}^{1\ldots6}, \bar{\psi}^{1,2,3,4,5}, \bar{\eta}^{1,2,3}, \bar{\phi}^{1,2,3,4,5,6,7,8}\}. \tag{23}$$

The set of fermions : $\{y^{1\ldots6}, \omega^{1\ldots6} | \bar{y}^{1\ldots6}, \bar{\omega}^{1\ldots6}\}$ will be defined as the real fermions whilst all others will be complex. Antoniadis et. al [6] then formulated a set of rules which would be sufficient to define a heterotic string model, these rules state that we must define two things:

1). Basis vectors that specify boundary conditions for the internal free fermions.
2). Coefficients (one-loop phases) for each intersection of the basis vectors.

These rules themselves also have conditions on them which we need to define now.

4.1.1 Rules on the Basis Vectors

The model is generated by a set of basis vectors a_i, given as:

$$a_i = f^j, ..., f^n, \tag{24}$$

where any fermion that appears in the vector will have periodic boundary conditions and those fermions which don't appear will have anti-periodic boundary conditions, we can write this mathematically as:

$$a(f) = \begin{cases} 1 : \textit{Periodic} \\ 0 : \textit{Anti-periodic} \\ \frac{1}{2} : \textit{Twisted by a phase -i} \end{cases}$$

for each of the fermions f. We will now define the additive group Ξ:

$$\Xi = \sum_i m_i a_i \qquad \text{where; } m_i = 0,...,N_i \text{ -1.} \tag{25}$$

We will now state that the product, $N_i a_i = 0 mod 2$ and that we can define N_{ij} as the least common multiple of both N_i and N_j it will become clear as to why we have done this when we now lay out the following five rules on the basis vectors.

1). The basis vector $\bar{1}$ is present in the model (all the basis vectors are periodic.)
2). There exist an even number of real fermions
3). $\sum_i m_i\ a_i = 0 \iff m_i = 0\text{mod } N_i\ \forall$ i
4). $N_{ij} = 0\text{mod}4$ [or $a_i a_j = 0\text{mod}2$]
5). $N_i\ a_i\ a_i = 0\text{mod}8$ [or $a_i\ a_i = 0\text{mod}4$] .

4.1.2 Rules on the Co-efficients

Once the space of states is defined, we then need to specify the phases: C $\binom{b_i}{b_j}$ for all intersections of the basis vectors. The equations needed to calculate these co-effiecients are written as follows:

$$1).\quad C\begin{pmatrix} b_i \\ b_j \end{pmatrix} = \delta_{b_i} e^{\frac{2\pi i}{N_j}} = \delta_{b_i} e^{\frac{\pi i}{2}(b_i b_j)} \cdot e^{\frac{2\pi i}{N_i} m}$$

$$2).\quad C\begin{pmatrix} b_i \\ b_i \end{pmatrix} = e^{\frac{-i\pi}{4} b_i b_j} C\begin{pmatrix} b_i \\ \vec{1} \end{pmatrix}$$

$$3).\quad C\begin{pmatrix} b_i \\ b_j \end{pmatrix} = e^{\frac{i\pi}{2} b_i b_j} C\begin{pmatrix} b_j \\ b_i \end{pmatrix}^*$$

$$4).\quad C\begin{pmatrix} b_i \\ b_j + b_k \end{pmatrix} = \delta_{b_i} C\begin{pmatrix} b_i \\ b_j \end{pmatrix} C\begin{pmatrix} b_i \\ b_k \end{pmatrix}.$$

And we should also state the GSO projection that we will be using in order to determine the surviving massless states in the spectrum, the GSO projections were put forward in the 1977 paper by Gliozzi, Scherk and Olive [3]. The projection can be expressed as follows:

$$e^{i\pi b_i \cdot F_\alpha} |S>_\alpha = \delta_\alpha C\begin{pmatrix} \alpha \\ b_i \end{pmatrix}^* |S>_\alpha . \tag{26}$$

Where the $|S>_\alpha$ is a state in the sector 'α'. 'α' sector that we are analysing, and 'b_i' are the basis vectors that we are using to do the projection from the sector 'α'. Finally we will define δ_α for the sector α as:

$$\delta_\alpha = \begin{cases} 1 \text{ If } \alpha(\psi^\mu) = 0 \\ -1 \text{ If } \alpha(\psi^\mu) = 1 \\ 0 \text{ Otherwise} \end{cases}$$

5 NAHE Basis Free Fermionic Models

The free fermionic formulation that we have looked into so far produces many models with GUT symmetry, one that produces many realistic physical models with three generations is the so called NAHE set. Here we will look at the NAHE set of basis vectors which can be used to produce the 16 + $\bar{1}6$ that we require by enhancing the gauge group. The NAHE set was first formulated by Nanopoulos, Antoniadis, Hagelin, Ellis in the 1988 paper [10], the set of basis vectors break the *SO(44)* gauge group down to E_6 x E_8 x $SO(4)^3$ x $U(1)^2$. From this we will show that we can get the 16 + $\bar{1}6$ representation out of this. The NAHE set consists of the following basis vectors:

$$\nu_1 = \vec{1} = \{\psi^\mu, \chi^{1,\dots,6}, y^{1,\dots,6}, \omega^{1,\dots,6} | \bar{y}^{1,\dots,6}, \bar{\omega}^{1,\dots,6}, \bar{\eta}^{1,2,3}, \bar{\psi}^{1,2,3,4,5}, \bar{\phi}^{1,\dots,8}\} \tag{27a}$$

$$\nu_2 = \vec{S} = \{\psi^\mu, \chi^{1,2}, \chi^{3,4}, \chi^{5,6}\} \tag{27b}$$

$$\nu_3 = \vec{b}_1 = \{\psi^\mu, \chi^{1,2}, y^{3,4}, y^{5,6} | \bar{y}^{3,4}, \bar{y}^{5,6}, \bar{\eta}^1, \bar{\psi}^{1,2,3,4,5}\} \tag{27c}$$

$$\nu_4 = \vec{b}_2 = \{\psi^\mu, \chi^{3,4}, y^{1,2}, \omega^{5,6} | \bar{y}^{1,2}, \bar{\omega}^{5,6}, \bar{\eta}^2, \bar{\psi}^{1,2,3,4,5}\} \tag{27d}$$

$$\nu_5 = \vec{b}_3 = \{\psi^\mu, \chi^{5,6}, \omega^{1,2}, \omega^{3,4} | \bar{\omega}^{1,2}, \bar{\omega}^{3,4}, \bar{\eta}^2, \bar{\psi}^{1,2,3,4,5}\}. \tag{27e}$$

(27f)

We will note that for the initial set of basis vectors gives us the *SO(44)* gauge group, these 44 internal free fermions represent the correction to the conformal anomaly that we encountered earlier. We will start initially with the following set of basis vectors:

$$\mathcal{B} = \{\vec{1}, \vec{S}, \vec{b_1}\}, \tag{28}$$

where;

$$\vec{b_1}^R = \{\bar{\psi}^{1,2,3,4,5} \bar{\eta}^1 \bar{y}^{3,4,5,6}\} = 1$$
$$\vec{b_1}^R = \{\bar{\eta}^{2,3} \bar{y}^{1,2} \bar{\omega}^{1,2,3,4,5,6} \bar{\phi}^{1,\dots,8}\} = 0.$$

We can use equation(26) to perform the GSO projection and see which states survive and can then analyse the resulting gauge group. This is done as follows:

$$e^{i\pi b_1 \cdot F_{NS}} \{\psi^\mu \bar{\phi}^a_{\frac{1}{2}} \bar{\phi}^b_{\frac{1}{2}}\} |S>_0 = -|S>_0 \ . \tag{29}$$

And we conclude that all others are projected out, we can also note that we have enhanced the gauge group: *SO(44)* $\rightarrow$ *SO(16)* x *SO(28)*. Now we will add another of our basis vectors and repeat the process, here we find that when using:

$$\vec{b_2}^R = \{\bar{\psi}^{1,2,3,4,5}\bar{\eta}^2\bar{y}^{1,2}\bar{\omega}^{5,6}\} = 1$$
$$\vec{b_2}^R = \{\bar{\eta}^{1,3}\bar{y}^{3,4,5,6}\bar{\omega}^{1,2,3,4}\bar{\phi}^{1,\dots,8}\} = 0.$$

With the GSO projection condition:

$$e^{i\pi b_2\cdot F_{NS}}\{\psi^\mu\bar{\phi}^a_{\frac{1}{2}}\bar{\phi}^b_{\frac{1}{2}}\}|S>_0 = -|S>_0 \ . \tag{30}$$

Which leads us to the states:

$$\psi^\mu_{\frac{1}{2}}|0>_L \otimes\{\bar{\psi}^{1,2,3,4,5}\}\{\bar{\psi}^{1,2,3,4,5}\}^*|0>_R = -|S>_0$$
$$\psi^\mu_{\frac{1}{2}}|0>_L \otimes\{\bar{\eta}^2\bar{\omega}^{5,6}\bar{y}^{1,2}\}\{\bar{\eta}^2\bar{\omega}^{5,6}\bar{y}^{1,2}\}^*|0>_R = -|S>_0$$
$$\psi^\mu_{\frac{1}{2}}|0>_L \otimes\{\bar{\eta}^1\bar{y}^{3,4,5,6}\}\{\bar{\eta}^1\bar{y}^{3,4,5,6}\}^* = -|S>_0$$
$$\psi^\mu_{\frac{1}{2}}|0>_L \otimes\{\bar{\eta}^3\bar{\omega}^{1,2,3,4}\bar{\phi}^{1,\dots,8}\}\{\bar{\eta}^3\bar{\omega}^{1,2,3,4}\bar{\phi}^{1,\dots,8}\}^* = -|S>_0$$

And so all of the states listed above survive the GSO projection and we get another enhancement to the gauge group, which is as follows:

SO(44) $\rightarrow$ *SO(16)* x *SO(28)* $\rightarrow$ *SO(10)* x *SO(6)*^{2} x *SO(22)* .

Immediately we can see that we have an *SO(10)* gauge symmetry now which is good news for our standard model states, but we can continue further with our enhancements annd apply the final basis vector from this NAHE set.

$$\vec{b_3}^R = \{\bar{\psi}^{1,2,3,4,5}\bar{\eta}^3\bar{\omega}^{1,2,3,4}\} = 1$$
$$\vec{b_3}^R = \{\bar{\eta}^{1,2}\bar{y}^{1,\dots,6}\bar{\omega}^{5,6}\bar{\phi}^{1,\dots,8}\} = 0$$

and again we compute the GSO projection and again we will find that the surviving states will have $-|S>_0$ the same form. therefore the states which survive are given by:

$$\psi^\mu_{\frac{1}{2}}|0>_L \otimes\{\bar{\psi}^{1,2,3,4,5}\}\{\bar{\psi}^{1,2,3,4,5}\}^*|0>_R = -|S>_0$$
$$\psi^\mu_{\frac{1}{2}}|0>_L \otimes\{\bar{\eta}^2\bar{\omega}^{5,6}\bar{y}^{1,2}\}\{\bar{\eta}^2\bar{\omega}^{5,6}\bar{y}^{1,2}\}^*|0>_R = -|S>_0$$
$$\psi^\mu_{\frac{1}{2}}|0>_L \otimes\{\bar{\eta}^1\bar{y}^{3,4,5,6}\}\{\bar{\eta}^1\bar{y}^{3,4,5,6}\}^* = -|S>_0$$
$$\psi^\mu_{\frac{1}{2}}|0>_L \otimes\{\bar{\eta}^3\bar{\omega}^{1,2,3,4}\}\{\bar{\eta}^3\bar{\omega}^{1,2,3,4}\}^* = -|S>_0$$
$$\psi^\mu_{\frac{1}{2}}|0>_L \otimes\{\bar{\phi}^{1,\dots,8}\}\{\bar{\phi}^{1,\dots,8}\}^* = -|S>_0 \ .$$

Again we have a gauge enhancement which we will write as:

SO(44) $\rightarrow$ *SO(10)* x *SO(6)*^{2} x *SO(22)* $\rightarrow$ *SO(10)* x *SO(6)*^{3} x *SO(16)* .

5.1 Construction of Realistic Models using NAHE

We will now briefly take a look at the realistic models that we can obtain from breaking *SO(10)*. First we will state explicitly that the *SO(10)* gauge group is generated by the fermions labelled : $\bar{\psi}^{1,2,3,4,5}$, as *SO(10)* is an example of an *SO(2n)* group where n=5. So the physical states that we can obtain are given by:

$$\psi^{\mu}_{\frac{1}{2}}|0>_L \otimes\{\bar{\psi}^{1,2,3,4,5}\}\{\bar{\psi}^{1,2,3,4,5}\}^{*}|0>_R \; .$$

We could for example pick a charge configuration for this group of fermions in order to examine breaking patterns.
If we choose, say: $\alpha = 11100$ then we will obtain:

$$\psi^{\mu}_{\frac{1}{2}}|0>_L \otimes\{\bar{\psi}^{1,2,3}\}\{\bar{\psi}^{1,2,3}\}^{*}|0>_R \; - SO(6)$$
$$\psi^{\mu}_{\frac{1}{2}}|0>_L \otimes\{\bar{\psi}^{4,5}\}\{\bar{\psi}^{4,5}\}^{*}|0>_R \; - SO(4) \; .$$

This is the Pati-Salam breaking of $SO(10) \rightarrow SO(6) \times SO(4)$
If for example we choose the configuration as: $\beta = \frac{1}{2}\frac{1}{2}\frac{1}{2}\frac{1}{2}\frac{1}{2}$ then we will obtain some combination of the states where the fermions may or may not be conjugates:

$$\psi^{\mu}_{\frac{1}{2}}|0>_L \otimes\{\bar{\psi}^{1,2,3,4,5}\}^{(*)}\{\bar{\psi}^{1,2,3,4,5}\}^{(*)}|0>_R \; .$$

We will solve this issue by using the GSO projection, which again for us here returns the $-|S>$ and so now we can see that from the three possible combinations of the conjugates that we could obtain, the only one that survives the projection is:

$$\psi^{\mu}_{\frac{1}{2}}|0>_L \otimes\{\bar{\psi}^{1,2,3,4,5}\}\{\bar{\psi}^{1,2,3,4,5}\}^{*}|0>_R \; .$$

Which gives the breaking $SO(10) \rightarrow$ x $SU(5) \times U(1)$ to the flipped-*SU(5)* gauge group.
We can also see by inspection, that if after we have applied α to our group we then apply β then we obtain the SLM gauge group. The SLM gauge group is of the form: *SU(3)* x *U(1)* x *SU(2)* x *U(1)*.

5.2 An Extension to NAHE set

We will now move on to an extension of the NAHE set of basis vectros, our aim here is to again enhance the gauge group following the same method as before. The surviving vacuum after we have completed the projections with the basis vectors above is given by:

$$\left[\binom{4}{0}+\binom{4}{2}+\binom{4}{4}\right]\left\{\binom{2}{0}\left[\binom{5}{0}+\binom{5}{2}+\binom{5}{4}\right]\binom{1}{0}+\binom{2}{2}\left[\binom{5}{1}+\binom{5}{3}+\binom{5}{5}\right]\binom{1}{1}\right\}.$$

Along with the vacuum's conjugate term:

$$\left[\binom{4}{1}+\binom{4}{3}\right]\left\{\binom{2}{0}\left[\binom{5}{0}+\binom{5}{2}+\binom{5}{4}\right]\binom{1}{1}+\binom{2}{2}\left[\binom{5}{1}+\binom{5}{3}+\binom{5}{5}\right]\binom{1}{0}\right\}.$$

We will define a vector $\vec{X} = \{\vec{0}|\bar{\psi}^{1,2,3,4,5}\bar{\eta}^1\bar{\eta}^2\bar{\eta}^3\}$ this is what we will use to extend the NAHE set of basis vectors. We will see that however we fix our GSO projecection coefficient we will always lose the CPT conjugate term, hence one of the consequences of introducing $\vec{X}$ is that for any specific GSO coefficient we obtain half the number of states.

We will show this explicitly once for $\vec{X}$ acting on the surviving vacuum, the signs that will be assigned as subscripts to the fermions in the will indicate the charge they are carrying with respect to the GSO condition. We will also state before we start that we have fixed the GSO coefficient to be positive for this.

$$\vec{b}_1 = \{\psi^\mu, \chi^{1,2}, y^{3,4}, y^{5,6}|\bar{y}^{3,4}, \bar{y}^{5,6}, \bar{\eta}^1, \bar{\psi}^{1,2,3,4,5}\}$$
$$\vec{X} = \{\vec{0}|\bar{\psi}^{1,2,3,4,5}\bar{\eta}^{1,2,3}\} \ .$$

The effect this has on the vacuum is:

$$\left[\binom{4}{0} + \binom{4}{2} + \binom{4}{4}\right]_+ \left\{\binom{2}{0}_+ \left[\binom{5}{0} + \binom{5}{2} + \binom{5}{4}\right]_+ \binom{1}{0}_+ + \binom{2}{2}_+ \left[\binom{5}{1} + \binom{5}{3} + \binom{5}{5}\right]_- \binom{1}{1}_- \right\}.$$

Where both of the representations here are positive overall and hence survive, compared with:

$$\left[\binom{4}{1} + \binom{4}{3}\right]_+ \left\{\binom{2}{0}_+ \left[\binom{5}{0} + \binom{5}{2} + \binom{5}{4}\right]_+ \binom{1}{1}_- + \binom{2}{2}_+ \left[\binom{5}{1} + \binom{5}{3} + \binom{5}{5}\right]_- \binom{1}{0}_+ \right\}.$$

Where both of the representations here are negative overall with respect to the GSO coefficient and hence are projected out. The same thing happens for the other basis vectors in the NAHE set.

Now we will analyse the $\vec{X}$-sector to determine if there are any enhancements to the gauge group.

We must first check the form of the states that will be produced on analysis of this sector, we will use the Virasoro mass constraint and find that we can obtain states of the form: $\psi^\mu|0> \otimes|0>$. The projection goes as follows:

Sector analysis $\vec{X} = \{\vec{0}\vert\bar{\psi}^{1,2,3,4,5}\bar{\eta}^{1,2,3}\}$		
Projection Vector	Enhancement	GSO Projection
$\vec{1}$	$\psi^\mu_- \left[\binom{8}{0} + \binom{8}{2} + \binom{8}{4} + \binom{8}{6} + \binom{8}{8}\right]_+$	$e^{i\cdot\pi(\vec{1}\cdot\vec{F}_X)}\vert S>_X = -\vert S>$
$\vec{S}$	$\psi^\mu_- \left[\binom{8}{0} + \binom{8}{2} + \binom{8}{4} + \binom{8}{6} + \binom{8}{8}\right]_+$	$e^{i\cdot\pi(\vec{S}\cdot\vec{F}_X)}\vert S>_X = -\vert S> *$
$\vec{b_1}$	$\psi^\mu_- \left[\binom{6}{0} + \binom{6}{2} + \binom{6}{4} + \binom{6}{6}\right]_+ \left[\binom{2}{0} + \binom{2}{2}\right]$	$e^{i\cdot\pi(\vec{b_1}\cdot\vec{F}_X)}\vert S>_X = -\vert S> *$
$\vec{b_2}$	$\psi^\mu_- \left\{ \left[\binom{5}{0} + \binom{5}{2} + \binom{5}{4}\right] \binom{1}{0} \left[\binom{2}{0}\right] + \left[\binom{5}{1} + \binom{5}{3} + \binom{5}{5}\right] \binom{1}{1} \left[\binom{2}{2}\right] \right\}$	$e^{i\cdot\pi(\vec{b_2}\cdot\vec{F}_X)}\vert S>_X = -\vert S> *$
$\vec{b_3}$	$\psi^\mu_- \left\{ \left[\binom{5}{0} + \binom{5}{2} + \binom{5}{4}\right] \binom{1}{0}\binom{1}{0}\binom{1}{0} + \left[\binom{5}{1} + \binom{5}{3} + \binom{5}{5}\right] \binom{1}{1}\binom{1}{1}\binom{1}{1} \right\}$	$e^{i\cdot\pi(\vec{b_2}\cdot\vec{F}_X)}\vert S>_X = -\vert S> *$

Where here any of the GSO conditions that have been marked with a star (*) indicate that this was our choice of projection and is now fixed due to us making it so. If we count the states that we have found then we can see that in the coefficient $\left[\binom{4}{0} + \binom{4}{2} + \binom{4}{4}\right]$ we have a total of 1+10+5 states. We also see that the states are split into the sum of two parts and so in total we have 16 + $\bar{1}$6 states, and now we will analyse any affect this has on the gauge symmetry. The gauge group we had before was given by:

$$SO(10) \times U(1)^3 \times SO(4)^3 \times SO(16). \tag{31}$$

The Enhancements are presented in the following table:

Gauge group enhancement for the sector: $\vec{X} = \{\vec{0}\|\bar{\psi}^{1,2,3,4,5}\bar{\eta}^{1,2,3}\}$			
$\bar{\psi}^{1,\dots,5} : SO(10)$	$\bar{\eta}^{1,2,3} : U(1)^3$	$\bar{\phi}^{1,\dots,8} : SO(16)$	$\bar{y}^i, \bar{\omega}^i : SO(4)^3$
16	$\frac{1}{2}\ \frac{1}{2}\ \frac{1}{2}$	1	1
$\overline{16}$	$-\frac{1}{2} - \frac{1}{2} - \frac{1}{2}$	1	1

So the system does not transform under the last two gauge groups. We will now need to impose a condition on the *U(1)* generators such that there are three linear, orthogonal combinations of them and requiring that two of the combinations produce trivial charges and one does not. We will make the choice:

$$U(1)_z : U(1)_1 + U(1)_2 + U(1)_3 = \frac{3}{2}$$
$$U(1)_y : U(1)_1 - U(1)_2 + = 0$$
$$U(1)_x : U(1)_1 + U(1)_2 - 2U(1)_3 = 0$$

So the *SO(10)* group that does transform will do so with a combination of one of the *U(1)* charges, it will pick the non-trivially charged *U(1)*. The combination $SO(10) \times U(1)_z$ has a rank of 6 and dimensionality equal to 45 + 1 + 16 + $\bar{1}$6 which is equal to 78. These two properties also represent the gauge group E_6. Therefore we can conclude that this extension to the NAHE set has enhanced our gauge group to:

$$SO(10) \times U(1)^3 \times SO(4)^3 \times SO(16) \to E_6 \times U(1)^2 \times SO(4)^3 \times SO(16). \tag{32}$$

It can also be shown that by using the same method as described above, adding the basis vector: $\vec{Z} = \{\vec{0}|\bar{\phi}^{1,\cdots,8}\}$ enhance the $SO(16) \to E_8$. and so we finish this extended NAHE section with the final gauge group:

$$E_6 \times U(1)^2 \times E_8 \times SO(4)^3. \tag{33}$$

This concludes the section on the NAHE set, where we saw that we could obtain four full 16 representations of *SO(10)* and then enhance the gauge symmetries that we found. All of this was from breaking an *SO(44)* gauge group which represented the basis vectors in our set.

6 The String Model

The main goal of this paper is to analyse models with the standard-like subgroup *SU(3)* x *SU(2)* x $U(1)^2$, which arise from the unbroken *SO(10)* GUT symmetry. In the free fermionic construction, the *SO(10)* symmetry has 12 basis vectors associated with its classification. These are the first 12 vectors listed below, and then in order to produce Standard-Like Models a further 2 basis vectors are required (the final two vectors in the list). It should be noted that the two additional basis vectors break the gauge symmetry to the Pati-Salam (*SO(6)* x *SO(4)*) and to the Flipped-*SU(5)* subgroups. And because the Standard-Like Models incorporates both of these then they will be included in the overall set of basis vectors.

We are specifically looking to enhance the untwisted sector which is given by:

$$\bar{\psi}^{1,2,3,4,5}, \bar{\eta}^{1,2,3}, \bar{\phi}^{1,2,3,4,5,6,7,8}. \tag{34}$$

Which has the associated gauge-group:

$$\begin{gathered} SU(3) \times U(1) \times SU(2) \times U(1) \times U(1)_1 \times U(1)_2 \times U(1)_3 \times SU(2) \times U(1) \times \\ SO(4) \times SU(2) \times U(1) \times U(1) \times U(1). \end{gathered}$$

Where the groups pair to the basis vectors in the following way:

$$\bar{\psi}^{1,2,3} = SU(3) \times U(1), \quad \bar{\psi}^{4,5} = SU(2) \times U(1) \tag{35}$$
$$\bar{\eta}^{1,2,3} = U(1)_1 \times U(1)_2 \times U(1)_3, \quad \bar{\phi}^{1,2} = SU(2) \times U(1) \tag{36}$$
$$\bar{\phi}^{3,4} = SO(4), \quad \bar{\phi}^{5,6} = SU(2) \times U(1) \tag{37}$$
$$\bar{\phi}^{7} = U(1), \quad \bar{\phi}^{8} = U(1). \tag{38}$$

This model, which we will be analysing, was constructed in the paper by Faraggi, Rizos and Sonmez (2018) [1] and is generated using the following set of basis vectors. We will use these throughout the rest of the paper in order to perform projections in a similar manner to those that we did explicitly earlier in the paper.

$$\nu_1 = \vec{1} = \{\psi^\mu, \chi^{1,...,6}, y^{1,...,6}, \omega^{1,...,6} | \bar{y}^{1,...,6}, \bar{\omega}^{1,...,6}, \bar{\eta}^{1,2,3}, \bar{\psi}^{1,2,3,4,5}, \bar{\phi}^{1,...,8}\} \tag{39a}$$

$$\nu_2 = \vec{S} = \{\psi^\mu, \chi^{1,2}, \chi^{3,4}, \chi^{5,6}\} \tag{39b}$$

$$\nu_{2+i} = \vec{e}_i = \{y^i, \omega^i | \bar{y}^i, \bar{\omega^i}\}, i = 1, ..., 6 \tag{39c}$$

$$\nu_9 = \vec{b}_1 = \{\chi^{3,4}, \chi^{5,6}, y^{3,4}, y^{5,6} | \bar{y}^{3,4}, \bar{y}^{5,6}, \bar{\eta}^1, \bar{\psi}^{1,2,3,4,5}\} \tag{39d}$$

$$\nu_{10} = \vec{b}_2 = \{\chi^{1,2}, \chi^{5,6}, y^{1,2}, y^{5,6} | \bar{y}^{1,2}, \bar{y}^{5,6}, \bar{\eta}^2, \bar{\psi}^{1,2,3,4,5}\} \tag{39e}$$

$$\nu_{11} = \vec{z}_1 = \{0 | \bar{\phi}^{1,2,3,4}\} \tag{39f}$$

$$\nu_{12} = \vec{z}_2 = \{0 | \bar{\phi}^{5,6,7,8}\} \tag{39g}$$

$$\nu_{13} = \alpha = \{0 | \bar{\psi}^{4,5}, \bar{\phi}^{1,2}\} \tag{39h}$$

$$\nu_{14} = \beta = \{0 | \bar{\psi}^{1,2,3,4,5}, \bar{\eta}^{1,2,3}, \bar{\phi}^{1,2,5,6} = \frac{1}{2}, \bar{\phi}^{3,4,7} = 1, \bar{\phi}^8 = 0\}. \tag{39i}$$

6.1 The string Spectrum

We are looking for massless spacetime vector bosons and preserving N=1 supersymmetry in the spectrum of this model. We will look at the conditions that we impose in order to obtain only states that are massless and then progress to using the a tree diagram method to determine any gauge enhancements on the sectors which produce massless states. We will state the set of sectors which could produce massless spacetime vector bosons, this being the requirement for an enhancement of the gauge symmetry. This setof sectors was calculated and presented in the paper by Faraggi et. al (2018) [1].
The Virasoro condition for sectors which produce physical states is given as:

$$M_L^2 = -\frac{1}{2} + \frac{\xi_L \cdot \xi_L}{8} + N_L = -1 + \frac{\xi_R \cdot \xi_R}{8} + N_R = M_R^2 \geq 0. \tag{40}$$

Where we can see that the vacuum is split into its left and right moving parts, and we have imposed that the masses of the states must exceed or equal zero. $N_{L/R}$ are the sums over the left/right moving oscillators which are acting on the left/right vacuum respectively.
We will now define the matrix **G**, which contains all the sectors which could produce spacetime vector bosons and therefore could produce enhancements to the gauge symmetry:

$$
G = \left\{
\begin{array}{cccc}
x & z_1 & z_2 & z_1 + z_2 \\
x + 2\beta & z_1 + x + 2\beta & x + z_2 + 2\beta & x + z_1 + z_2 + 2\beta \\
 & & & \\
\alpha & \alpha + z_1 & \alpha + z_2 & \alpha + z_1 + z_2 \\
\alpha + 2\beta & \alpha + x + z_1 & z_2 + \alpha + 2\beta & z_1 + z_2 + x + \alpha + 2\beta \\
\alpha + x & z_1 + x + \alpha + 2\beta & x + \alpha + 2\beta & z_2 + x + \alpha + 2\beta \\
 & & & \\
\pm\beta & z_1 \pm \beta & z_2 \pm \beta & z_1 + z_2 \pm \beta \\
x \pm \beta & x + z_1 \pm \beta & x + z_2 \pm \beta & x + z_1 + z_2 \pm \beta \\
 & & & \\
\alpha \pm \beta & z_1 + \alpha \pm \beta & z_2 + \alpha \pm \beta & z_1 + z_2 + \alpha \pm \beta \\
x + \alpha \pm \beta & x + z_1 + \alpha + \pm\beta & x + z_2 + \alpha \pm \beta & x + z_1 + z_2 + \alpha \pm \beta
\end{array}
\right\}. \tag{41}
$$

There are 36 sectors here in total which could produce massless spacetime vector bosons which could lead an enhancement of the gauge symmetry. The total set is split up into four distinct sections, where the top two rows are the sectors that don't enhance the *SO(10)* and the following nine rows do. They are arranged in the following ways respectively to their row in G. 3-5 enhance the symmetry to the Pati-Salam, rows 6-7 enhance it to Flipped-*SU(5)* and the last two rows 8-9, break *SO(10)* to the Standard-Like Models with subgroup *SU(3)* x *SU(2)* x *U(1)*^{2}.

6.2 Tree Diagram Method

In this section we will look into this tree diagram method which is used to analyse enhancement sectors in the model. The method still includes the projections that were done earlier but they cover all possible bases for the answer. The states are written explicitly at each tier and then by following a specific pathway to the bottom of the tree we can then analayse the specific state and judge whether the gauge group has been enhanced, also we can write down the conditions for any particular enhancements in terms of the GSO projection coefficients.
The point of this project was again to analyse sectors which can enhance the gauge symmetry from the untwisted sector, to do this we will analyse sectors that produce massless spacetime vector bosons. We will, in analogy to bosonic string theory, need to apply a Virasoro mass constraint. This is done as follows:

$$
\left[M_L^{\vec{X}}\right]^2 = -\frac{1}{2} + \frac{\alpha_L \cdot \alpha_L}{8} + N_L = -1 + \frac{\alpha_R \cdot \alpha_R}{8} + N_R = \left[M_R^{\vec{X}}\right]^2 \geq 0. \tag{42}
$$

We see here how we have used the fact that the heterotic string is constructed with the left and right movers completely decoupled, and treated independent of each other. We first calculate the dot product of the basis vectors in the equation and then we find the values of $N_{L/R}$. The quantity N_L tells us the sum of the oscillators acting on the left or the right vacuum. For example if $N_L = 1$ then this tells us we either need one bosonic oscillator acting on this side of the state or two fermionic oscillators. This mass condition must always be greater than or equal to zero as we require that the theory produces only physical states, specifically we look for sectors that contain massless spacetime vector bosons and so we will require equation(42) to equal zero. Once we have determined the form of the states that we are going to use we can begin using the tree method, the representation at the very top of the tree will be equal to the most general combinatoric notation of the fermions we include and any oscillators that are present. We can see that we may possibly require an oscillator acting on the right vacuum, recalling that fermionic oscillators carry $\frac{1}{2}$ worldsheet charge then we can add integer numbers of these oscillators to the right vacuum depending on the value of the sum N_R. We will also say that we use specific fermions to make our oscillators, we use ones which do not occur in the sector representation. We will also see later in the paper that there are exceptions to this, were different oscillators are required, this will be covered in detail at the point they are needed.

We will make clear the notation that we are going to need when using these trees in order to analyse the enhancement sectors. First we will establish that we are still using the combinatoric notation as we have done throughout the report. We will make note of certain features of the tree in an example that will follow.
We will see the tree is comprised of nodes and branches, at the end of the branches are the spinorial representations of the state which may have been broken. Along the branch there will always be a term representing the basis vector that we are using to do the projection and associated with it, a sign. This sign indicates the form of the GSO coefficient for the projection. We use all of this information to then determine the composition of the state at the end of a node. We continue in this fashion for every possibility of the sector, until we can either make no more difference or we have used up all of the basis vectors.

We will now cover the breaking of the states at the nodes and we will now list three rules that concisely describe the process of breaking the states:
1). Does the projection vector 'see' any of the fermions listed in our sector?
-If no: assign a positive charge.
-If yes: We will assign either a positive or negative charge.
2). If the projector does 'see' fermions from the sector we assign:
-Positive charge for an even combination of charges with integer boundary conditions.
-Negative charge for a negative combination of charges with integer boundary conditions.
3). We must always conserve the combination of the negative charges i.e. something which have an odd combination must break in a way were its constituent parts form a sum that is odd. Along with the total number of the state being conserved. Example:

$$\binom{6}{D} \rightarrow \binom{5}{E}\binom{1}{D} \text{ (or vice versa).}$$

When we arrive at the endpoint of a leg of the tree we can then analyse the state that we have, in doing this we can determine if we have enhanced the gauge symmetry or not. We will now move on to an example sector.

7 The $\vec{X}$ sector

Here we will begin as explained before, we will first state the sector that we are investigating: $\vec{X} = \{\vec{0}|\bar{\psi}^{1,2,3,4,5}\bar{\eta}^{1,2,3}\}$. So when we put this into the Virasoro mass condition we find that $N_L = \frac{1}{2}$ and $N_R = 0$ and so we get states of the form:

$$\rightarrow \psi^{\mu}|0>_L \otimes |0>_R \ .$$

So we state explicitly for this example that there are eight right moving fermions and an oscillator with spacetime index acting on the left vacuum meaning we have produced spacetime vector bosons here and hence there could be an enhancement to the sector. This presents itself as: $\psi^{\mu}\binom{8}{ALL}$ which is the most general form. We will then move on to do our first projection. This will be with the unit vector: $\vec{1}$, this vector is required for completeness by the ABK rules and is such that all fermions are periodic. We now for the first time have branching in our tree, this is due to the $\pm$ degeneracy in the GSO condition on our system, and we must assign the positive-negative states accordingly. The $\vec{X}$ sector is of the same form as that of the NAHE set that we examined earlier in the report, we will state that here this sector is a composite vector consisting of:

$$\vec{X} = 1 + S + \sum_{i=1}^{6}(e_i + z_1 + z_2) = \{\vec{0}|\bar{\psi}^{1,2,3,4,5}\bar{\eta}^{1,2,3}\}. \tag{43}$$

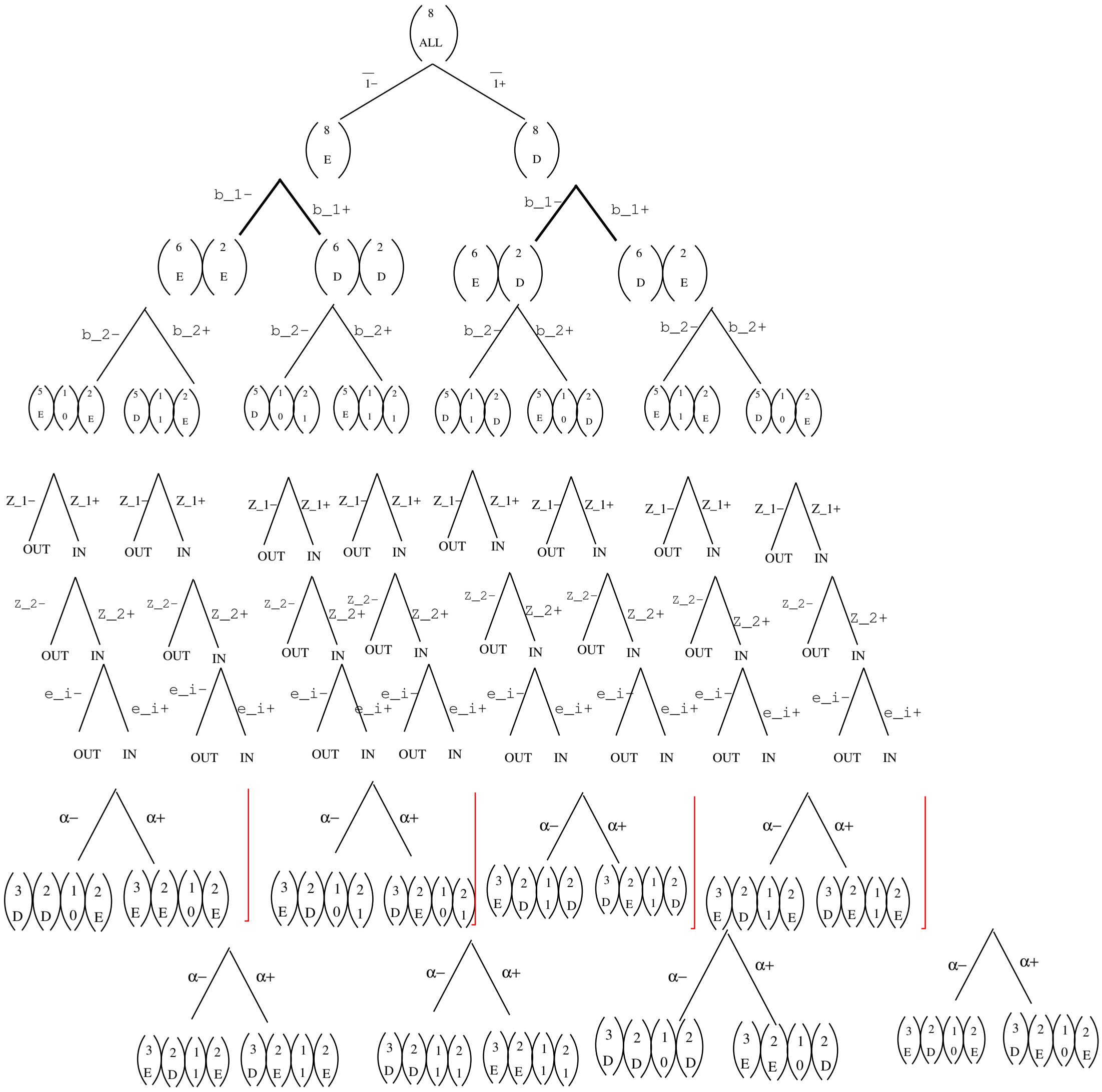

Figure 2: This is part of the tree diagram for analysis of the $\vec{X}$ sector.

It is first important to make a couple of things clear about the diagrams, they have proved a very useful calculational tool for the analysis of enhancement sectors in these models, but they do have some downsides. Firstly, they are very cumbersome to produce due to their size. This has certain consequences, for instance along with every term at a node on the tree the state does also have a CPT (charge, parity and time) conjugate term associated with it. But in order to get a meaningfull amount of information on a diagram these had to be omitted, but it must be emphasised here that they will be used. Also the oscillator which should be acting on the state which we found by calculation using the Virasoro mass constraint is omitted. This is partly because of the extra space this term would take up, but also the software package that was used to construct these trees, namely "XFig" did not accomodate their input. But as long as we accompany any of these tree diagrams with the necessary information to complete the full picture we can continue to use it.

In this figure we have included an example of the procedure that we are following to complete the analysis on the sector. We see that once we get to the alpha projection on the sector we need to truncate the diagram which we do with the red vertical lines, all of the states here are still under the same projection from α. It is worth noting as well that there is still the β projection to be done on this sector but a we can see it would be almost impossible to fit this onto the same diagram in this report. We will elaborate on finding the final states under the β projection next, to do this we will choose one specific path down the tree and then perform the β projection for one of the given paths.

In order for us to understand the form of the β projection we need to refer back the ABK rules, we will use this to pick out the combinations of the states that are possible under this projection. Making note of the form of β from equation (39i), we see that this vector will not change the form of the state but it will determine how the state can be charged. So we use the modular invariance constraints to see that $N_{ij} b_i \cdot b_j = 0 \mod 4$ and hence any result of the β projection that will be positive after the projection must have some combination of either; 0, 4,8,... negative charges. Whereas in a similar fashion any result after the β projection that will be negative after the projection will have either; 2,6,12,... negative charges. We will see more clearly when we perform this explicitly how these conditions come about.

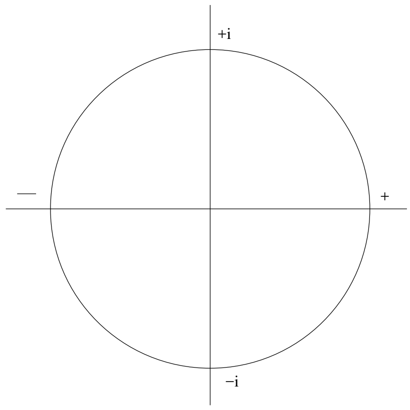

Figure 3: Visual representation of the modular constraint on the charge composition of a state, under $\frac{1}{4}$ charges.

So for our example we have that after some projections in a specific path we ended up with the alpha projection that left us with the state:

$$\left[\binom{3}{D}\binom{2}{D}\binom{1}{0}\binom{1}{0}\binom{1}{0}\right]. \tag{44}$$

And when we expand this and write out the state and express its combinatoric representation explicitly we find:

$$\left[\binom{3}{1}+\binom{3}{3}\right]\binom{2}{1}\binom{1}{0}\binom{1}{0}\binom{1}{0}+\left[\binom{3}{0}+\binom{3}{2}\right]\binom{2}{1}\binom{1}{1}\binom{1}{1}\binom{1}{1}. \tag{45}$$

The second term in the above equation is the CPT conjugate for the state which must be included. And so suppose for example we would like to find the positive β projection after the negative α projection then, from our state that we had we will need the negatives to be either: 0,4 or 8. And so with this information we get:

$$\alpha_-\beta_+ = \left[\binom{3}{3}\binom{2}{1}\binom{1}{0}\binom{1}{0}\binom{1}{0}+\binom{3}{0}\binom{2}{1}\binom{1}{1}\binom{1}{1}\binom{1}{1}\right]. \tag{46}$$

And this is our final state for particular choices in this particular sector. Now we must examine the state and see if this state we have obtained enhances the gauge symmetry or not. We will do this in a similar way to what we did before, the composition of the states can be shown in the following table:

Gauge group enhancement for the sector: $\vec{X} = \{\vec{0}\vert\bar{\psi}^{1,2,3,4,5}\bar{\eta}^{1,2,3}\}$				
$\bar{\psi}^{1,2,3}$: $SU(3)$	$tr(\bar{\psi}^{1,2,3}) : U(1)$	$\bar{\psi}^{4,5} : SU(2)$	$tr(\bar{\psi}^{4,5}) : U(1)$	$\bar{\eta}^{1,2,3}$: $U(1)_1 \times U(1)_2 \times U(1)_3$
$\bar{1}$	-$\frac{3}{2}$	2	0	$\frac{1}{2}$ $\frac{1}{2}$ $\frac{1}{2}$
1	$\frac{3}{2}$	2	0	$-\frac{1}{2}-\frac{1}{2}-\frac{1}{2}$

So we can see that the state does not transform under *SU(3)* but does under *SU(2)*.
If we take the *SU(2)* $\times$ *U(1)* group we see that this is a rank 3 group with dimensionality 3+1+(2+2) = 8. Where the brackets denote the doublet representation of the transformed *SU(2)* group. After some inspection of the results we can say that we have the following enhancement of the untwisted unbroken subgroup of SO(10):

$$SU(3) \times U(1) \times SU(2) \times U(1) \to SU(3)^2 \times U(1)^4.$$

This was an example of one of the gauge enhancements that is possible in this sector, there are however more. The rest will be provided in the results section that will follow.
Lastly we must now state the conditions for us to get this, we will follow the specific path of the tree stating the GSO conditions as we go in order to have exact constraints. So in our example if we require that from this sector we want to this gauge enhancement then we will require the following:

$$(\vec{1} = \text{-}),\ (\vec{b_1} = +),\ (\vec{b_2} = \text{-}),(\vec{z_1} = \vec{z_2} = \vec{e_i} = +),\ (\vec{\alpha} = \text{-}\ ),\ (\vec{\beta} = +).$$

We will need to make use of a proof regarding the GSO projection coefficients in order to complete the next part of the result as the vector $\vec{X}$ is a composite basis vector.

With these conditions we can now go on to fix the GSO conditions in the following way:

GSO projections for the sector $\vec{X} = \{\vec{0}\vert\bar{\psi}^{1,2,3,4,5}\bar{\eta}^{1,2,3}\}$		
Projection Vector	Possible GSO Constraint	Our GSO Constraint
$\vec{1}$	$\delta_1 {}_{-} c\binom{1}{X}_{\pm}$	$\delta_1 {}_{-} c\binom{1}{X}_{+}$
$\vec{S}$	$\delta_S {}_{-} c\binom{S}{X}_{\pm}$	$\delta_S {}_{-} c\binom{S}{X}_{-}$
$\vec{b_1}$	$-\delta_{b_1} {}_{-} c\binom{b_1}{X}_{\pm}$	$-\delta_{b_1} {}_{-} c\binom{b_1}{X}_{+}$
$\vec{b_2}$	$-\delta_{b_2} {}_{-} c\binom{b_2}{X}_{\pm}$	$-\delta_{b_2} {}_{-} c\binom{b_2}{X}_{-}$
$\vec{z_1}$	$\delta_{z_1} {}_{-} c\binom{z_1}{X}_{\pm}$	$\delta_{z_1} {}_{-} c\binom{z_1}{X}_{-}$
$\vec{z_2}$	$\delta_{z_2} {}_{-} c\binom{z_2}{X}_{\pm}$	$\delta_{z_2} {}_{-} c\binom{z_2}{X}_{-}$
$\vec{e_i}$	$\delta_{e_i} {}_{-} c\binom{e_i}{X}_{\pm}$	$\delta_{e_i} {}_{-} c\binom{e_i}{X}_{-}$
$\vec{\alpha}$	$-\delta_{\alpha} {}_{-} c\binom{\alpha}{X}_{\pm}$	$-\delta_{\alpha} {}_{-} c\binom{\alpha}{X}_{-}$
$\vec{\beta}$	$-\delta_{\beta} {}_{-} c\binom{\beta}{X}_{\pm}$	$-\delta_{\beta} {}_{-} c\binom{\beta}{X}_{+}$

$$c\begin{pmatrix} 1 \\ X \end{pmatrix} = c\begin{pmatrix} e_i \\ X \end{pmatrix} = c\begin{pmatrix} z_i \\ X \end{pmatrix} = \begin{pmatrix} \beta \\ X \end{pmatrix} = 1$$
$$c\begin{pmatrix} b \\ X \end{pmatrix} = c\begin{pmatrix} \alpha \\ X \end{pmatrix} = -1.$$

This section has provided us with a full worked example for how the results that will be presented will all be calculated by hand.
And so we can now write the new enhanced gauge symmetry for the sector under these conditions as:

$$SU(2) \times U(1) \rightarrow SU(3).$$

Which gives an enhancement on the unbroken untwisted sector as:

$$SU(3) \times U(1) \times SU(3) \times U(1)_1 \times U(1)_2 \times U(1)_3 \times SU(2) \times U(1) \times$$
$$SO(4) \times SU(2) \times U(1) \times U(1) \times U(1).$$

8 Results

In this penultimate chapter we will state the results of the possible enhancements to the untwisted sector of the unbroken *SO(10)* subgroup, we will use the tree diagram method that we have shown previously, but we will not include the diagrams here.
We will split this section, as was done in **G** up into four groups, according to how they break the *SO(10)* symmetry of the model. The First set of results will correspond to sectors which do not enhance *SO(10)* subgroup. Next will be the results from some of the sectors which break the *SO(10)* symmetry to the Pati-Salam group, the sectors for breaking *SO(10)* to the flipped-*SU(5)* group were not completed in this report but can be completed in exactly the same way as we have done here. Finally the results for of the sectors where the *SO(10)* subgroup is broken to the Standard-Like Model will be presented. For any of the sectors which require an oscillator acting on the right vacuum , we will state that the oscillator acting will be: $\{\bar{\phi}\} = \{\bar{\psi}^{1,2,3}\}$, except for the final result which will be explained at that point.

We now look to the first set of results:

- The $\vec{X}$ sector

  $\vec{X} = \{\vec{0}|\bar{\psi}^{1,2,3,4,5}\bar{\eta}^{1,2,3}\}$

  States of the form: $\psi^\mu|0>_L \otimes|0>_R$.

Sector condition
$(X\|\vec{e_i})$=$(X\|z_k)$=+

Enhancement Condition	**Resulting Enhancement**
$(X\|\alpha) = (X\|\beta) = +\|S>$	$SU(3) \times SU(2) \times U(1)^5 \rightarrow SU(4) \times SU(2)^2 \times U(1)^3$
$(X\|\alpha) \neq (X\|\beta) = -\|S>$	$SU(3) \times SU(2) \times U(1)^5 \rightarrow SU(4) \times SU(2)^2 \times U(1)^3$
$(X\|\alpha) \neq (X\|\beta) = +\|S>$	$SU(3) \times SU(2) \times U(1)^5 \rightarrow SU(3) \times SU(3) \times U(1)^4$
$(X\|\alpha) = (X\|\beta) = -\|S>$	$SU(3) \times SU(2) \times U(1)^5 \rightarrow SU(5) \times U(1)^5$

For example the state that appears third in the table would require fixing the GSO conditions as follows:

$$c\binom{1}{X} = c\binom{e_i}{X} = c\binom{z_i}{X} = c\binom{\beta}{X} = 1 \quad (47)$$

$$c\binom{b_i}{X} = c\binom{\alpha}{X} = -1 \quad (48)$$

Where $i = 1, ..., 6$ and $k = 1, 2$.

- The $\vec{X} + 2\vec{\beta}$ sector

$\vec{X} + 2\vec{\beta} = \{\vec{0}|\bar{\phi}^{1,2,5,6}\}$

States of the form: $\psi^{\mu}|0>_L \otimes \{\bar{\phi}\}_{\frac{1}{2}}|0>_R$

Sector condition
$(\vec{X} + 2\vec{\beta}|\vec{b_1})$=$(\vec{X} + 2\vec{\beta}|b_2)$=+

Enhancement Condition	**Resulting Enhancement**
$(\vec{X} + 2\vec{\beta}|\alpha) \neq (\vec{X} + 2\vec{\beta}|\beta) = +i|S>$	*SU(2)* × *U(1)* × *SU(2)* × *U(1)* → *SU(3)* × *SU(2)* × *U(1)*

- The $\vec{z_1} + \vec{z_2} + \vec{X} + 2\vec{\beta}$ sector

$\vec{z_1} + \vec{z_2} + \vec{X} + 2\vec{\beta} = \{\vec{0}|\bar{\phi}^{3,4,7,8}\}$

States of the form: $\psi^{\mu}|0>_L \otimes \{\bar{\phi}\}_{\frac{1}{2}}|0>_R$

Sector condition
$(\vec{z_1} + \vec{z_2} + \vec{X} + 2\vec{\beta}|\vec{b_1})$=$(\vec{X} + 2\vec{\beta}|b_2)$=+
$(\vec{z_1} + \vec{z_2} + \vec{X} + 2\vec{\beta}|\vec{z_1})\neq(\vec{X} + 2\vec{\beta}|b_2)$
$(\vec{z_1} + \vec{z_2} + \vec{X} + 2\vec{\beta}|\vec{z_2})\neq(\vec{X} + 2\vec{\beta}|\alpha)$

Enhancement Condition	**Resulting Enhancement**
$(\vec{z_1} + \vec{z_2} + \vec{X} + 2\vec{\beta}|\alpha) = (\vec{X} + 2\vec{\beta}|\beta) = -i|S>$	*SU(3)* × *SU(2)* × *U(1)*^{3} → *SU(4)* × *U(1)*^{3}

- The $\vec{z_1}$ sector

$\vec{z_1} = \{\vec{0}|\bar{\phi}^{1,2,3,4}\}$

States of the form: $\psi^{\mu}|0>_L \otimes \{\bar{\phi}\}_{\frac{1}{2}}|0>_R$

Sector condition
$(\vec{z_1}\|\vec{b_1})$=$(\vec{z_1}\|b_2)$=-
$(\vec{z_1}\|\vec{z_1})\neq(\vec{z_1}\|z_2)$

Enhancement Condition	Resulting Enhancement
$(\vec{z_1}\|\alpha) \neq (\vec{z_1}\|\beta) =$	*SU(2)* $\times$ *SU(2)* $\times$ *U(1)*$^2 \to$ *SU(3)* $\times$ *SU(2)* $\times$ *U(1)*
$(\vec{z_1}\|\alpha) = (\vec{z_1}\|\beta) =$	*SU(2)* $\times$ *SU(2)* $\times$ *U(1)*$^2 \to$ *SU(4)* $\times$ *SU(2)* $\times$ *U(1)*

Now we will move on to the results from the sector which breaks the *SO(10)* to *SO(6)* $\times$ *SO(4)*:

- The $\vec{\alpha}$ sector

  $\vec{\alpha} = \{\vec{0}|\bar{\psi}^{4,5}\bar{\phi}^{1,2}\}$

  States of the form: $\psi^\mu|0>_L \otimes\{\bar{\phi}\}_{\frac{1}{2}}|0>_R$

Sector condition
$(\vec{\alpha}\|\vec{b_1})$=$(\vec{\alpha}\|b_2)$

Enhancement Condition	Resulting Enhancement
$(\vec{\alpha}\|\vec{z_2}) = (\vec{\alpha}\|\vec{\beta}) = +i$	*SU(2)* $\times$ *SU(2)* $\times$ *U(1)*$^2 \to$ *SU(3)* $\times$ *SU(2)* $\times$ *U(1)*
$(\vec{\alpha}\|\vec{z_2}) \neq (\vec{\alpha}\|\vec{\beta}) = -i$	*SU(3)* $\times$ *SU(2)*^{2} $\times$ *U(1)*$^2 \to$ *SU(4)* $\times$ *SU(2)* $\times$ *U(1)*

- The $\vec{\alpha}+\vec{z_1}$ sector

  $\vec{\alpha}+\vec{z_1} = \{\vec{0}|\bar{\psi}^{4,5}\bar{\phi}^{3,4}\}$

  States of the form: $\psi^\mu|0>_L \otimes\{\bar{\phi}\}_{\frac{1}{2}}|0>_R$

Sector condition
$(\vec{\alpha}+\vec{z_1}\|\vec{b_1})$=$(\vec{\alpha}+\vec{z_1}\|b_2)$
$(\vec{\alpha}+\vec{z_1}\|\vec{\alpha})$=$(\vec{\alpha}+\vec{z_1}\|\beta)$

Enhancement Condition	Resulting Enhancement
$(\vec{\alpha}+\vec{z_1}\|\vec{\alpha}) = (\vec{\alpha}+\vec{z_1}\|\vec{\beta}) = -$	*SU(3)* $\times$ *SU(2)*^{2} $\times$ *U(1)*$^3 \to$ *SU(4)* $\times$ *SU(2)* $\times$ *U(1)*^{2}

- The $\vec{\alpha}+\vec{z_2}$ sector

  $\vec{\alpha}+\vec{z_2} = \{\vec{0}|\bar{\psi}^{4,5}\bar{\phi}^{1,2,5,6,7,8}\}$

  States of the form: $\psi^\mu|0>_L \otimes|0>_R$

Sector condition
$(\vec{\alpha}+\vec{z_2}\|\vec{b_1})$=$(\vec{\alpha}+\vec{z_2}\|b_2)$
$(\vec{\alpha}+\vec{z_2}\|\vec{\alpha})$=$(\vec{\alpha}+\vec{z_2}\|\beta)$

Enhancement Condition	**Resulting Enhancement**
$(\vec{\alpha}+\vec{z_2}\|\vec{\alpha}) = (\vec{\alpha}+\vec{z_1}\|\vec{\beta}) = -$	*SU(2)*$^3 \times$ *U(1)*$^3 \rightarrow$ *SU(4)* $\times$ *SU(2)* $\times$ *U(1)*^{4}

Now we look at the sectors which break the *SO(10)* symmetry to that of SLM.

- The $\alpha + \beta$ Sector
  $\vec{\alpha}+\vec{\beta} = \{\vec{0}|\bar{\psi}^{1,2,3}\bar{\eta}^{1,2,3}\bar{\phi}^{5,6} = \frac{1}{2}, \bar{\psi}^{4,5}\bar{\phi}^{1,2} = -\frac{1}{2}, \bar{\phi}^{3,4,7} = 1\}$

States of the form: $\psi^{\mu}|0>_L \otimes \{\bar{\phi}\}_{\frac{1}{4}}|0>_R$.

This sector has far more degeneracy than the others, as we can see the state requires an oscillator to act on it. This particular sector is different to the other that we have encountered as we need to perform the addition of two basis vectors that have half integer boundary conditions. We will do this explicitly to show the requirements for the this tree. We will use equations (39h, 39i) for the basis vectors, and now we will evaluate the Virasoro mass condition.

$$\alpha_L \cdot \alpha_L = 0 \quad and \quad \alpha_R \cdot \alpha_R = 6. \tag{49}$$

Putting these values into the Virasoro mass condition equation we see that in order to obtain massless states we must have the sum of oscillators acting on the left: $N_L = \frac{1}{2}$ and the sum of oscillators acting on the right: $N_R = \frac{1}{4}$. This is how we obtained the form of our states. The different thing about this state is how the right moving oscillator is charged, with it being $\frac{1}{4}$ we see that instead of the fermions involved being from those outside of the enhancement sector they are in fact from the set of basis vectors that make up the sector. We also need to take into account the sign of the boundary conditions of the fermions in the sector as well, for example the oscillator:

$$\{\bar{\phi}\} = \{\bar{\psi}^{1,2,3}\}^{(}*). \tag{50}$$

So a $\frac{1}{2}$ charged oscillator which is defined by $\frac{1}{2}$ boundary conditions fulfills the Virasoro requirements and we can continue with the tree. We see that this oscillator is the conjugate, this is because its boundary condition is positive whilst if it were negative then it would not be a conjugate oscillator. These trees are far more complicated and are left here, but in principle would be completed in the same manner as earlier sectors were.

9 Conclusions

So now that we have listed all the results that were obtained for the report we will briefly conclude by talking about how we could move on from here, and discuss further what we found.
The paper began with a brief overview of the modern physics that we use to describe the standard model, this included an investigation into the Lorentz gauge group *SO(10)* and how states from the standard model can fit into a representation of it. This was important as it gave us some insight into the relevance of this group. Then set up the next part of the paper by reviewing the bosonic string theory and we can extend that specific formalism by employing supersymmetry, specifically by making use of the symmetries of the worldsheet. Then we covered the construction of the heterotic string which would become our choice of supersymmetric string for use in this analysis. We reviewed the free fermioninc formulation in greater detail and encounted the set of rules which would allow us to build a consistent theory directly in four dimensions, these were the ABK rules. From here we went on to look at the NAHE set of basis vectors which was our first encounter with realistic models, we did analysis of gauge group enhancements and laid the ground-work for the techniques that we would make use of later on in the paper.
Next we moved on to the specific model that we studied, this was the Standard-Like Model, straight away we talked through the basis vectors that generate these models and specifically the necessity of specific vectors. From here we looked at the string spectrum obtained from these models, and we made it clear that we were going to be analysing sectors which produced massless spacetime vectors bosons. The sectors which did this can have their gauge symmetry enhanced, and it was this that we were to study.
Once we established that we wanted to study the gauge symmetry of the untwisted sector we then needed a mechanism to do this, and we found one in the use of tree diagrams. We detailed the methodology behind these diagrams and presented an example sector for clarity, it was using this method that we obtained the results for this paper.
Finally we listed the results of the projections in the given sectors and judged if the symmetry had been enhanced or not.
To conclude the report we will remark how the results we have obtained could then be used further, we can say that for any of the tree diagrams that we made, we followed a specific path down the tree fixing GSO constraints as we go. Then for every sector we could pick one enhancement produced by one specific state, fix the GSO constraints and use these in the next sector and so on until we have a consistent Standard-Like Model as we did in equation (47)(48). We could extend this to build a consistent untwisted sector of a SLM by fixing these conditions and then eliminating any of the other paths from the other sectors which did not adhere to these constraints.

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